Title: Certifying long-range quantum correlations through routed Bell experiments

Speakers: Stefano Pironio

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Abstract: In a recent paper, Chaturvedi et al considered the interesting idea of routed Bell experiments. These are Bell experiments where Bob can measure his quantum particles at two distinct locations, one close to the source and another far away. This can be accomplished in the lab by using a switch that directs Bob's quantum particle either to the nearby measurement device or to the distant one, depending on a classical input chosen by Bob. Chaturvedi et al argue that there exists in such experiments a tradeoff between short-range and long-range correlations and that high-quality CHSH tests close to the source (which are achievable with current technology) lower the requirements for witnessing nonlocality faraway from the source, and in particular increase their tolerance to particle losses. We critically review their results and present a simple counterexample to it. We then introduce a class of hybrid quantum-classical models, which we refer to as "short-range quantum models". These models suitably capture the tradeoff between short-range and long-range correlations in routed Bell experiments. Using our definition, we explore new nonlocal tests in which high-quality short-range correlations lead to weakened conditions for long-range tests. Although we do find improvements, they are significantly smaller than those claimed by CVP.

Long-range quantum correlations in routed Bell experiments

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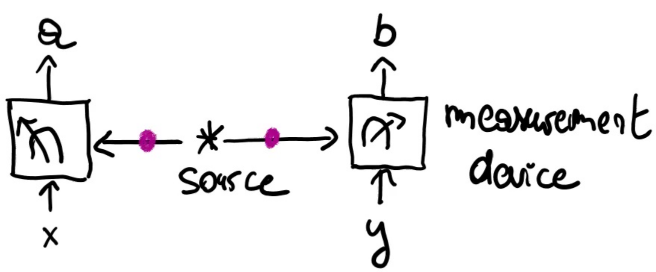
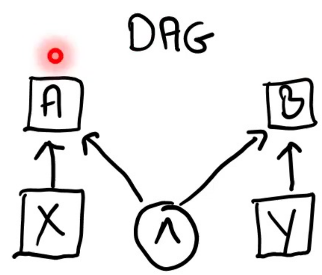
Joint work with Edwin Peter Lobo and Jef Pauwels

Causal Inference & Quantum Foundations Workshop – April 20, 2023

The Bell scenario





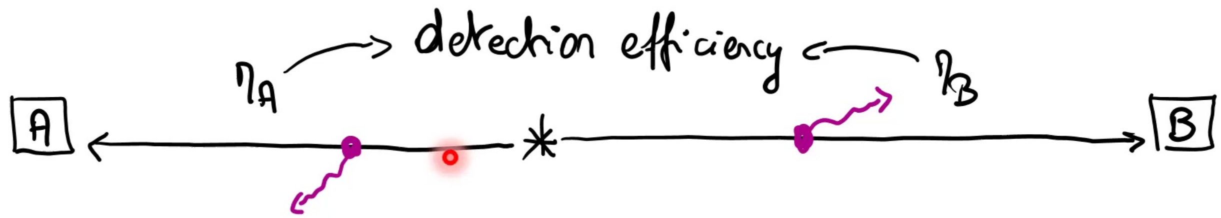
$CHSH = \sum_{a,b,x,y}(-1)^{a+b+xy}P(ab|xy) \leq 2$ for classical models

$\leq 2\sqrt{2}$ for quantum models

→ Assuming the causal structure, we can certify quantumness from the data alone (device-independence)

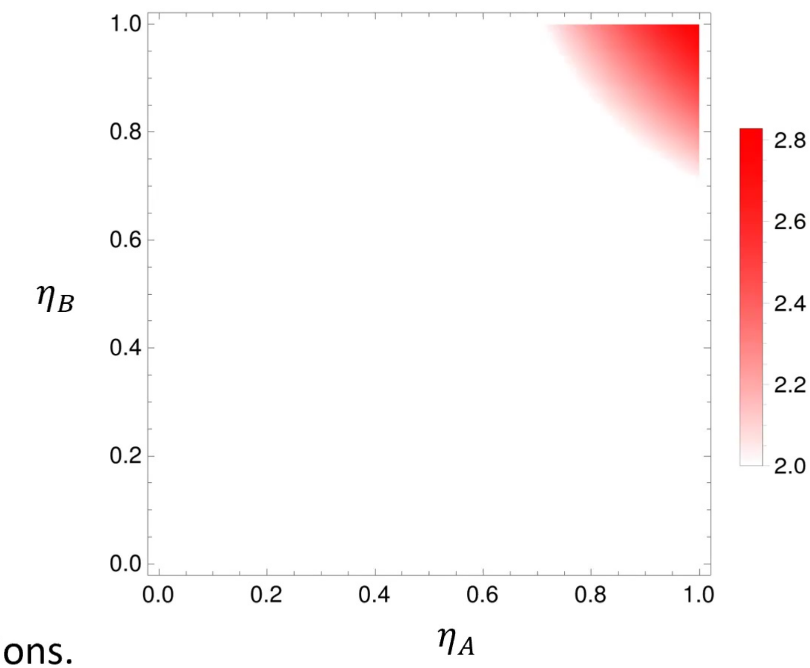
A value $CHSH > 2$ actually provides much more information on the underlying quantum model (self-testing)
This can be used to certify the correct behavior of entire quantum info protocols, e.g., DIQKD.

Quantum correlations are strongly affected by photon losses



Losses lower the CHSH value:

$$CHSH = 2\sqrt{2}\,\eta_A\eta_B + 2\,(1-\eta_A)(1-\eta_B)$$



Losses increase with the distance
exponentially in fibers and quadratically in free-space !

→ In practice, we cannot certify long-range quantum correlations.

1. Chaturvedi, Viola, Pawlowski's proposal

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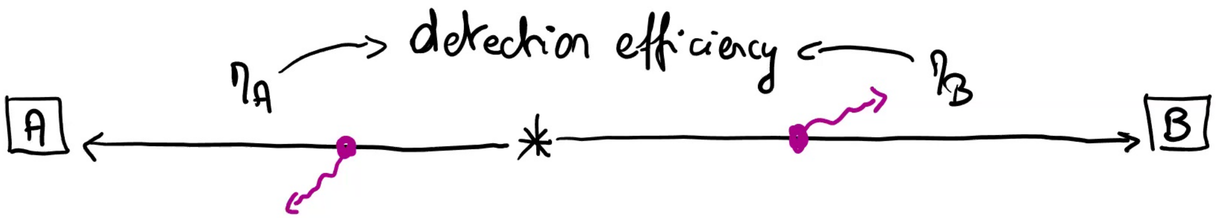

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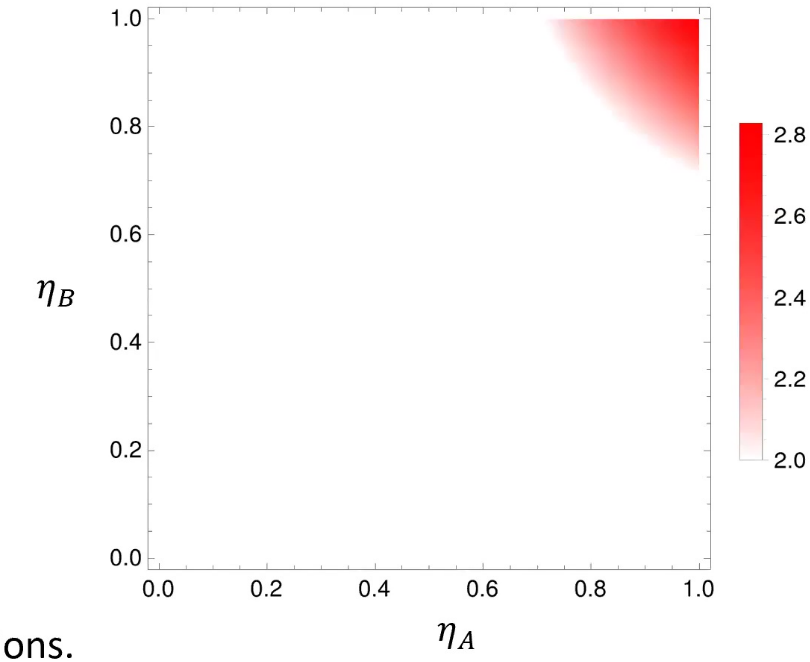

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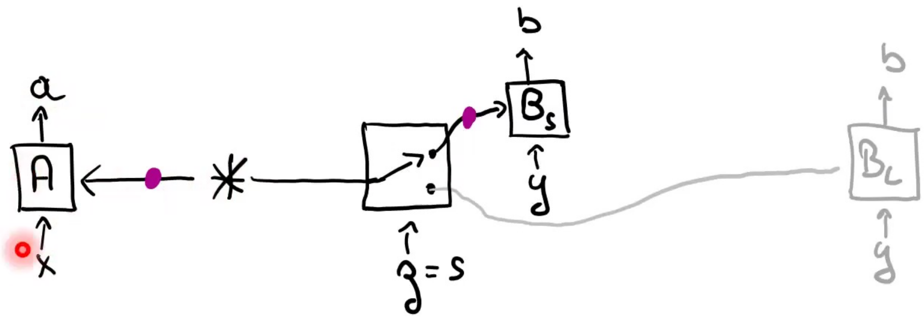
Routed Bell experiments – Chaturvedi, Viola, Pawlowski, arXiv:2211.14231



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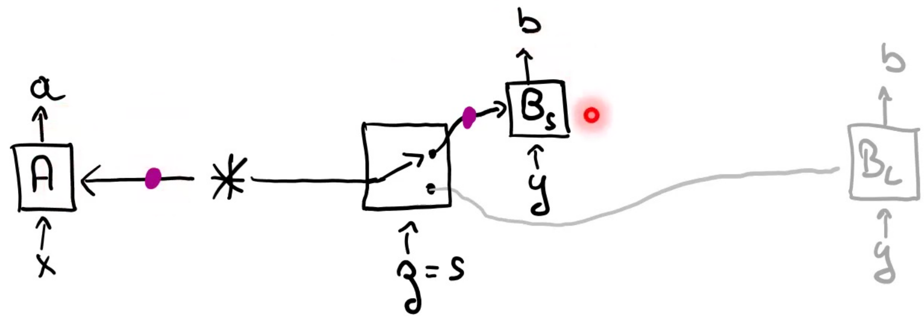

We can observe two CHSH quantities:

- short-path CHSH: $CHSH_S = \sum_{a,b,x,y}(-1)^{a+b+xy} P(ab|xy, z = s)$
- long-path CHSH: $CHSH_l = \sum_{a,b,x,y}(-1)^{a+b+xy} P(ab|xy, z = l)$

According to the usual analysis, the condition for ruling out classical models and certifying long-range quantum correlations between A and B_L is $CHSH_L > 2$.

CVP argue that a violation $CHSH_S > 2$ in the short-path CHSH test weakens the condition for ruling out classical models in the long-path CHSH test.
Specifically, they derive the following trade-off for any model where B_L is classical:

$$CHSH_L \leq \sqrt{8 - CHSH_S^2} \quad \text{if } CHSH_S > 2$$

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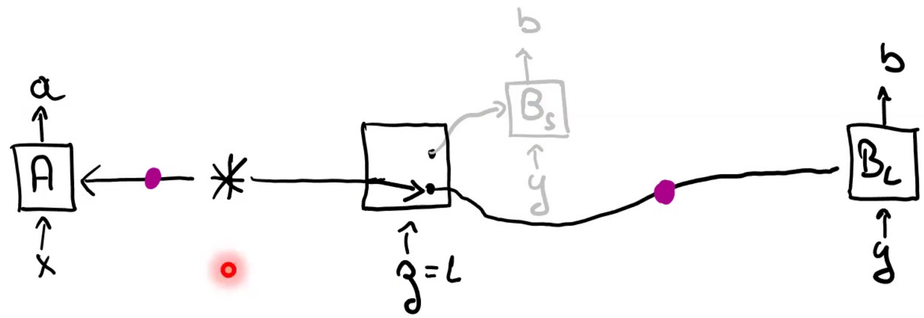

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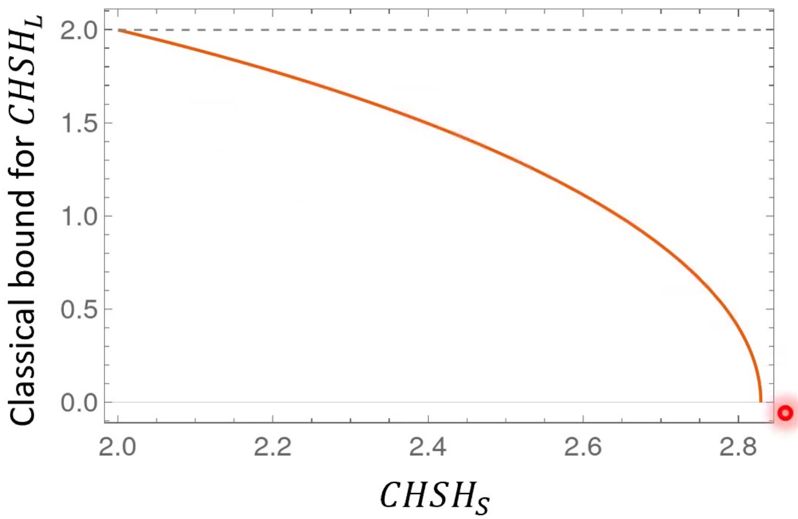
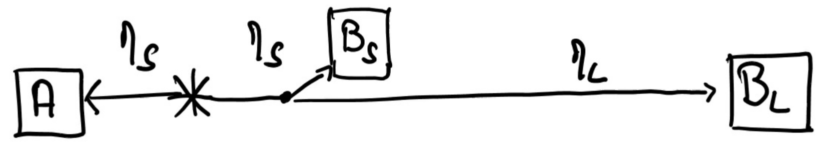
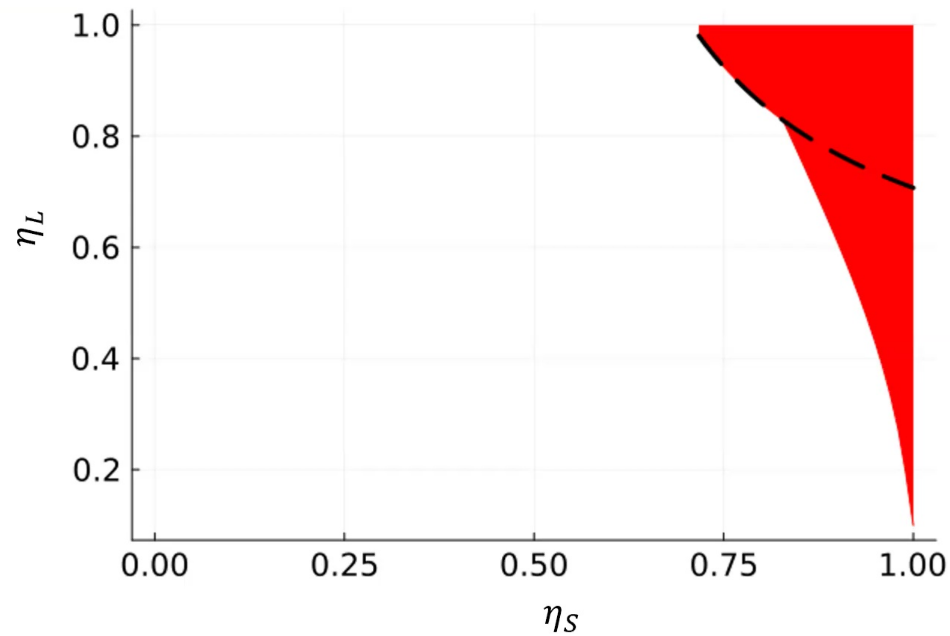


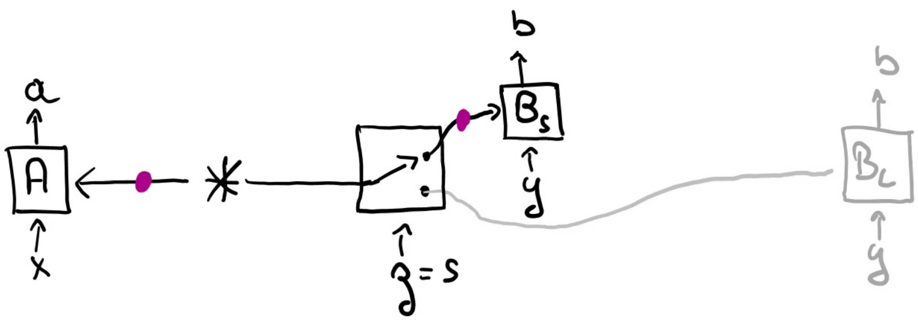
For $CHSH_S = 2\sqrt{2} - \epsilon,\ CHSH_L \leq \sqrt{\epsilon}$!!

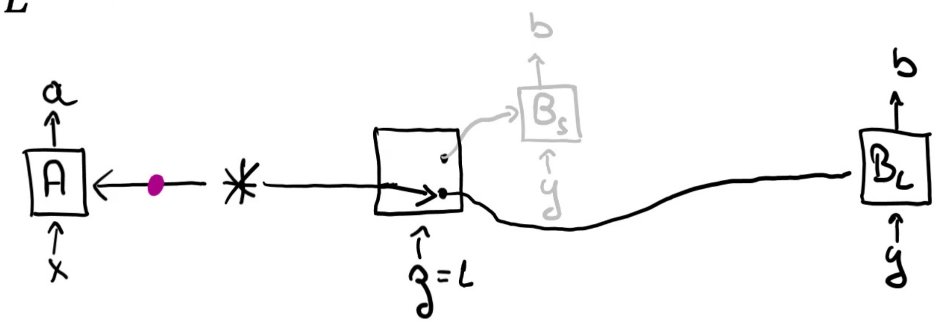




By performing high-quality CHSH tests close to the source (which are achievable with current technology), we can significantly extend the range over which genuine quantum correlations can be certified

Intuition behind the $CHSH_S - CHSH_L$ trade-off





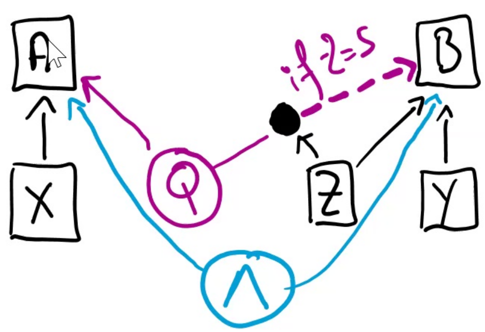
$CHSH_S > 2$

→ a quantum common cause is required

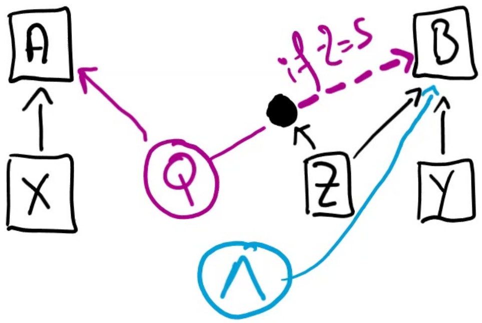
$CHSH_L < 2$

→ could a classical common cause be sufficient, without any quantum particle actually reaching B_L?

This leads us to consider a hybrid quantum-classical causal model



Intuition behind the $CHSH_S \; - CHSH_L$ trade-off



What is the maximal value of $CHSH_L$ given some $CHSH_S > 2$?

Assume $CHSH_S > 2$

→ • A approximately implements Pauli observables on a two-qubit maximally entangled state and is weakly correlated to any other degrees of freedom in the universe, in particular to Λ

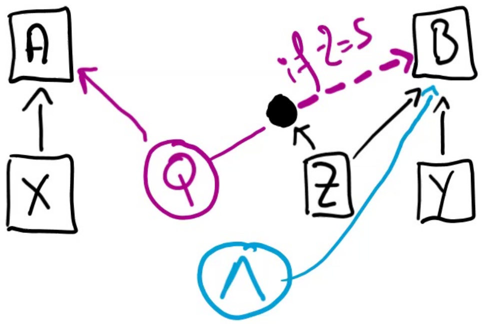
$$P(ab|xy, z = L) \simeq P(a|x)P(b|y, z = L)$$

• A's output are locally partly random

$$P(ab|xy, z = L) \leq \left(\frac{1}{2} + c\right) P(b|y, z = L)$$

$$\rightarrow \; CHSH_L \leq \sqrt{8 - CHSH_S^2}$$

Intuition behind the $CHSH_S - CHSH_L$ trade-off



What is the maximal value of $CHSH_L$ given some $CHSH_S > 2$?

Assume $CHSH_S = 2\sqrt{2}$

→ • A implements Pauli observables on a two-qubit maximally entangled state and is independent of any other degrees of freedom in the universe, in particular of Λ

$$P(ab|xy, z = L) = P(a|x)P(b|y, z = L)$$

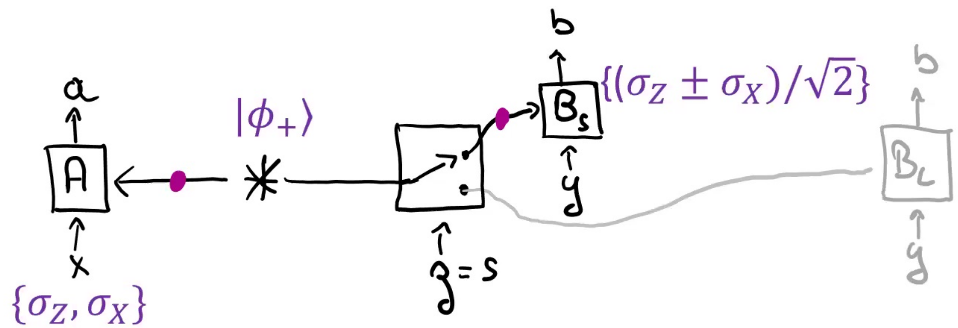
• A's output are locally uniform

$$P(ab|xy, z = L) = \frac{1}{2}\, P(b|y, z = L)$$

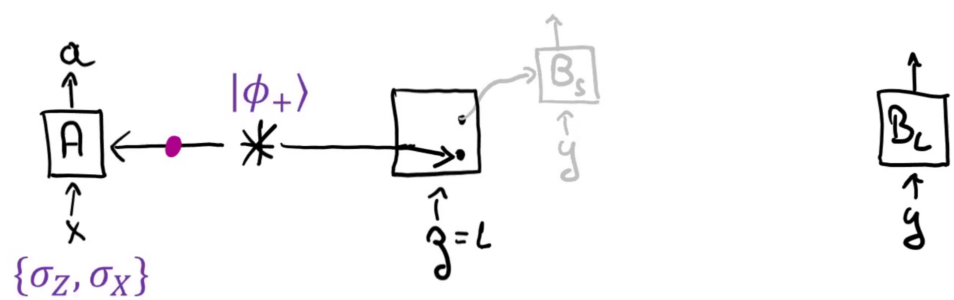
1. Chaturvedi, Viola, Pawlowski's proposal

2. A counterexample

A counterexample where $CHSH_L = 2$ for $CHSH_S = 2\sqrt{2}$ for B_L classical



$CHSH_S = 2\sqrt{2}$



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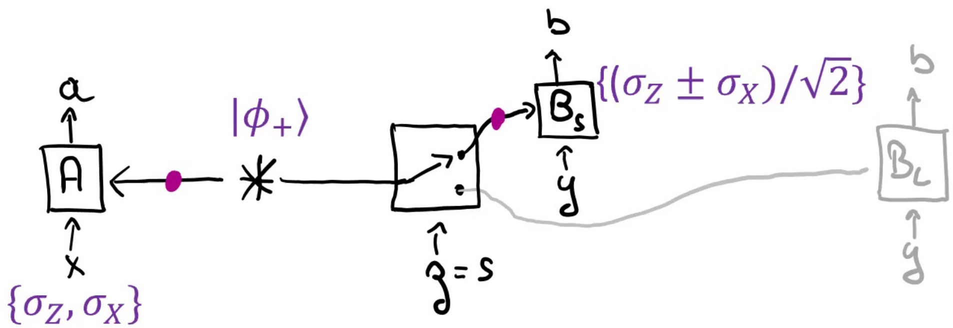

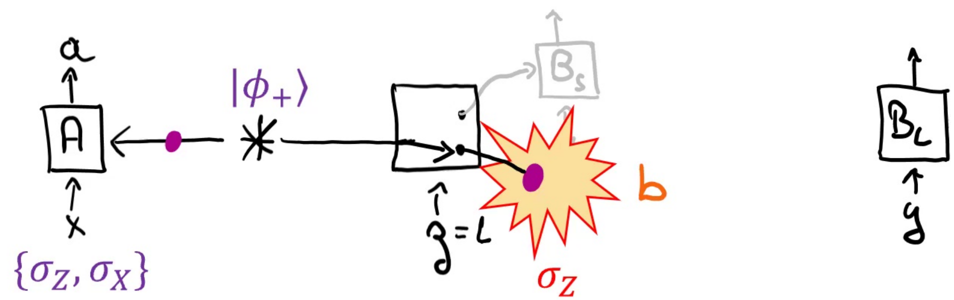

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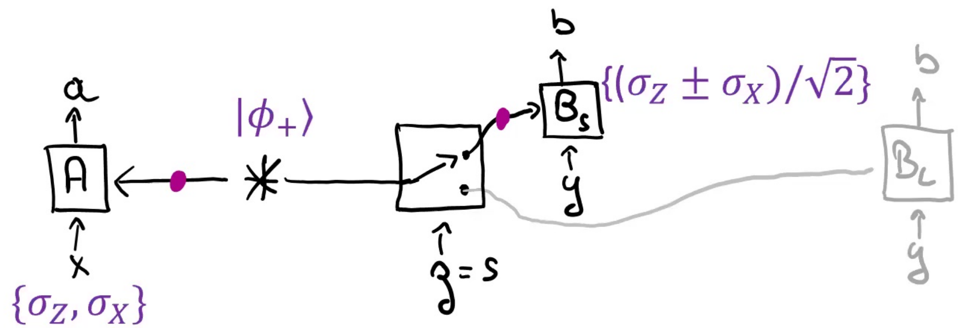

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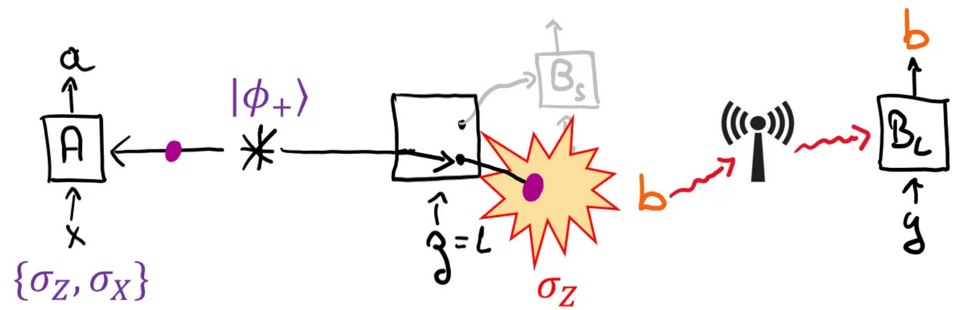

$x = 0, y = 0 \;\rightarrow x = \sigma_z, y = \sigma_Z \rightarrow P(a = b) = 1$

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$x = 1, y = 0 \;\rightarrow x = \sigma_x, y = \sigma_Z \rightarrow P(a, b) = 1/4$

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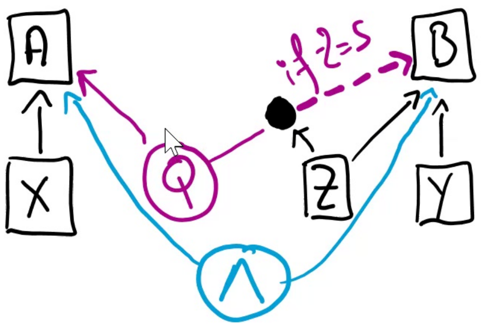
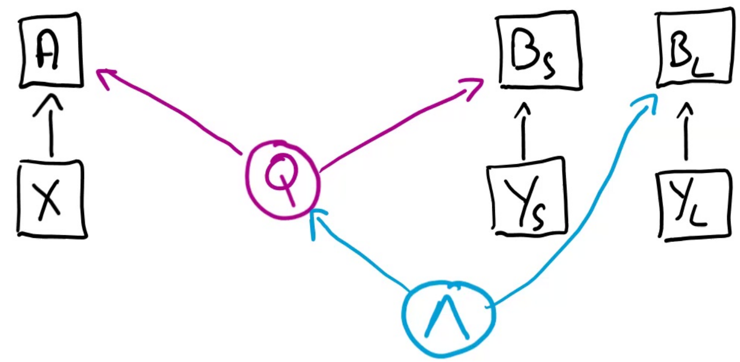
$\rightarrow$ $CHSH_L = 2$



$\Longleftrightarrow$

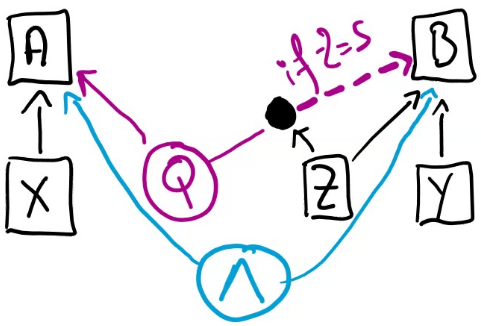


$$P(ab|xy, z = s) = \sum_{b_l} \tilde{P}(abb_l|xyy_l)$$

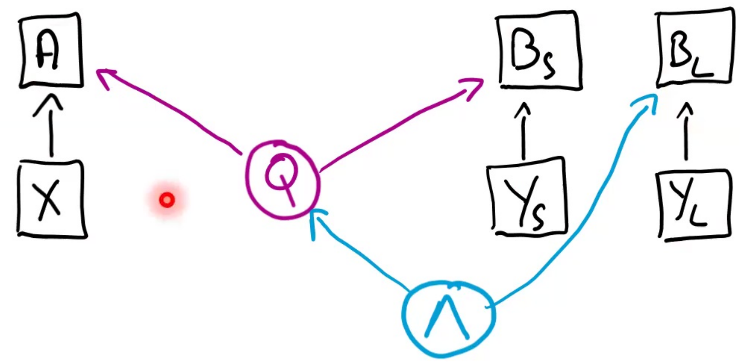
$$P(ab|xy, z = l) = \sum_{b_S} \tilde{P}(ab_S b|xy_s y)$$

$$\tilde{P}(ab_s b_l|xy_s y_l)$$

Chaturvedi *et al* implicitly assume a tripartite causal structure (but the experiment is bipartite!)
In doing so, they add on top of a hypothesis of classicality, an implicit monogamy assumption.
It is this implicit assumption of monogamy, rather than B_L being classical, that leads to the S/L tradeoff.



$\Leftrightarrow$



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$$\tilde{P}(ab_sb_l|xy_sy_l)$$

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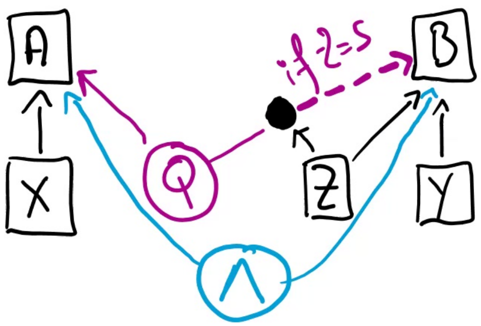

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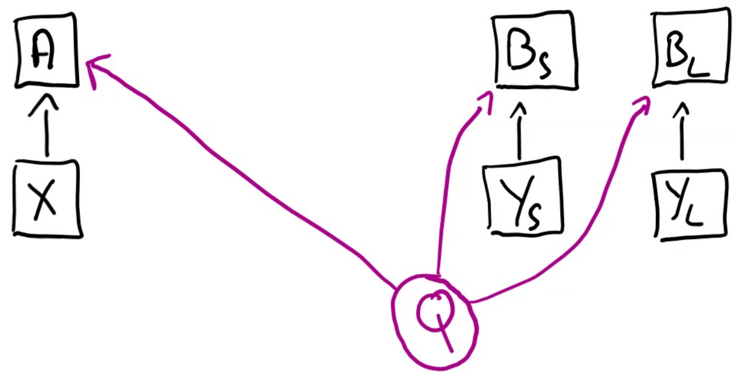

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In doing so, they add on top of a hypothesis of classicality, an implicit monogamy assumption.
It is this implicit assumption of monogamy, rather than B_L being classical, that leads to the S/L tradeoff.

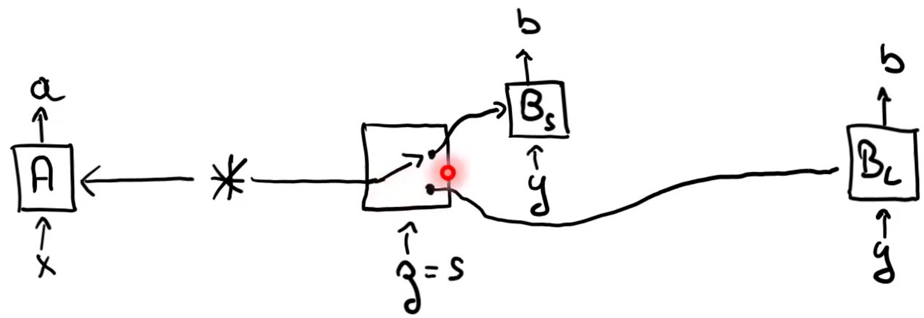
The relation $CHSH_L \leq \sqrt{8 - CHSH_S^2}$ is also true in a fully quantum model [Toner, Verstraete, quant-ph/0611001]

1. Chaturvedi, Viola, Pawlowski's proposal

2. A counterexample

3. How to formulate general models that exhibit only short-range quantum correlations, i.e., where the distant device B_L is classical ?

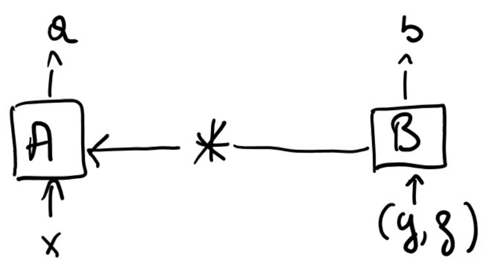
Characterization of quantum correlations in routed Bell scenarios



$$p(ab|xyz) = \mathrm{tr}\left[(I \otimes C_z)(\rho_{AB}) M_{a|x} \otimes M_{b|y}\right]$$

where C_z are the CPTP maps describing the transmission on the short or long path channels

Or equivalently, defining $M'_{b|yz} = C_L^{\dagger}\left(M_{b|y}\right)$



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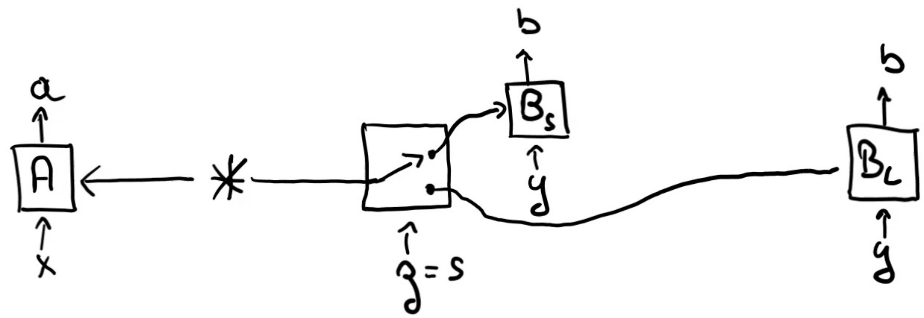
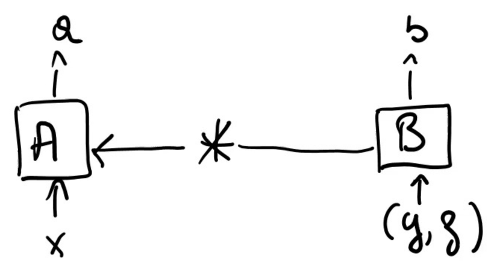


$$p(ab|xyz) = \mathrm{tr}\left[(I \otimes C_z)(\rho_{AB}) M_{a|x} \otimes M_{b|y}\right]$$

We can now define *short-range quantum correlations* as those where C_L is entanglement-breaking.

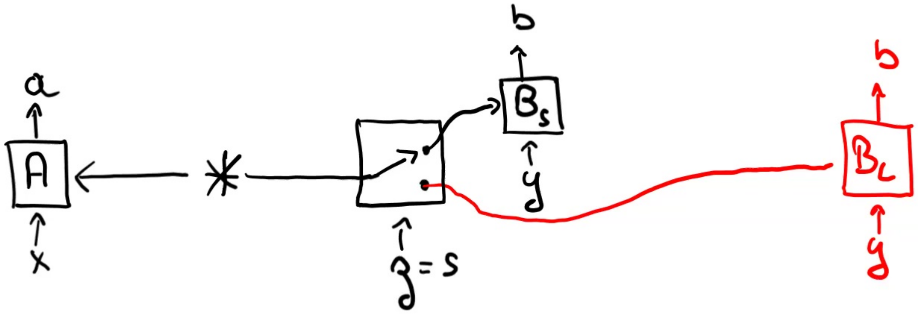
Its adjoint $C_L^\dagger$ maps the operators $\{M_{b|y}\} \to \left\{C_L^\dagger\left(M_{b|y}\right)\right\}$ to a set of jointly-measurable operators.

[Pusey arXiv:1502.03010]

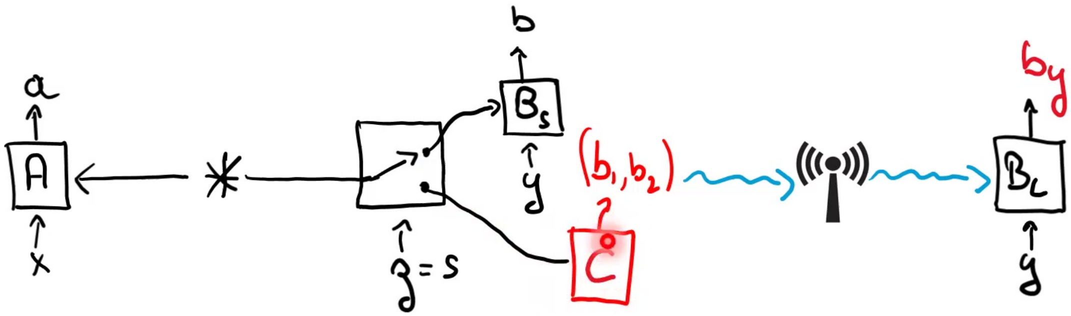


$$p(ab|xyz) = \mathrm{tr}\left[\rho_{AB}\, M_{a|x} \otimes M'_{b|yz}\right]$$

where $\left\{M'_{b|y,z=L}\right\}$ are jointly-measurable



The effective measurements $\{M'_{b|y=0,z=L}\}$ and $\{M'_{b|y=1,z=L}\}$ are jointly-measurable
$\Leftrightarrow$ there exists a *single* measurement $\{C_{b_1 b_2}\}$ that returns a pair (b_1, b_2) of outcome for $y = 0$ & $y = 1$.



The correlations $p(ab|xyz) = \mathrm{tr}\big[\rho_{AB}\, M_{a|x} \otimes M'_{b|yz}\big]$ where $\big\{M'_{b|y,z=L}\big\}$ are jointly-measurable can be characterized through a standard application of the NPO hierarchy.

1. Chaturvedi, Viola, Pawlowski's proposal

2. A counterexample

3. How to formulate general models that exhibit only short-range quantum correlations, i.e., where the distant device B_L is classical ?

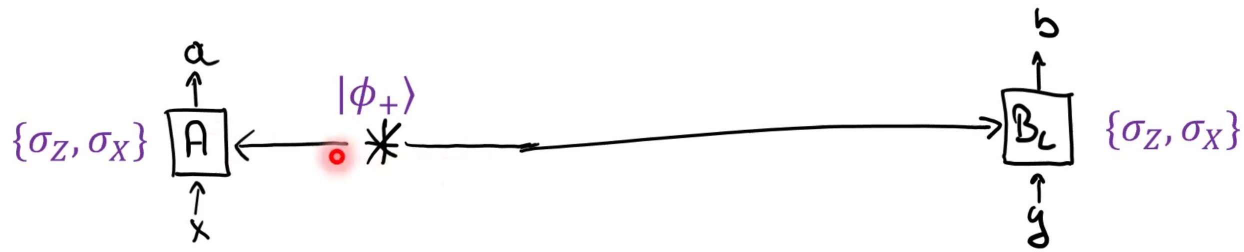
4. Do Bell violations in the short-path weaken the conditions for ruling out classical models in the long-path?

[redacted]

The classical bound of the long-path CHSH does not decrease if the short-path CHSH is violated.

But, we found other inequalities where this happens.

<u>Example:</u>



$$ZX_L = \sum_{abk} (-1)^{a+b} P(ab|x = k, y = k)$$

Then

$ZX_L \leq 2$ for classical model

$ZX_L \leq 2$ for quantum models (achieved by the above correlations)

[redacted]

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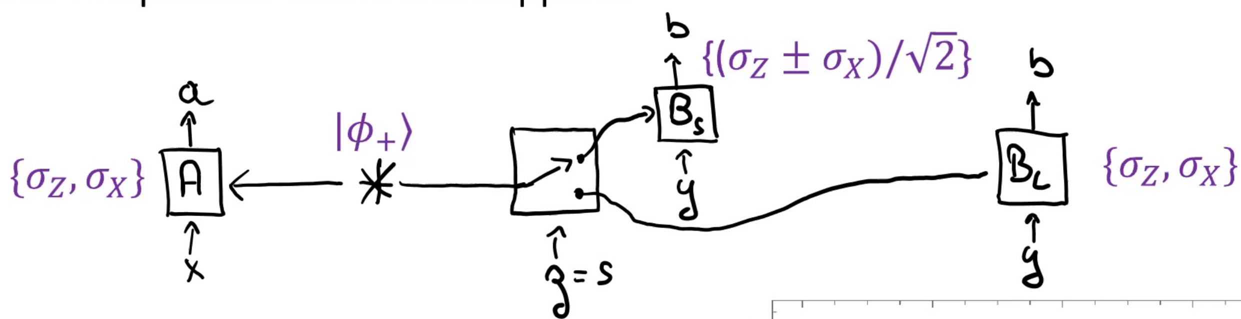
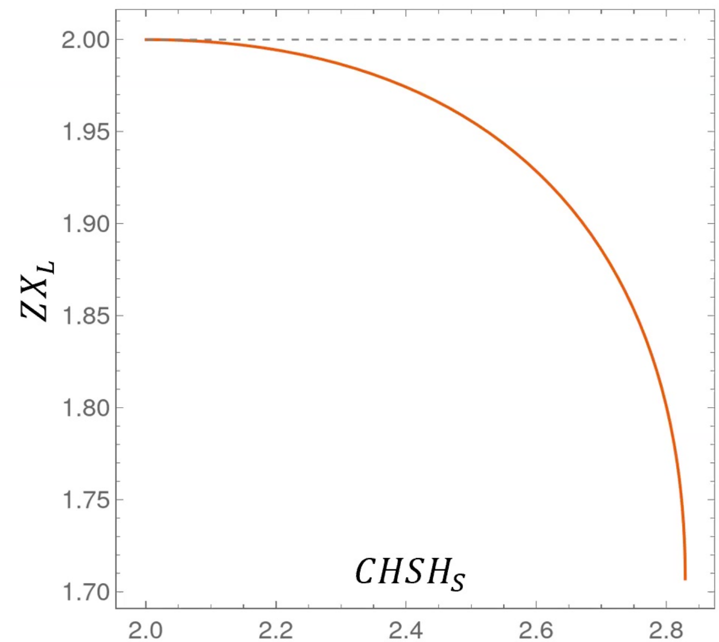


$$ZX_L = \sum_{abk} (-1)^{a+b} P(ab|x=k, y=k)$$

Then

$$\frac{CHSH_S^2}{4} - \frac{CHSH_S \times ZX_L}{2} + \frac{ZX_L^2}{2} \leq 1$$



Interestingly, if we trust that the measurement devices do the σ_Z, σ_x measurements the quantity

$$ZX_L = \sum_{abk} (-1)^{a+b} P(ab|x = k, y = k)$$

is an entanglement-witness:

$$ZX_L \leq \sqrt{2} \quad \text{for separable states}$$
$$ZX_L \leq 2 \quad \text{for entangled states}$$

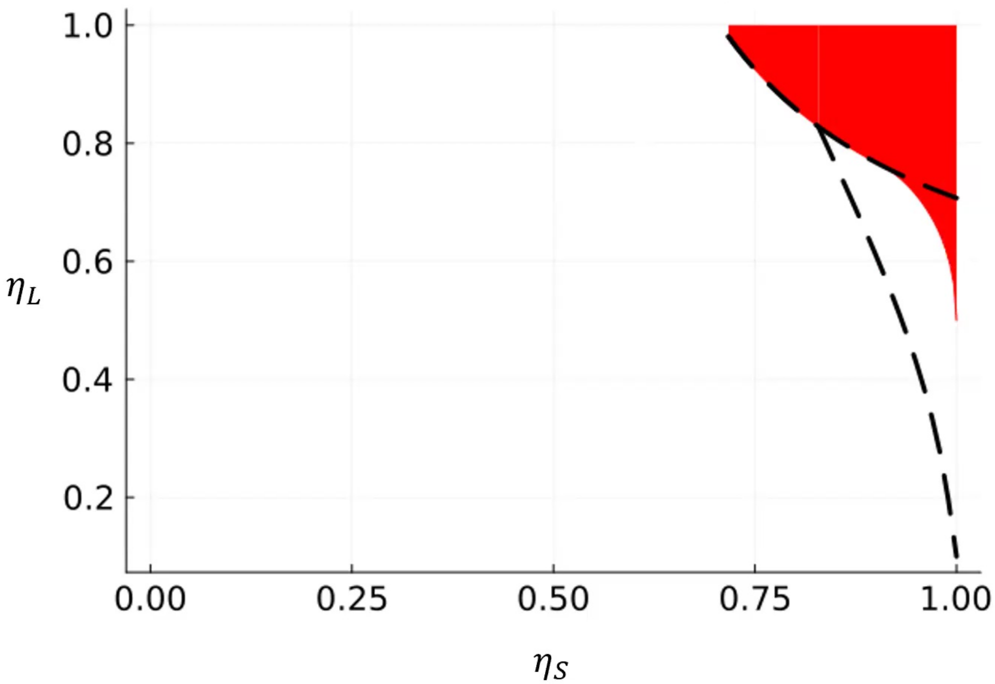
But it is not a device-independent (DI) entanglement witness,
i.e., if we don't trust the measurement devices

$$ZX_L \leq 2 \quad \text{for separable states}$$

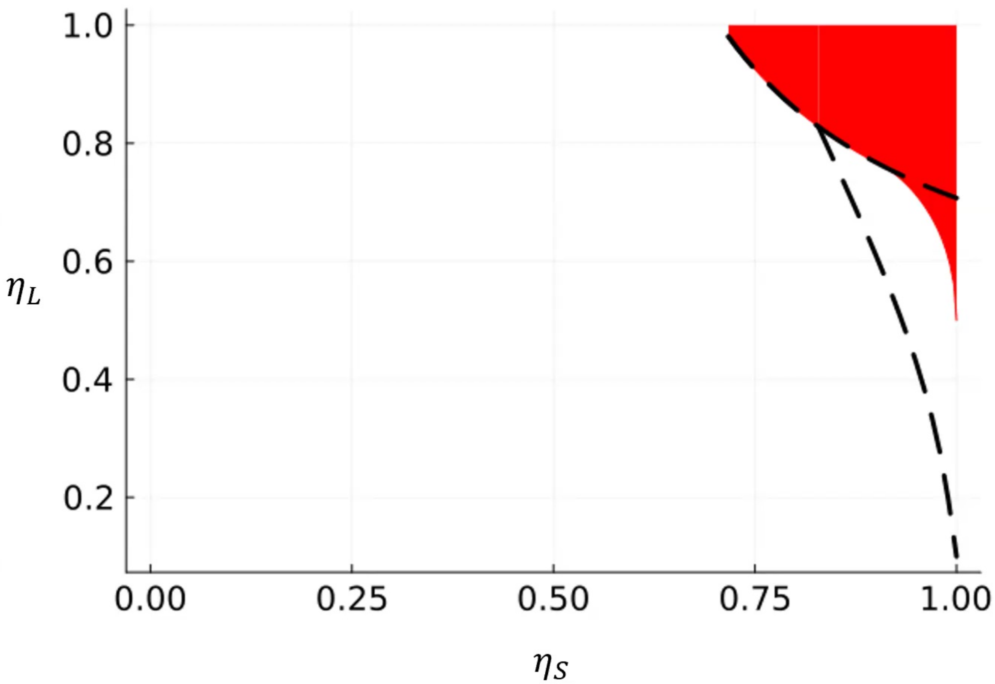
By embedding it in a routed Bell experiment, we can turn ZX_L into a DI entanglement-witness.

In particular if $CHSH_S = 2\sqrt{2}$ → $ZX_L \leq \sqrt{2}$ for short-range quantum correlations

Improving detection efficiency tolerance with routed Bell tests



Improving detection efficiency tolerance with routed Bell tests



For regular Bell tests, there exists a lower bound on the detection efficiency:

$$\eta_L > 1/|Y|$$

The same lower-bounds applies for routed Bell scenarios.

This is because $|Y|$ lossy measurements are always jointly-measurable if $\eta_L \leq 1/|Y|$

[Masini *et al,* in preparation]

Conclusion

- Routed Bell experiments are interesting
  - Form a causal perspective: they are represented by hybrid quantum/classical models, where some latent notes (the switch) output either quantum or classical systems.

    We showed how to model this using the concept of joint-measurability.

  - We showed that they do weaken the conditions to certify quantumness among distant parties.

    However, not to the extent originally announced in Chaturvedi *et al.*

- So far, we have analyzed the simplest scenario (binary variables, only a single switch)

  There is still much to be explored!