Title: A quantum tale of causes and effects

Speakers: Rafael Chaves

Collection: Causal Inference & Quantum Foundations Workshop

Date: April 19, 2023 - 2:00 PM

URL: https://pirsa.org/23040117

Abstract: Explaining the natural world through cause-and-effect relations is the fundamental principle of science. Although a classical theory of causality has been recently introduced, enabling us to model causation across diverse research fields, it is crucial to examine which aspects of it require modification or abandonment to also comprehend causality in the quantum world. To address this question, we will investigate paradigmatic scenarios, including the double slit, Bell's theorem and generalizations to quantum networks, also exploring recent experimental advancements.

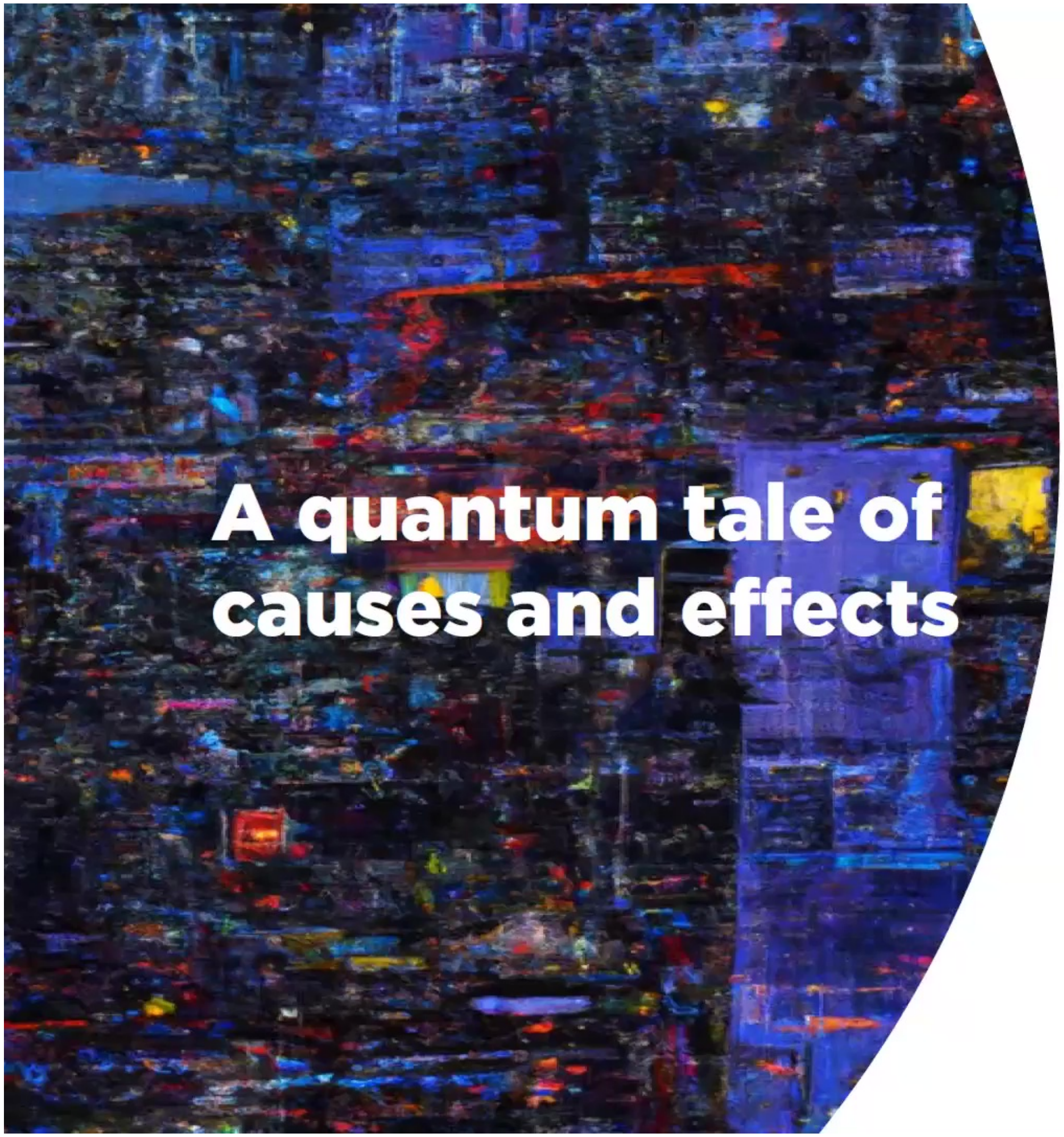
A quantum tale of causes and effects

INTERNATIONAL INSTITUTE OF PHYSICS
Federal University of Rio Grande do Norte

Rafael Chaves

Causal Inference & Quantum Foundations Workshop 2023

Quantum Information and Quantum Matter Group
www.iip.ufrn.br/qiqm



A quantum tale of causes and effects

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

MEC
MINISTÉRIO DA EDUCAÇÃO

INCT-IQ
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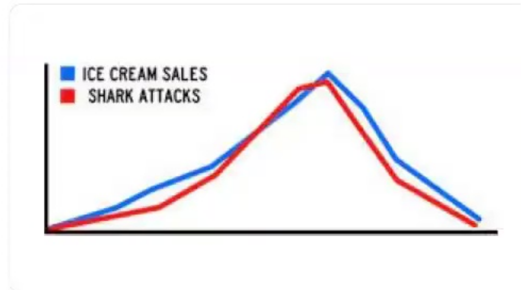
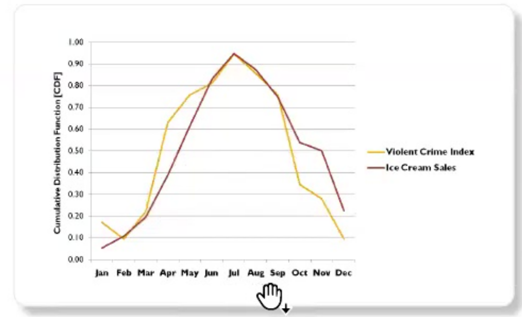
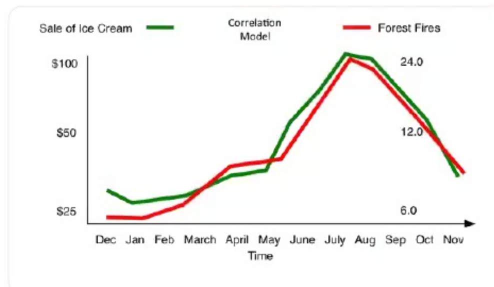
serrapilheira

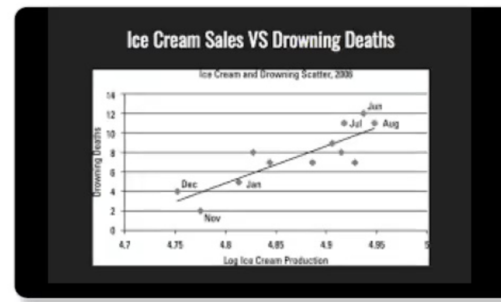
SIMONS FOUNDATION

Don't blame the ice cream

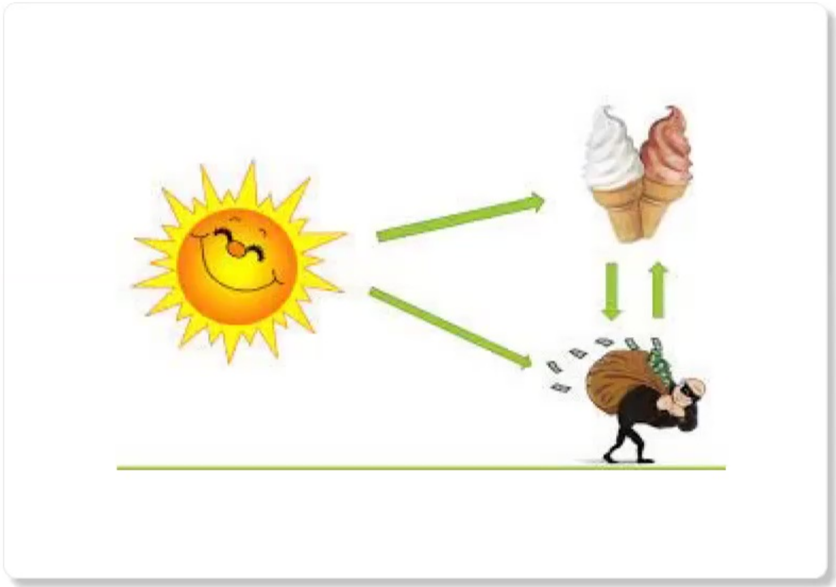








Correlation does not imply causation!



"**Correlation** supersedes **causation**, and science can advance even without coherent models, unified theories, or really any mechanistic explanation at all.

....

Correlation is enough. We can stop looking for models. We can analyze the data without hypotheses about what it might show. We can throw the numbers into the biggest computing clusters the world has ever seen and let statistical algorithms find patterns where **science cannot**.

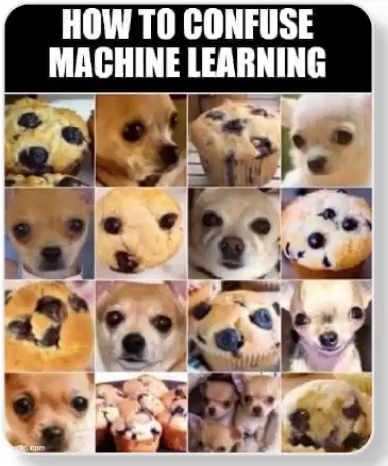
Correlation does not imply causality but…

"Very large databases have to contain **arbitrary correlations.** These correlations appear only due to the size, not the nature, of data… **most correlations are spurious**. **Too much information** tends to behave like **very little information**. The scientific method can be enriched by computer mining in immense databases, but not replaced by it."

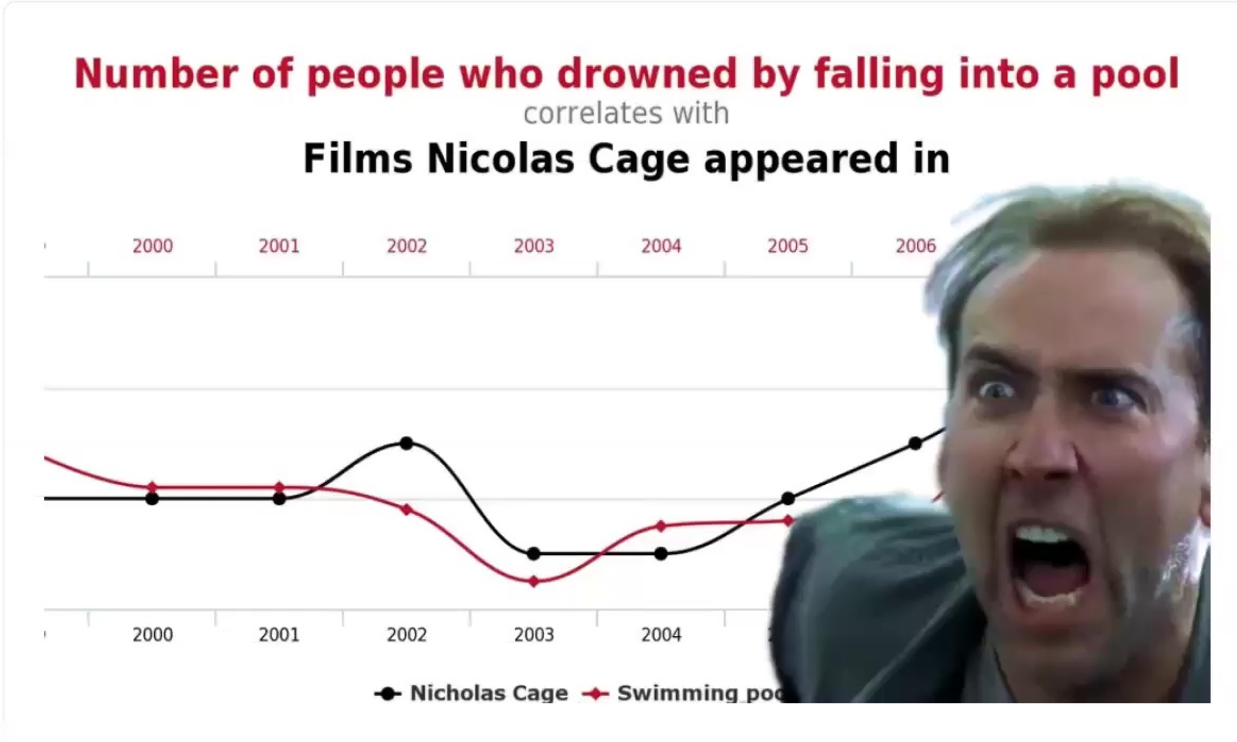
The Deluge of Spurious Correlations in Big Data

Cristian S. Calude & Giuseppe Longo

Foundations of Science 22 (3):595-612 (2016) Copy BibTeX



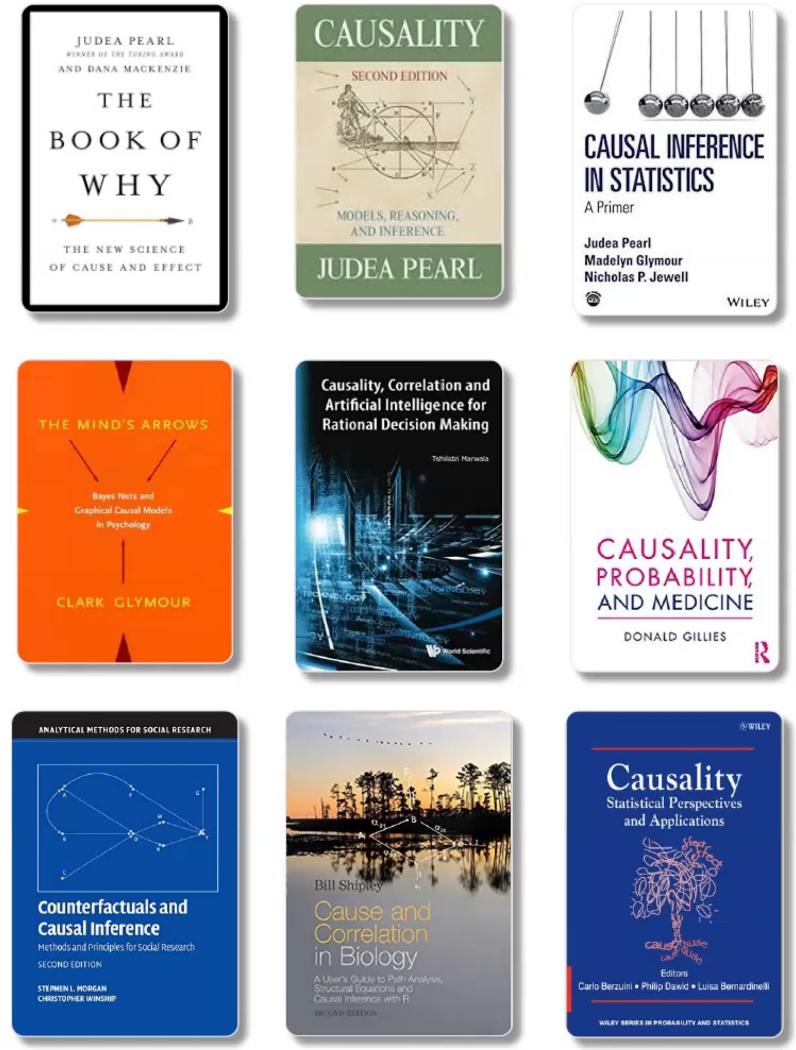
If we look enough, we will always find **patterns** where there is only **noise**...



Data cannot be interpreted in a theoretical vacuum!

We should start with an **hyphotesis** and only then generate the date to **confirm** or **falsify** it!

Causality Theory



Outline

- **DAGs and the Language of Causality**
- Double slit experiment
- Quantifying quantum causality
- Quantum networks

“If an improbable coincidence has ocurred, there must exist direct influence and/or a common cause.“

Reichenbach‘s principle:

no correlation without causation.

"If an improbable coincidence has ocurred, there must exist direct influence and/or a common cause."

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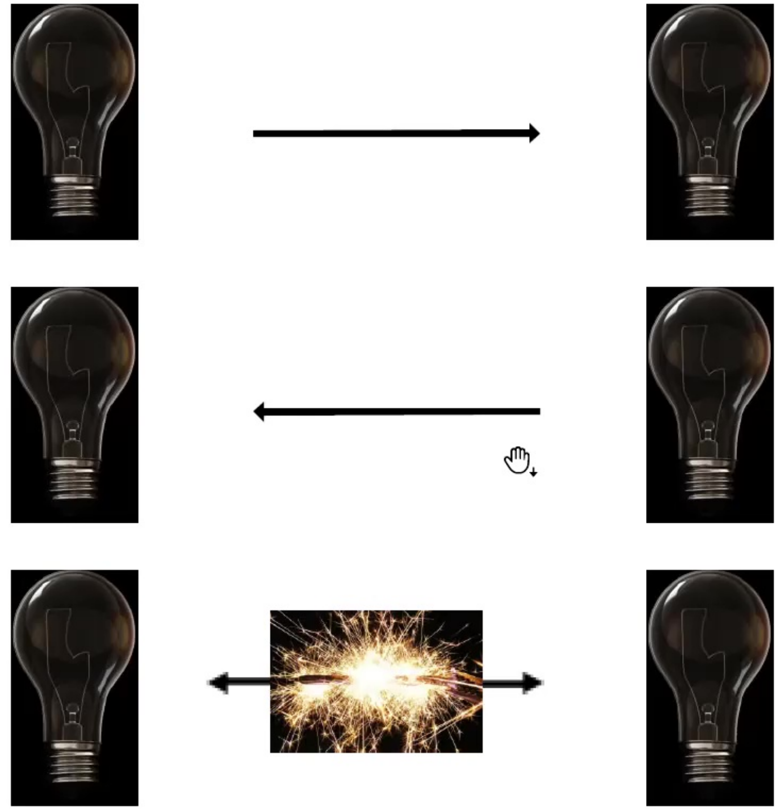
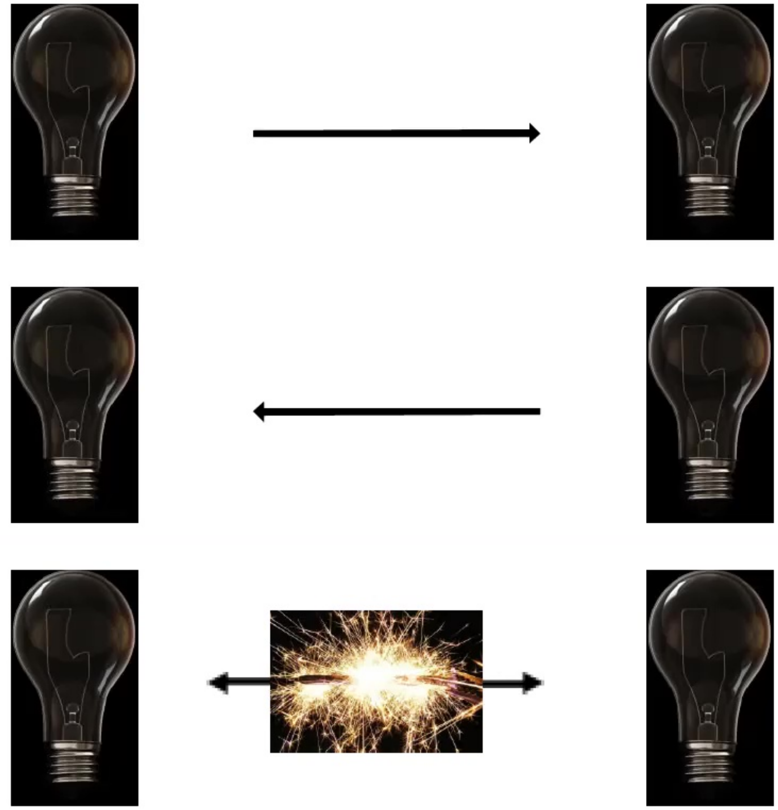


“If an improbable coincidence has ocurred, there must exist direct influence and/or a common cause.“



Reichenbach‘s principle:

no correlation without causation.

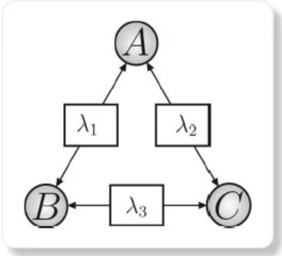
Task: Infer **causal relationships** from **observational** (statistical) data.

DAGs: Representing causal relations

For n variables $\boldsymbol{X_1}, \dots, \boldsymbol{X_n}$, the causal relationships are encoded in a **causal structure**, represented by a **directed acyclic graph** (DAG), with ith variable being a deterministic

$$\mathbf{x_i = f_i(pa_i, u_i)}$$

of its parents $\mathbf{pa_i}$ and jointly independent noise variables $\mathbf{u_i}$



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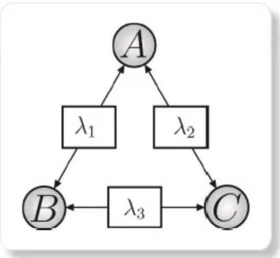


- Causal relationships are encoded in the **conditional**
- **independencies** (CIs) implied by the DAG

$$p(\lambda_1, \lambda_2) = p(\lambda_1)p(\lambda_2)$$
$$p(A, B|\lambda_1) = p(A|\lambda_1)p(B|\lambda_1)$$
$$\ldots$$

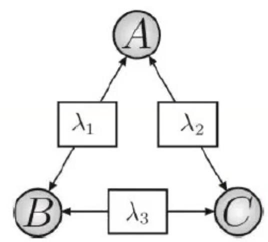
Conditional independencies hold information about causation!

[See J. Pearl, Causality]

Conditional independencies: Uncovering causal relations **Part 1**

Is a given probability distribution compatible with a presumed *causal structure*?

Example: Is a given $p(\lambda_1, \lambda_2, \lambda_3, A, B, C)$ compatible with



$$p(\lambda_1, \lambda_2) = p(\lambda_1)p(\lambda_2)$$

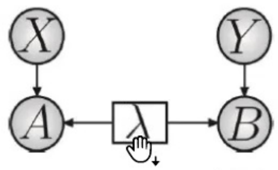
$$p(A, B|\lambda_1) \cancel{=} p(A|\lambda_1)p(B|\lambda_1)$$

➢ If the the full probability distribution (of all nodes in a DAG) is available, CIs hold all information required to solve the compatibility problem

However…

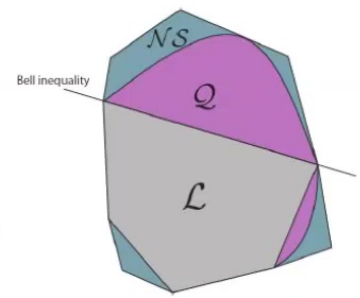
Bell Inequalities: Uncovering causal relations **Part 2**

- Usually and for a variety of reasons not all variables in a DAG are observable, i.e., not all CIs are available from empirical data



$$p(x,y,\lambda) = p(x)p(y)p(\lambda)$$
$$p(a|x,y,\lambda) = p(a|x,\lambda)$$
$$p(a,b|\lambda) = p(a|\lambda)p(b|\lambda)$$

$$\begin{aligned} p(a,b|x,y) &= \sum_\lambda p(a,b,\lambda|x,y) \\ &= \sum_\lambda p(a,b|x,y,\lambda)p(\lambda|x,y) \\ &= \sum_\lambda p(a|x,\lambda)p(b|y,\lambda)p(\lambda) \end{aligned}$$



Pic from **[Rev. Mod. Phys. 86, 419 (2014)]**

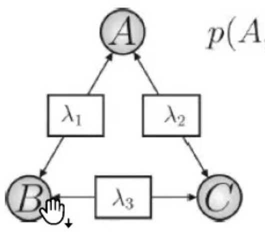
- CIs impose non-trivial constraints on the level of the observable variables, for example, Bell inequalities.

- In quantum mechanics non commuting observables cannot be jointly observed

Marginal scenario: subset of variables that are (jointly) observable

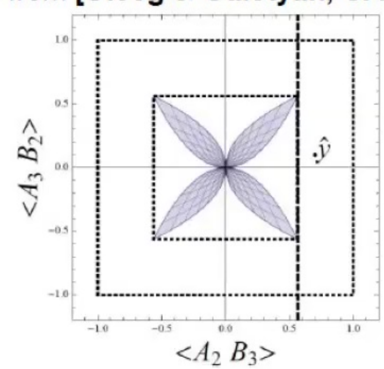
The challenge: Uncovering causal relations **Part 3**

- Describe marginals compatible with DAGs
- The observable probability dist. contains the full information required for that...
- ..very difficult, non-convex sets (algebraic geometry methods required, see for instance **[Geiger & Meek, UAI 1999]**)



$$p(A,B,C) = \int d\lambda_1 d\lambda_2 d\lambda_3 p(\lambda_1)p(\lambda_2)p(\lambda_3)\, p(A|\lambda_1,\lambda_2)p(B|\lambda_1,\lambda_3)p(C|\lambda_2,\lambda_3)$$

Picture from **[Steeg & Galstyan, UAI 2011]**



[Chaves et al, Uncertainty in Artificial Intelligence (UAI 2014)]
[Chaves, Phys. Rev. Lett. 116, 010402 (2016)]
[Lee, Spekkens, Journal of Causal Inference 5 (2017)]
[Wolfe, Spekkens, Fritz, J. Causal Inference 7 (2019)]
[Kela et al, IEEE Transactions on Information Theory 66, 339 (2019)]

Outline

- DAGs and the Language of Causality
- **Double slit experiment**
- Quantifying quantum causality
- Quantum networks

PHYSICAL REVIEW LETTERS **120**, 190401 (2018)

Causal Modeling the Delayed-Choice Experiment

Rafael Chaves, Gabriela Barreto Lemos, and Jacques Pienaar
International Institute of Physics, Universidade Federal do Rio Grande do Norte, Campus Universitario, Lagoa Nova, Natal, Rio Grande do Norte 59078-970, Brazil

PHYSICAL REVIEW A **100**, 022111 (2019)

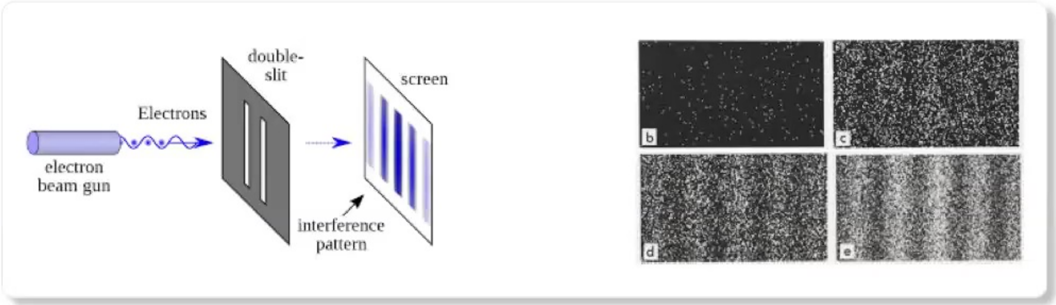
Device-independent test of a delayed choice experiment

Emanuele Polino,[1] Iris Agresti,[1] Davide Poderini,[1] Gonzalo Carvacho,[1] Giorgio Milani,[1] Gabriela Barreto Lemos,[2,3] Rafael Chaves,[2,4,*] and Fabio Sciarrino[1,5,†]

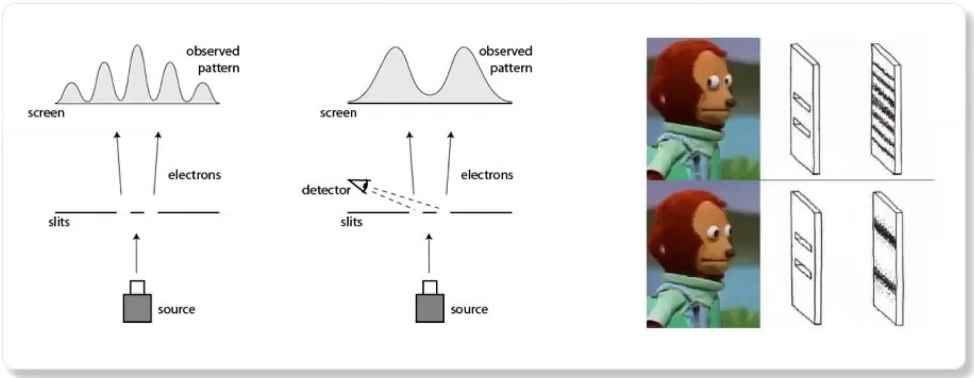
The double slit experiment

I will take just this one experiment, which has been designed to contain all of the mystery of quantum mechanics... Any other situation in quantum mechanics, it turns out, can always be explained by saying, 'You remember the case of the experiment with the two holes? It's the same thing'.

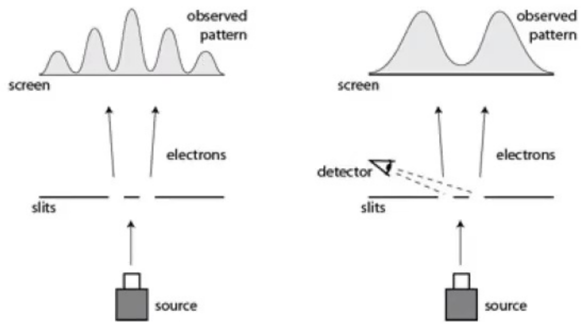
Richard Feynman



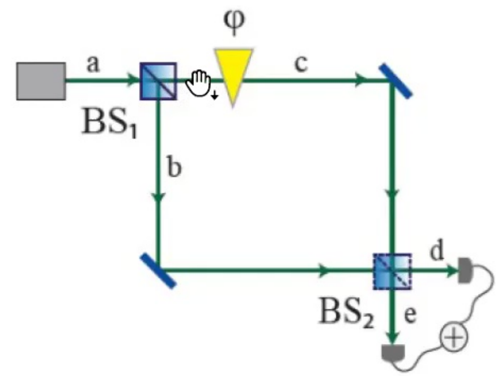
Wave-particle duality?



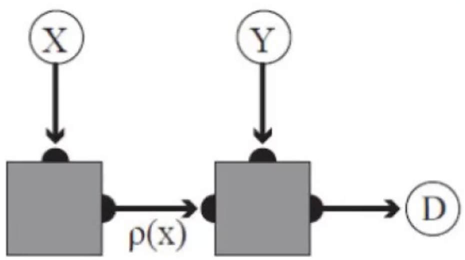
The delayed choice version



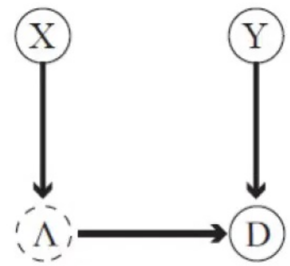
Double-Slit



Mach-Zender Interferometer



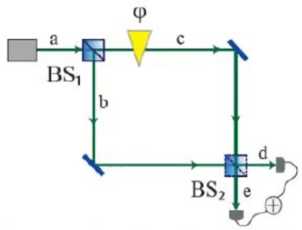
Prepare and Measure

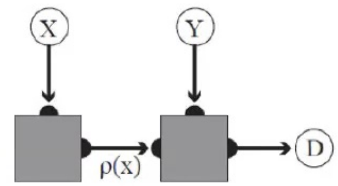


Causal Structure

Can this causal model explain the observed statistics?

The delayed choice version





$$|\Psi(x,0)\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + e^{i\phi_x}|10\rangle\right) \quad \Rightarrow \quad p(d|x,0) = 1/2$$

$$|\Psi(x,1)\rangle = \cos(\tfrac{\varphi_x}{2})|01\rangle - i\sin(\tfrac{\varphi_x}{2})|10\rangle \quad \Rightarrow \quad p(1|x,1) = 1 - p(0|x,1) = \sin^2(\tfrac{\phi_x}{2})$$

$$\boxed{p(d|y,\lambda) = \textstyle\sum_{u_D} p(d|y,\lambda,u_D)p(u_D)}$$

$i)\quad p(u_D = 0) = p(u_D = 1) = 1/2,$

$ii)\quad p(\lambda = 0|x) = 1 - p(\lambda = 1|x) = \cos^2 \frac{\phi_x}{2}$

$iii)\quad p(d|y = 0, \lambda, u_D) = p(d|y = 0, u_D) = \delta_{d,u_D},$

$iv)\quad p(d|y = 1, \lambda, u_D) = p(d|y = 1, \lambda) = \delta_{d,\lambda},$

Can this causal model explain the observed statistics?

YES!!!

If we give up on wave-particle concepts, the double-slit experiment does have a classical explanation.

The delayed choice version

If we slightly change the experiment, a classical model with the same dimension constraints cannot explain the data.

Non-classicality!

$$I_{DW} = \langle D_{00} \rangle + \langle D_{01} \rangle + \langle D_{10} \rangle - \langle D_{11} \rangle - \langle D_{20} \rangle \leq 3$$

$$I_Q = 1 + 2\sqrt{2}$$

$$\min R_{Y \to \Lambda} = \max \left[\frac{I - 3}{4}, 0 \right]$$

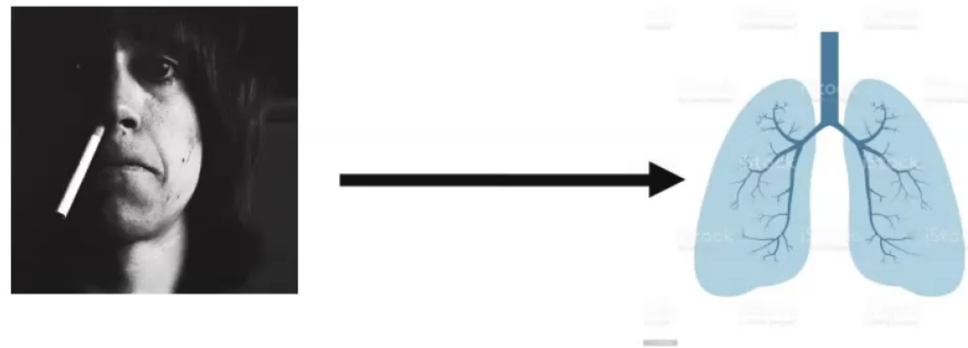
Outline

- DAGs and the Language of Causality
- Double slit experiment
- **Quantifying quantum causality**
- Quantum networks

Common causes

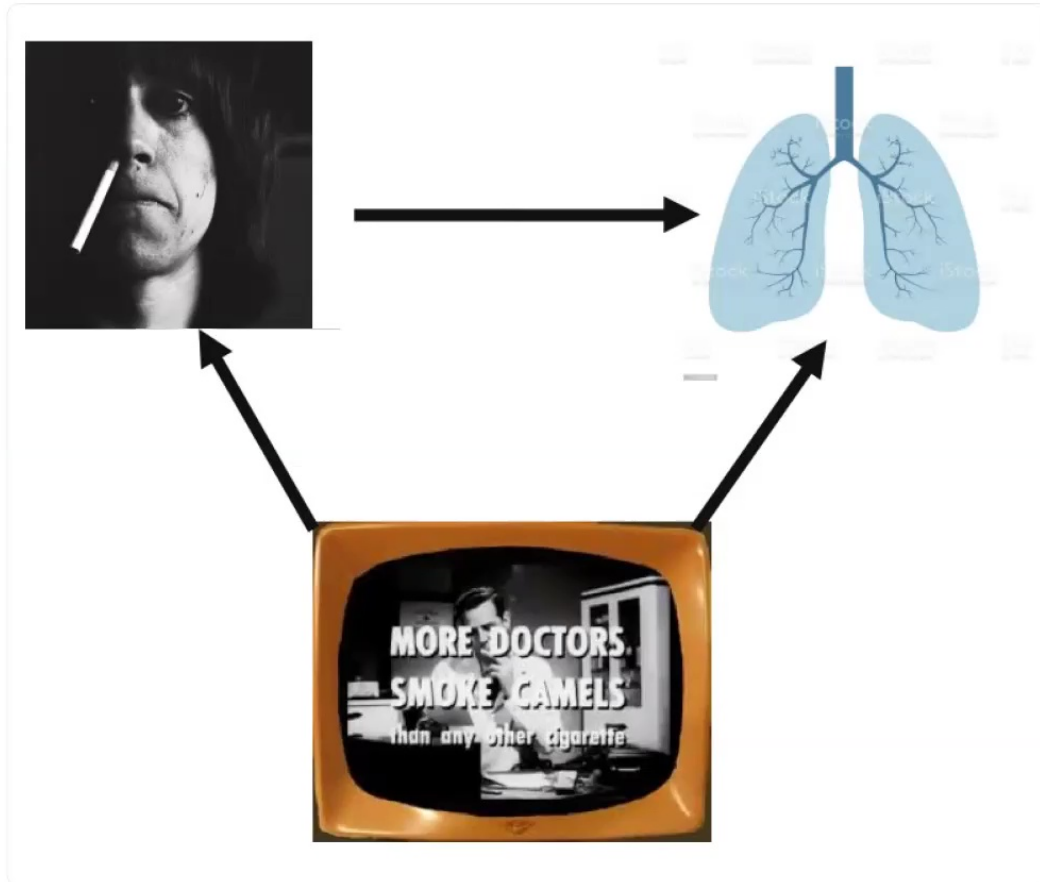
X

Causal Influences

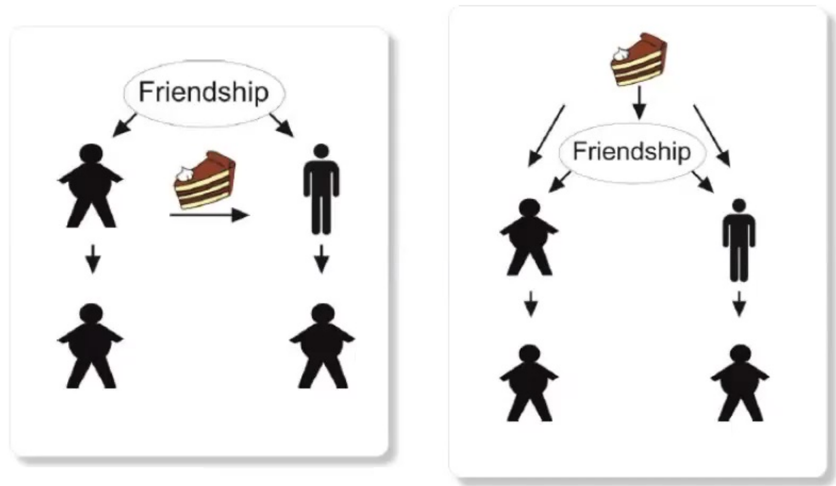


Does smoking cause cancer?

Common causes
X
Causal Influences



Common causes
X
Causal Influences

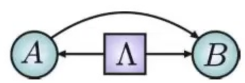


Is obesity contagious?

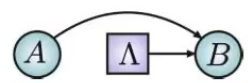
What about quantum causality/interventions?

Does A have some causal influence over B, or all the correlations between A and B are mediated via the common ancestor?

Intervention

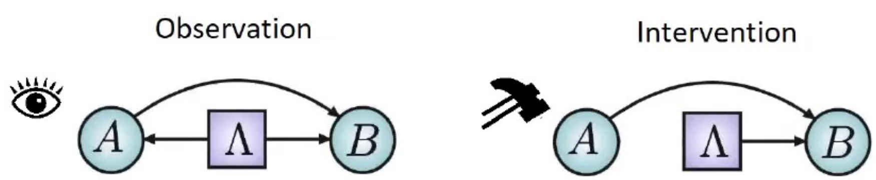


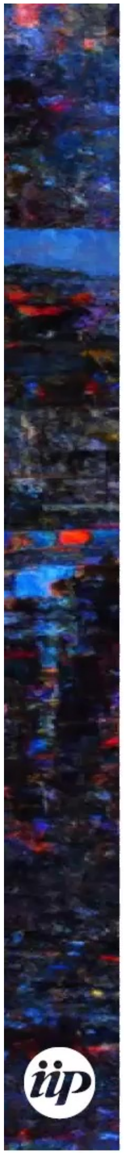
$$p(b|a) = \sum_{\lambda} p(b|a,\lambda)p(\lambda|a)$$



$$p(b|\mathrm{do}(a)) = \sum_{\lambda} p(b|a,\lambda)p(\lambda)$$

$$p(b|a) \neq p(b|\mathrm{do}(a))$$





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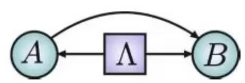
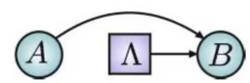




$$p(b|a) = \sum_{\lambda} p(b|a,\lambda)p(\lambda|a) \qquad p(b|\mathrm{do}(a)) = \sum_{\lambda} p(b|a,\lambda)p(\lambda)$$

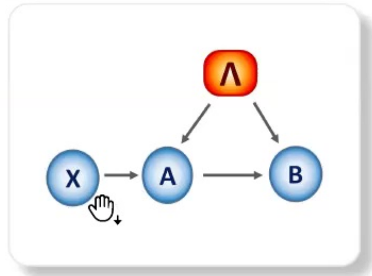
$$p(b|a) \neq p(b|\mathrm{do}(a))$$

Measure of causality

$$\mathrm{ACE}_{A\to B} = \sup_{a,a',b} |p(b|\mathrm{do}(a)) - p(b|\mathrm{do}(a'))|$$

Instrumental variables

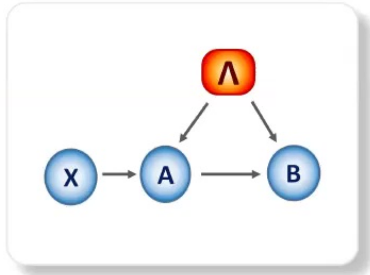
Can we do it with observational data only, i.e., without interventions? Yep, use an instrumental variable X.



Empirical data is encoded in the distribution $p(a,b|x)$

Instrumental variables

Can we do it with observational data only, i.e., without interventions? Yep, use an instrumental variable X.

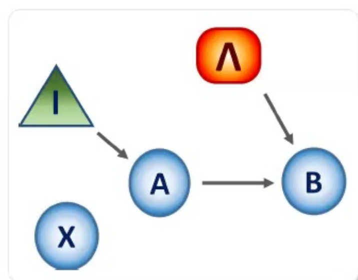


Empirical data is encoded in the distribution $p(a,b|x)$

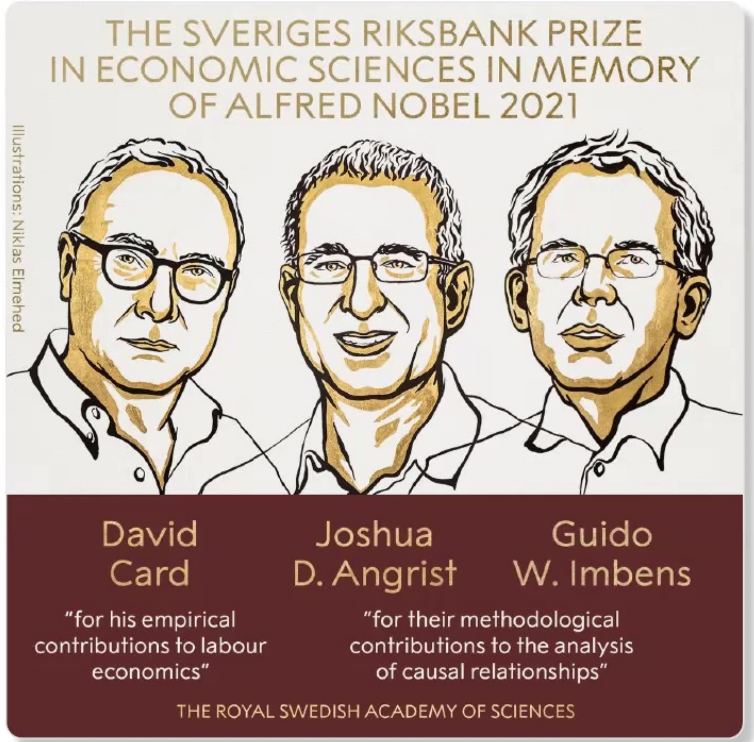
Can estimate causal influence in a device independent way, e.g, the average causal effect (ACE).

Balke & Pearl JASA 1997

$$\mathrm{ACE}_{A\to B} = \sup_{a,a',b} |p(b|\mathrm{do}(a)) - p(b|\mathrm{do}(a'))|$$



$$\mathrm{ACE}_{A\to B} \geq 2p(a=0,b=0|x=0) - 2 + p(a=1,b=1|x=0) + p(b=1|x=1)$$



Identification of causal effects using instrumental variables

JD **Angrist**, GW Imbens, DB Rubin - Journal of the American …, 1996 - Taylor & Francis

We outline a framework for causal inference in settings where assignment to a binary treatment is ignorable, but compliance with the assignment is not perfect so that the receipt of treatment is nonignorable. To address the problems associated with comparing subjects …

☆ 99 Citado por 6370 Artigos relacionados Todas as 26 versões

What about quantum causality/interventions?

nature physics — ARTICLES
https://doi.org/10.1038/s41567-017-0008-5

Quantum violation of an instrumental test

Rafael Chaves[1]*, Gonzalo Carvacho[2], Iris Agresti[2], Valerio Di Giulio[2], Leandro Aolita[3], Sandro Giacomini[2] and Fabio Sciarrino[2]*

PHYSICAL REVIEW LETTERS 125, 230401 (2020)

Quantifying Causal Influences in the Presence of a Quantum Common Cause

Mariami Gachechiladze,[1,*] Nikolai Miklin,[2,†] and Rafael Chaves[3,4]

PHYSICAL REVIEW LETTERS 120, 140408 (2018)

Editors' Suggestion

Quantum Steering Beyond Instrumental Causal Networks

R. V. Nery,[1] M. M. Taddei,[1] R. Chaves,[2] and L. Aolita[1,3]

Exclusivity graph approach to Instrumental inequalities

Davide Poderini[1], Rafael Chaves[2,3], Iris Agresti[1], Gonzalo Carvacho[1], Fabio Sciarrino[1]

COMMUNICATIONS PHYSICS

ARTICLE

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Experimental device-independent certified randomness generation with an instrumental causal structure

Quantifying Quantum Causal Influences

Lucas Hutter,[1] Rafael Chaves,[2,3] Ranieri Vieira Nery,[2] George Moreno,[2,4] and Daniel Jost Brod[1]

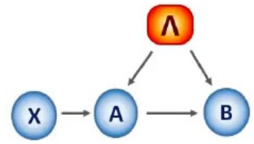
SCIENCE ADVANCES | RESEARCH ARTICLE

PHYSICS

Experimental test of quantum causal influences

Iris Agresti[1], Davide Poderini[1], Beatrice Polacchi[1], Nikolai Miklin[2,3], Mariami Gachechiladze[4], Alessia Suprano[1], Emanuele Polino[1], Giorgio Milani[1], Gonzalo Carvacho[1], Rafael Chaves[5]*, Fabio Sciarrino[1]*

Quantum causal influences



- In the simplest scenario all correlations are classical **[Henson,Lal,Pusey NJP 2014]**

$$p(a,b|x) = \sum_{\lambda} p(a|x,\lambda)p(b|a,\lambda)p(\lambda)$$

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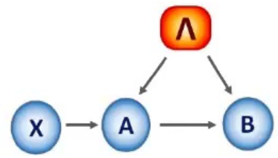
Quantum causal influences



- **Quantum ACE**

$$p_Q(a,b|x) = \mathrm{Tr}\left[(M_a^x \otimes M_b^a)\varrho\right]$$

$$\varrho = v|\phi^+\rangle\langle\phi^+| + (1-v)\mathbb{1}/4$$
$$|\phi^+\rangle = (1/\sqrt{2})(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$p_Q(b|\mathrm{do}(a)) = p_Q(b|\mathrm{do}(a')) = \mathrm{Tr}\left[(M_b^a)\mathbb{1}/2\right] = 1/2$$
$$\mathrm{ACE}_{A\to B} = 0$$

- Classical ACE

$$\mathrm{ACE}_{A\to B} \geq 2p(a=0,b=0|x=0) - 2 + p(a=1,b=1|x=0) + p(b=1|x=1)$$

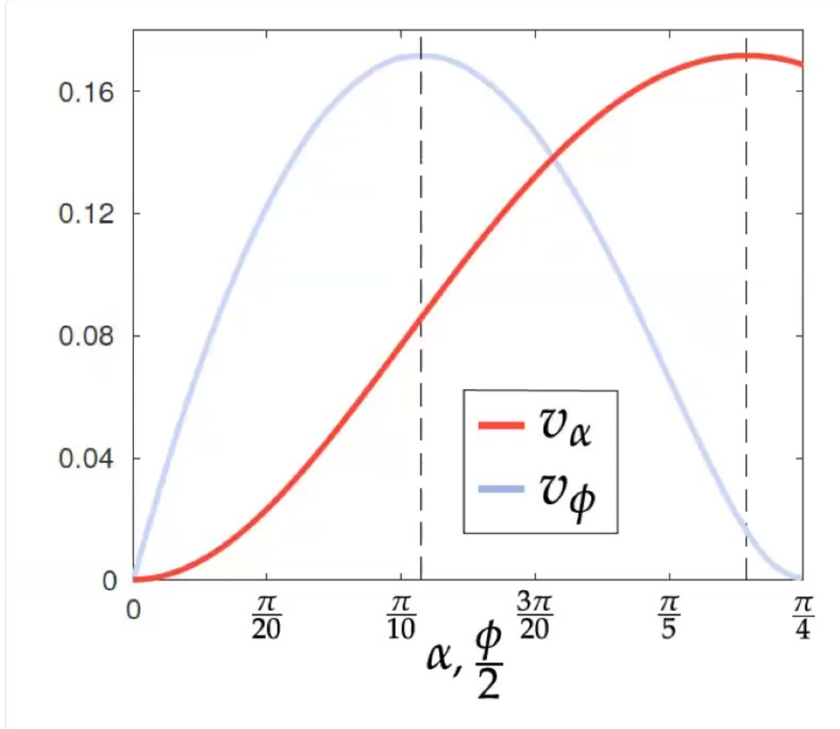
$$\mathrm{ACE}_{A\to B} \gtrsim 0.91v - 0.75$$

Quantum effects can lead to an overestimation of causal influences!

Quantifying causal influences

Result 1. *Every pure entangled state can generate correlations that violate the classical bound on ACE. Moreover, entanglement is necessary but not sufficient for such violations.*

- Even though **no Bell inequality** can be violated, we still can witness the **non-classicality** of the correlations.
- A quantum common source leads to na **overestimation** of causal influences if the classical bounds are used.

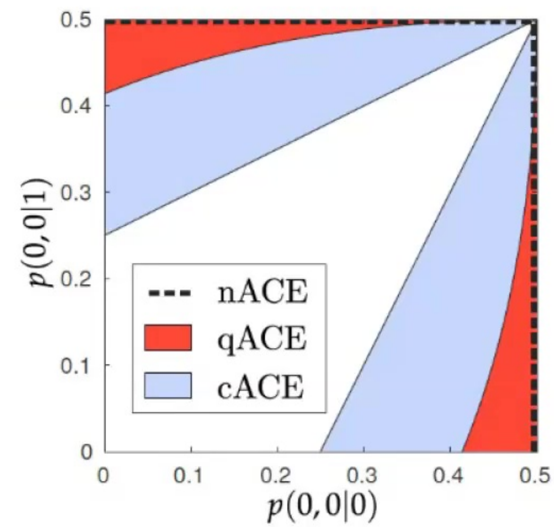


Quantifying quantum causal influences

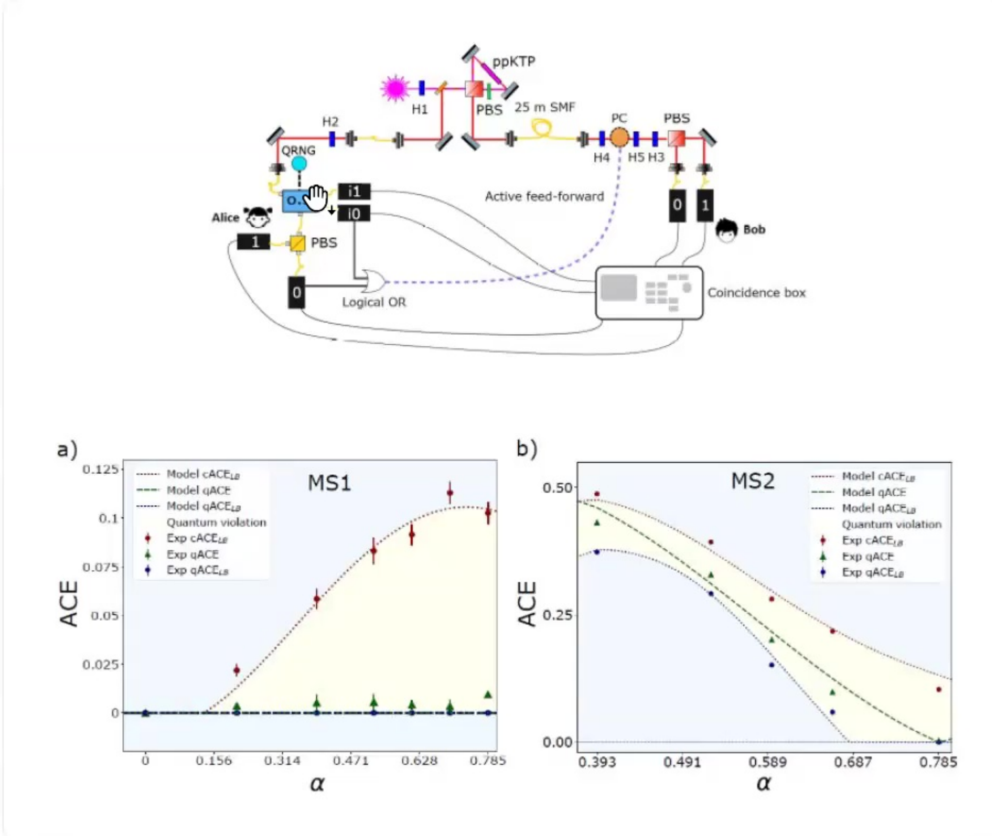
Result 3. *In the instrumental scenario with dichotomic measurements* qACE *is lower bounded as*

$$\mathrm{qACE}_{A\to B} \geq \sum_{x=0.1} \left(p(0,0|x) + p(1,1|x)\right) + \zeta - 1, \quad (11)$$

$$\zeta = \max_{\pm} \sqrt{\prod_{a=0,1} \left(1 \pm \sum_{x=0,1} (-1)^x \left(p(a,0|x) - p(a,1|x)\right)\right)}.$$



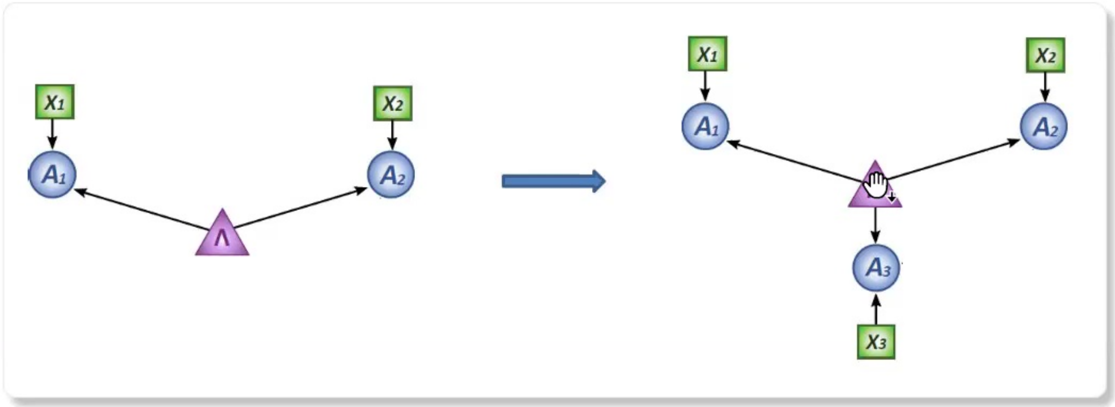
Experimental test of quantum causal influences



Outline

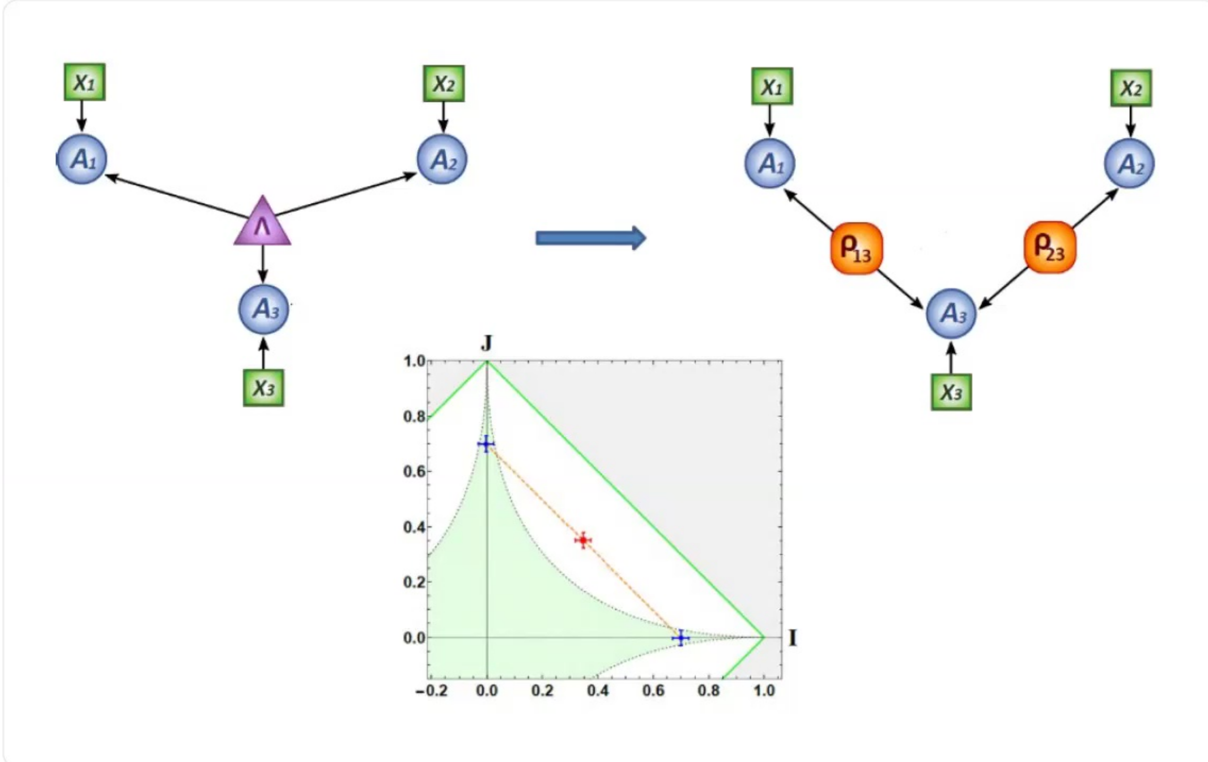
- DAGs and the Language of Causality
- Double slit experiment
- Quantifying quantum causality
- **Quantum networks**

Beyond Bell



Quantum networks can have much more interesting topologies than the **“single source connects all”** scenario!

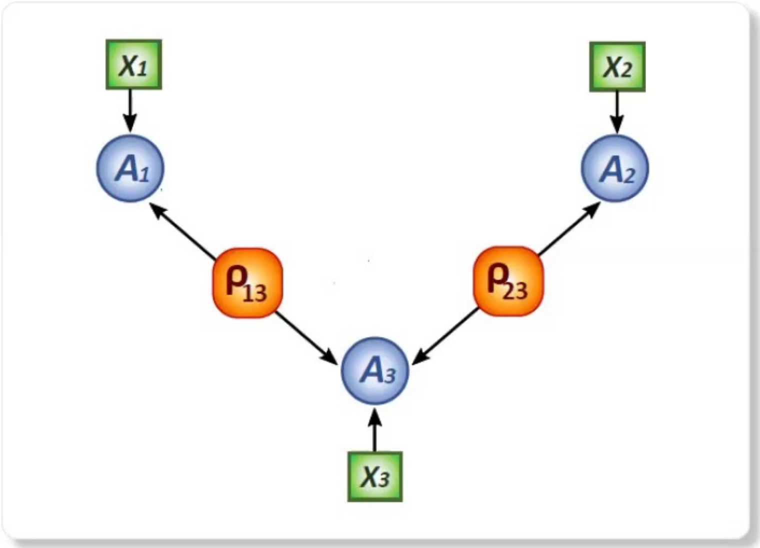
Beyond Bell



Bilocality: the independence of the sources is explicility taken into account

[Branciard, Gisin, Pironio, PRL 104, 170401 (2010)]

Unlocking new features with quantum networks



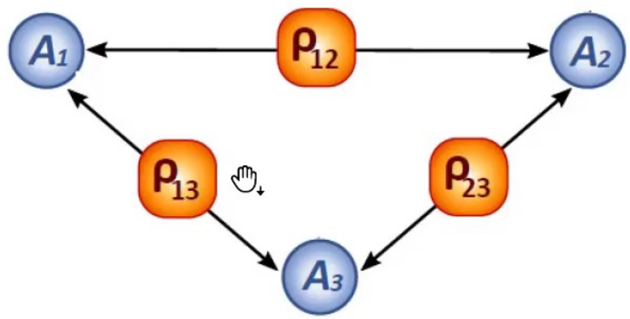
Non-localitity activation of measurements
[Pozas et al, PRL 123, 140503 (2019)

Self-testing quantum theory
[Weilenmann, Colbeck, PRL 125, 060406 (2020)]

Proving the need of complex numbers
[Renou et al,Nature 600, 625-629 (2021)

Full network nonlocality
[Pozas,Gisin,Tavakoli ,PRL 128, 010403 (2022)

Unlocking new features with quantum networks

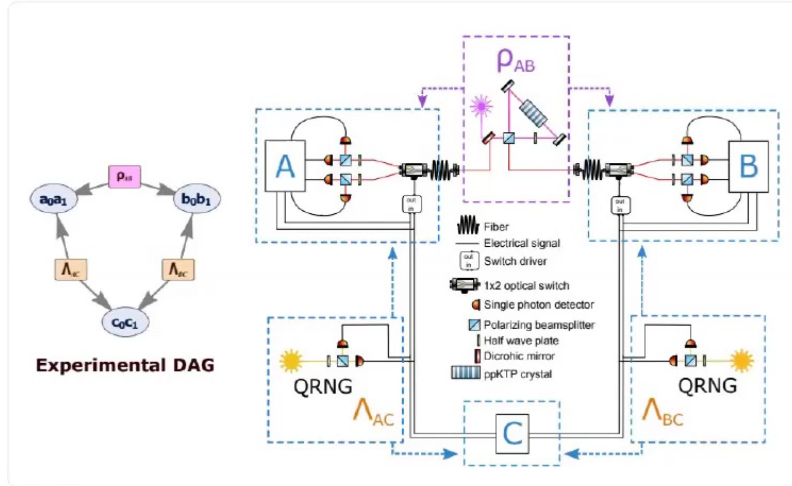


Non-localitity without inputs
[Fritz, NJP 14, 103001 (2012)]
[Renou et al, PRL 123, 140401 (2019)]

Genuine Multipartite non-locality in a network
[Suprano et al, PRX Quantum 3, 030342 (2022)]

Quantifying measurement dependence in Bell's theorem
[Chaves et al, PRX Quantum 3, 040323 (2021)]

Experimental realization of the “Fritz” distribution



[Polino et al, Nat Comm14, 909 (2023)

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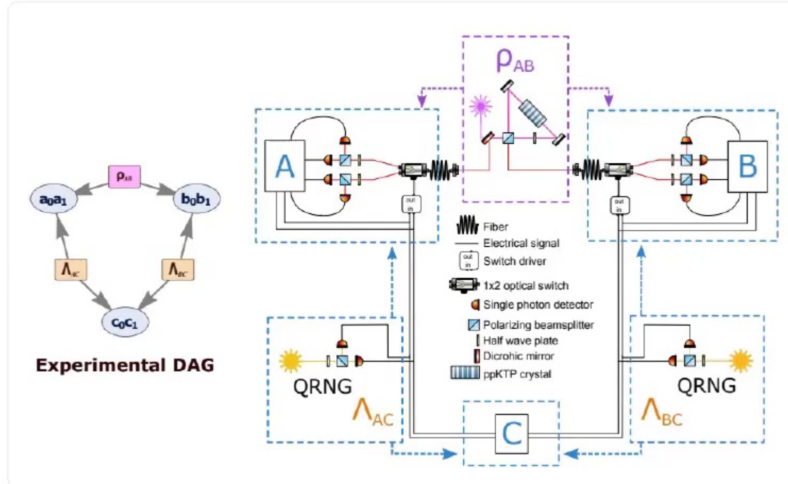
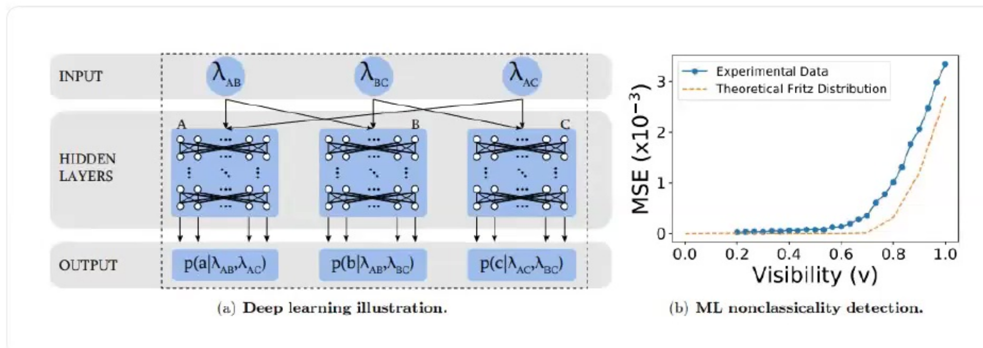


[Polino et al, Nat Comm14, 909 (2023)]

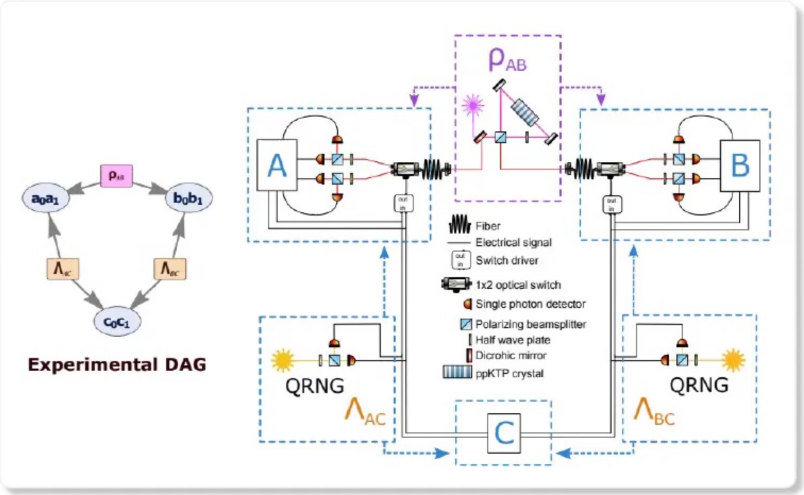


(a) **Deep learning illustration.**

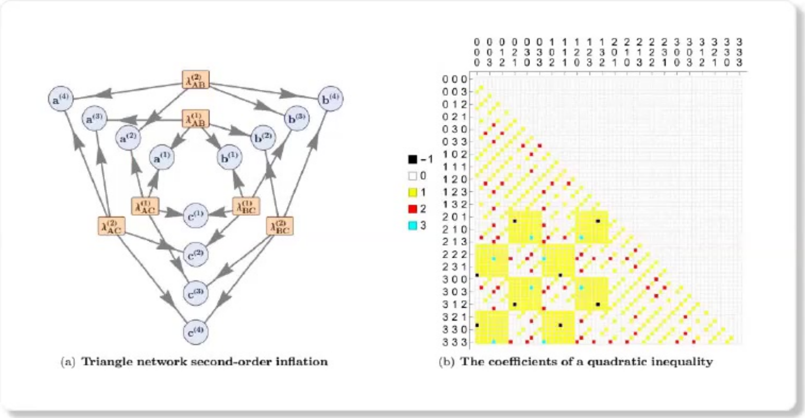
(b) **ML nonclassicality detection.**

[Krivachy et al, npj Quantum Inf 6, 70 (2020)]

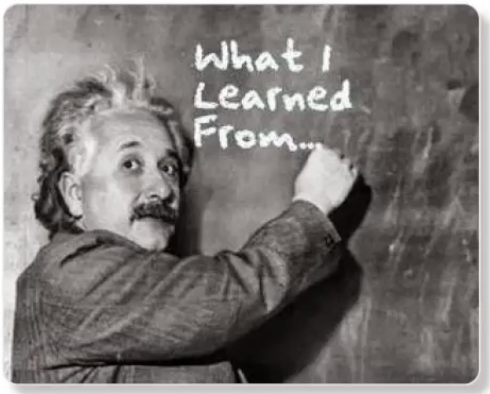
Experimental realization of the “Fritz” distribution



[Polino et al, Nat Comm14, 909 (2023)]



[Wolfe, Spekkens Fritz, J. Causal Inference 7, 2019]



Take-Home Messages

Causality theory provides a fairly unexplored framework

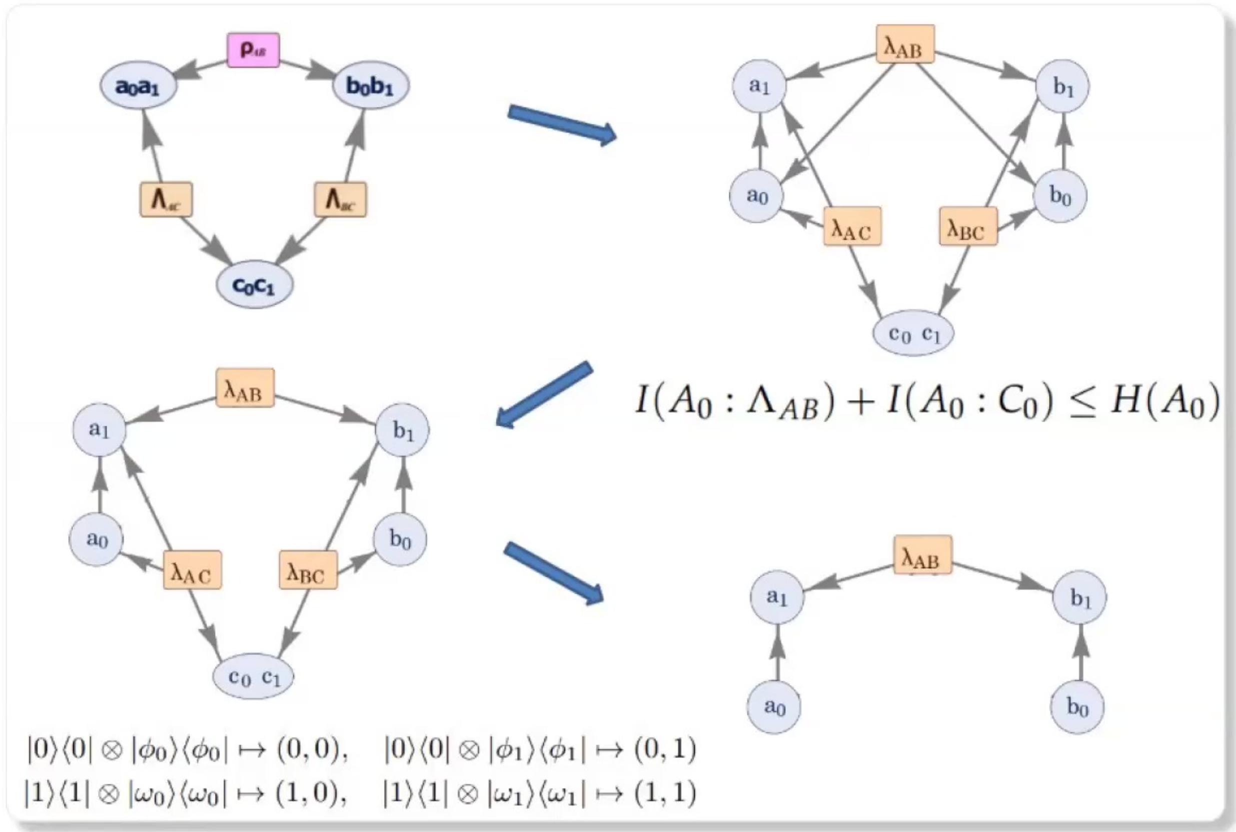
Causal analysis of the double slit experiment shows its classicality

Interventions and the quantification of causal influences allow for new method to detect non-classicality

Causal networks reveal new quantum features

Come to Paraty! Registration is open!

Embedding Bell in a Triangle



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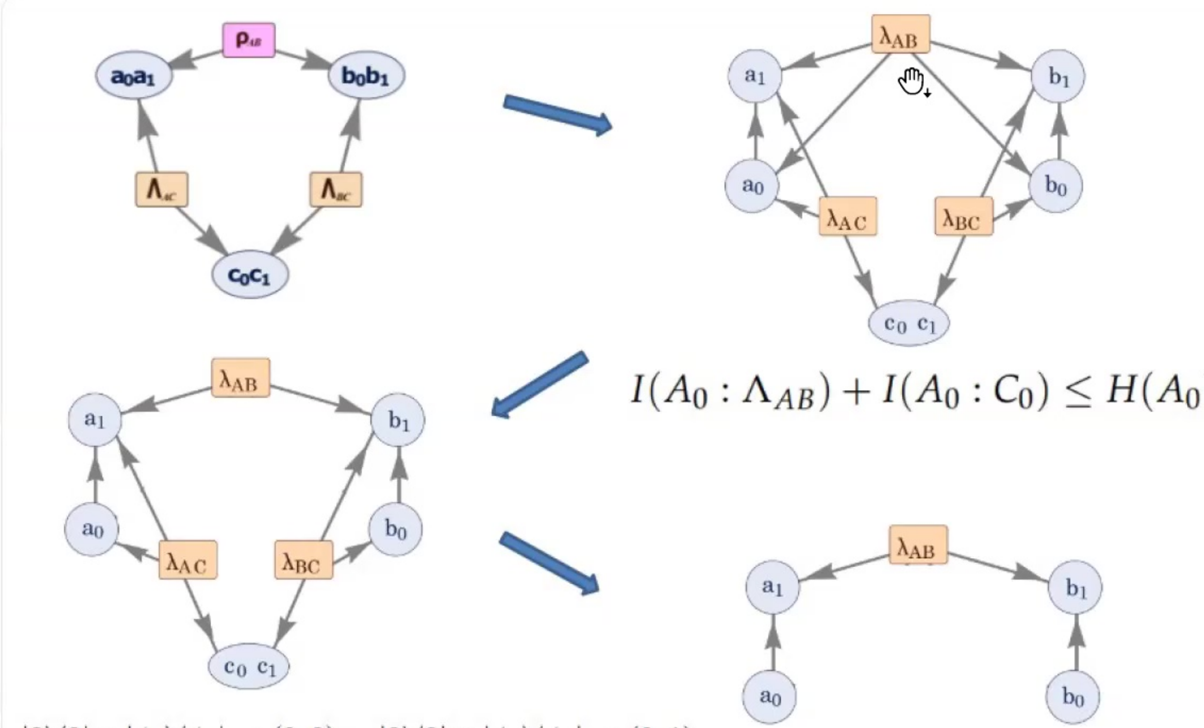
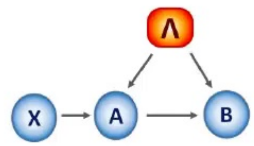


$|0\rangle\langle 0| \otimes |\phi_0\rangle\langle\phi_0| \mapsto (0,0), \quad |0\rangle\langle 0| \otimes |\phi_1\rangle\langle\phi_1| \mapsto (0,1)$

$|1\rangle\langle 1| \otimes |\omega_0\rangle\langle\omega_0| \mapsto (1,0), \quad |1\rangle\langle 1| \otimes |\omega_1\rangle\langle\omega_1| \mapsto (1,1)$

[Fritz, NJP 14, 103001 (2012)]

Quantum causal influences



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$$p(b|do(a)) = \mathrm{tr}\left[(\mathbb{1} \otimes N_b^a)\rho_{AB}\right] = \mathrm{tr}\left[N_b^a \rho_B\right]$$

- Do the classical bounds on ACE still apply?

$$\mathrm{ACE}_{A\to B} = \max_{a,a',b}\Big(p(b|do(a)) - p(b|do(a'))\Big)$$

$$\begin{aligned}\mathrm{ACE}_{A\to B} \geq & 2p(a=0,b=0|x=0) - 2 \\ & + p(a=1,b=1|x=0) + p(b=1|x=1)\end{aligned}$$

The delayed choice version

If we slightly change the experiment, a classical model with the same dimension constraints cannot explain the data.

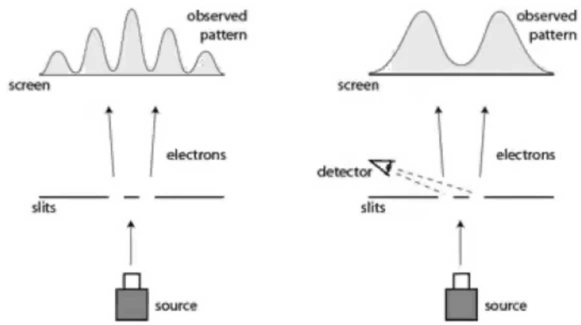
Non-classicality!

$$I_{DW} = \langle D_{00}\rangle + \langle D_{01}\rangle + \langle D_{10}\rangle - \langle D_{11}\rangle - \langle D_{20}\rangle \leq 3$$

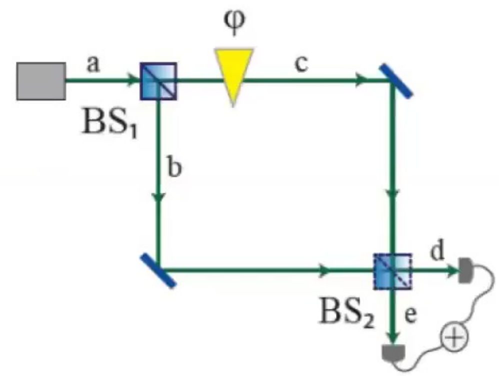
$$I_Q = 1 + 2\sqrt{2}$$

$$\min R_{Y\to\Lambda} = \max\left[\frac{I-3}{4}, 0\right]$$

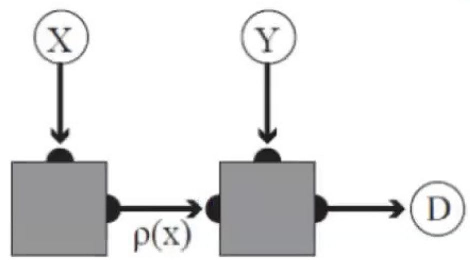
The delayed choice version



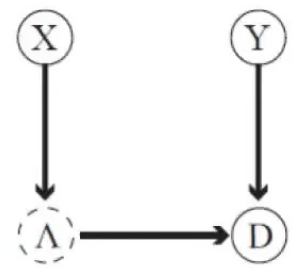
Double-Slit



Mach-Zender Interferometer



Prepare and Measure



Causal Structure

Can this causal model explain the observed statistics?