Title: A Hierarchy of Multi-Party Nonlocal Effects

Speakers: Peter Bierhorst

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Abstract: According to recent new definitions, a multi-party behavior is genuinely multipartite nonlocal (GMNL) if it cannot be modeled by measurements on an underlying network of bipartite-only nonlocal resources, possibly supplemented with local (classical) resources shared by all parties. Three experimental results published in 2022 provide initial evidence, subject to postselection-related assumptions, for the existence of behaviors meeting these definitions of GMNL. The new definitions of GMNL differ on whether to allow entangled measurements upon, and/or superquantum behaviors among, the underlying bipartite resources when classifying behaviors asonly bipartite nonlocal. I will discuss the interrelationships of these choices in three-party quantum networks, and present a behavior in the simplest nontrivial multi-partite measurement scenario (3 parties, 2 measurement settings, and 2 outcomes) that (A) cannot be simulated in a bipartite network prohibiting both entangled measurements and superquantum resources, (B) can be simulated with bipartite-only quantum states allowing for an entangled quantum measurement (indicating an approach to device independent certification of entangled measurements with fewer settings than in previous protocols), and surprisingly (C) can be simulated with bipartite-only superquantum states (Popescu-Rohrlich boxes) while maintaining a prohibition on entangled measurements. It turns out that other behaviors previously studied as device-independent witnesses of entangled measurements can also be simulated in the manner of (C), posing a challenge to a theory-independent understanding of entangled measurements as an observable phenomenon distinct from bipartite nonlocality.

A Hierarchy of Multi-Party Nonlocal Effects

arXiv:2301.12081

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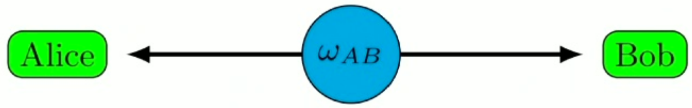
Bipartite Nonlocality



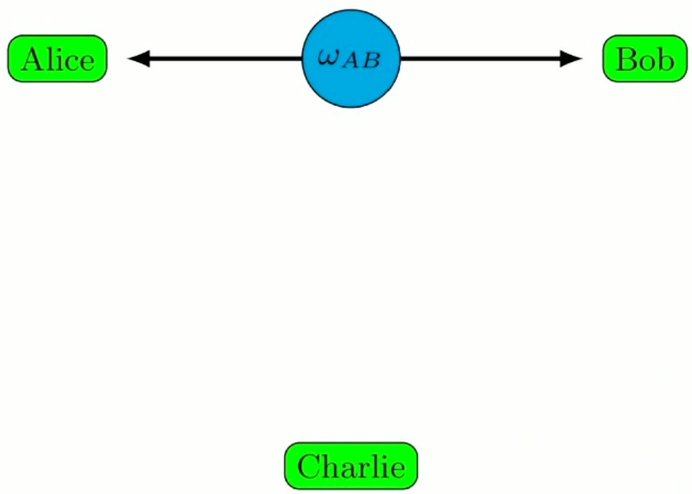
- 2022 Physics Nobel Prize for experiments in 1970s, 1980s, 1990s
- Loophole-Free Demonstrations of Bell nonlocality in 2015

Hensen *et al.* Nature 526:682, Shalm *et al.* PRL 115:250402,
Giustina *et al.* PRL 115:250401

- Random Numbers certified by impossibility of FTL signaling

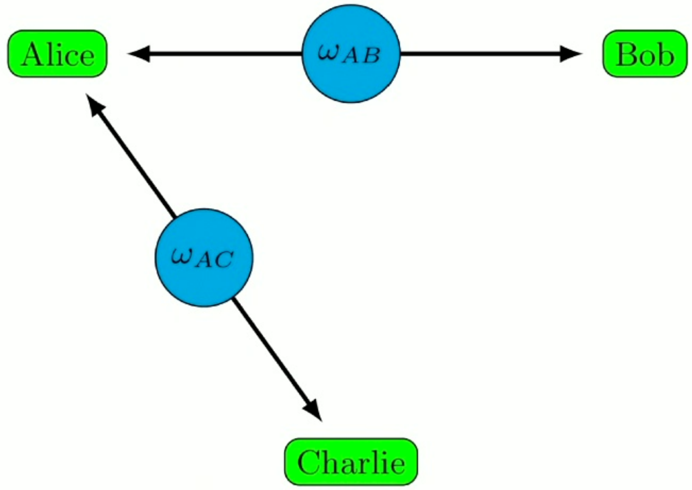
Bierhorst *et al.* Nature 556:223 (2018)

Tripartite Experiments



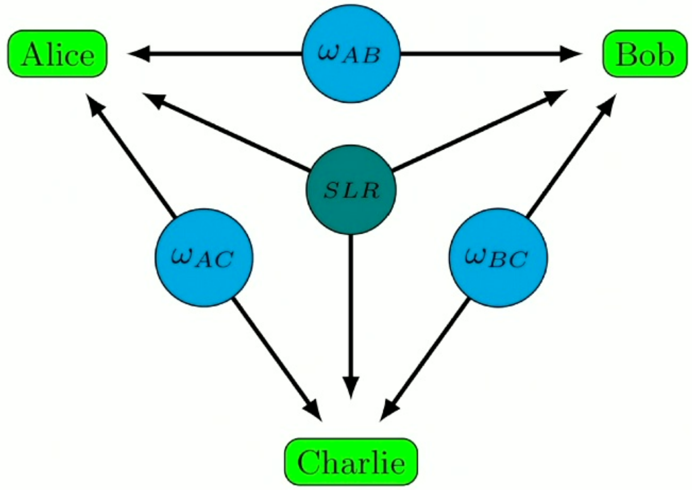
- The three-party behavior $P(ABC|XYZ)$ is nonclassical, but not in a *genuinely multipartite* way:
$P(ABC|XYZ) = P(AB|XY)P(C|Z)$
($X, Y, Z =$ inputs, $A, B, C =$ observed outputs)
- Old definition of GMNL: $P(ABC|XYZ)$ does not decompose into a convex combination of 2 vs. 1 splits

Tripartite Experiments



- For parallel Bell experiments, $P(ABC|XYZ)$ is (mis?)classified as GMNL according to old definition

New Definitions of Genuine Multiparty Nonlocality



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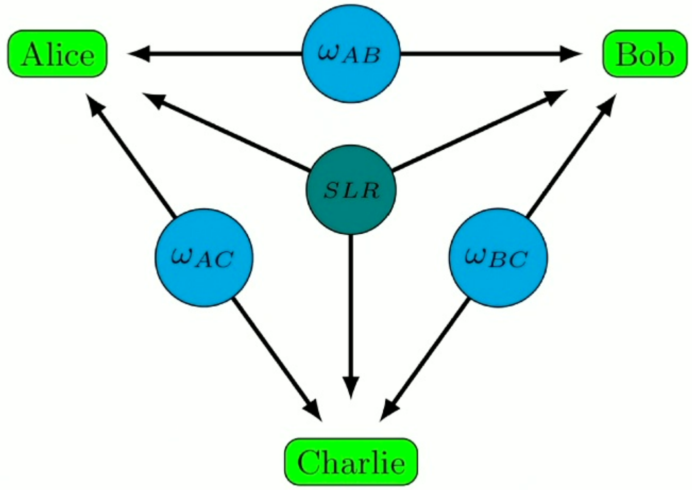

New definitions say $P(ABC|XYZ)$ is GMNL if it *cannot* be obtained from an underlying network of bipartite nonclassical resources ω_{PQ}, allowing also a tripartite classical resource SLR

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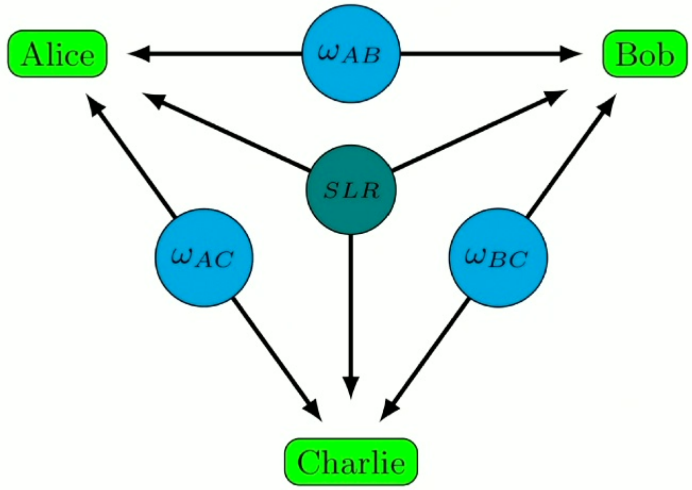

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Variants: what sort of ω_{PQ} resources (quantum/superquantum)?

What kind of measurements (separate/entangled)?

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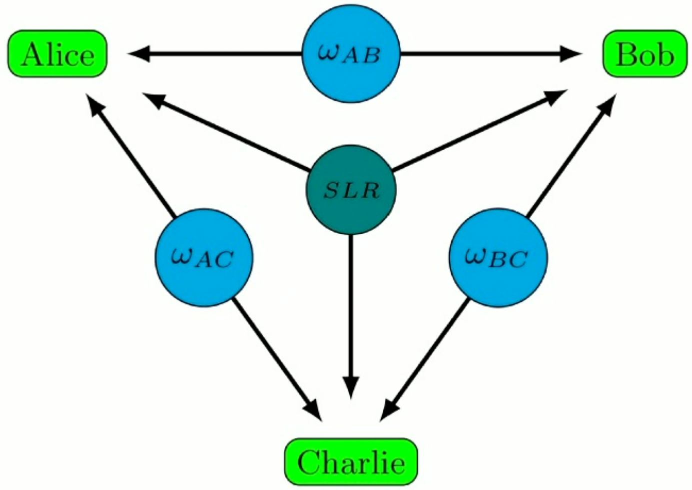

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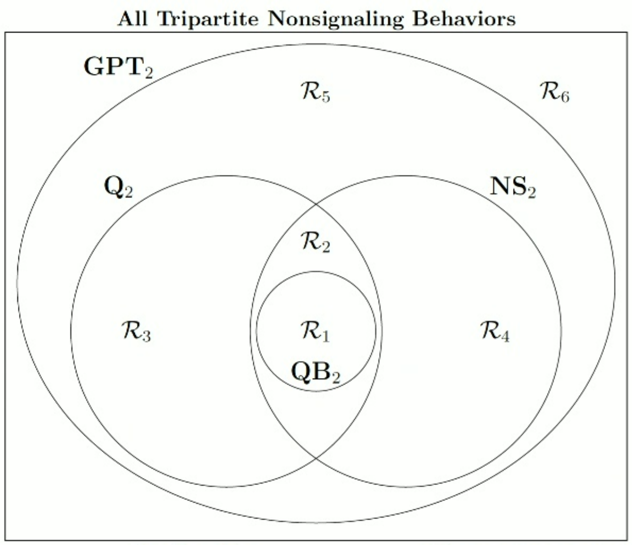
Interest: foundations, device-independent certification of effects

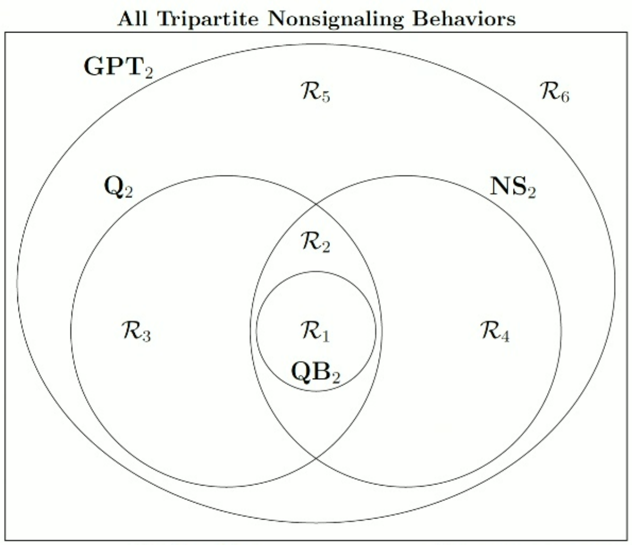
Different Variants of New Definitions of GMNL



- GPT_2^C: Most general definition of GMNL; that of Coiteux-Roy, Wolfe, Renou [PRL 127:200401(2021)]
- Quantum correlations in $\mathcal{R}_6$ exist and can be tested

Behavior Class	Superquantum Bipartite Sources	Entangled Measurements
QB_2	No	No
NS_2	Yes	No
Q_2	No	Yes
GPT_2	Yes	Yes

Different Variants of New Definitions of GMNL



- Q_2^C: Definition of GMNL in Schmid, Fraser, Kunjwal, Sainz, Wolfe, Spekkens [arXiv:2004.09194(2020)]
- A correlation in Q_2^C device-independently witnesses the presence of a three-way entangled state

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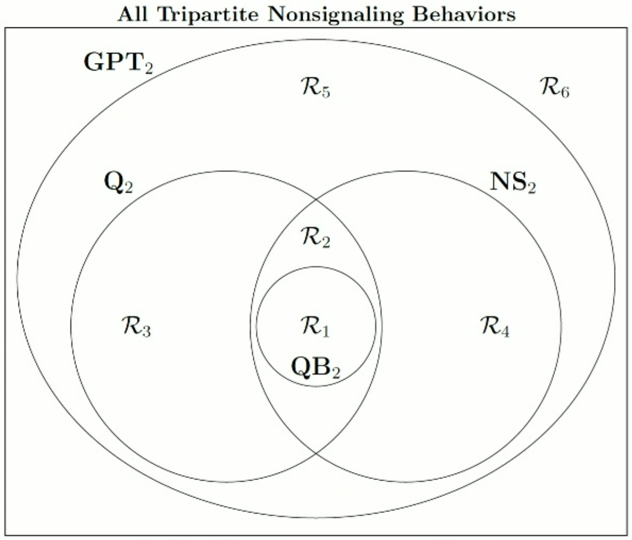
Different Variants of New Definitions of GMNL



- NS_2^C: Definition of GMNL in Bierhorst [PRA 104:012210(2021)]
- Argument: A nonsignaling "box" is the most abstract manifestation of (just) bipartite nonlocality

Behavior Class	Superquantum Bipartite Sources	Entangled Measurements
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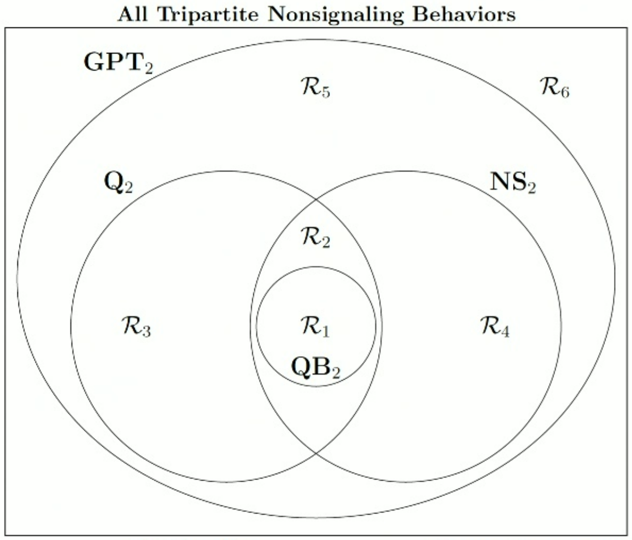
Today's focus: QB_2



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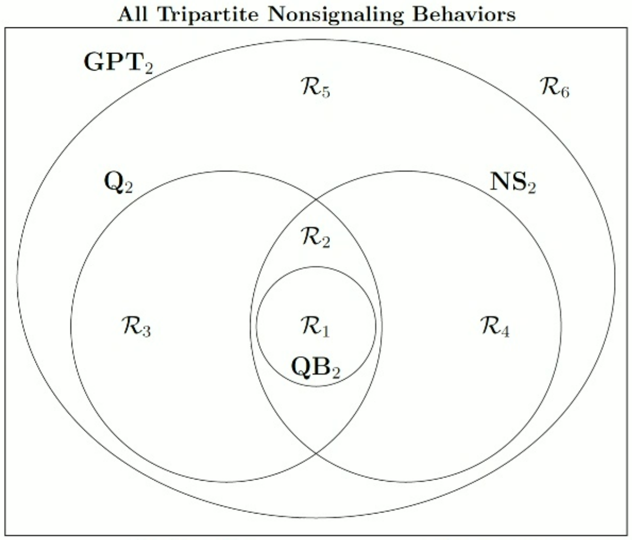

- QB_2^C: Correlations in Q_2 outside QB_2 witnesses presence of an entangled measurement

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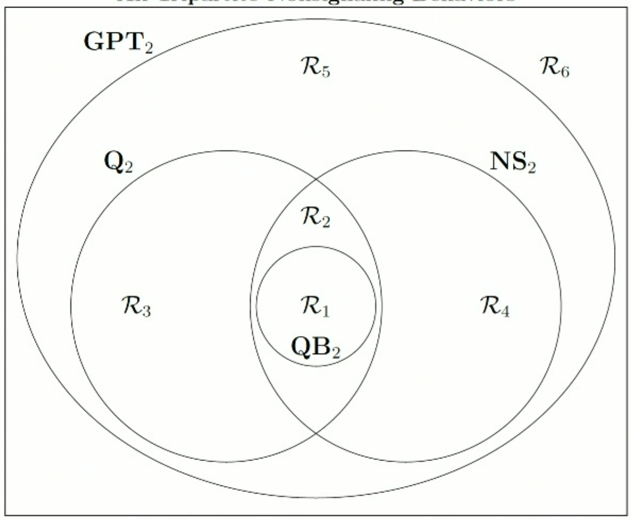
Today's focus: QB_2

All Tripartite Nonsignaling Behaviors



- QB_2^C: Correlations in Q_2 outside QB_2 witnesses presence of an entangled measurement
- Argument: Device-independent witness of entangled measurements is an inherently three-plus party phenomenon

Behavior Class	Superquantum Bipartite Sources	Entangled Measurements
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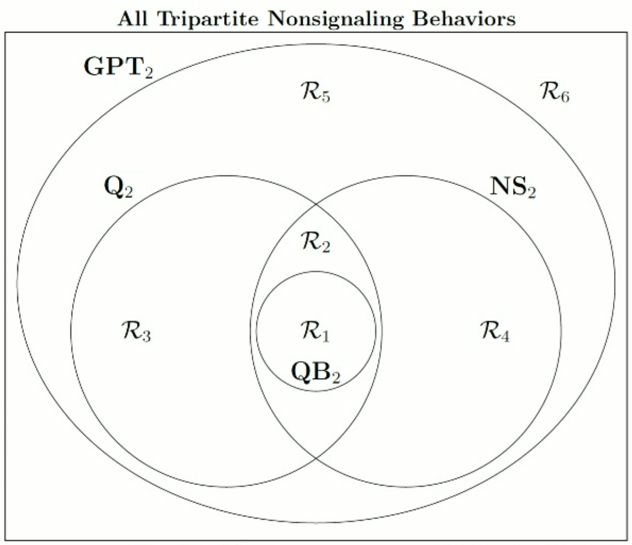
What Was Known



- $\mathcal{R}_3 \cup \mathcal{R}_2$ non-empty by Rabello *et al.* [PRL 107:050502(2011)]

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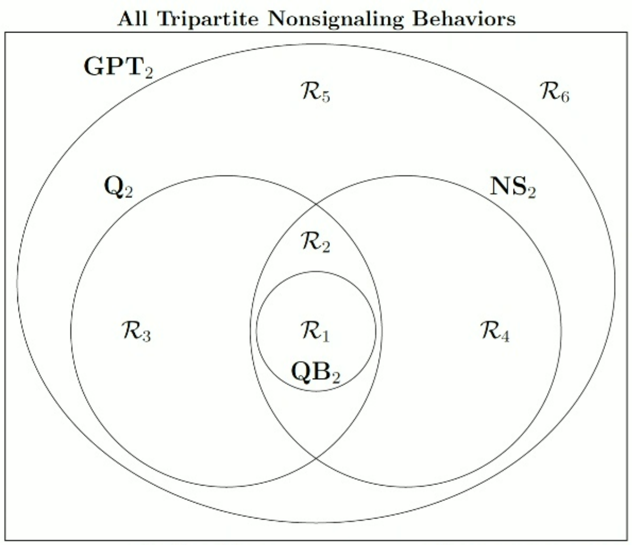

- $\mathcal{R}_3 \cup \mathcal{R}_2$ non-empty by Rabello *et al.* [PRL 107:050502(2011)]
- Behavior uses entanglement swapping so should be in $\mathcal{R}_3$ – entanglement swapping is impossible in boxworld

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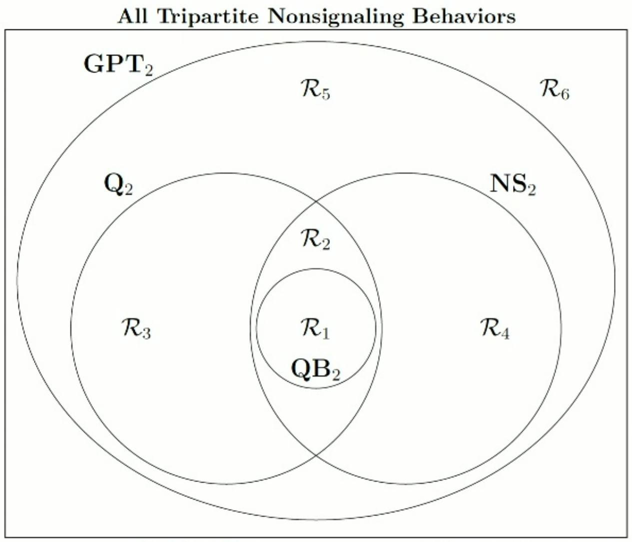

- $\mathcal{R}_3 \cup \mathcal{R}_2$ non-empty by Rabello *et al.* [PRL 107:050502(2011)]
- Behavior uses entanglement swapping so should be in $\mathcal{R}_3$ – entanglement swapping is impossible in boxworld
- Rabello approach requires at least 3 measurement settings for one party

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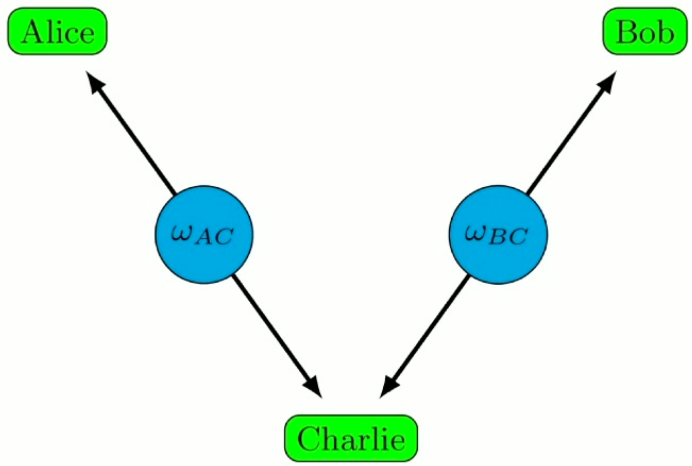
Our Q_2 Scenario Requiring an Entangled Mesaurement



- Alice and Bob always perform CHSH measurements
- On setting $Z = 0$, Charlie performs a Bell basis entangled measurement

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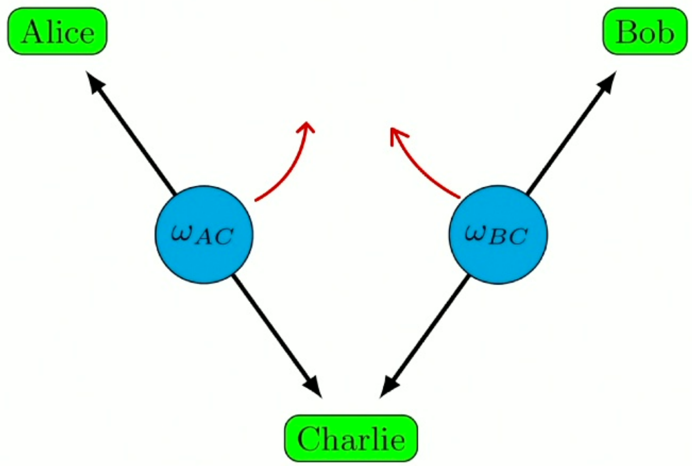

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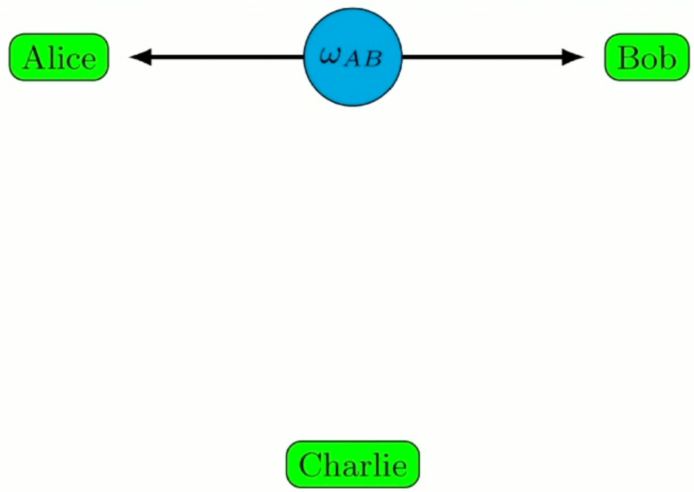

- Alice and Bob always perform CHSH measurements
- On setting $Z = 0$, Charlie performs a Bell basis entangled measurement
- Charlie "succeeds" and observes $C = 0$ with probability $1/4$
- Alice and Bob maximally violate CHSH inequality

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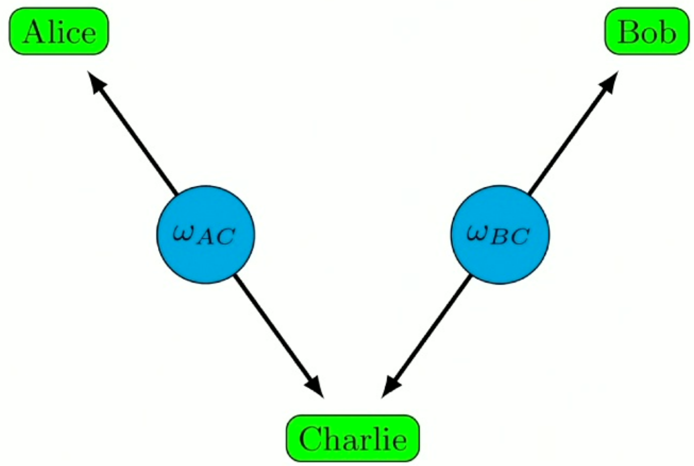

- Alice and Bob always perform CHSH measurements
- On setting $Z = 1$, Charlie ignores Bob and measures a direction aligned with one of Alice's directions, leading to perfect correlation with Alice when she measures this direction

Results

Theorem 1. There is a behavior $P(ABC|XYZ)$ in Q_2 with binary input and output random variables satisfying the conditions $P(C=0|Z=0)>0$, $P_{Z=0,C=0}(AB|XY)$ maximally violates the CHSH inequality, and $P(A=C|X=0,Z=1)=1$. Furthermore, no behavior in QB_2 can satisfy these conditions.

Setting for impossibility proof:

- in QB_2, measured state is of form $\rho = \rho_{AB}\otimes\rho_{BC}\otimes\rho_{AC}$, where each ρ_{PQ} is a positive trace-one operator
- Quantum probabilities are given by the formula $\text{Prob}(i) = \text{tr}[(M_i \otimes I)\rho]$. A measurement is *not* entangled if the measurement operators can be written in the form $M_i = \sum_x M^x_{\text{System 1}} \otimes N^x_{\text{System 2}}$, with $M^x_{\text{System 1}}$ and $N^x_{\text{System 2}}$ positive. This encompasses quantum boxes and wirings.

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Technical aspects of proof:

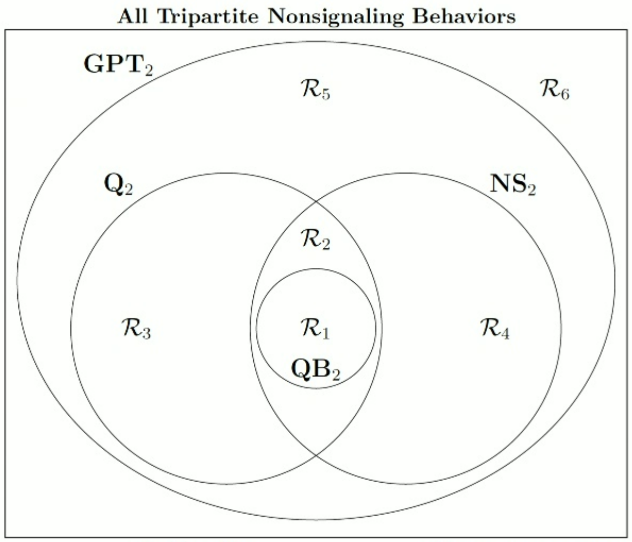
- Easy to reduce problem to pure states, and ignore shared local randomness
- Hard to reduce from POVMs to PVMs for Alice and Bob (needed for self-testing); dilation approach of Peres (1993)
- Since we are in QB_2, we can violate CHSH with A-B link, but this means Alice only measures Bob-shared portion
- Requires care to properly rewind this constraint to the joint state prior to conditioning on Charlie's measurement outcome

Results

Application: device-independent certification that Charlie is using an entangled measurement in simplest-possible (3,2,2) scenario

Important for testing hardware in a quantum network

What Was Known



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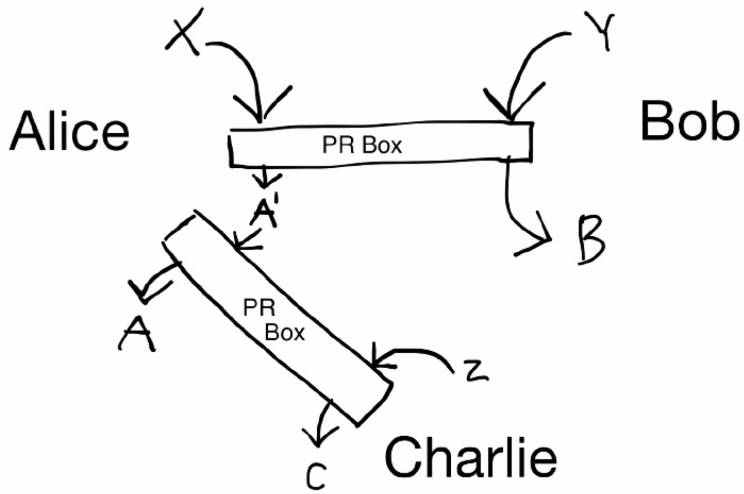
Results

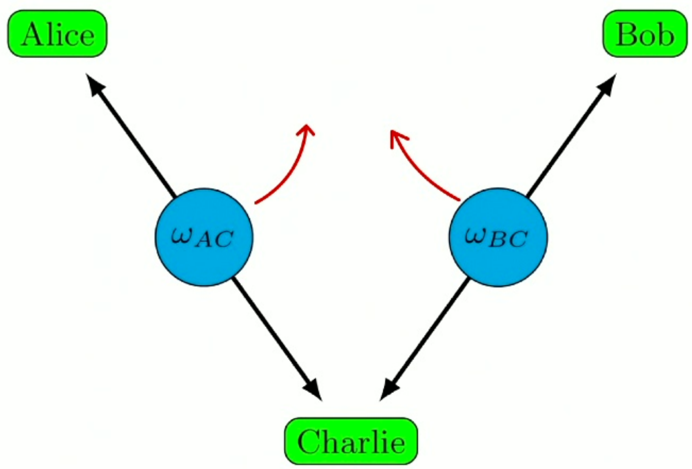
Theorem 2. There exists a behavior in NS_2 meeting the conditions of Theorem 1.



PR Box: $\mathrm{Prob}(AB|XY) = (1/2)\delta_{A\oplus B, XZ}$

Results

- What about Rabello?
- This is a stronger result, ruling out not just QB_2, but also unentangled measurements on tripartite states
- Charlie has 3 measurement settings, 4 outcomes



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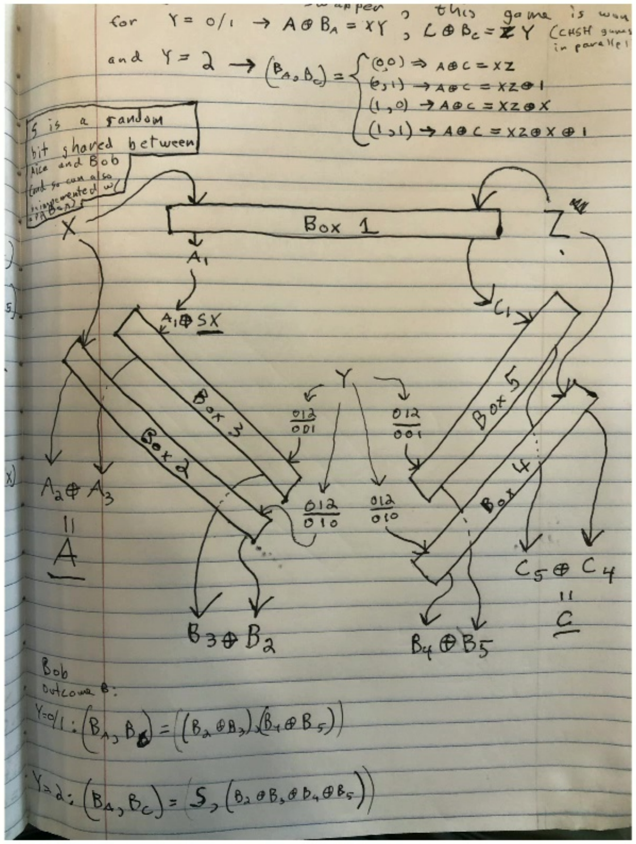
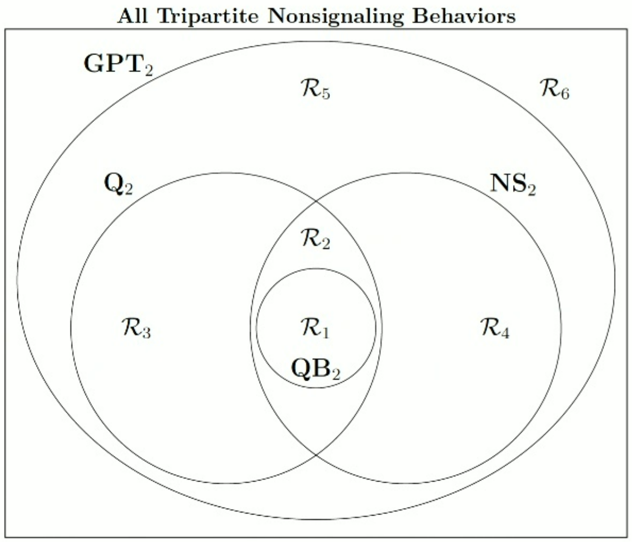


Future Work



- Is $\mathcal{R}_3$ nonempty? Is there a theory-independent notion of an entangled measurement?
- Is $\mathcal{R}_5$ nonempty? NS_2 and GPT_2 may align
- Give a second look to PR box simulations
- Best robust testable constraint for (3,2,2)
- Better experiments to witness $\mathcal{R}_6$ correlations – different setup? Possible with re-analysis of un-postselected data?

Thank You

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A Hierarchy of Multi-Party Nonlocal Effects
with Jitendra Prakash. arXiv:2301.12081

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