Title: Causal Scenarios: the Interesting, the Boring and the Elusive

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Collection: Causal Inference & Quantum Foundations Workshop

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Abstract: I will sketch the current state of play with classifying causal scenarios (aka DAGs with latent variables). Some are interesting: the classical correlations are constrained by non-trivial inequalities such as Bell's. Some are boring: the classical correlations are constrained only by observable conditional independencies. Some we still don't know. Along the way I will mention joint work with Joe Henson, Ray Lal, Shashaank Khanna, Marina Ansanelli and Elie Wolfe, and disjoint work by Robin Evans.
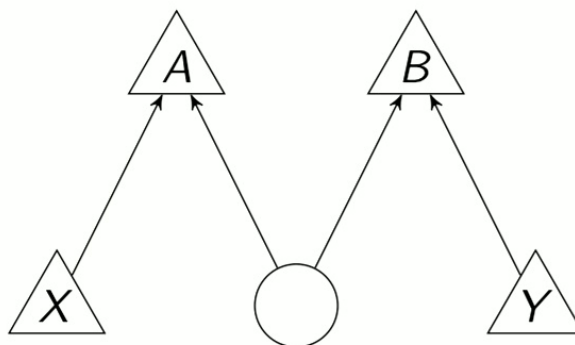
Causal scenarios:
the Interesting, the Boring, and the Elusive

Matthew F. Pusey
with Joe Henson, Ray Lal, Shashaank Khanna, Marina
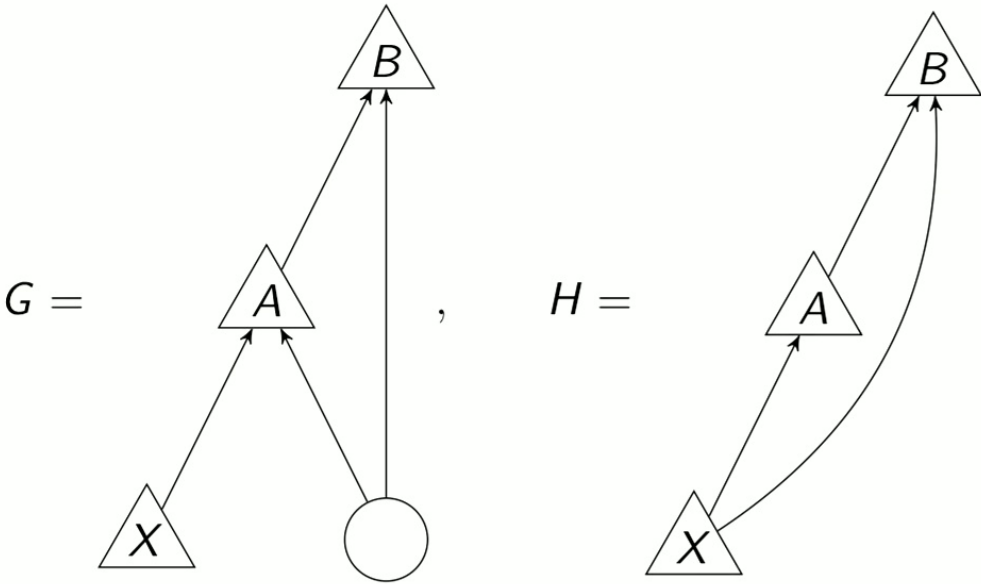Ansanelli and Elie Wolfe

April 18, 2023

The Interesting

$$\mathcal{C} = \{p(a,b,x,y) = \sum_\lambda p(a|x,\lambda)p(b|y,\lambda)p(x)p(y)p(\lambda)\}$$
$$\mathcal{I} = \{p(a,b,x,y) \quad | \quad p(x,y) = p(x)p(y), p(a,x,y)p(x) = $$
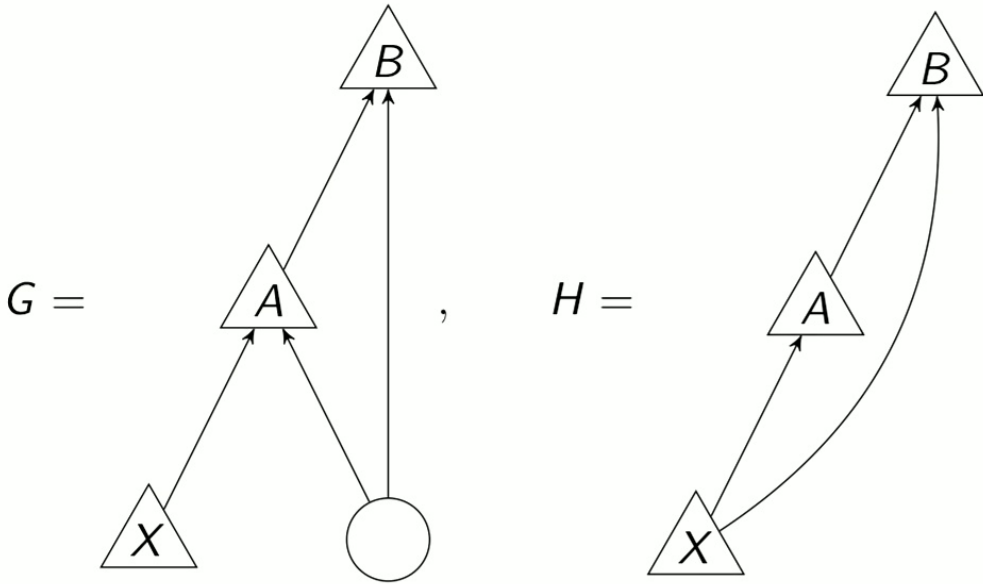$$p(a,x)p(x,y), p(b,x,y)p(y) = p(b,x)p(b,y)\}$$
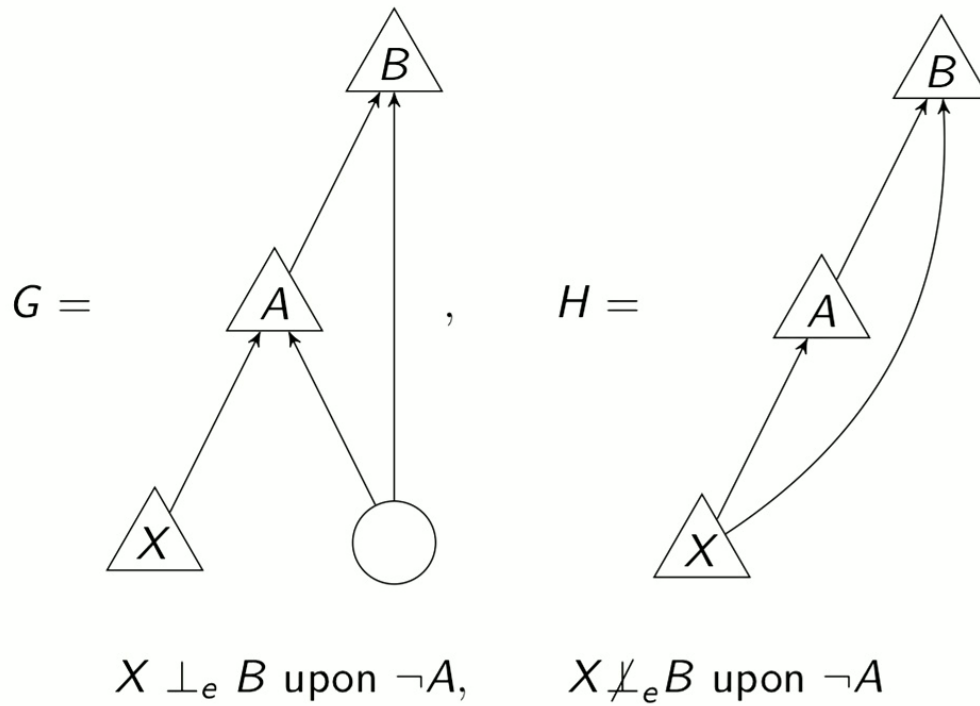
First step for interestingness

Are G's d-separations achievable in latent-free?

- ▶ No $\implies$ interesting (Evans arXiv:2209.06534)
- ▶ Yes, call such a latent-free H

Achievable in latent-free

$$G = \qquad \text{[graph with } X, A, B, \text{ and circle node]} \qquad , \qquad H = \qquad \text{[graph with } X, A, B \text{]}$$

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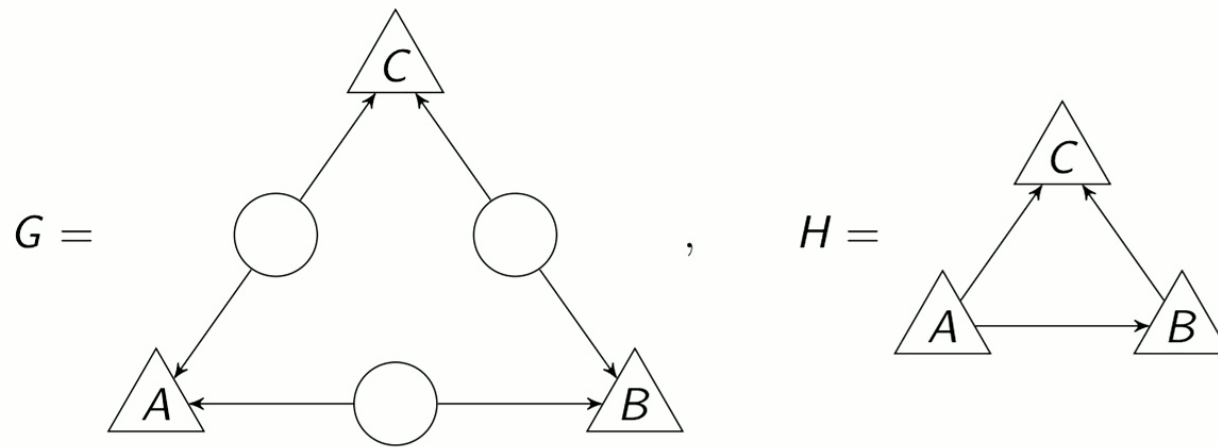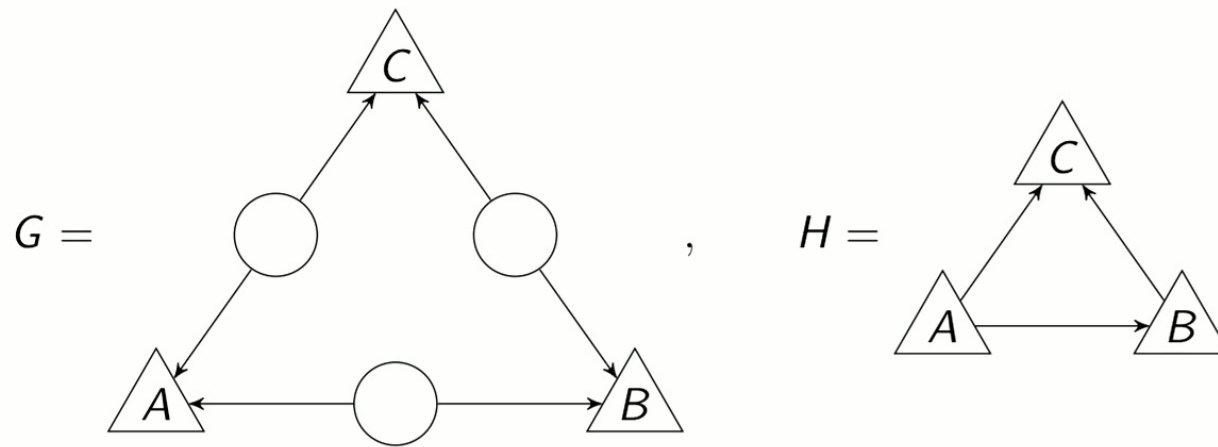
Achievable in latent-free

Achievable in latent-free

$G =$, $H =$

$X \perp_e B$ upon $\neg A$, $X \not\perp_e B$ upon $\neg A$

Finkelstein et. al. arXiv:2107.07087

e-separations match too

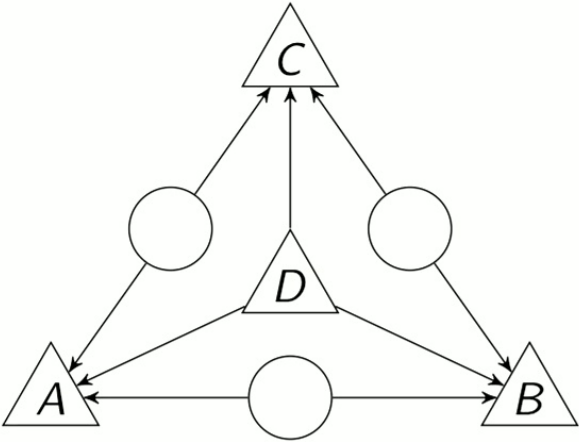$$G = \qquad , \qquad H =$$

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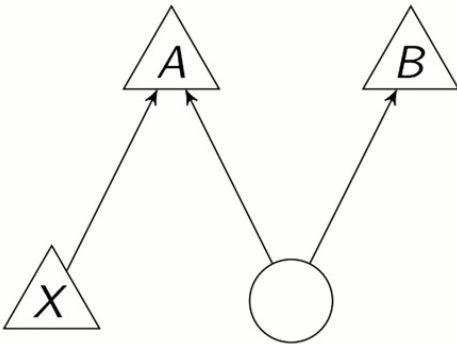
$$\{(0,0,0),(1,1,1)\}$$

Fraser arXiv:1902.07091

Time-saver: reduction
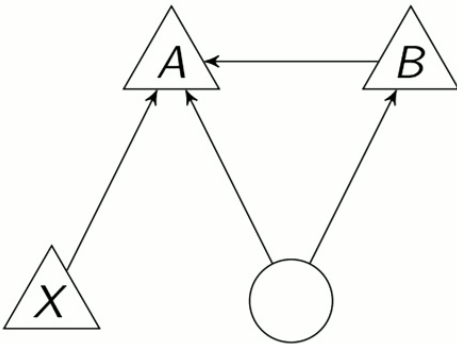

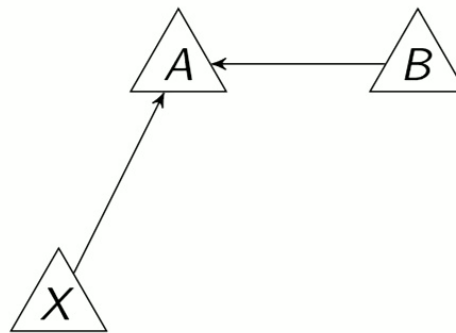Henson et. al. arXiv:1405.2572

The Boring

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Allowed steps

▶ Add an edge $X \to Y$ where $\mathrm{pa}(X) \subseteq \mathrm{pa}(Y)$ and $\mathrm{pa}(X)$ contains an unobserved node.

▶ Remove an edge.

Finally, remove all unobserved nodes.

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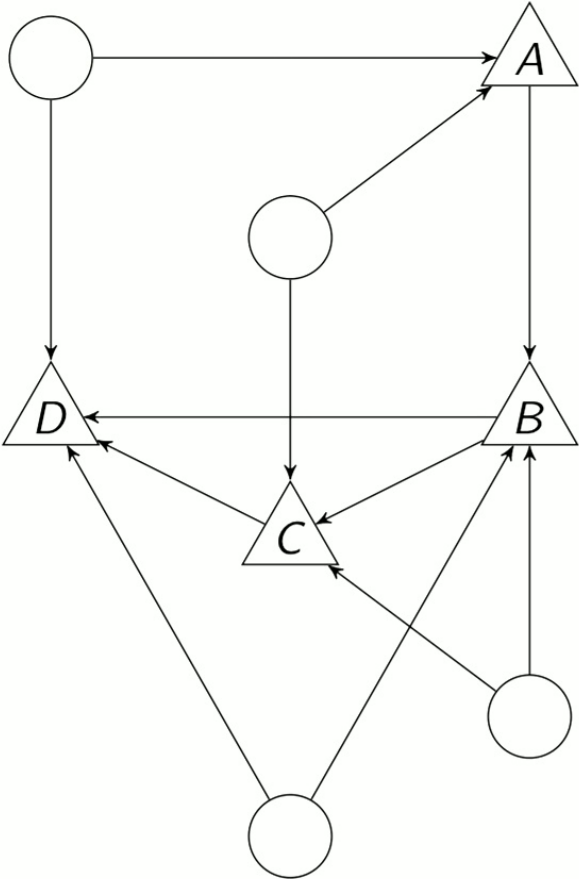- ▶ Remove an edge.

Finally, remove all unobserved nodes.
If G transforms to H then $\mathcal{C}_H \subseteq \mathcal{C}_G$.
Since $\mathcal{C}_H = \mathcal{I}_H$, if $\mathcal{I}_G = \mathcal{I}_H$ then $\mathcal{I}_G \subseteq \mathcal{C}_G$, i.e. G is boring.

Results for 4 observed nodes

Step	Remaining mDAGs
Enumeration	2809
$\mathcal{C} = \mathcal{I}$ criterion	996
Reduction	87
e-separation	54
Fraser's algorithm	3

The Elusive

The Elusive