

Title: Separation of quantum, spatial quantum and approximate quantum correlations

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Abstract: Quantum nonlocal correlations are generated by implementation of local quantum measurements on spatially separated quantum subsystems. Depending on the underlying mathematical model and the dimension of the underlying Hilbert spaces, various notions of sets of quantum correlations can be defined. This talk is devoted to the separations of some of these sets via simple ideas in quantum information theory, namely self-testing and entanglement embezzlement.

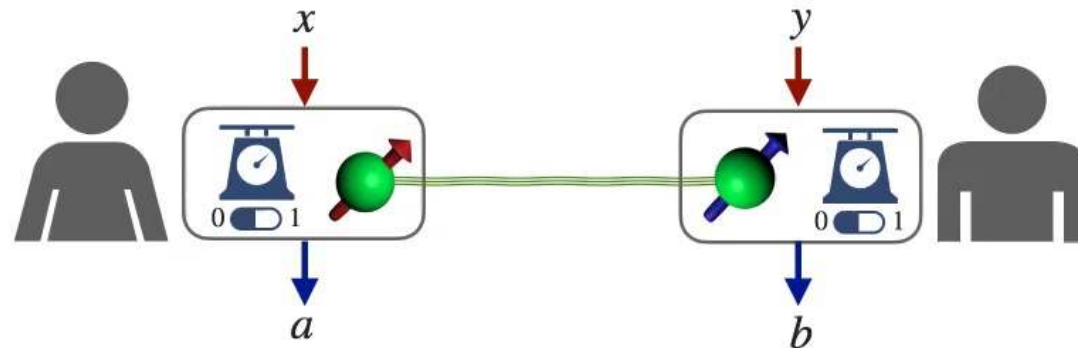
# Separation of quantum, spatial quantum and approximate quantum correlations

Salman Beigi

*Causal Inference & Quantum Foundations Workshop  
April 17-21, 2023*

[arXiv:2004.11103](https://arxiv.org/abs/2004.11103)

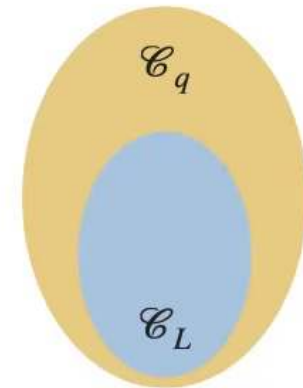
## Bell's setup



❖ Classical theory:  $p(a, b | x, y) = \sum_{\lambda} p(\lambda) p(a | x, \lambda) p(b | y, \lambda)$

❖ Quantum theory:  $p(a, b | x, y) = |\langle \psi | M_a^x \otimes N_b^y | \psi \rangle|^2$

❖ Bell-CHSH shows that  $\mathcal{C}_L \neq \mathcal{C}_q$



# Hierarchy of Quantum Correlations

$$p(a, b | x, y) = |\langle \psi | M_a^x \otimes N_b^y | \psi \rangle|^2$$

Vector in  $\mathcal{H} \otimes \mathcal{H}$       Operators acting on a Hilbert space  $\mathcal{H}$

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- ❖ Quantum correlations  $\mathcal{C}_q$  : **finite dimensional**  $\mathcal{H}$
- ❖ Spatial quantum correlations  $\mathcal{C}_{qs}$  : **infinite dimensional**  $\mathcal{H}$
- ❖ Approximate quantum correlations  $\mathcal{C}_{qa}$  : **topological closure**

$$\mathcal{C}_{qa} = \overline{\mathcal{C}_{qs}} = \overline{\mathcal{C}_q}$$

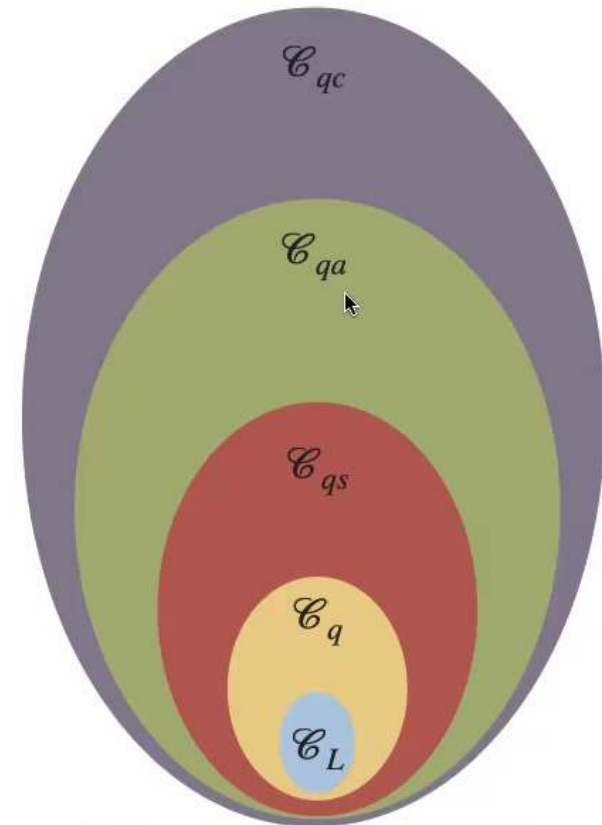
- ❖ **Commuting** quantum correlations  $\mathcal{C}_{qc}$  :

$$p(a, b | x, y) = \langle \psi | M_a^x N_b^y | \psi \rangle \quad M_a^x N_b^y = N_b^x M_a^y$$

# Tsirelson's problem

$$\mathcal{C}_L \subseteq \mathcal{C}_q \subseteq \mathcal{C}_{qs} \subseteq \mathcal{C}_{qa} \subseteq \mathcal{C}_{qc}$$

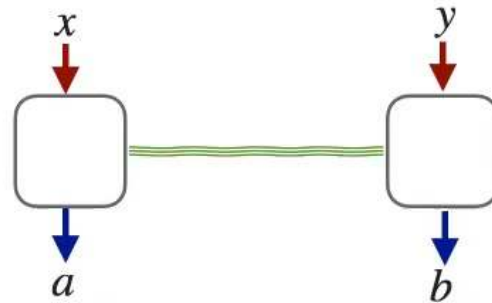
- ❖ Bell-CHSH (1964):  $\mathcal{C}_L \neq \mathcal{C}_q$
- ❖ Slofstra (2020):  $\mathcal{C}_{qa} \neq \mathcal{C}_{qs}$ 
  - Improved by Dykema et al and Musat & Rordam
  - Simplified by Coladangelo (2019)
- ❖ Coladangelo and Stark (2018):  $\mathcal{C}_q \neq \mathcal{C}_{qs}$
- ❖ Ji, Natarajan, Vidick, Wright, Yuen (2020):  $\mathcal{C}_{qa} \neq \mathcal{C}_{qc}$ 
  - $\text{MIP}^* = \text{RE}$



All inclusions are strict

**Our results:** Improvements in  $\mathcal{C}_q \neq \mathcal{C}_{qs}$  and  $\mathcal{C}_{qs} \neq \mathcal{C}_{qa}$

## Some details



❖  $\mathcal{C}_*^{(n_A, n_B, m_A, m_B)}$ : set of correlations with  $x \in [n_A]$ ,  $y \in [n_B]$  and  $a \in [m_A]$ ,  $b \in [m_B]$

- ❖ Slofstra (2020):  $\mathcal{C}_{qa} \neq \mathcal{C}_{qs}$  for parameters (185, 235, 8, 2)
- ❖ Dykema et al and Musat & Rordam improved to (5, 5, 2, 2)
- ❖ Coladangelo (2019) gave a simple proof for (5, 6, 3, 3)
- ❖ Coladangelo and Stark (2018):  $\mathcal{C}_q \neq \mathcal{C}_{qs}$  for (4, 5, 3, 3)

Theorem:  $\mathcal{C}_{qa} \neq \mathcal{C}_{qs}$  for (4, 4, 3, 3)

Theorem:  $\mathcal{C}_q \neq \mathcal{C}_{qs}$  for (4, 4, 2, 2)

## Proof idea (I): self-testing

- ❖ Can self-test the maximally entangled state with the CHSH game
- ❖ Can self-test any two-qubit state with a **tilted CHSH game**

$$\beta \langle A_0 \rangle + \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \leq 2 + \beta$$

- ❖ Quantum violation  $\sqrt{8 + 2\beta^2}$  is a self-test for

$$\left\{ \begin{array}{l} |\varphi_\alpha\rangle = \frac{1}{\sqrt{1 + \alpha^2}}(|00\rangle + \alpha|11\rangle) \\ A_0 = \sigma_z, \quad A_1 = \sigma_x \\ B_0 = \cos \mu \sigma_z + \sin \mu \sigma_x, \quad B_1 = \cos \mu \sigma_z - \sin \mu \sigma_x \end{array} \right. \quad \left\{ \begin{array}{l} \alpha = \tan \theta \\ \tan \mu = \sin(2\theta) = \sqrt{\frac{4 - \beta^2}{4 + \beta^2}} \end{array} \right.$$

- ❖ Maximum violation guarantees that up to local isomorphisms the shared state equals

$$|\varphi_\alpha\rangle \otimes |\psi'\rangle$$

## Proof idea (I): self-testing

$$|\psi\rangle = \frac{1}{\sqrt{C}} \left( \dots + \alpha^3 |-3, -3\rangle + \underbrace{\alpha^2 |-2, -2\rangle + \alpha |-1, -1\rangle}_{\sim |00\rangle + \alpha |11\rangle} + \underbrace{|0,0\rangle + \alpha |1,1\rangle}_{\sim |00\rangle + \alpha |11\rangle} + \underbrace{\alpha^2 |2,2\rangle + \alpha^3 |3,3\rangle}_{\sim |00\rangle + \alpha |11\rangle} + \dots \right)$$

$$\Rightarrow |\psi\rangle \sim (|00\rangle + \alpha |11\rangle) \otimes |\psi'\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{C}} \left( \dots + \underbrace{\alpha^3 |-3, -3\rangle + \alpha^2 |-2, -2\rangle}_{\sim |00\rangle + \alpha |11\rangle} + \underbrace{\alpha |-1, -1\rangle + |0,0\rangle}_{\sim |00\rangle + \alpha |11\rangle} + \underbrace{\alpha |1,1\rangle + \alpha^2 |2,2\rangle}_{\sim |00\rangle + \alpha |11\rangle} + \alpha^3 |3,3\rangle + \dots \right)$$

$$\Rightarrow |\psi\rangle \sim (|00\rangle + \alpha |11\rangle) \otimes |\psi''\rangle$$

- ❖ Can play tilted CHSH in two different ways & these two games are NOT independent
- ❖ Gives the separation  $\mathcal{C}_{qs} \neq \mathcal{C}_q$  in  $(n_A, n_B, m_A, m_B) = (4, 4, 2, 2)$

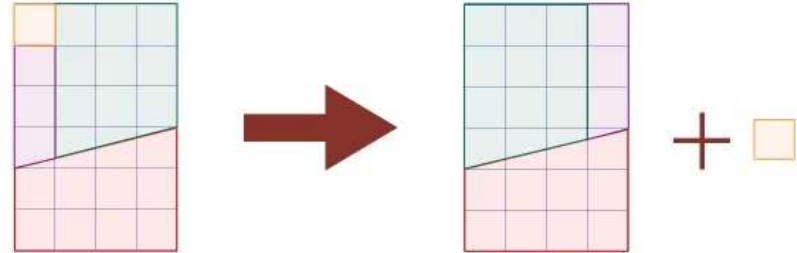
# Proof idea (II): entanglement embezzlement

[van Dam, Hayden '02]

- There is a state  $|\chi_n\rangle$  such that for any state  $|\varphi\rangle$

$$|\chi_n\rangle \sim |\varphi\rangle \otimes |\chi_n\rangle \quad \epsilon_n = O(1/\ln n)$$

$$|\chi_n\rangle = \frac{1}{\sqrt{H_n}} \sum_{j=1}^n \frac{1}{\sqrt{j}} |j, j\rangle$$



- Exact embezzlement is **impossible** even in infinite dimensions
- A signature of asymptotic approximation
- Can be used to prove the separation  $\mathcal{C}_{qa} \neq \mathcal{C}_{qc}$  in  $(n_A, n_B, m_A, m_B) = (4, 4, 3, 3)$

## Open problems



- ❖ Can we prove the separation  $\mathcal{C}_q \neq \mathcal{C}_{qs}$  for  $(3, 3, 2, 2)$ ?  
Conjectured in [Pal, Vertesi '10]
  
- ❖ Other signatures of asymptotic approximation?
  - Improve the separation  $\mathcal{C}_{qa} \neq \mathcal{C}_{qs}$ ?
  
- ❖ What about the networks?
  - Self-testing in networks?