Title: Observational Equivalences Between Causal Structures with Latent Variables

Speakers: Marina Maciel Ansanelli

Collection: Causal Inference & Quantum Foundations Workshop

Date: April 17, 2023 - 4:00 PM

URL: https://pirsa.org/23040108

Abstract: "If one only performs experiments involving passive observations, in general there are multiple causal structures that can explain the same set of distributions over the observed variables. In this case, we say that these causal structures are observationally equivalent. In this work, we explore all the known techniques for proving observational equivalence or inequivalence, as well as some original ones.
Even if the existing rules are not enough to achieve the full classification of the causal structures with four observed variables, our results get close to such classification and show that admitting inequality constraints is a generic feature among structures with four observed variables."

Observational Equivalences between (classical) Causal Structures

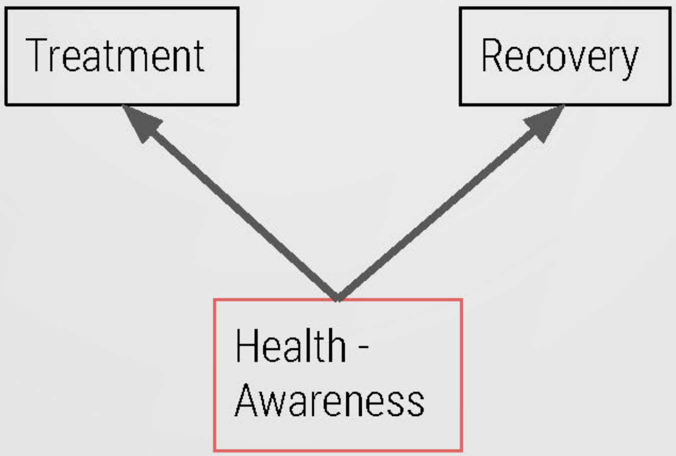
Marina Maciel Ansanelli

Joint work with Elie Wolfe and Robert Spekkens

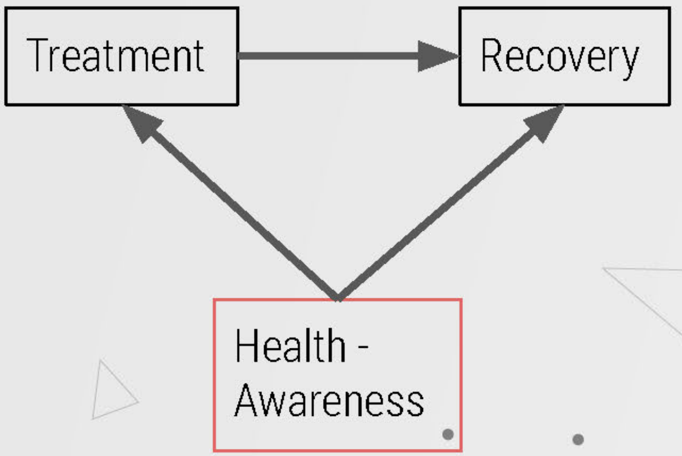
PERIMETER INSTITUTE

UNIVERSITY OF WATERLOO

Does the treatment cause
the recovery?



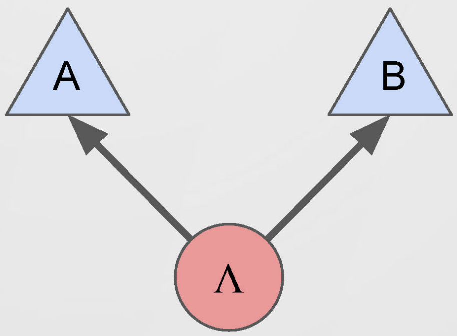
OR

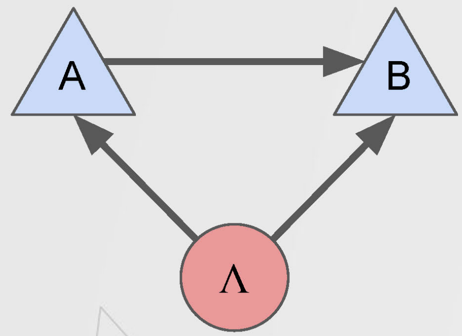


Passive observation only: Indistinguishable

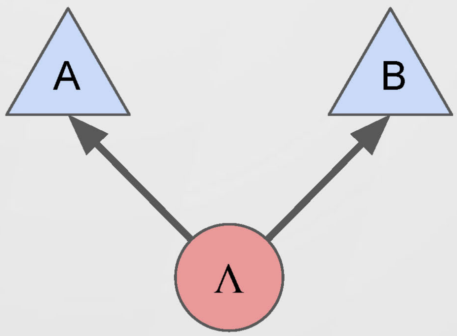
Passive observation only: Indistinguishable
Intervene on Treatment: Distinguishable

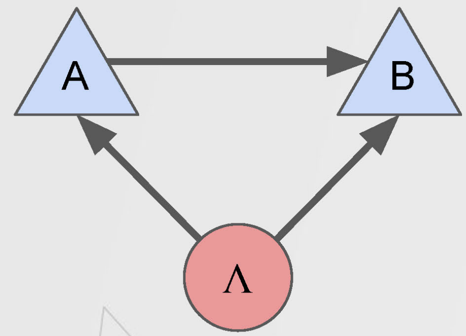
Observational and Interventional Equivalence





Observationally Equivalent ✅

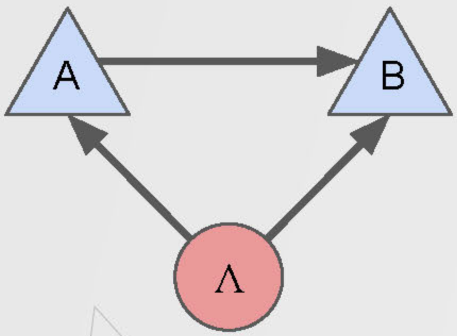


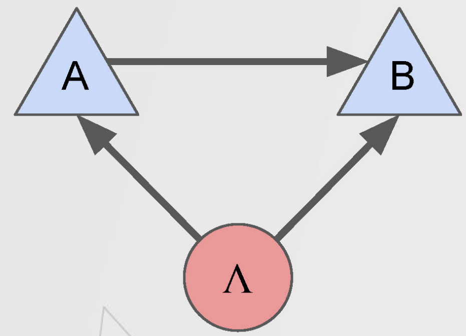


Observationally Equivalent ✅

Interventionally Inequivalent ❌

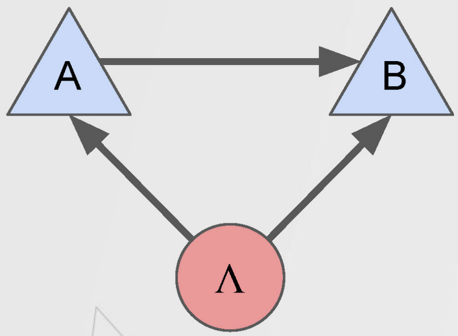

A
B


A
B
Λ



Observationally Equivalent ✅

- **A** and **B** perfectly correlated observationally
- Independent when intervene on **A**



Observationally Equivalent ✅

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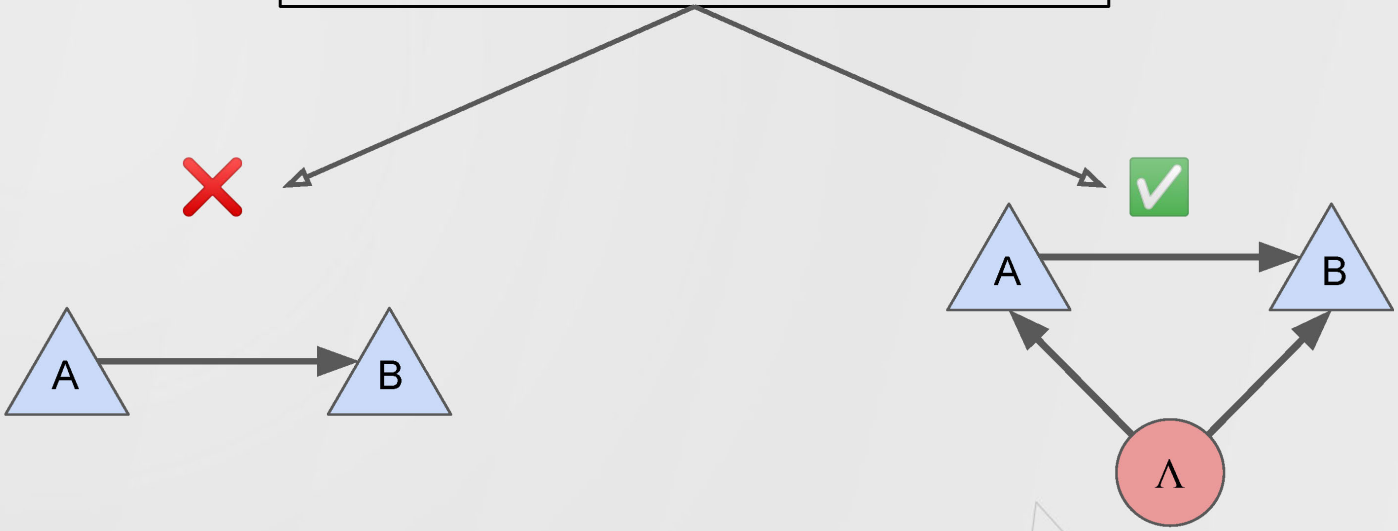

Observationally Equivalent ✅

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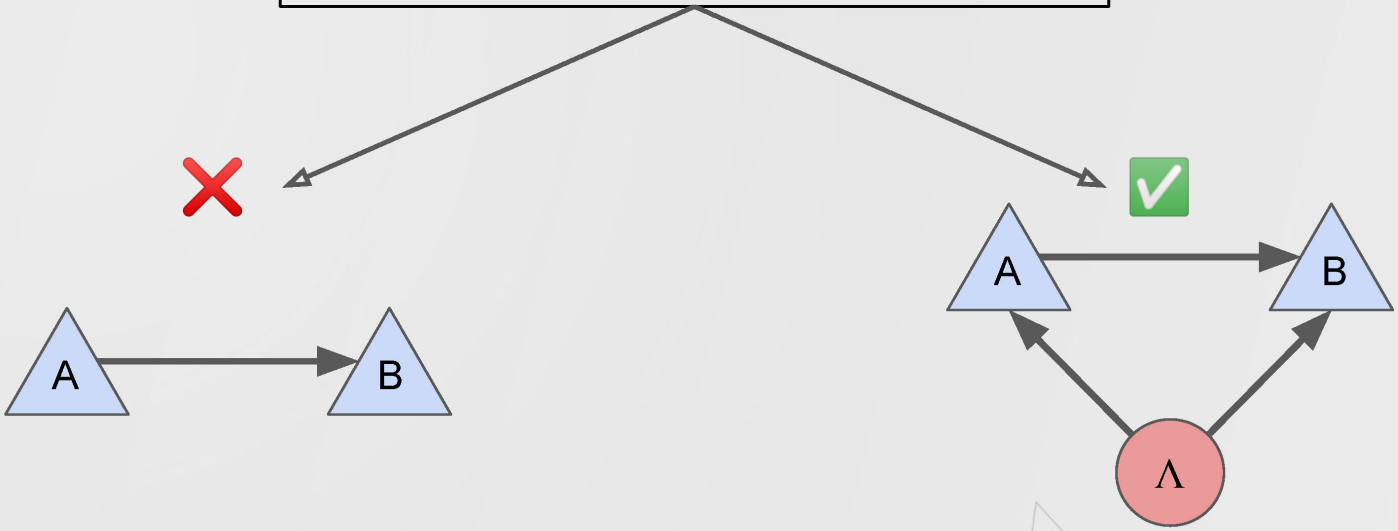
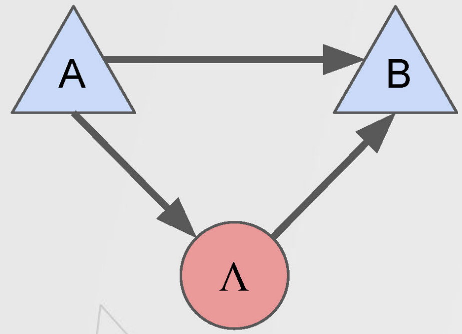
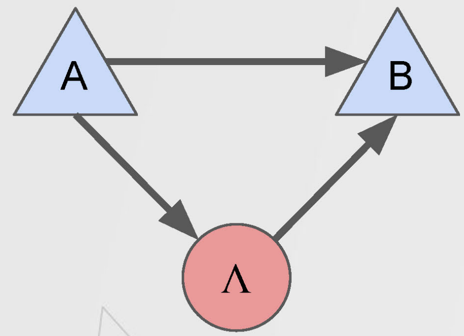


Observationally Equivalent ✅
Interventionally Inequivalent ❌

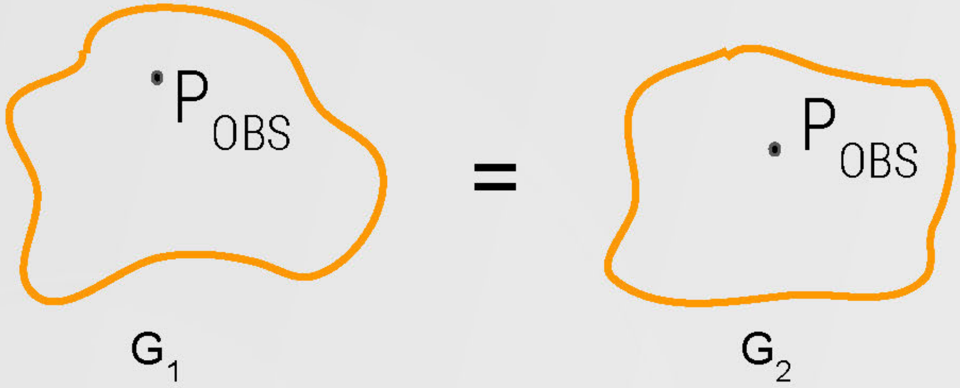


Observationally Equivalent ✅

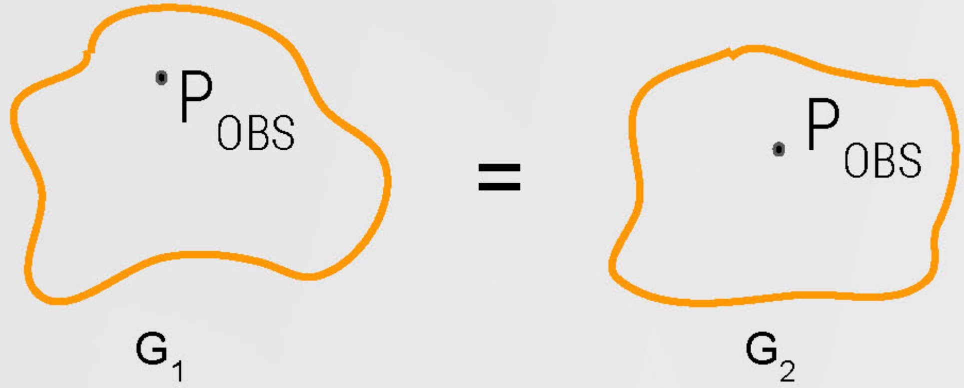


Observationally Equivalent ✅
Interventionally Equivalent ✅

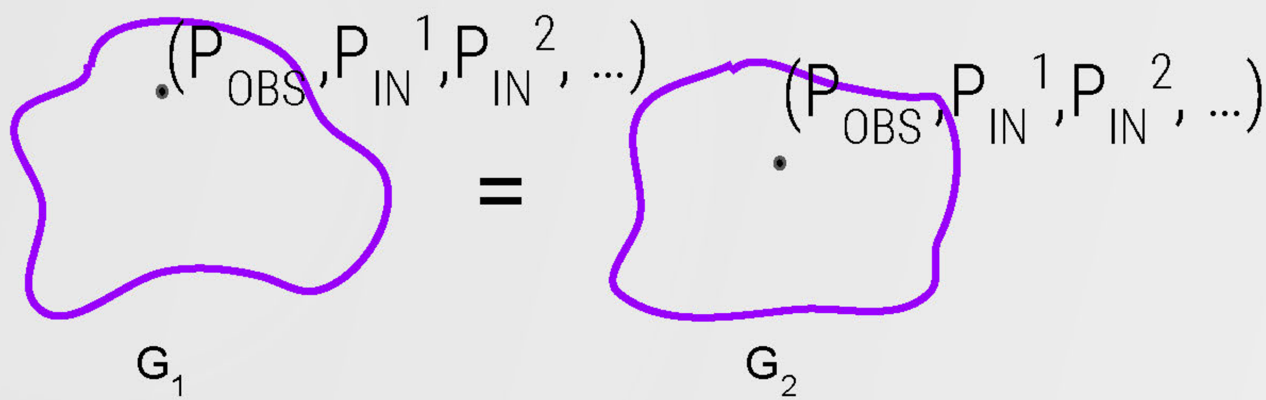
Observational Equivalence:



Observational Equivalence:



Interventional Equivalence:



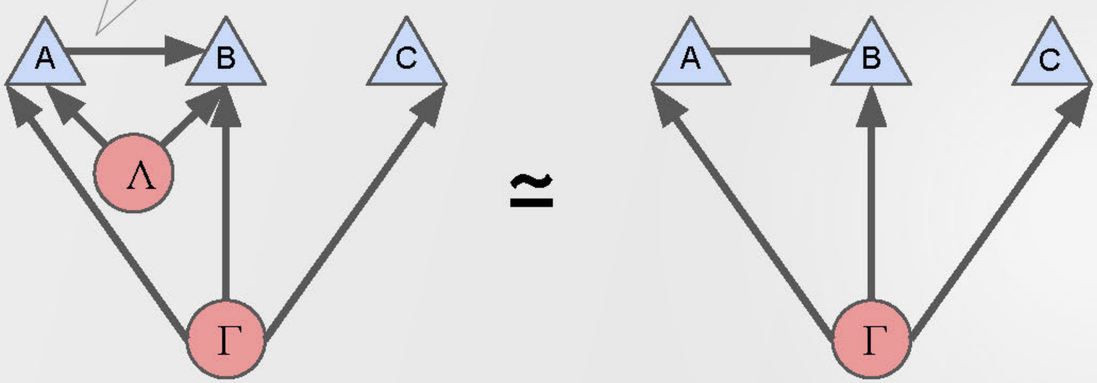
mDAGs

R.J. Evans: Graphs for Margins of Bayesian Networks (2016)

mDAGs

R.J. Evans: Graphs for Margins of Bayesian Networks (2016)

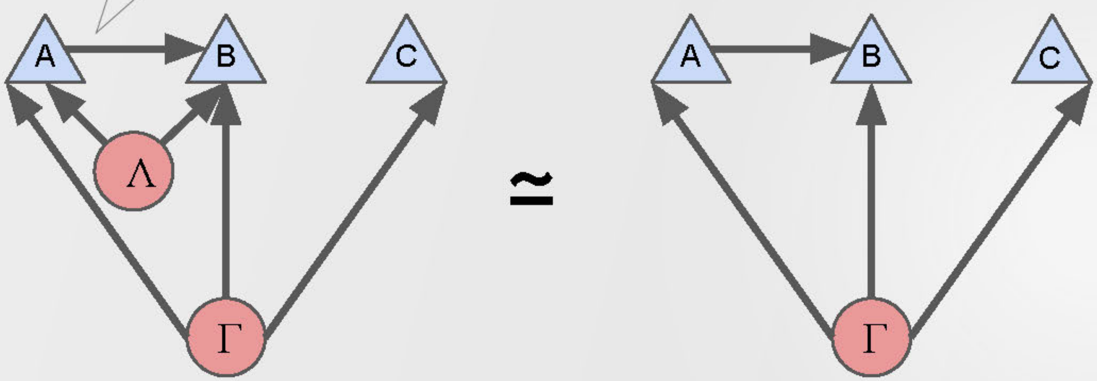
1. No - Redundancy



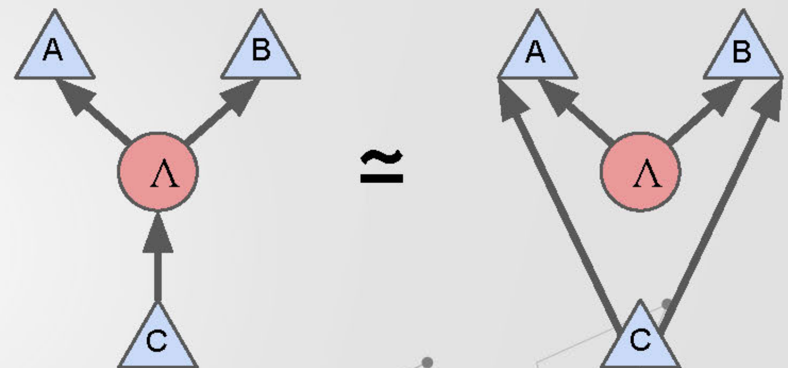
mDAGs

R.J. Evans: Graphs for Margins of Bayesian Networks (2016)

1. No - Redundancy



2. Exogenization



mDAGs

R.J. Evans: Graphs for Margins of Bayesian Networks (2016)

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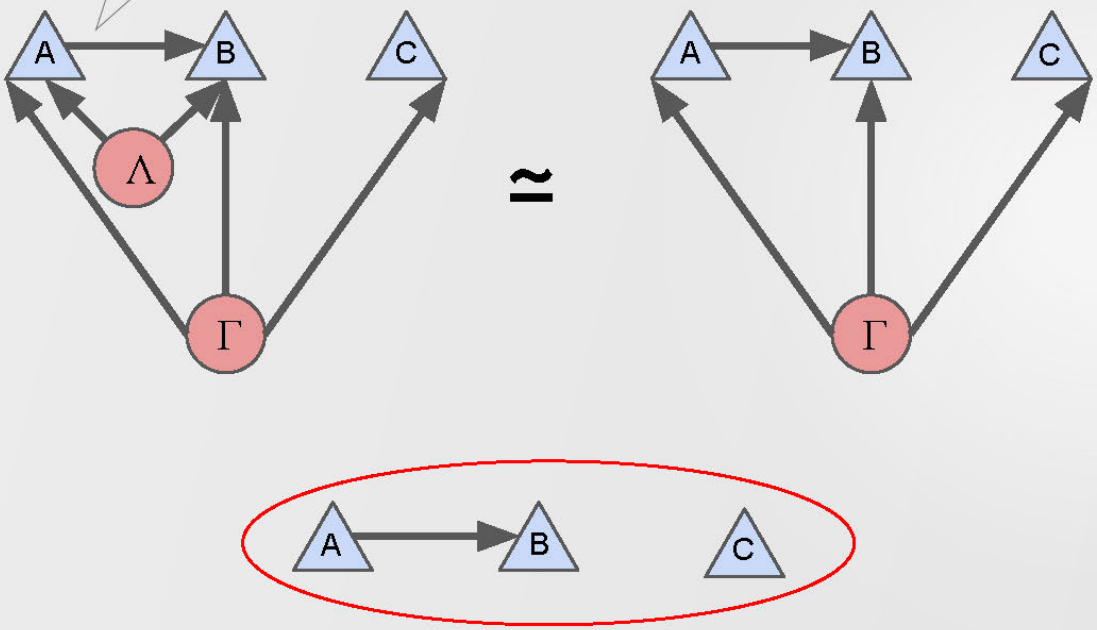
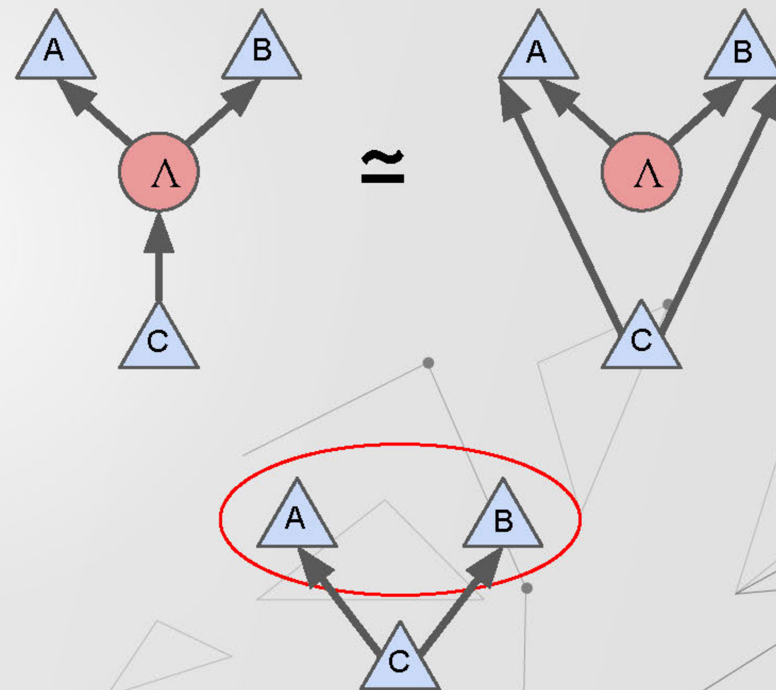
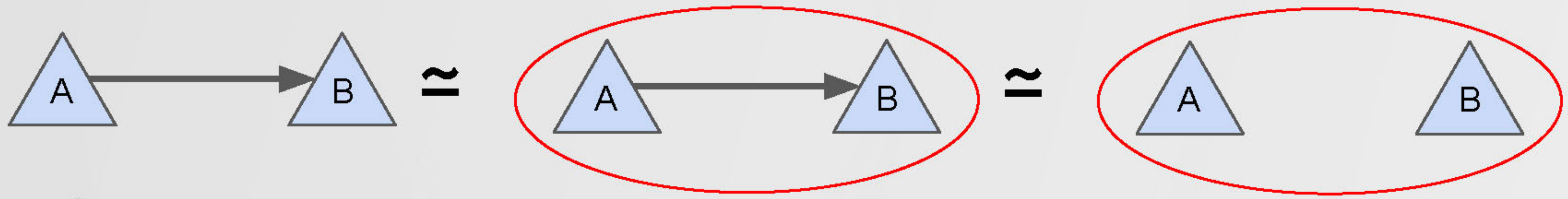


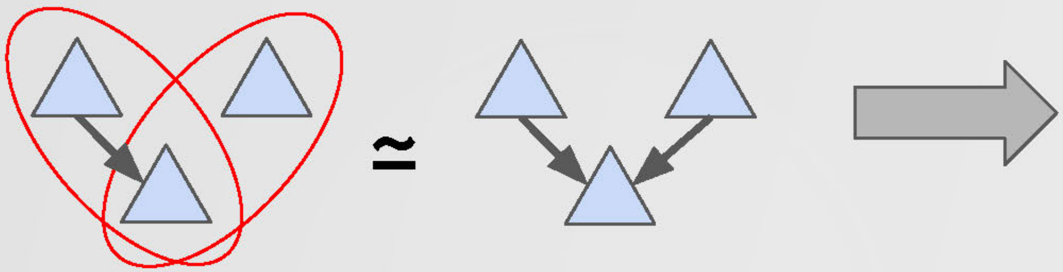
2. Exogenization



Interventionally Equivalent ⟺ Correspond to the same mDAG

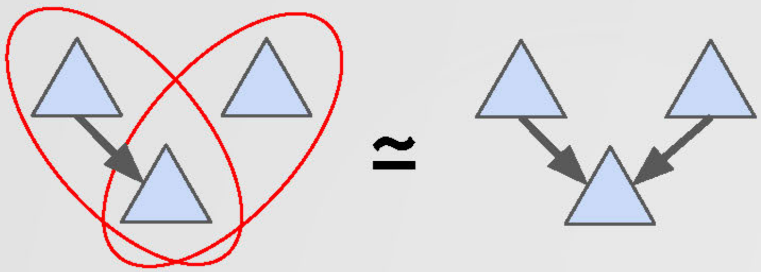
But there are different mDAGs that are observationally equivalent



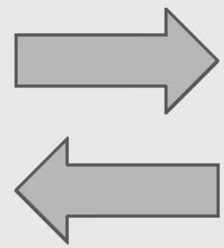


Equivalent to latent-free

No non-trivial Inequalities



Equivalent to latent-free



R. J. Evans - Latent-Free Equivalent mDAGs (2022)

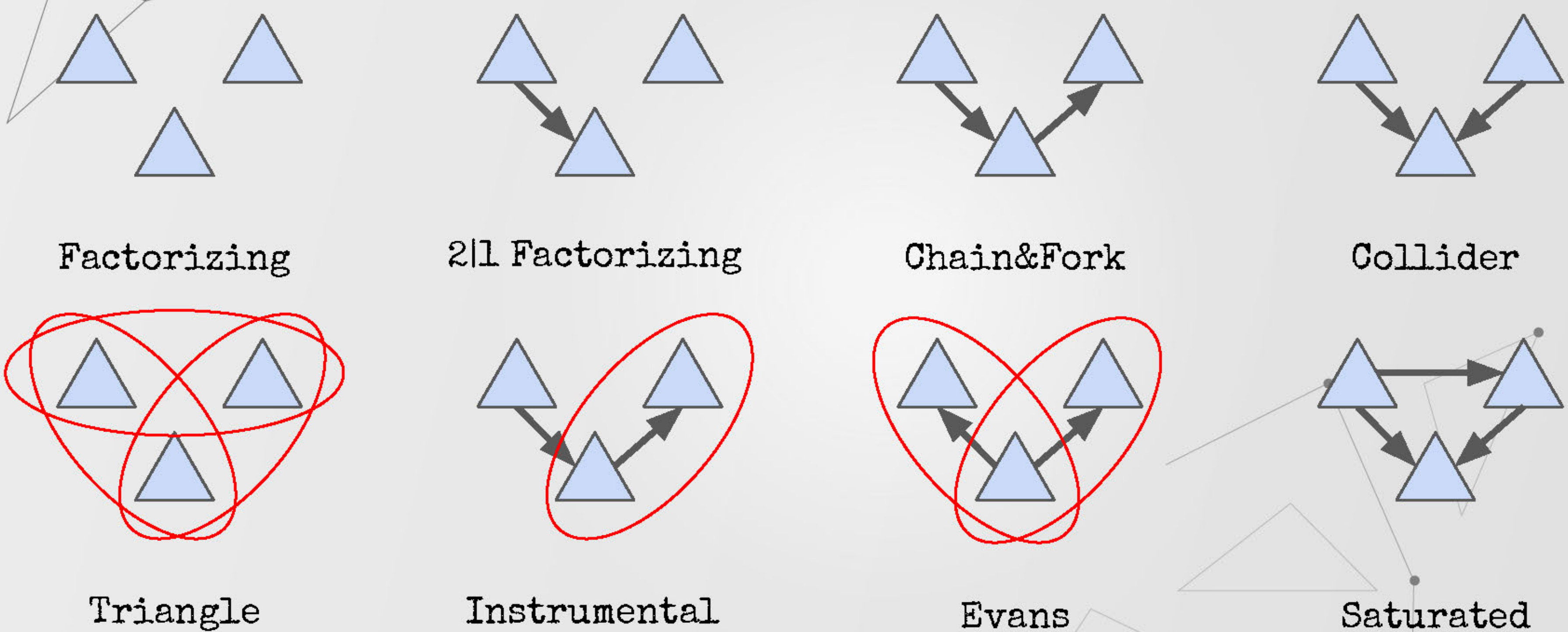
No non-trivial Inequalities

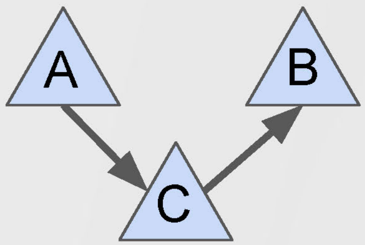
Case of 3 observed nodes

3 Observed Nodes

R.J. Evans: Graphs for Margins of Bayesian Networks (2016)

Total: 46 mDAGs

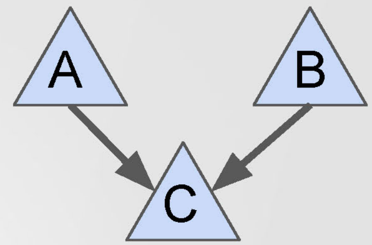




Chain&Fork

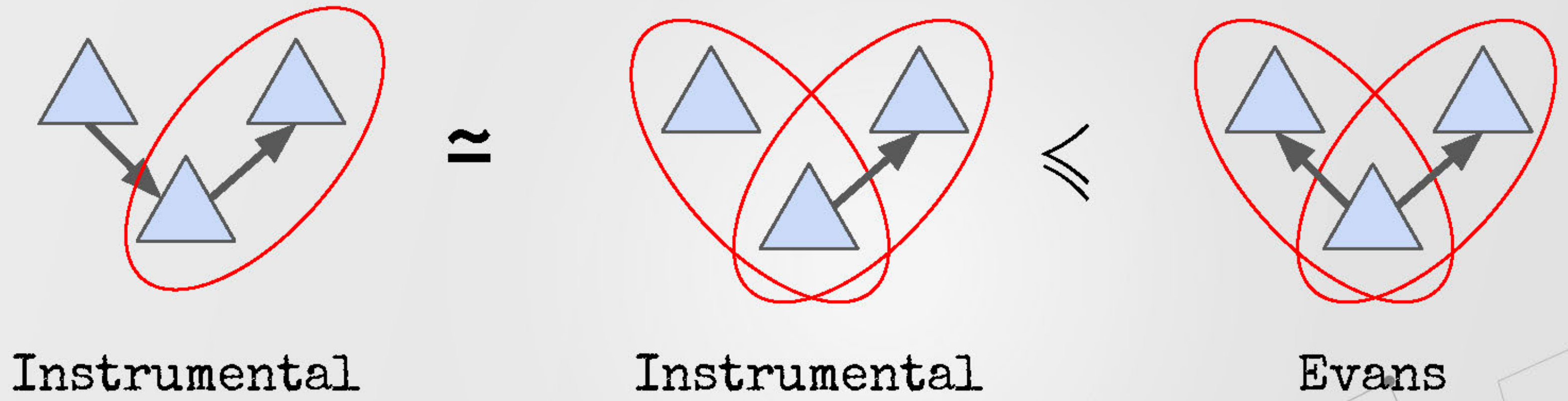
$$A \perp B \mid C$$

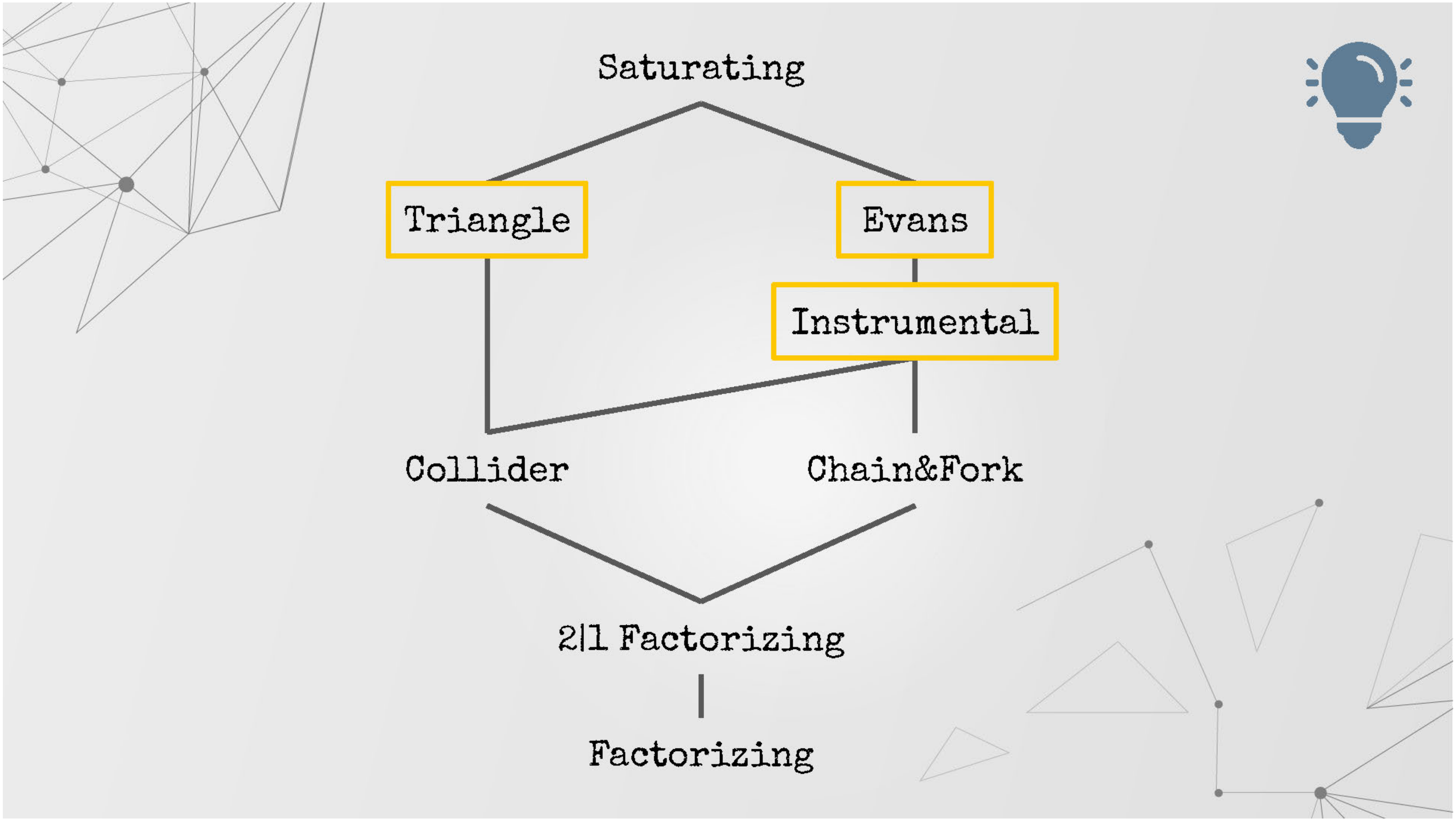
$$\neq$$

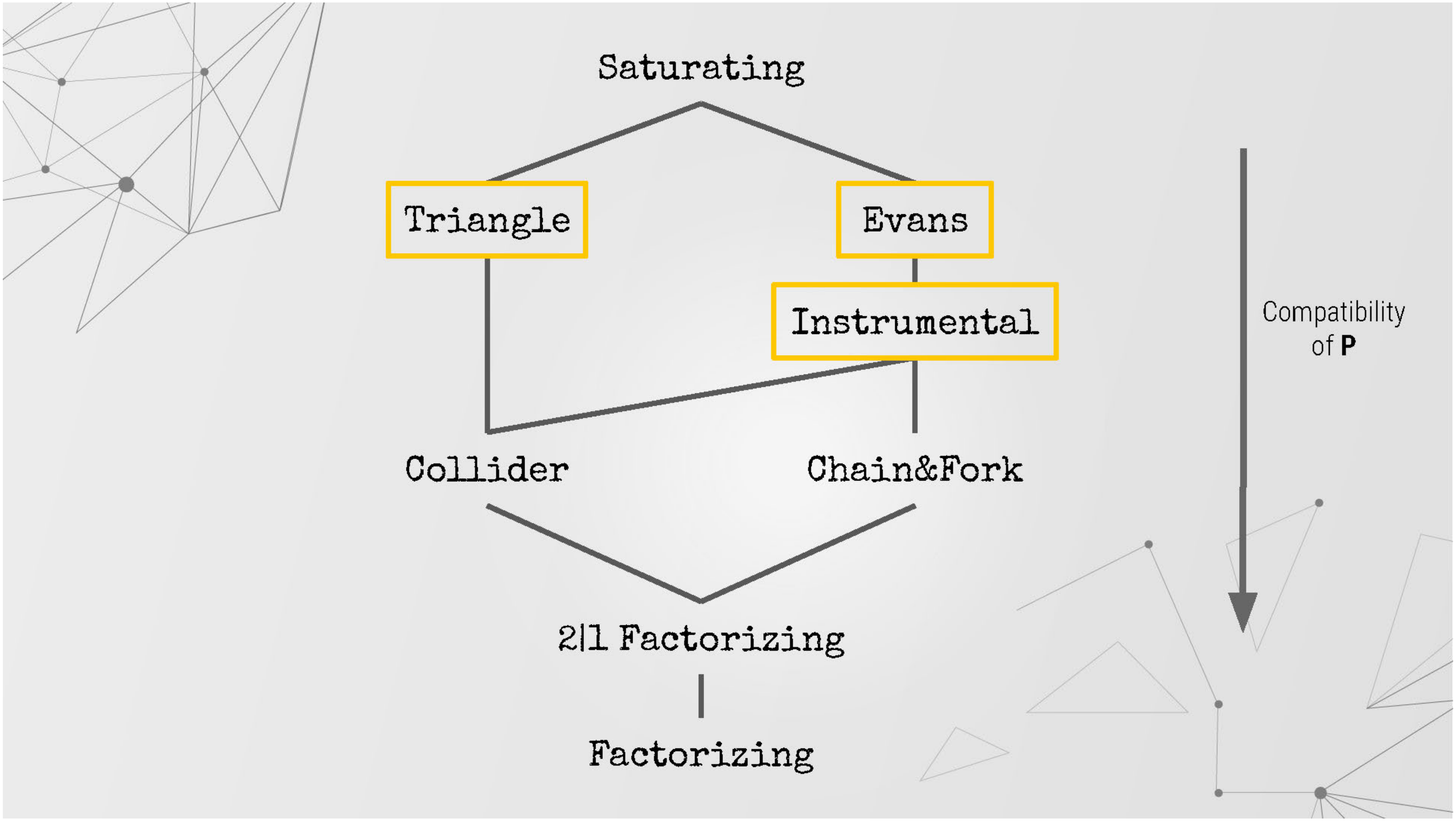


Collider

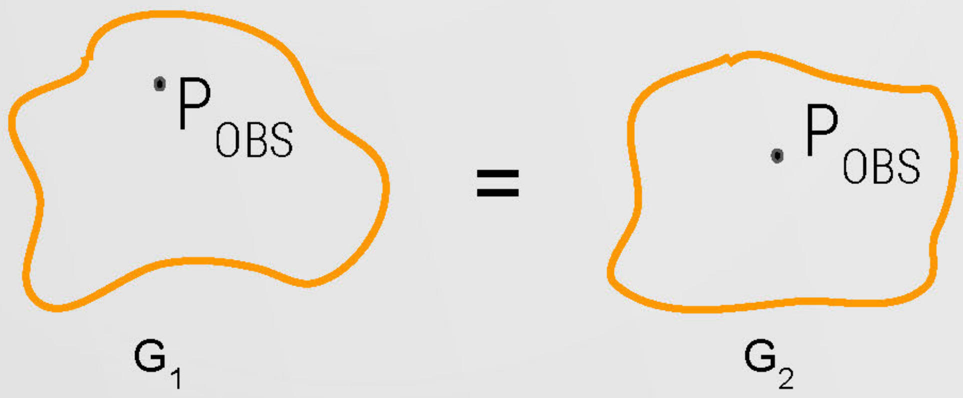
$$A \perp B$$


Instrumental
≃
Instrumental
≼
Evans

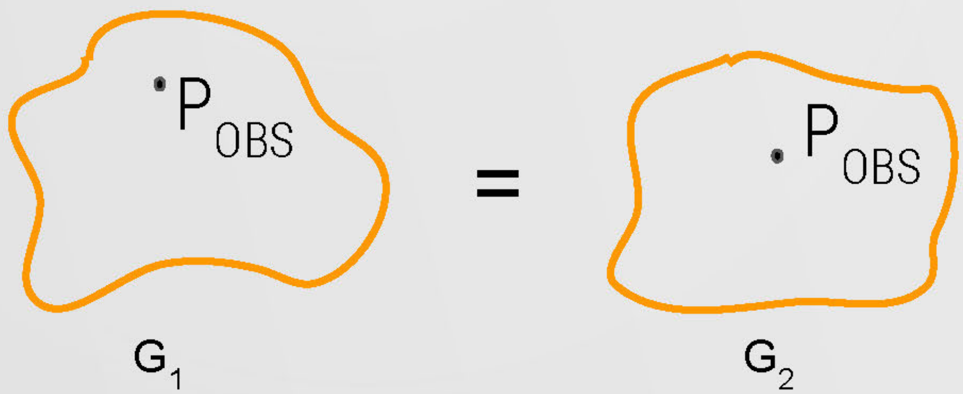

Saturating
Triangle
Evans
Instrumental
Collider
Chain&Fork
2|1 Factorizing
Factorizing


Saturating
Triangle
Evans
Instrumental
Collider
Chain&Fork
2|1 Factorizing
Factorizing
Compatibility of **P**

To find such equivalence classes, do we need to know all compatible distributions?

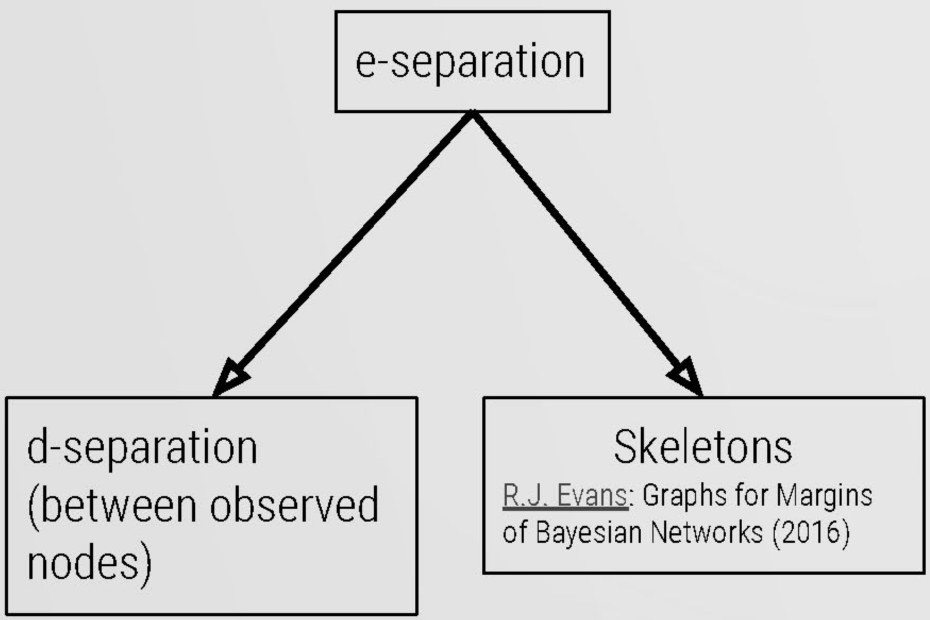


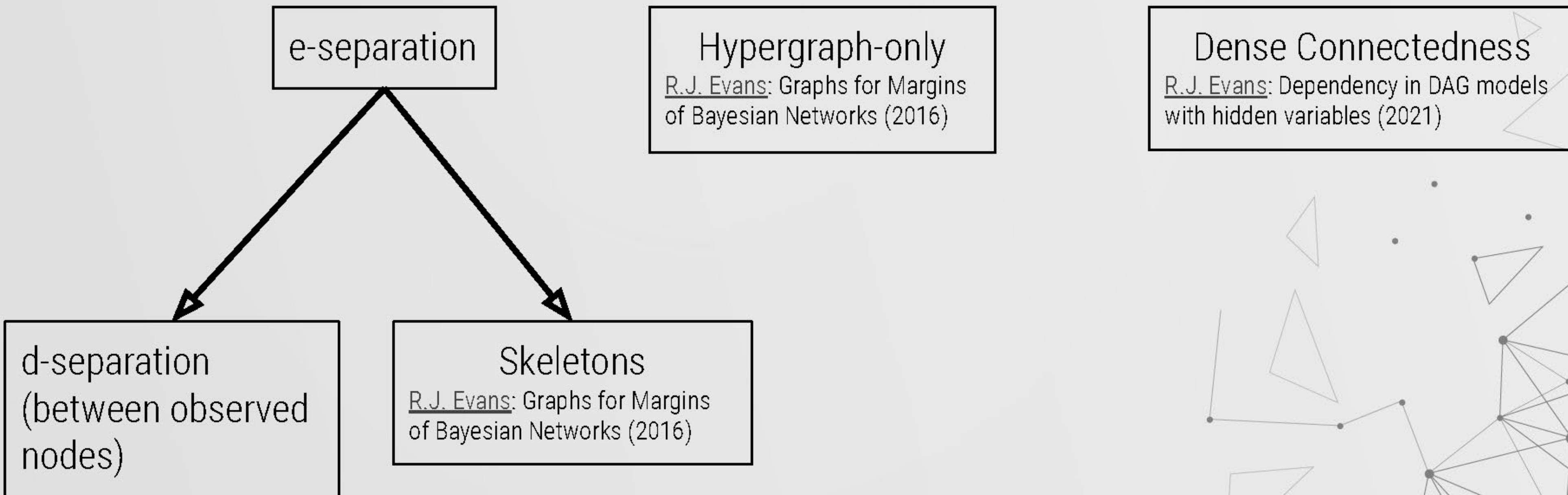
No! There are rules

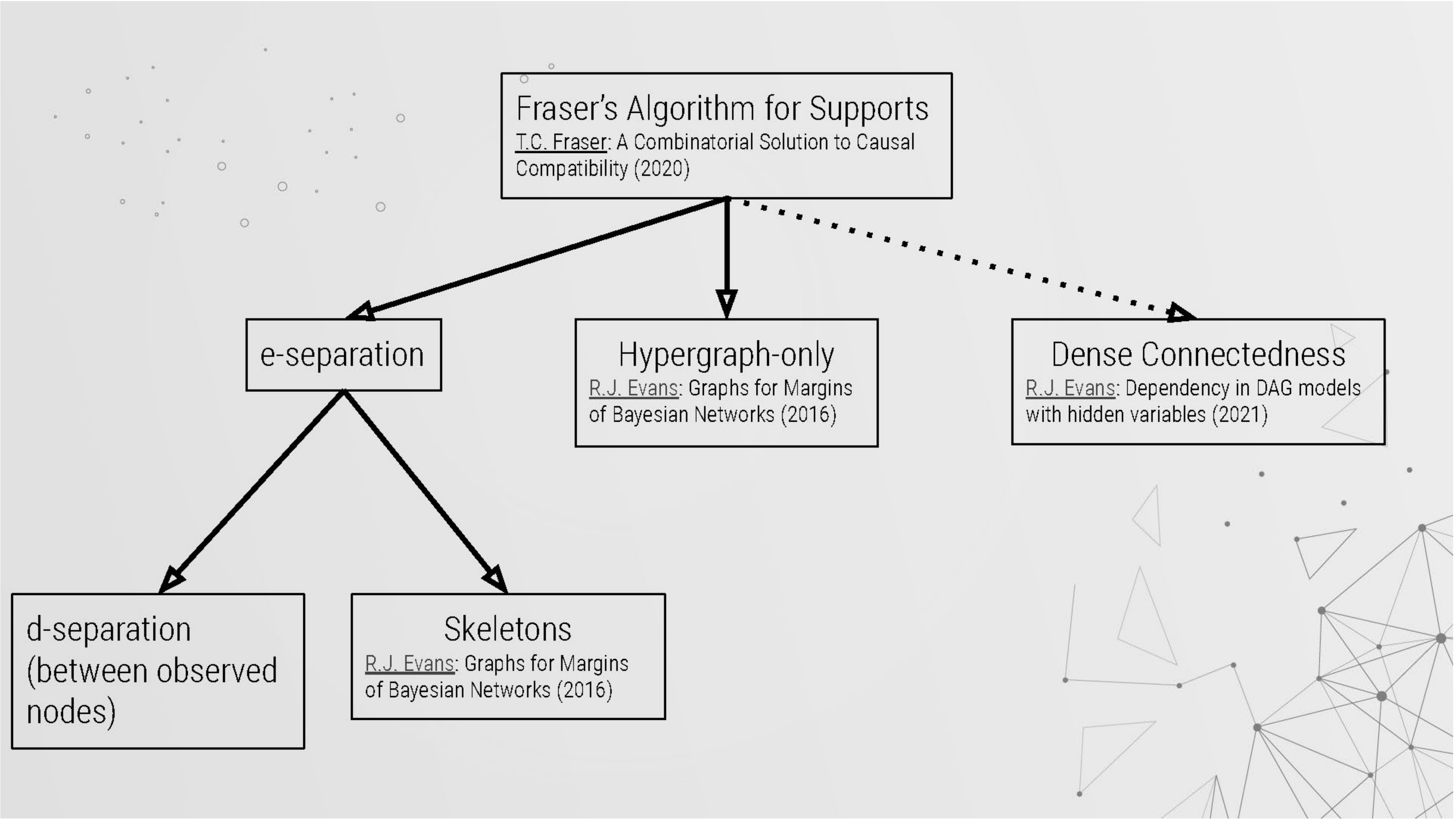


Rules to prove observational <u>inequivalence</u>

d-separation
(between observed
nodes)


e-separation
d-separation (between observed nodes)
Skeletons
R.J. Evans: Graphs for Margins of Bayesian Networks (2016)

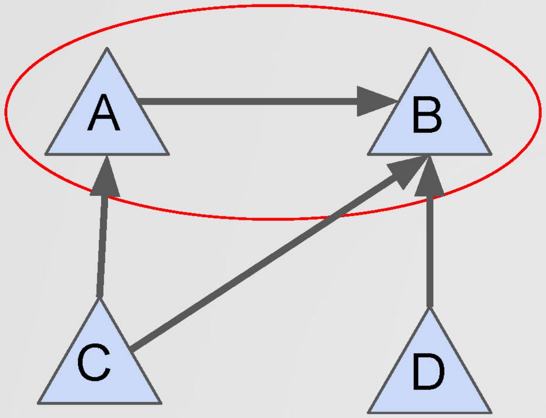

e-separation
d-separation (between observed nodes)
Skeletons
R.J. Evans: Graphs for Margins of Bayesian Networks (2016)
Hypergraph-only
R.J. Evans: Graphs for Margins of Bayesian Networks (2016)
Dense Connectedness
R.J. Evans: Dependency in DAG models with hidden variables (2021)


Fraser's Algorithm for Supports
T.C. Fraser: A Combinatorial Solution to Causal Compatibility (2020)
e-separation
Hypergraph-only
R.J. Evans: Graphs for Margins of Bayesian Networks (2016)
Dense Connectedness
R.J. Evans: Dependency in DAG models with hidden variables (2021)
d-separation (between observed nodes)
Skeletons
R.J. Evans: Graphs for Margins of Bayesian Networks (2016)

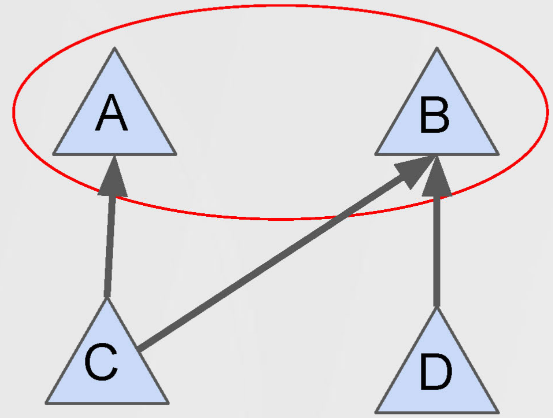
Rules to prove observational equivalence

Remove edges for free - HLP Proposition

$$\mathbf{pa}(A) \subseteq \mathbf{pa}(B)$$



$\simeq$



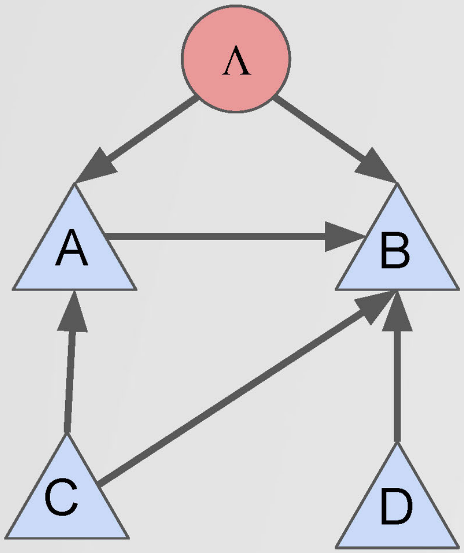
J.Henson, R.Lal and M. F. Pusey: Theory-independent limits on correlations from generalized Bayesian networks (2014)

Split Latents for Free - Evans Proposition

$$\mathbf{pa}(A) \subseteq \mathbf{pa}(B)$$

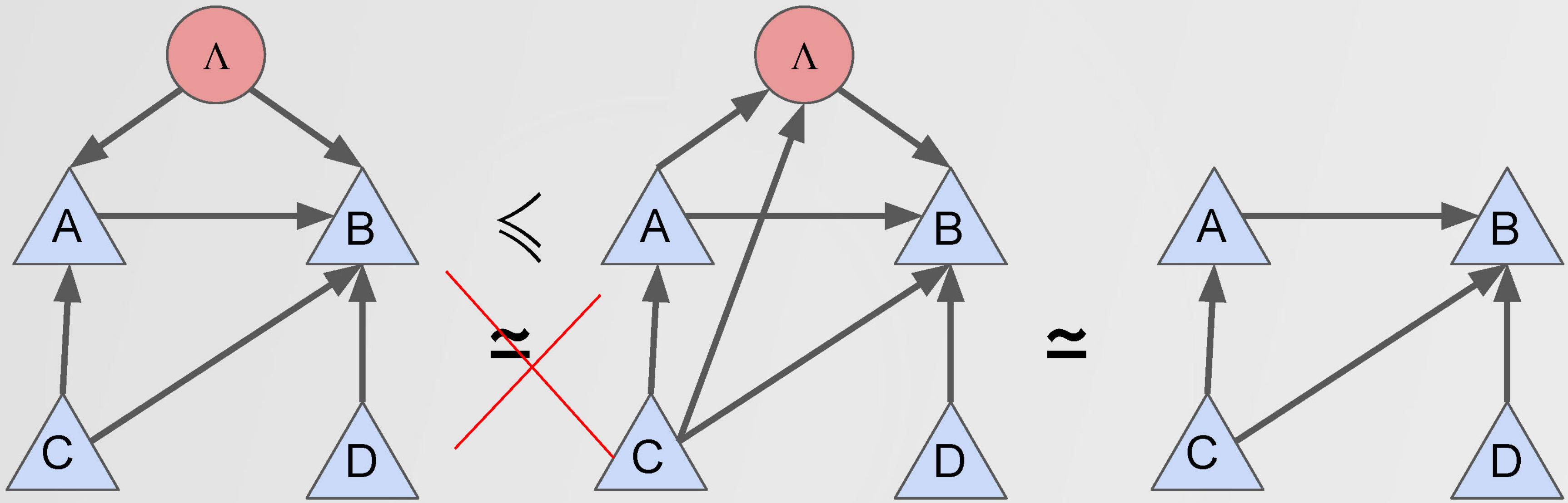
R.J. Evans: Graphs for Margins of Bayesian Networks (2016)

$$\mathrm{pa}(A) \subseteq \mathrm{pa}(B)$$



Errata in the video: the steering/Bayesian updating argument in general can only show dominance, not equivalence

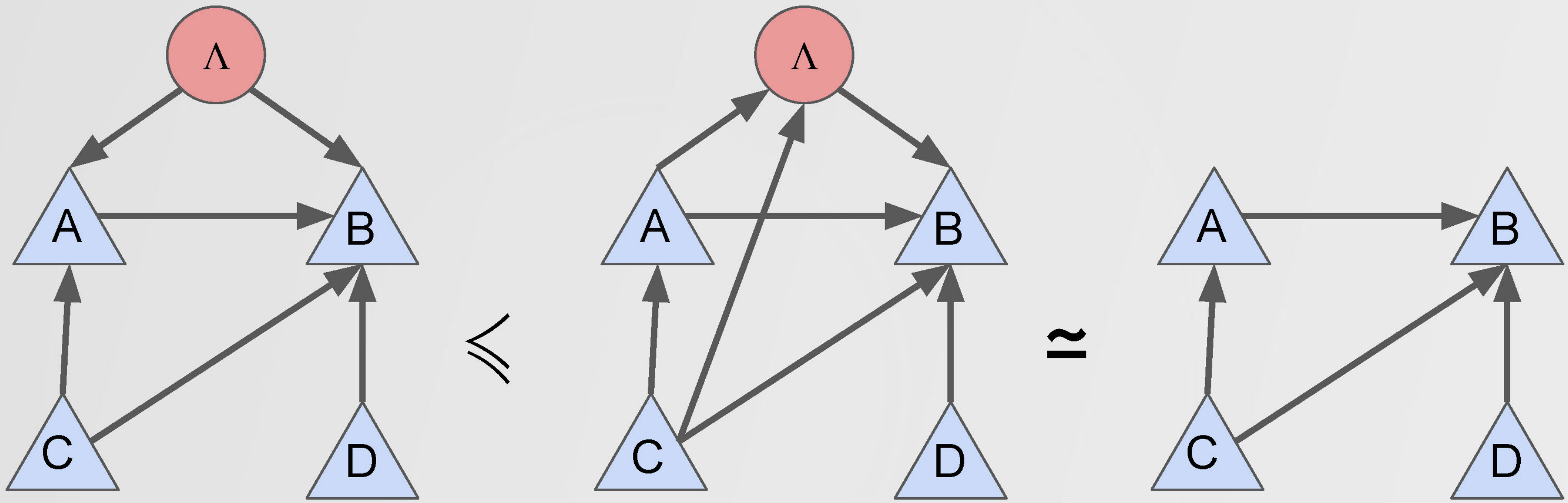
$$\mathbf{pa}(A) \subseteq \mathbf{pa}(B)$$



(a) (b) (c)

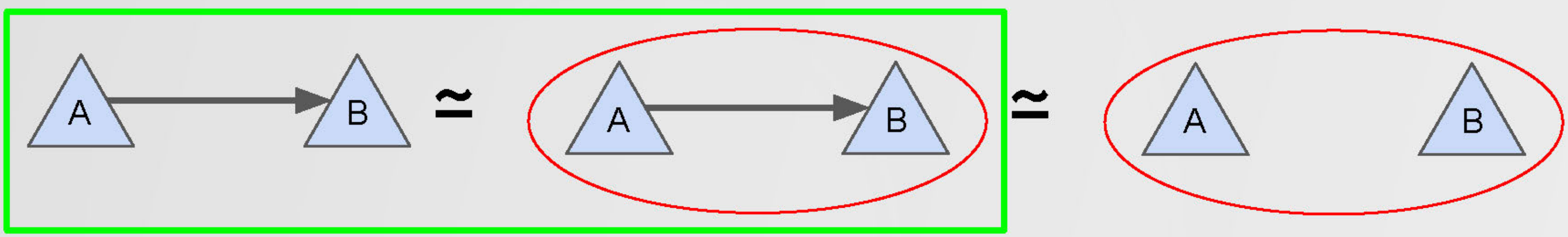
Corrected proof:

$$\mathbf{pa}(A) \subseteq \mathbf{pa}(B)$$

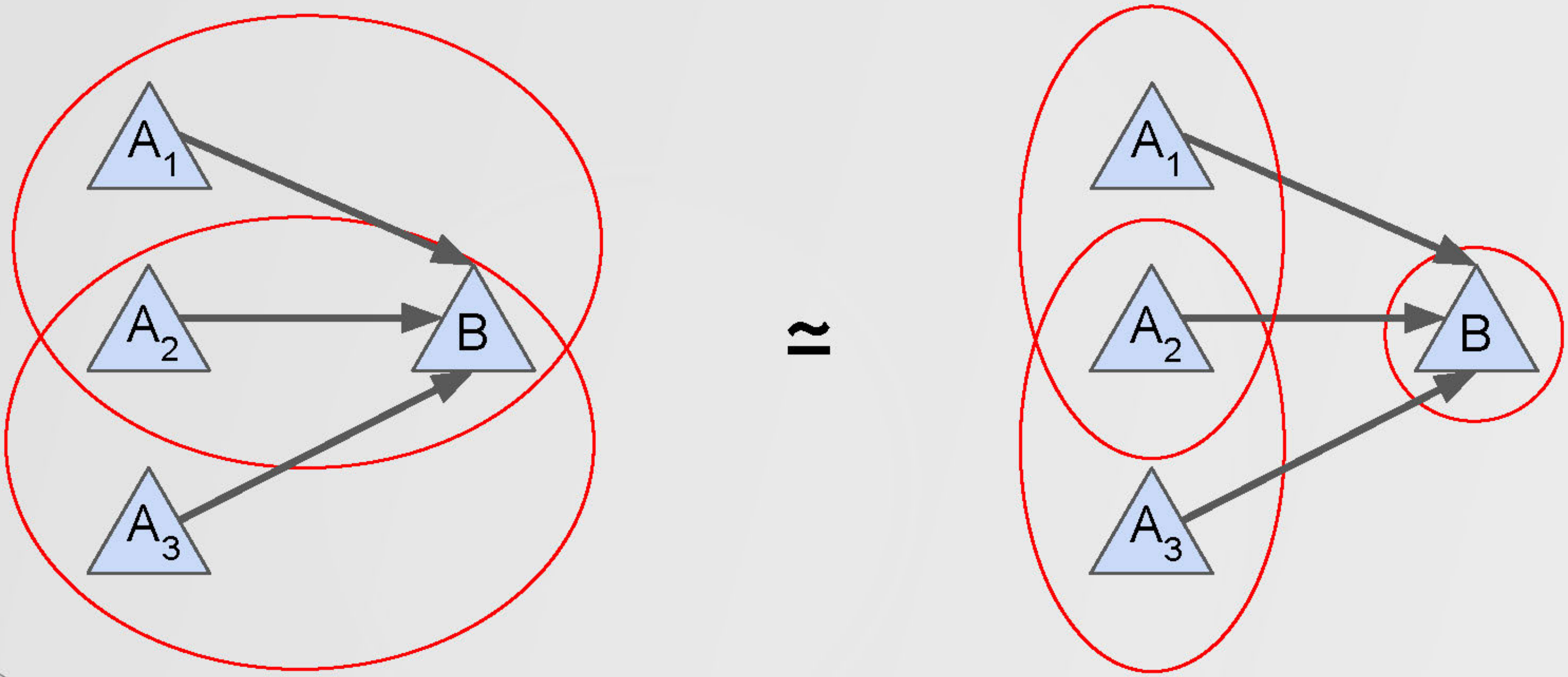


(a) (b) (c)

But clearly (c)⩽(a), so they are all equivalent


A
B
≃
A
B
≃
A
B

Generalization: Strong Face Splitting

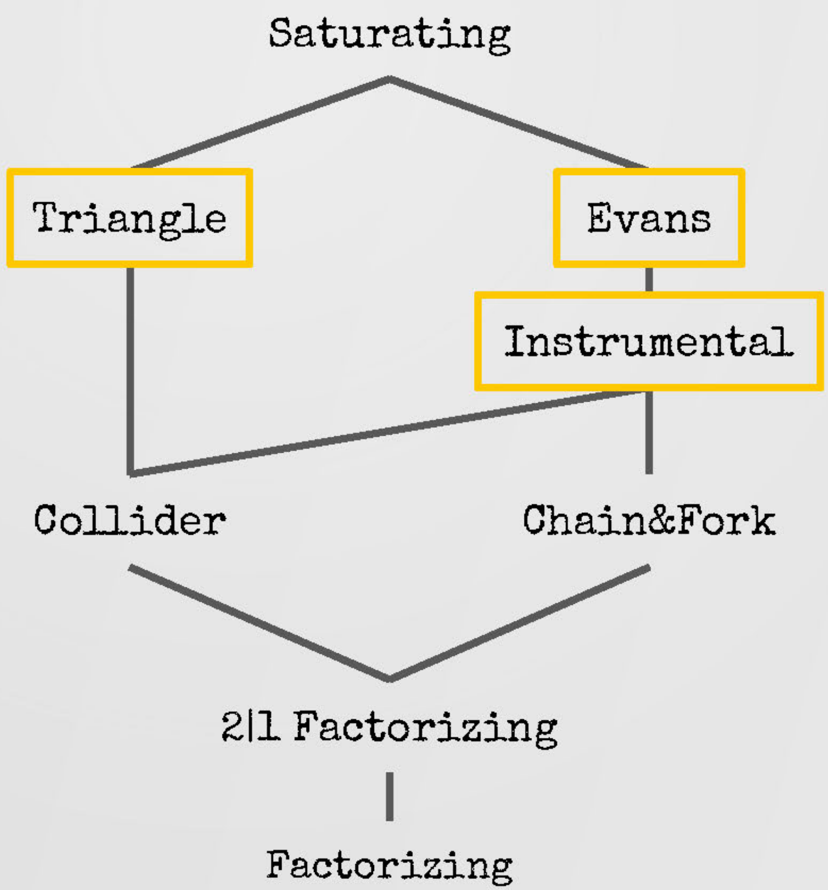


Applying the Rules

Number of mDAGs

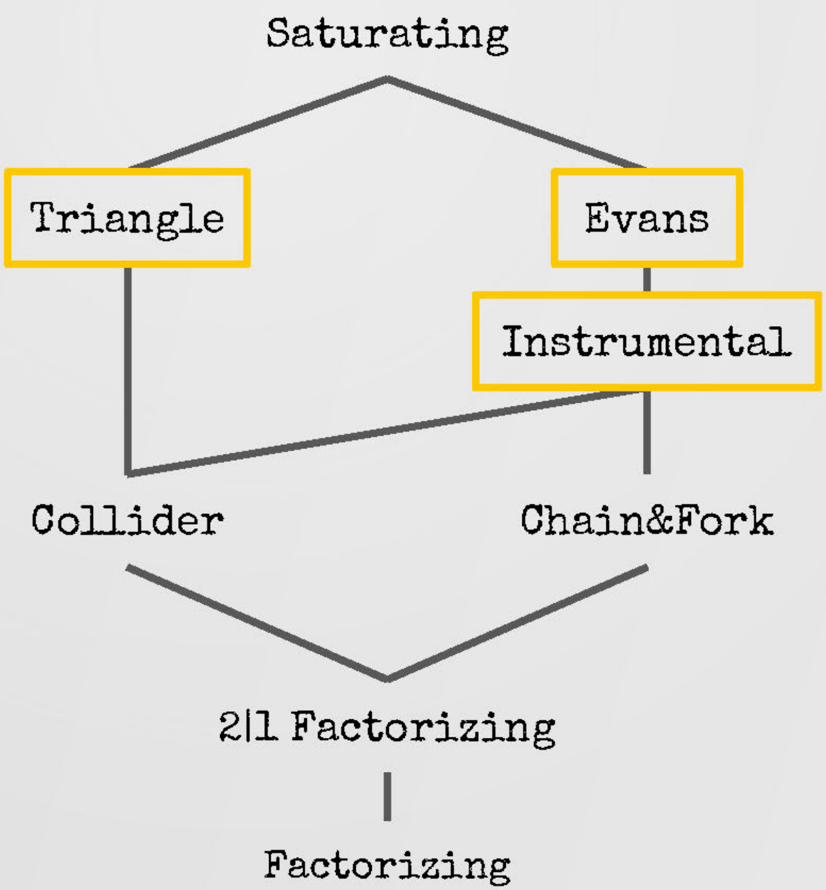
Number of Observables	2	3	4	5
Total Number of mDAGs (ignore labels)	4	46	2809	1718596

3 Observed Nodes



4 Observed Nodes

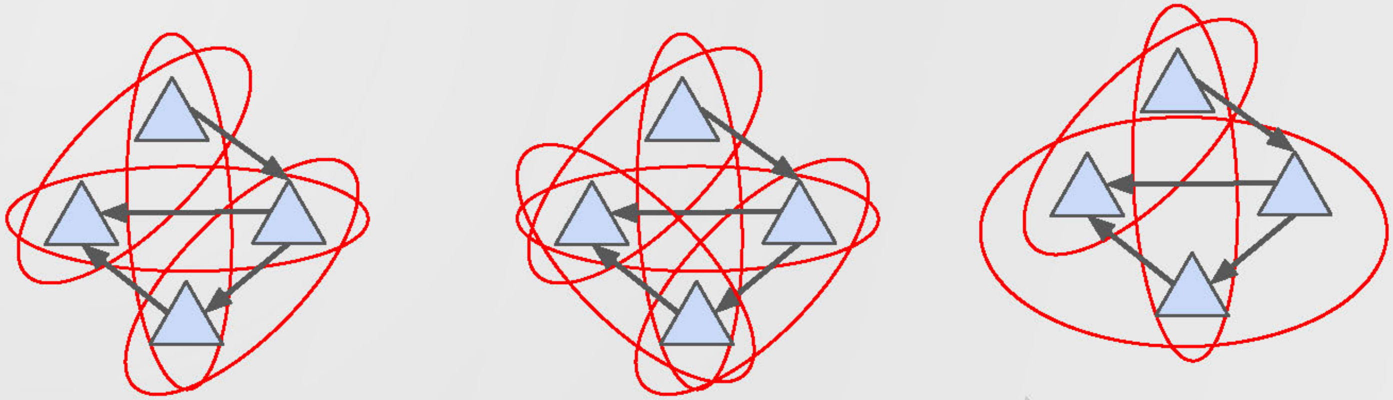
3 Observed Nodes



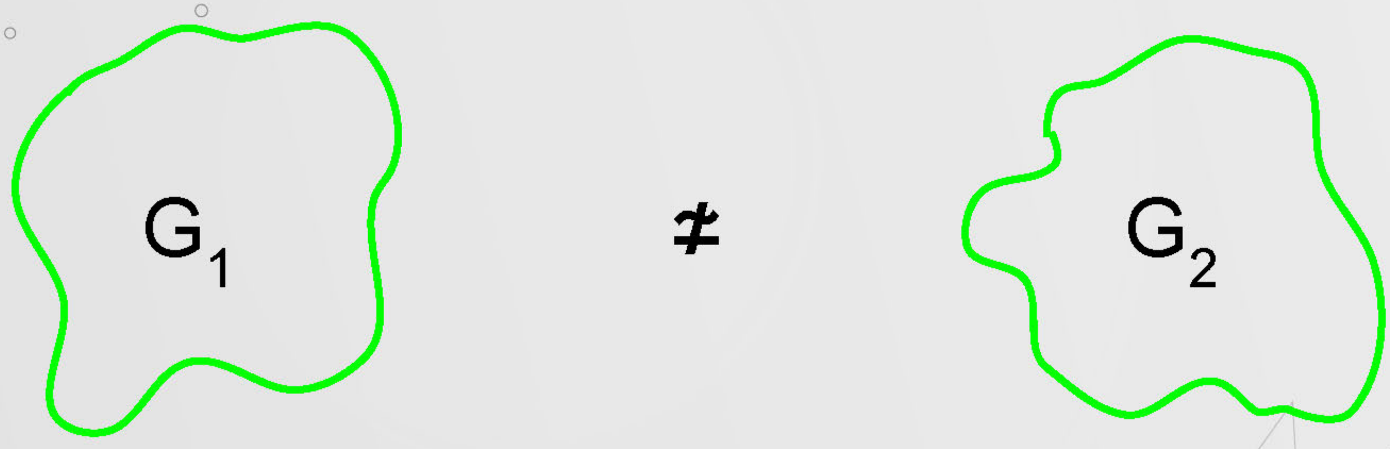
4 Observed Nodes

Rules to prove <u>equivalence</u> ⟹ Upper Bound: **396**

Rules to prove in<u>equivalence</u> ⟹ Lower Bound: **339**



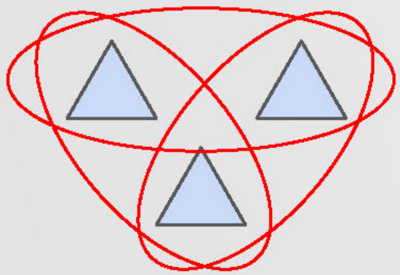
Are these saturated?



$\mathrm{dsep}(G_1) = \mathrm{dsep}(G_2)$

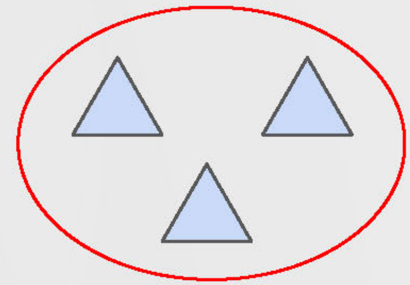
□

At least one of them has inequalities



Triangle

$\neq$

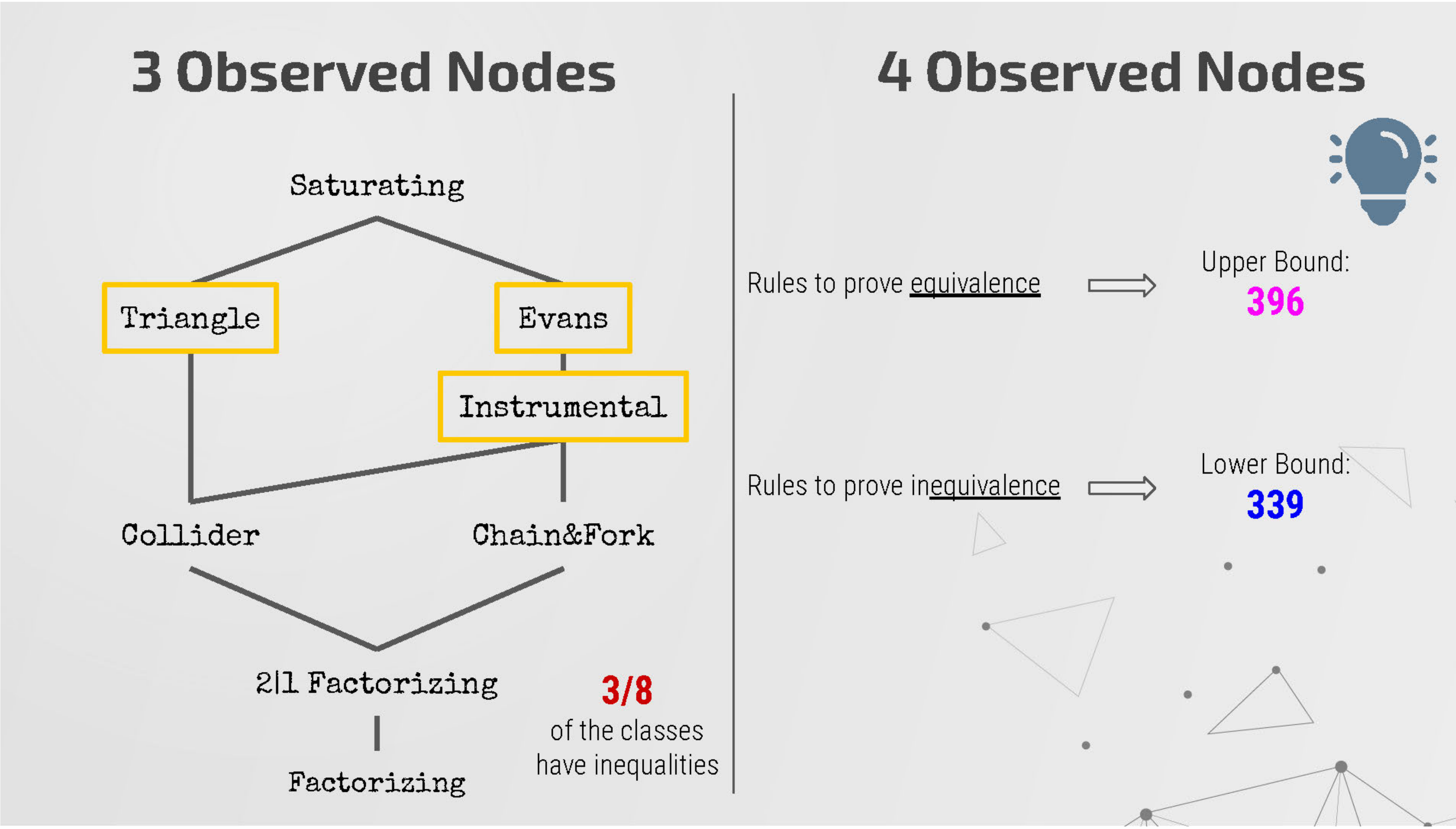


Saturated

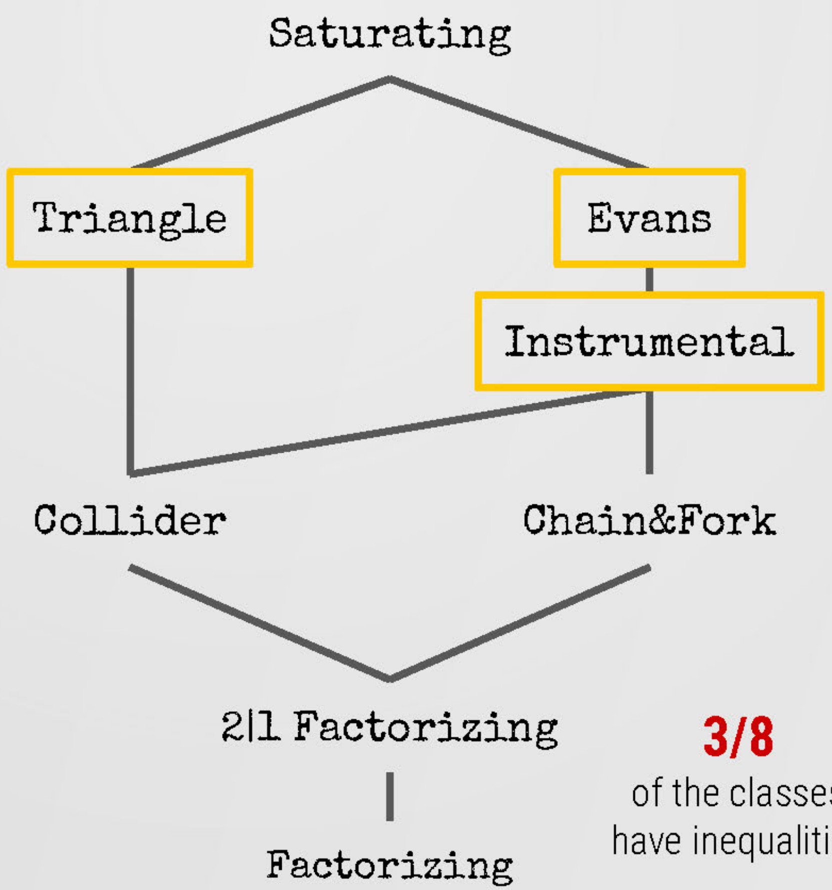
$\mathrm{dsep}(G_1) = \mathrm{dsep}(G_2)$

□

At least one of them has inequalities


3 Observed Nodes
Saturating
Triangle
Evans
Instrumental
Collider
Chain&Fork
2|1 Factorizing
Factorizing
3/8
of the classes
have inequalities
4 Observed Nodes
Rules to prove equivalence
Upper Bound:
396
Rules to prove inequivalence
Lower Bound:
339

3 Observed Nodes



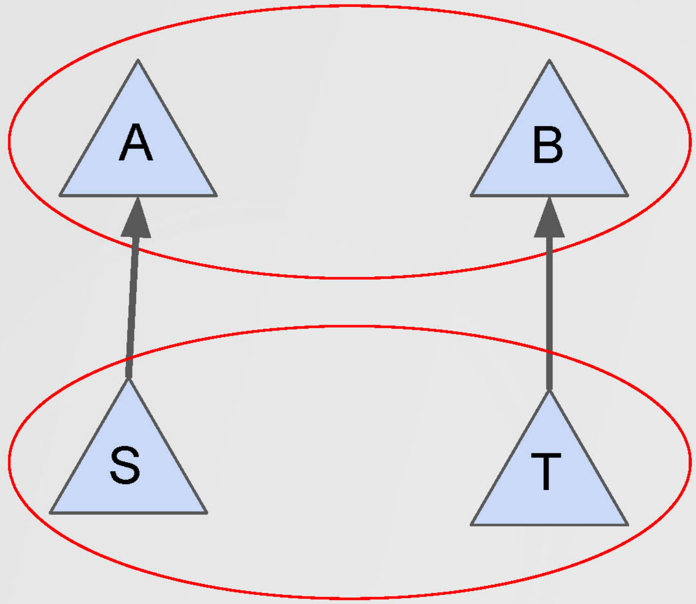
3/8 of the classes have inequalities

4 Observed Nodes

Rules to prove equivalence ⟹ Upper Bound: **396**

Rules to prove inequivalence ⟹ Lower Bound: **339**

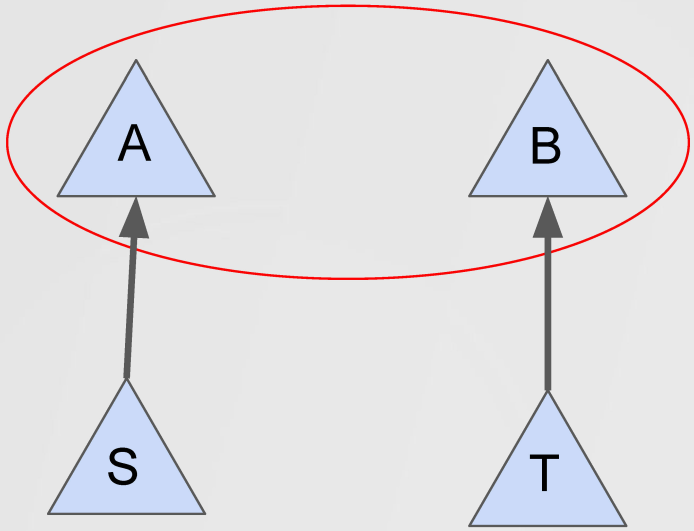
25 different d-separation patterns ⟹ At least **92,6%** of the classes have inequalities



Solved by d-separation

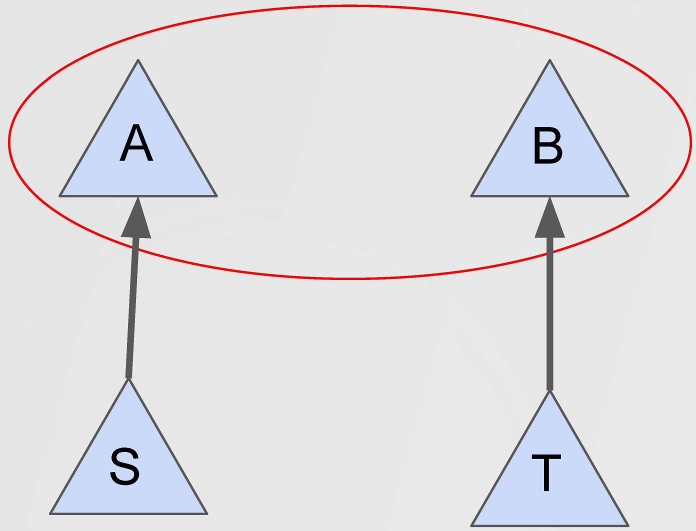
$$A \perp T \mid S$$

$$B \perp S \mid T$$



$$A \perp T$$
$$B \perp S$$
$$A \perp T \mid S$$
$$B \perp S \mid T$$

$$S \perp T$$
$$S \perp T \mid B$$
$$S \perp T \mid A$$



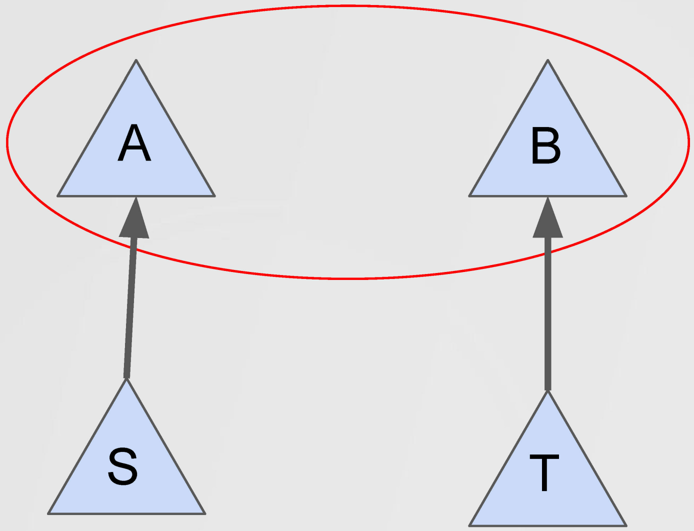
$$A \perp T$$
$$B \perp S$$
$$A \perp T \mid S$$
$$B \perp S \mid T$$

$$S \perp T$$
$$S \perp T \mid B$$
$$S \perp T \mid A$$

Every inequivalence is proved by d-separation between observed nodes

□

Solved by d-separation



Every inequivalence is proved by d-separation between observed nodes

□

Solved by d-separation

$$A \perp T$$
$$B \perp S$$
$$A \perp T \mid S$$
$$B \perp S \mid T$$

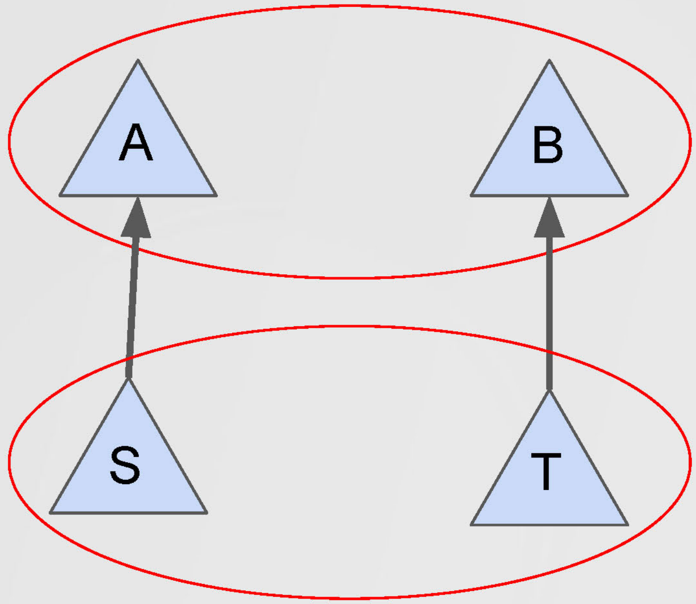
$$S \perp T$$
$$S \perp T \mid B$$
$$S \perp T \mid A$$

P such that:

- Violates Bell Inequality
- CI = set to the left

□

Classical explanations for **P** are all <u>fine-tuned</u>



$$A \perp T \mid S$$

$$B \perp S \mid T$$

Solved by d-separation

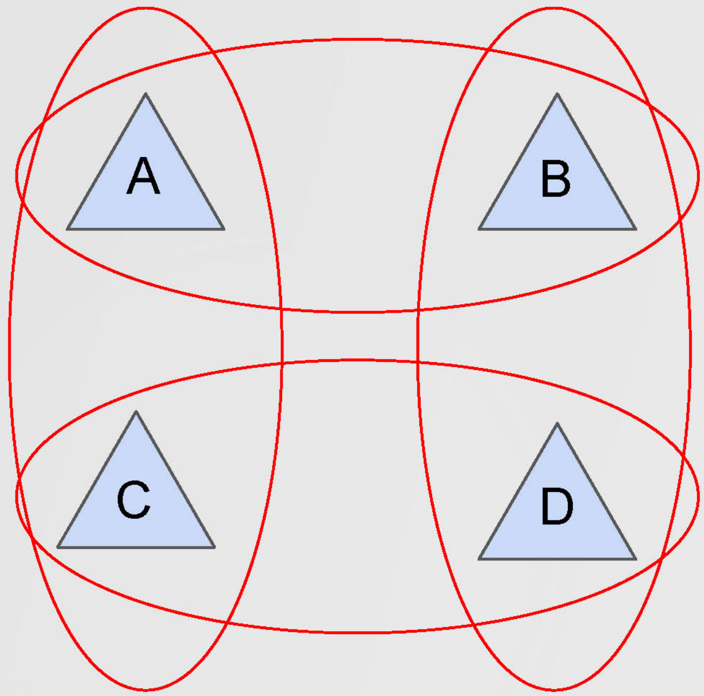
P such that:

- Violates Bell Inequality
- CI = no-signalling

□

Classical explanations for **P** are all fine-tuned

C.J. Wood and R.W. Spekkens: The lesson of causal discovery algorithms for quantum correlations (2012)



$$C \perp B$$
$$A \perp D$$

Solved by d-separation

P such that:

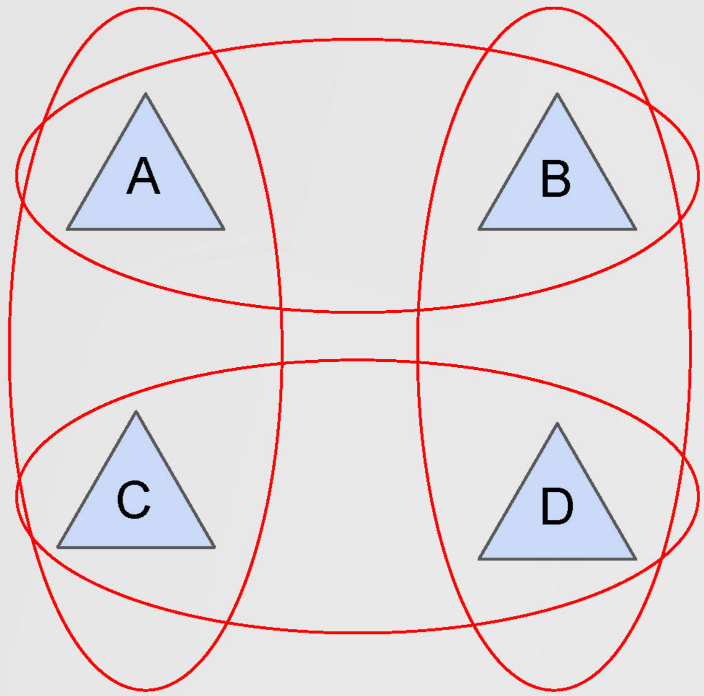
- Violates Square inequalities
- CI = $C \perp B, A \perp D$

□

Classical explanations for **P** are all <u>fine-tuned</u>

Only 18 classes

Solved by d-separation



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THANKS

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