Title: Correlations from joint measurements in boxworld and applications to information processing

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Collection: Causal Inference & Quantum Foundations Workshop

Date: April 17, 2023 - 3:30 PM

URL: https://pirsa.org/23040107

Abstract: Quantum measurements have been a central topic of research in quantum theory for many years. In the context of causal structures and communication over networks, we are often particularly interested in local measurements of subsystems of a multi-partite system and classical processing of their inputs and outcomes. Formally, this processing can often be described by means of maps that are known as wirings. These wirings are furthermore interesting for the analysis of generalized probabilistic theories, as they are shared by all of them. In this work, we explicitly characterise all possible mulitpartite measurements in the generalised probabilistic theory box-world for various numbers of parties n with systems characterised by n_i fiducial measurements (which can be thought of as inputs here) and n_o outcomes, for small n, n_i, n_o. This includes all n-party n_i-input, n_o-outcome wirings. For n > 2, we further classify these measurements into three classes: wirings, deterministic non-wiring type and non-deterministic non-wiring type measurements. We explore advantages of these different types of measurements over previous protocols in the context of non-locality distillation and state-distingishability. We further find examples of non-locality without entanglement (contrary to previous claims) and a relation of these measurements to classical process matrices.

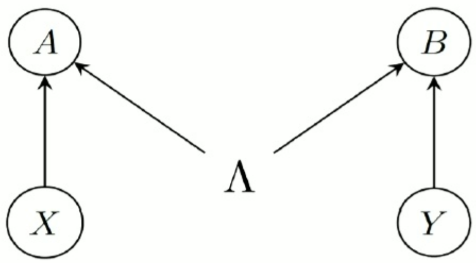
Correlations from joint measurements in boxworld and applications to information processing

MIRJAM WEILENMANN

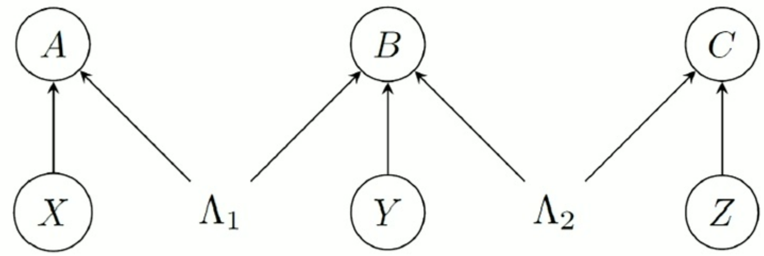
JOINT WORK WITH GIORGOS EFTAXIAS AND ROGER COLBECK

CAUSAL INFERENCE AND QUANTUM FOUNDATIONS WORKSHOP, APRIL 2023

MOTIVATION: CORRELATIONS IN NETWORKS



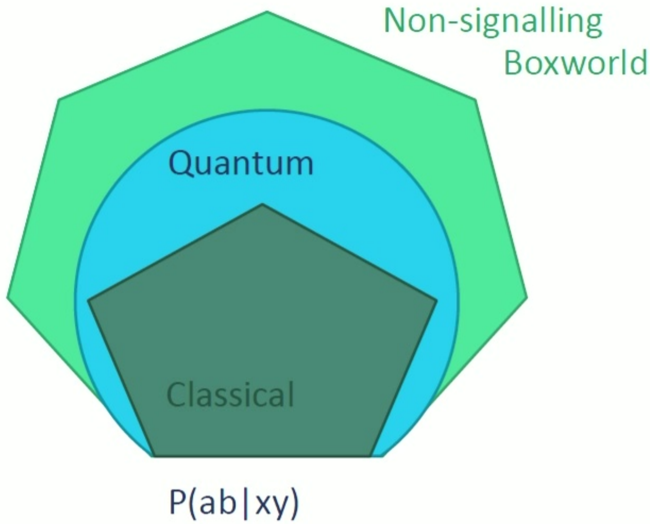
Bell causal structure

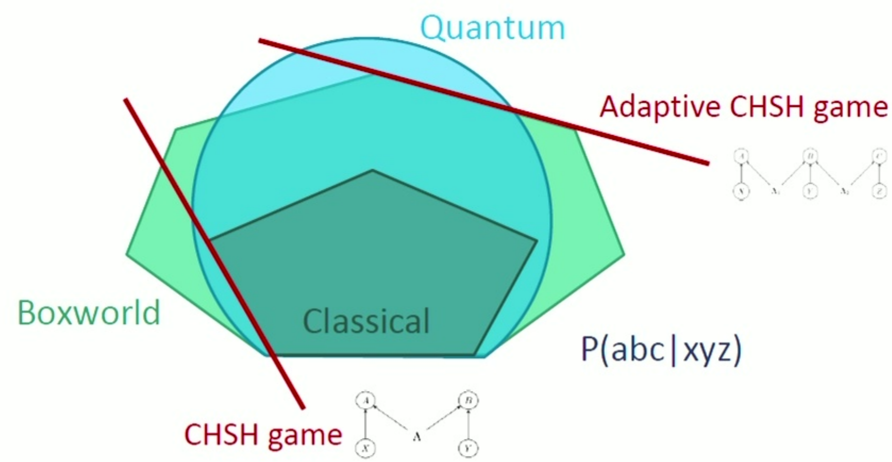


Bilocal causal structure

(Joint) measurements play an important role !

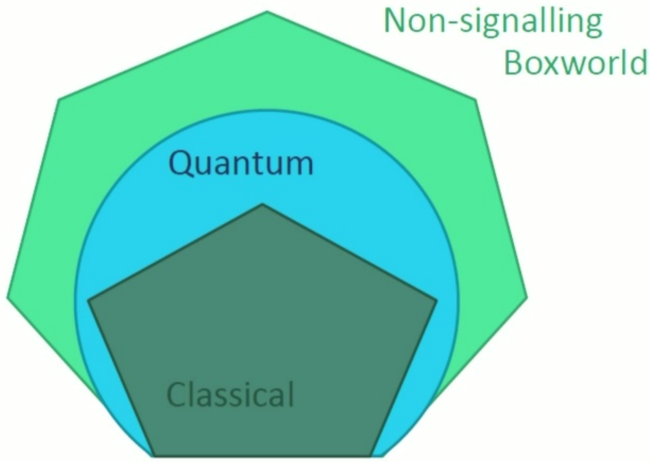
MOTIVATION: CORRELATIONS IN NETWORKS AND SELF-TESTING QUANTUM THEORY

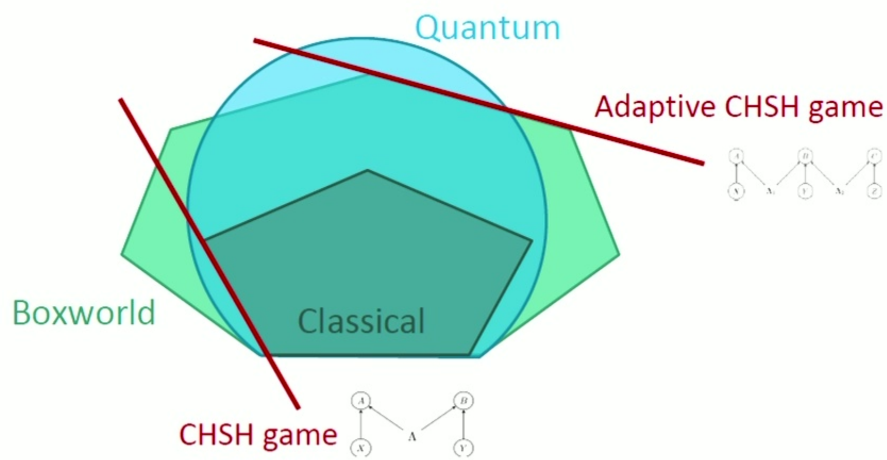




Tradeoff between states and measurements

MOTIVATION: CORRELATIONS IN NETWORKS AND SELF-TESTING QUANTUM THEORY



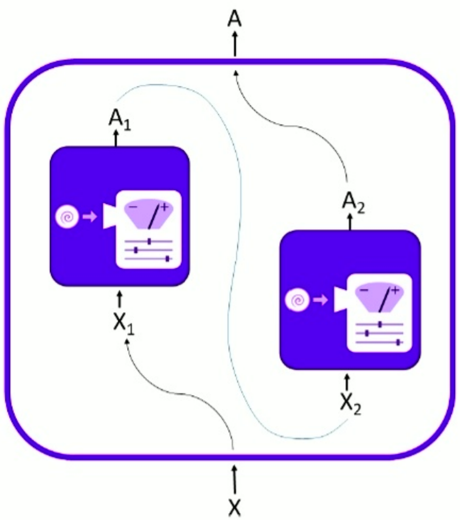


Tradeoff between states and measurements

Understanding measurements opens up avenues for quantum foundations:
Physical principles with respect to which quantum theory is optimal?

Weilenmann, Colbeck, Self-testing physical theories, PRL & PRA (2020).

A CLASS OF JOINT MEASUREMENTS: WIRINGS



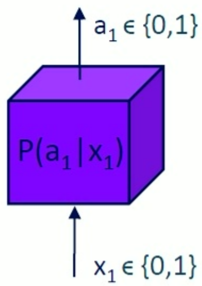
Extremal wiring measurements. For each *x*, set i = 1, for i ≤ *N:*

- Choose device (dependent on previous inputs and outcomes).
- Choose input as (deterministic) function of all previous inputs and outcomes.
- Increase i by one.

Output $a = f(x, \{x_i\}_i, \{a_i\}_i)$.

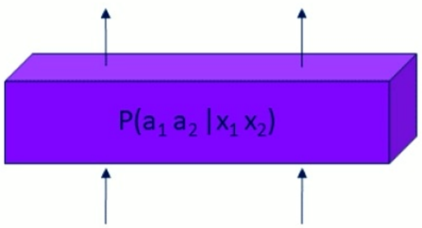
Added benefit: very easy to implement!

WIRINGS AS JOINT MEASUREMENTS IN BOXWORLD



$$\begin{array}{l} \quad\quad\quad s_1 \quad\quad\quad\quad s_2 \quad\quad\quad\quad s_3 \quad\quad\quad\quad s_4 \\ S_1 = \text{conv}(\ \{(1,0|1,0),\ (1,0|0,1),\ (0,1|1,0),\ (0,1|0,1)\}\ ) \end{array}$$

$$\begin{array}{l} E_1 = \text{conv}(\ \{(1,0|0,0),\ (0,1|0,0),\ (0,0|1,0),\ (0,0|0,1),\ (0,0|0,0),\ (1,1|0,0),\ (0,0|1,1)\}\ ) \\ \quad\quad\quad e_1 \quad\quad\quad I - e_1 \quad\quad\quad e_2 \quad\quad\quad I - e_2 \quad\quad\quad 0 \quad\quad\quad I \quad\quad\quad I \end{array}$$



$$S_2 = \{\ P(a_1 a_2 | x_1 x_2)\ |\ P \text{ non-signalling distribution}\ \}$$

$$E_2 = \{\ E(a_1 a_2 | x_1 x_2)\ |\ 0 \leq \langle\, E\, ,\, P\, \rangle \leq 1\ \ \forall\, P \in S_2\,\}$$

- For 2-party 2-input 2-output boxes there are only wirings.
- For 3-party 2-input 2-output boxes there is at least one other measurement.

Examples

$x_2=0$ $\quad$ $x_2=1$

$$s = \begin{bmatrix} 1/2\ 0 & 1/2\ 0 \\ 0\ 1/2 & 0\ 1/2 \\ 1/2\ 0 & 1/2\ 0 \\ 0\ 1/2 & 0\ 1/2 \end{bmatrix} \begin{matrix} x_1=0 \\ \\ x_1=1 \\ \\ \end{matrix}$$

$x_2=0$ $\quad$ $x_2=1$

$$e = \begin{bmatrix} 1\ 1 & 0\ 0 \\ 1\ 1 & 0\ 0 \\ 0\ 0 & 0\ 0 \\ 0\ 0 & 0\ 0 \end{bmatrix} \begin{matrix} x_1=0 \\ \\ x_1=1 \\ \\ \end{matrix}$$

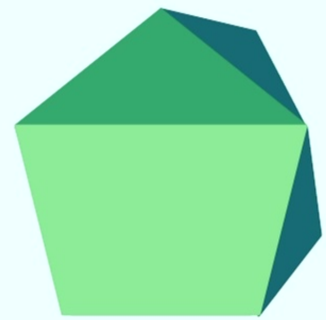
Barrett, Information processing in generalized probabilistic theories, PRA 75 (2007).
Short, Popescu & Gisin, Entanglement swapping for generalized nonlocal correlations, PRA 73 (2006).
Short & Barrett, Strong nonlocality: A trade-off between states and measurements, NJP 12 (2010).

JOINT MEASUREMENTS IN BOX-WORLD

$E_{(n,m,k)} = \{ E \mid 0 \leq \langle E, P\rangle \leq 1, \forall P(a_1 \dots a_n \mid x_1 \dots x_n)$ non-signalling, $a_i \in \{1, \dots, k\}, x_i \in \{1, \dots, m\} \}$

Example: extremal measurements for 3-party 2-input 2-output boxes

- Wiring types
- Deterministic measurement effects (beyond wirings)
- Non-deterministic measurement effects (beyond wirings)



- All measurement effects can be found as subeffects of the representations of the unit effect.

n=2	n=3
sub-effects of 12 units	sub-effects of 710760 units

$$\begin{array}{c c} & \quad x_2=0 \quad\quad x_2=1 \\ \begin{array}{c} x_1=0 \\ \\ x_1=1 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

JOINT MEASUREMENTS IN BOX-WORLD

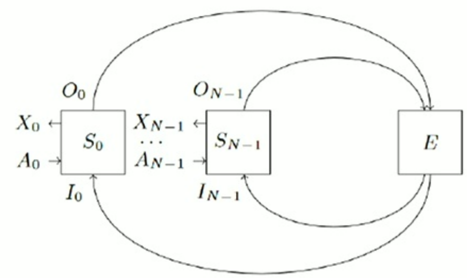
$E_{(n,m,k)} = \{ E \mid 0 \leq \langle E, P \rangle \leq 1, \forall P(a_1 \ldots a_n \mid x_1 \ldots x_n)$ non-signalling, $a_i \in \{1, \ldots, k\}$, $x_i \in \{1, \ldots, m\} \}$

Example: extremal measurements for 3-party 2-input 2-output boxes

- Wiring types
- Deterministic measurement effects (beyond wirings)
- Non-deterministic measurement effects (beyond wirings)

- Fun fact: unit effects correspond to logically consistent classical processes.

Baumeler & Wolf, The space of logically consistent classical processes without causal order, NJP 18 (2016).



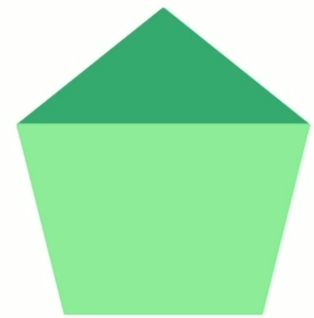
ENUMERATING DETERMINISTIC MEASUREMENTS

$x_2=0$ $x_2=1$

$x_1=0$, $x_1=1$

$$\begin{bmatrix}1\,1 & 0\,0\\ 1\,1 & 0\,0\\ 0\,0 & 0\,0\\ 0\,0 & 0\,0\end{bmatrix} \longrightarrow \begin{bmatrix}1\,1 & 0\,0\\ 0\,0 & 1\,1\\ 0\,0 & 0\,0\\ 0\,0 & 0\,0\end{bmatrix}$$

p(10|00)+p(11|00) = p(10|01)+p(11|01)



$$\begin{bmatrix}1\,1 & 0\,0\\ 1\,1 & 0\,0\\ 0\,0 & 0\,0\\ 0\,0 & 0\,0\end{bmatrix} \longrightarrow \begin{bmatrix}0\,0 & 1\,1\\ 0\,0 & 1\,1\\ 0\,0 & 0\,0\\ 0\,0 & 0\,0\end{bmatrix} \longrightarrow \begin{bmatrix}0\,0 & 1\,0\\ 0\,0 & 1\,0\\ 0\,0 & 0\,1\\ 0\,0 & 0\,1\end{bmatrix}$$

n_O	2	3	4
classes $n_I = 2$	7	44	523
classes $n_I = 3$	7	48	-
classes $n_I = 4$	7	-	-
total $n_I = 2$	82	8930	2977858
total $n_I = 3$	248	43400	-
total $n_I = 4$	562	-	-

- Generalises to higher dimensions (and more parties)
- Easy to generate (avoiding vertex enumeration)

JOINT MEASUREMENTS IN BOX-WORLD

Examples of measurements beyond wirings in the 3-party case

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \\ 0.5 & 0.5 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$e\,,\ I - e$ *

$f\,,\ I - f$

*Essentially equivalent to measurement from: Short & Barrett, Strong nonlocality: A trade-off between states and measurements, NJP 12 (2010).

JOINT MEASUREMENTS IN BOX-WORLD

Examples of measurements beyond wirings in the 3-party case

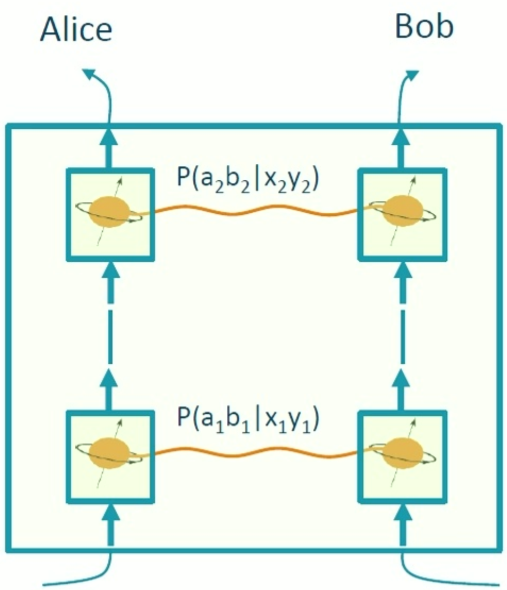
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \\ 0.5 & 0.5 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$e, I - e$ *　　　　　$f, I - f$

*Essentially equivalent to measurement from: Short & Barrett, Strong nonlocality: A trade-off between states and measurements, NJP 12 (2010).

APPLICATIONS: NON-LOCALITY DISTILLATION



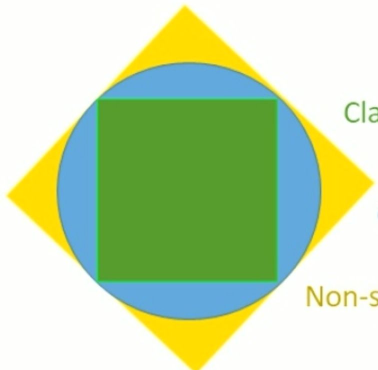
Goal:

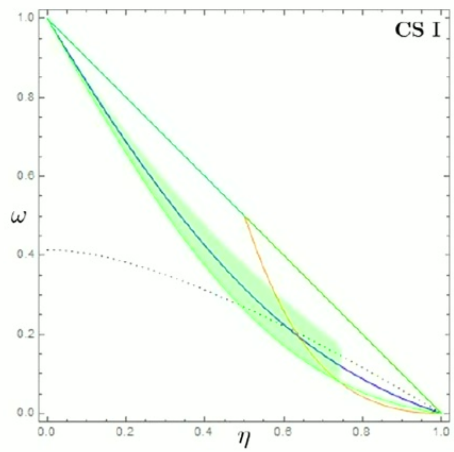
CHSH(P(ab|xy)) > max {CHSH(P($a_1b_1|x_1y_1$)), CHSH(P($a_2b_2|x_2y_2$))}

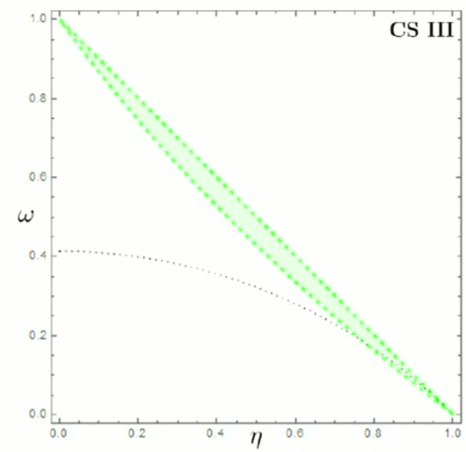
$$P(ab|xy)= \sum_{a_1a_2x_1x_2b_1b_2y_1y_2} e_x^{a}(a_1a_2|x_1x_2)\, e_y^{b}(b_1b_2|y_1y_2)\, P(a_1b_1|x_1y_1)\, P(a_2b_2|x_2y_2)$$

Forster, Winkler & Wolf, Distilling nonlocality, PRL 102 (2009).

APPLICATIONS: WIRINGS FOR NON-LOCALITY DISTILLATION







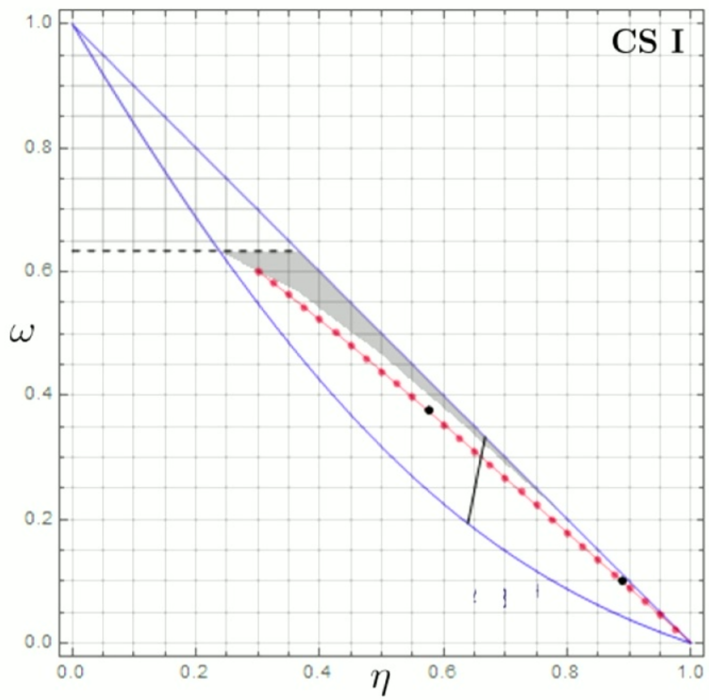
$x_1 = x_2 = x\oplus 1$, $x_3 = (x\oplus 1)a_1 \vee (x\oplus 1)a_2$, $a = a_1a_3 \vee a_2a_3 \vee (a_1\oplus 1)(a_2\oplus 1)(a_3\oplus 1)$,

$y_1 = y_2 = y$, $y_3 = yb_1 \vee yb_2 \vee (y\oplus 1)(b_1\oplus 1)(b_2\oplus 1)$,

$b = (y\oplus 1)b_1b_3 \vee (y\oplus 1)b_2b_3 \vee y(b_1\oplus 1)b_3 \vee y(b_2\oplus 1)b_3 \vee (y\oplus 1)(b_1\oplus 1)(b_2\oplus 1)(b_3\oplus 1) \vee y(b_1\oplus 1)(b_2\oplus 1)b_3$.

→ Valid in any GPT (including quantum theory) .

APPLICATIONS: WIRINGS FOR NON-LOCALITY DISTILLATION



Example: ping-pong protocol
(from sequential optimisation)

- $x_1 = x, \quad x_2 = x\,a_1, \quad a = a_1 \oplus a_2 \oplus 1$
  $y_1 = y, \quad y_2 = y \oplus b_1 \oplus 1, \quad b = b_1 \oplus b_2 \oplus 1.$
- $x_1 = x, \quad x_2 = x \oplus a_1, \quad a = a_1 \oplus a_2 \oplus 1$
  $y_1 = y, \quad y_2 = y\,(b_1 \oplus 1), \quad b = b_1 \oplus b_2 \oplus 1.$

- Further improvements by sequential use of 3-copy protocols

Brassard, Buhrman, Linden, Methot, Tapp & Unger, A limit on nonlocality in any world in which communication complexity is not trivial, PRL 96 (2006).
Brunner & Skrzypczyk, Non-locality distillation and post-quantum theories with trivial communication complexity, PRL 102 (2009).

APPLICATIONS: POST-QUANTUM NONLOCALITY WITHOUT ENTANGLEMENT

Boxworld example

$$
\begin{aligned}
s_1 &= t_3 \otimes t_4 \otimes t_2 \\
s_2 &= t_1 \otimes t_4 \otimes t_2 \\
s_3 &= t_4 \otimes t_2 \otimes t_3 \\
s_4 &= t_4 \otimes t_2 \otimes t_1 \\
s_5 &= t_1 \otimes t_1 \otimes t_3 \\
s_6 &= t_1 \otimes t_3 \otimes t_1 \\
s_7 &= t_3 \otimes t_1 \otimes t_4 \\
s_8 &= t_3 \otimes t_1 \otimes t_2
\end{aligned}
$$

$$
\begin{aligned}
\langle e_1, s \rangle &= P(110|000) \\
\langle e_2, s \rangle &= P(011|001) \\
\langle e_3, s \rangle &= P(111|010) \\
\langle e_4, s \rangle &= P(100|100) \\
\langle e_5, s \rangle &= P(001|000) \\
\langle e_6, s \rangle &= P(010|001) \\
\langle e_7, s \rangle &= P(101|010) \\
\langle e_8, s \rangle &= P(000|100).
\end{aligned}
$$

$t_1 = (1, 0 \mid 1, 0)$, $t_2 = (1, 0 \mid 0, 1)$, $t_3 = (0, 1 \mid 1, 0)$, $t_4 = (0, 1 \mid 0, 1)$

- Generalisation of quantum nonlocality without entanglement
- Disproving previous boxworld observations

Bennett, DiVincenzo, Fuchs, Mor, Rains, Shor, Smolin, Wootters, Quantum Nonlocality without Entanglement, PRA (1999).
Bhattacharya, Saha, Guha, Banik, Nonlocality without entanglement: quantum theory and beyond, Phys. Rev. Research (2020).

SUMMARY AND OUTLOOK

- Characterising and enumerating measurements in boxworld (including wirings!).
- Applications of wirings and advantages of measurements beyond wirings.
- More general understanding of measurements in GPTs including entangled **and** separable measurements?
- Understanding towards the physical principles underlying quantum theory ?

Thank you for your attention!

References: Phys. Rev. Lett. 130, 100201, 2023 and ArXiv: 2209.04474.