Title: Chern-Simons, time-reversal, bordism, and Smith homomorphisms.

Speakers: Matthew Yu

Series: Quantum Fields and Strings

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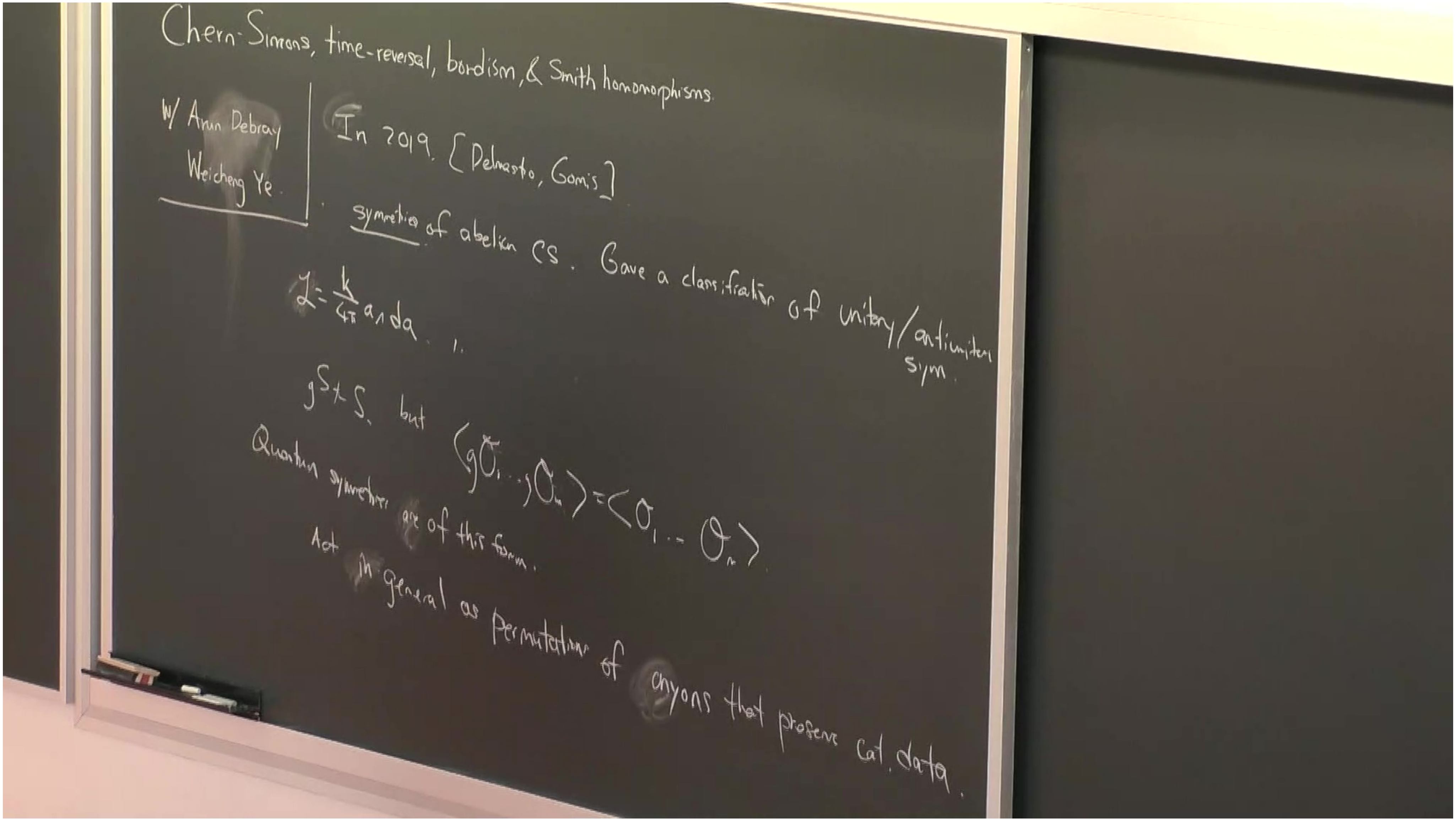
Abstract: Chern-Simons theories can possess 't Hooft anomalies for time-reversal symmetry. In this talk, we will discuss abelian Chern-Simons theories with an interesting time-reversal symmetry algebra and compute the classification of the anomaly, which is captured by a bordism group. In order to get the classification correct I will explain the argument we used that utilized the Smith homomorphism for bordism groups. In order to compute the value of the anomaly we study the Atiyah-Hirzebruch spectral sequence and find a way to trivialize each layer of the anomaly. I will also discuss the generating manifold for the corresponding bordism group, and how it arises from the Smith homomorphism. The knowledge of this manifold allows one to compute the partition function by using the data that characterizes the Chern-Simons theory and the symmetry actions; this gives a robust check of our answer for the anomaly. This is based on work in progress with Arun Debray and Weicheng Ye.

Zoom link: TBA


Chern-Simons, time-reversal, bordism, & Smith homomorphisms.
W/ Arun Debray
Weicheng Ye.
In 2019 [Delmastro, Gomis]
symmetries of abelian CS. Gave a classification of unitary/antiunitary sym.
$\mathcal{L} = \frac{k}{4\pi} a \wedge da$
$gS \neq S$. but $\langle gO_1, \ldots, gO_n \rangle = \langle O_1 \ldots O_n \rangle$
Quantum symmetries are of this form.
Act in general as permutations of anyons that preserve cat. data.

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$\mathcal{L} = \frac{k}{4\pi} a \wedge da$, $\alpha \in A$ $\quad W_\alpha = \exp(i\alpha \oint a)$, $\theta_\alpha = e^{2\pi i h_\alpha}$.

α in $\{0, \dots k\}$ if k is even
$\{0, \dots 2k\}$ if k is odd.

$h_\alpha = \frac{\alpha^2}{2k}$.

${}^gS \neq S$. but $\langle g\mathcal{O}_1, \dots, g\mathcal{O}_n \rangle = \langle \mathcal{O}_1, \dots \mathcal{O}_n \rangle$.

Quantum symmetries are of this form.

Act in general as permutations of anyons that preserve cat. data.

For this talk I focus on T-reversal symmetry.

$$T: \begin{cases} a_0(x) \to a_0(-x^0) \\ a_i(x^0) \to -a_i(-x^0) \end{cases}$$

is not a sym of $\mathcal{L}$.

but you can make this into an

$\mathrm{Aut}(\mathcal{A})$.

$$\Theta_{(T\alpha)} = \Theta^*_{(T\alpha)} \iff h_\alpha \equiv -h_\alpha \bmod 1.$$

/antiunitary sym.

$e^{2\pi i h_\alpha}$.

$h_\alpha = \frac{\alpha^2}{2k}$

data.

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$$\Theta_{(T\alpha)} = \Theta^*_{(T\alpha)} \iff h_\alpha \equiv -h_\alpha \mod 1.$$

Classification: if you have $U(1)_k$.

for k s.t. $kp - q^2 = 1$. for some integers p and q.

Then it admits time-reversal.

$k=5$.

$\theta^*_{(T\alpha)} \Leftrightarrow h_\alpha \equiv -h_\alpha \bmod 1$

$T^2 = C$ is the algebra of time reversal for those $U(1)_k$ that solve the eq.

if k is odd, then $U(1)_k$ is spin theory. (Spin MTC).

favorite $(-1)^F$.

$T^2 = (-1)^F C$.

Jaume Asks: For $U(1)_k$ with $T^2 = (-1)^F C$ sym. What is the value of the anomaly?

$k=5 \to \nu = 1/5$. FQHE.

1: Give the classification of the anomaly for time reversal.

- Symmetry structure associated to $T^2 = (-1)^F C$.
  $\hookrightarrow$ a structure on manifolds.
- Computing some relevant bordism group $\ni$ anomaly
  $\hookrightarrow$ associated to invertible TFT.

2: Evaluate the "number" for the anomaly
In particular for $U(1)_5$ $\longrightarrow$ AHSS.

3: Writing down the partition function on the manifold that gen the bordism group.

1: Give the classification of the anomaly for time reversal.

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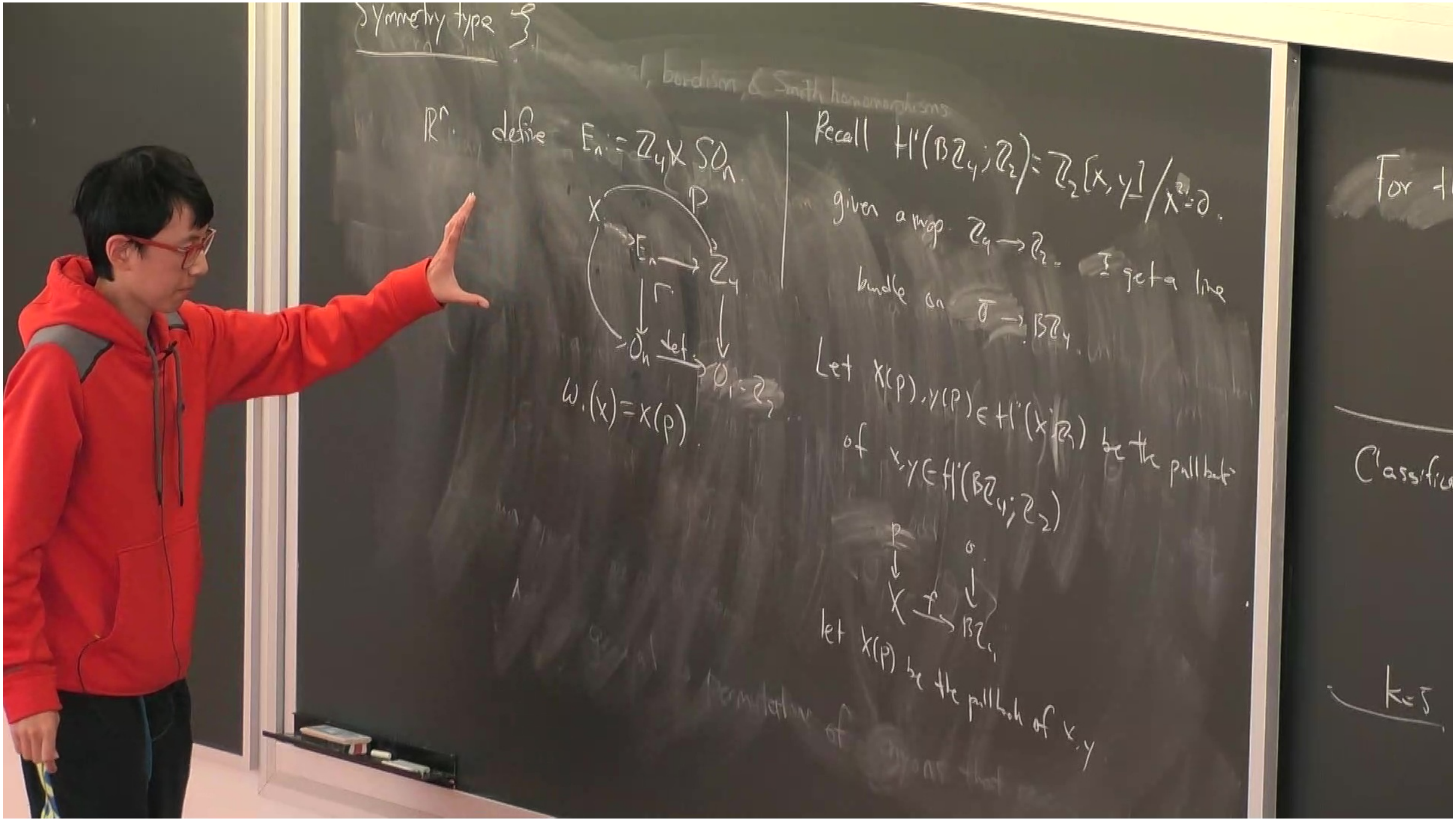
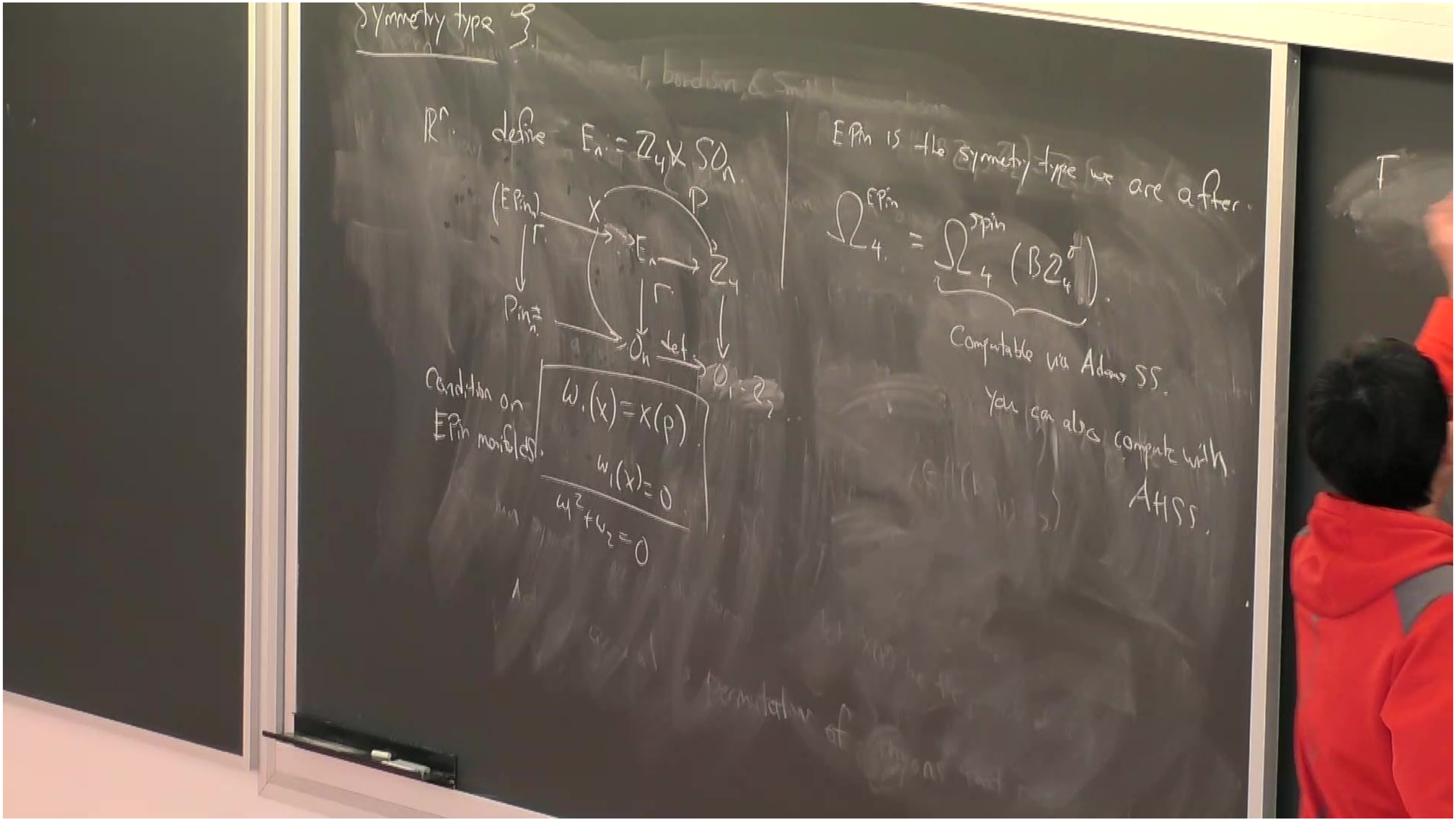
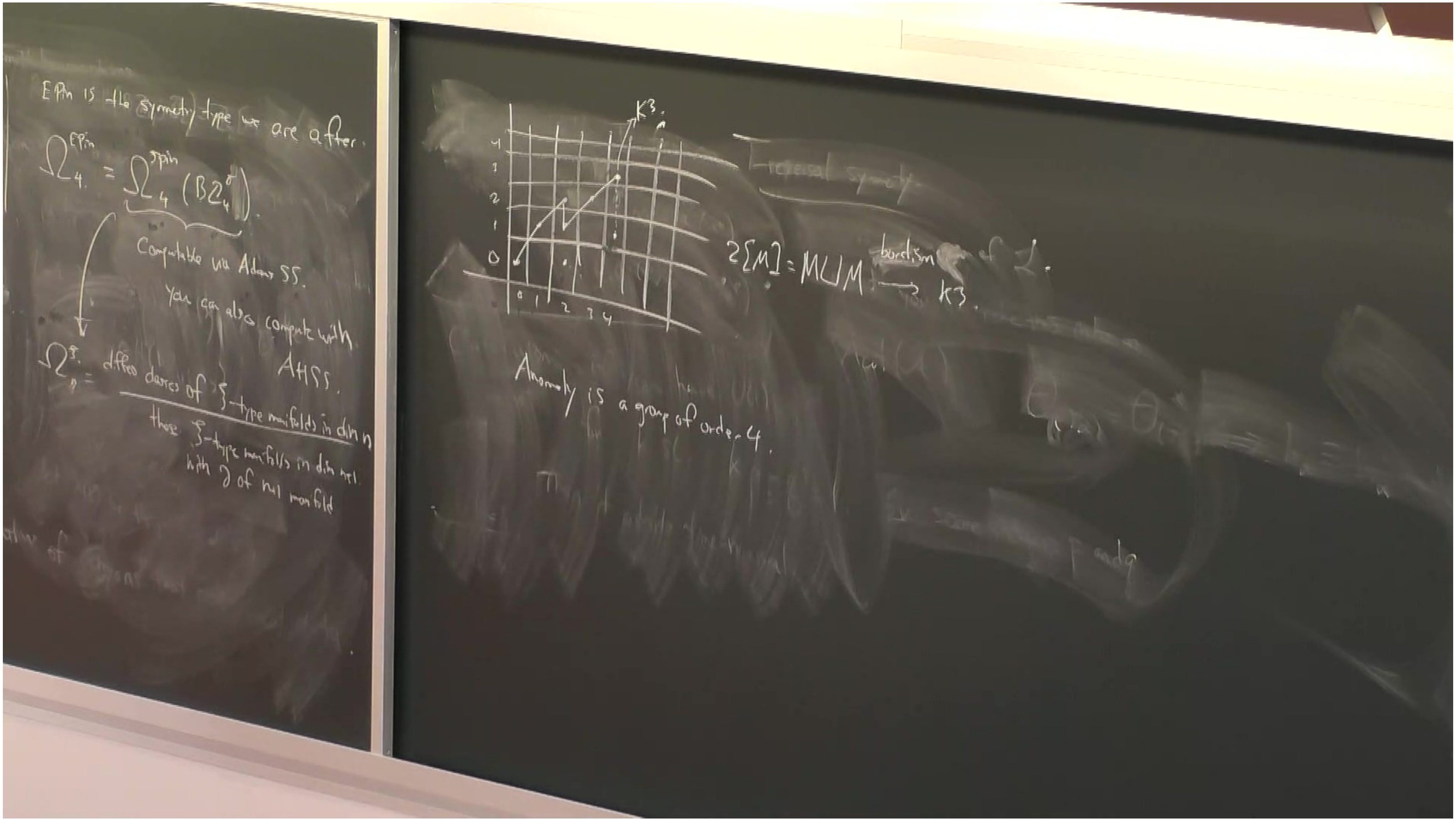

Symmetry type ξ
define E_n := Z_4 × SO_n
Recall H*(BZ_4; Z_2) = Z_2[x, y]/x^2 = 0.
given a map Z_4 → Z_2 ∃ get a line bundle on σ → BZ_4
Let x(P), y(P) ∈ H*(X; Z_2) be the pullback of x, y ∈ H*(BZ_4; Z_2)
let x(P) be the pullback of x, y
w_1(x) = x(P)
det


Symmetry type ξ
$\mathbb{R}^n$. define $E_n := \mathbb{Z}_4 \times SO_n$
EPin is the symmetry type we are after.
$\Omega_4^{EPin} = \Omega_4^{Spin}(B\mathbb{Z}_4^\sigma)$.
Computable via Adams SS.
You can also compute with AHSS.
Condition on EPin manifolds.
$\omega_1(x) = x(\rho)$
$\omega_1(x) = 0$
$\omega_1^2 + \omega_2 = 0$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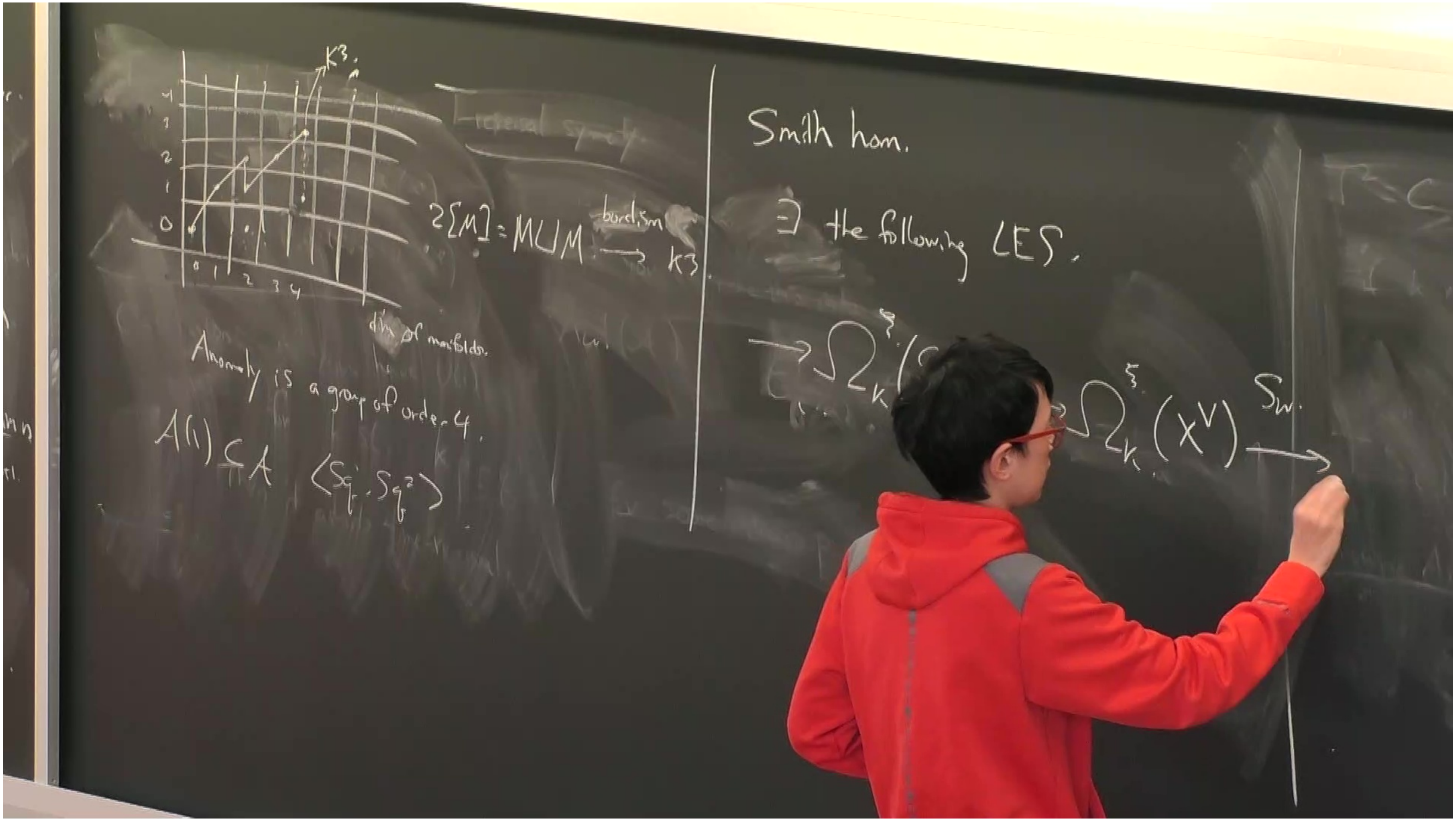
Computable via Adams SS.
You can also compute with AHSS.
Ω_n^ξ = diffeo classes of ξ-type manifolds in dim n / those ξ-type manifolds in dim n+1 with ∂ of n+1 manifold
K3
$2[M] = M \sqcup M \xrightarrow{\text{bordism}} K3$.
Anomaly is a group of order 4.


K3
dim of manifolds.
2[M] = M⊔M
bordism
→ K3
Anomaly is a group of order 4.
A(1) ⊆ A ⟨Sq¹, Sq²⟩
Smith hom.
∃ the following LES.
→ Ω_k^ξ (
Ω_k^ξ (X^V)
Sw.

Smith hom.

$\exists$ the following LES. X with two bundles V and W.

$$\cdots \to \Omega_k^{\xi}\left(S(W)^{p^*V}\right) \to \Omega_k^{\xi}\left(X^V\right) \xrightarrow{S_W} \Omega^{\xi}_{k-\mathrm{rank}(W)}\left(X^{V\oplus W}\right) \to \cdots$$

$S_W \to$ Smith hom.

$$\begin{array}{ccc} S\mathbb{Z}_2 = S(\sigma) & \longrightarrow & E\mathbb{Z}_2 = * \\ \downarrow & & \downarrow \\ B\mathbb{Z}_4 & \longrightarrow & B\mathbb{Z}_2 \end{array}$$

let $X = B\mathbb{Z}_4$. let $W = \sigma$. let V be trivial

$$\to \Omega_5^{Spin}(B\mathbb{Z}_2) \to \Omega_5^{Spin}(B\mathbb{Z}_4) \to \Omega_4^{Spin}(B\mathbb{Z}_4^{\sigma}) \to \Omega_4^{Spin}(B\mathbb{Z}_2)$$

$[M] = M \sqcup M \xrightarrow{\text{bordism}} k3$

$\exists$ the following LES. X with two bundles V and W.

$$\to \Omega_k^{\xi}(S(W)^{p^*V}) \to \Omega_k^{\xi}(X^V) \xrightarrow{S_W} \Omega^{\xi}_{k-\mathrm{rank}(W)}(X^{V\oplus W}) \to \cdots$$

Smith hom.

let $X = BZ_4$ let $W = \sigma$. let V be trivial

$$\to \Omega_5^{\mathrm{Spin}}(BZ_2) \to \Omega_5^{\mathrm{Spin}}(BZ_4) \to \Omega_4^{\mathrm{Spin}}(BZ_4^{\sigma}) \to \Omega_4^{\mathrm{Spin}}(BZ_2) \ldots$$

$$0 \to Z_4 \xrightarrow{\alpha} ? \xrightarrow{\rho} Z \ldots$$

$BZ_2 = S(\sigma) \to EZ_2 = *$

$\downarrow \qquad \downarrow$

$BZ_4 \to BZ_2$

α

Ω

$B\mathbb{Z}_4 \longrightarrow B\mathbb{Z}_2$

S_W → Smith hom.

$\longrightarrow \Omega^{\xi}_{k-\mathrm{rank}(W)}(X^{V\oplus W}) \longrightarrow \cdots$

e trivial

$\Omega^{Spin}_4(B\mathbb{Z}_4^{a}) \longrightarrow \Omega^{Spin}_4(B\mathbb{Z}_2)$

$\xrightarrow{\rho} \mathbb{Z}$

α is an isomorphism so we get

$$\Omega^{Spin}_4(B\mathbb{Z}_4^{\omega}) = \mathbb{Z}_4$$

1:

2:

3:

...e the eq.

...sm so we get

$\mathbb{Z}_H$

$$\Omega^4_{spin}(B\mathbb{Z}_4^\sigma).$$

$\mathbb{Z}^\sigma$	0	$\mathbb{Z}_2$				
$\mathbb{Z}_2^\sigma$	U	xU	$\cancel{yU}$	xyU	y^2U	
$\mathbb{Z}_2^\sigma$	U	xU	yU	xyU	$\cancel{y^2U}$	
0	–	–	–	–	–	
$\mathbb{Z}^\sigma$	0	$\mathbb{Z}_2$	0	$\mathbb{Z}_2$	0	$\cancel{\mathbb{Z}_2}$
	0	1	2	3	4	5

d_2 is Sq^2.

d_3 is $\beta\circ Sq^2$.

$$0\to\mathbb{Z}^\sigma\to\mathbb{Z}^\sigma\to\mathbb{Z}_2^\sigma\to 0$$

$$H^4(B\mathbb{Z}_4,\mathbb{Z}^\sigma)=0.$$

We find there is H^1 and H^3.

... is measuring an obstruction to:

$\rho : \mathbb{Z}_4 \to \mathrm{Aut}(\mathcal{A})$, $\quad \tilde{\rho} : \mathbb{Z}_4 \to \mathrm{Aut}(\tilde{\mathcal{A}}_\nu)$ $\quad \nu = 0, \dots, 15$

$\tilde{\mathcal{A}}_\nu$ is a minimal nondeg. extension of $\mathcal{A}$

in particular it is bosonic.

It was shown that for $U(1)_k$

Such an $\tilde{\mathcal{A}}$ is $U(1)_{4k}$

this has central charge 1.

[JFR] show that an extension always exists.

You can apply a procedure called Zesting to $\tilde{\mathcal{A}}$.

... obstruction to ...

$\rho: \mathbb{Z}_4 \to \mathrm{Aut}(\mathcal{A})$, $\tilde{\rho}: \mathbb{Z}_4 \to \mathrm{Aut}(\tilde{\mathcal{A}}_\nu)$ $\quad \nu = 0, \dots, 15$

It was shown that for $U(1)_k$

such an $\tilde{A}$ is $U(1)_{4k}$

$\hat{A}_\nu$ is a minimal modular extension of A.

in particular it is bosonic.

[JFR] show that an extension always exists.

this has central charge 1

You can apply a procedure called zesting to $\hat{A}$. A way of modifying fusion rules and can change the c by $1/2$.

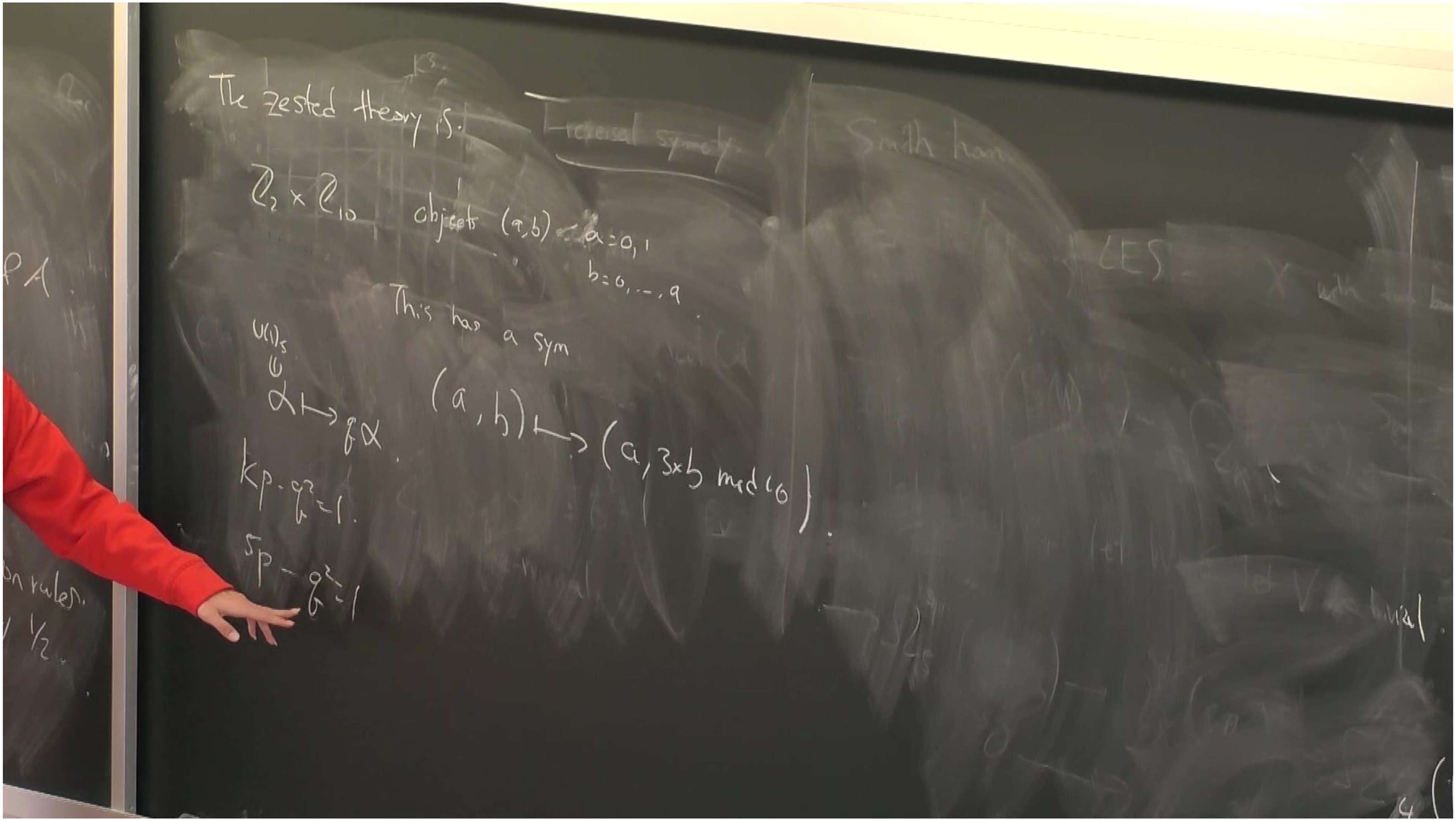
The zested theory is:

$\mathbb{Z}_2 \times \mathbb{Z}_{10}$ objects (a,b) $a=0,1$

$b=0,\dots,9$.

This has a sym

$$(a,b) \longmapsto (a, 3\times b \bmod 10).$$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b=0,...,9
This has a sym
(a,b) ↦ (a, 3×b mod 10)
Smith hom
LES

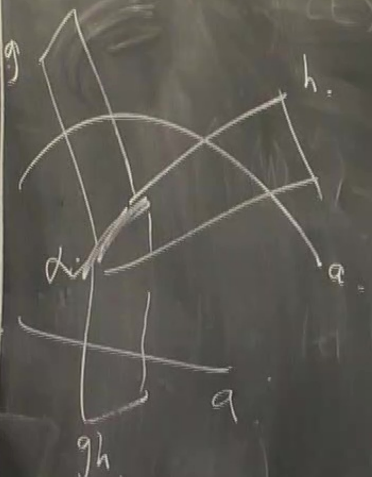
Internal symm.

(a,b) $a=0,1$
$b=0,\dots,9$

a sym

$h) \mapsto (a, 3\times b \bmod 10)$

The H^3 is an obstruction to extending sym frac. to $\mathbb{Z}_2\times\mathbb{Z}_{10}$.



is there a natural iso between $\rho_g\circ\rho_h \Rightarrow \rho_{gh}$.

So there is a natl. iso. $\eta_a(g,h)\in U(1)$.

b) $a = 0, 1$

$b = 0, \dots, 9$

sym

$\mapsto (a, 3 \times b \bmod 10)$

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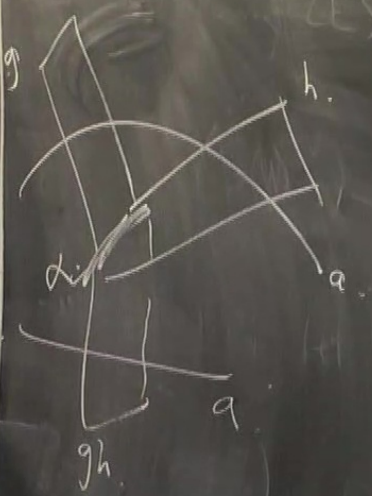

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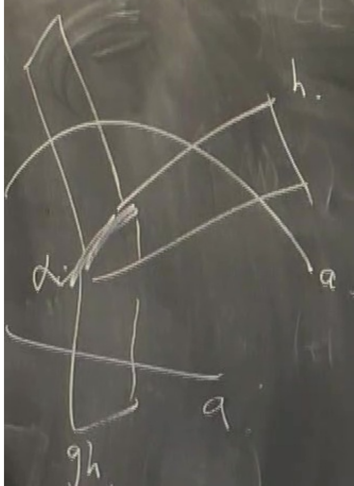
${}^h({}^g a) \Rightarrow {}^{gh}a$.

$$\exp\left(\frac{[g]+[h]-[g+h]}{4} \times a\right)$$

The $\cancel{H^3}$ is an obstruction to extending sym frac. to $\mathbb{Z}_2 \times \mathbb{Z}_{10}$.

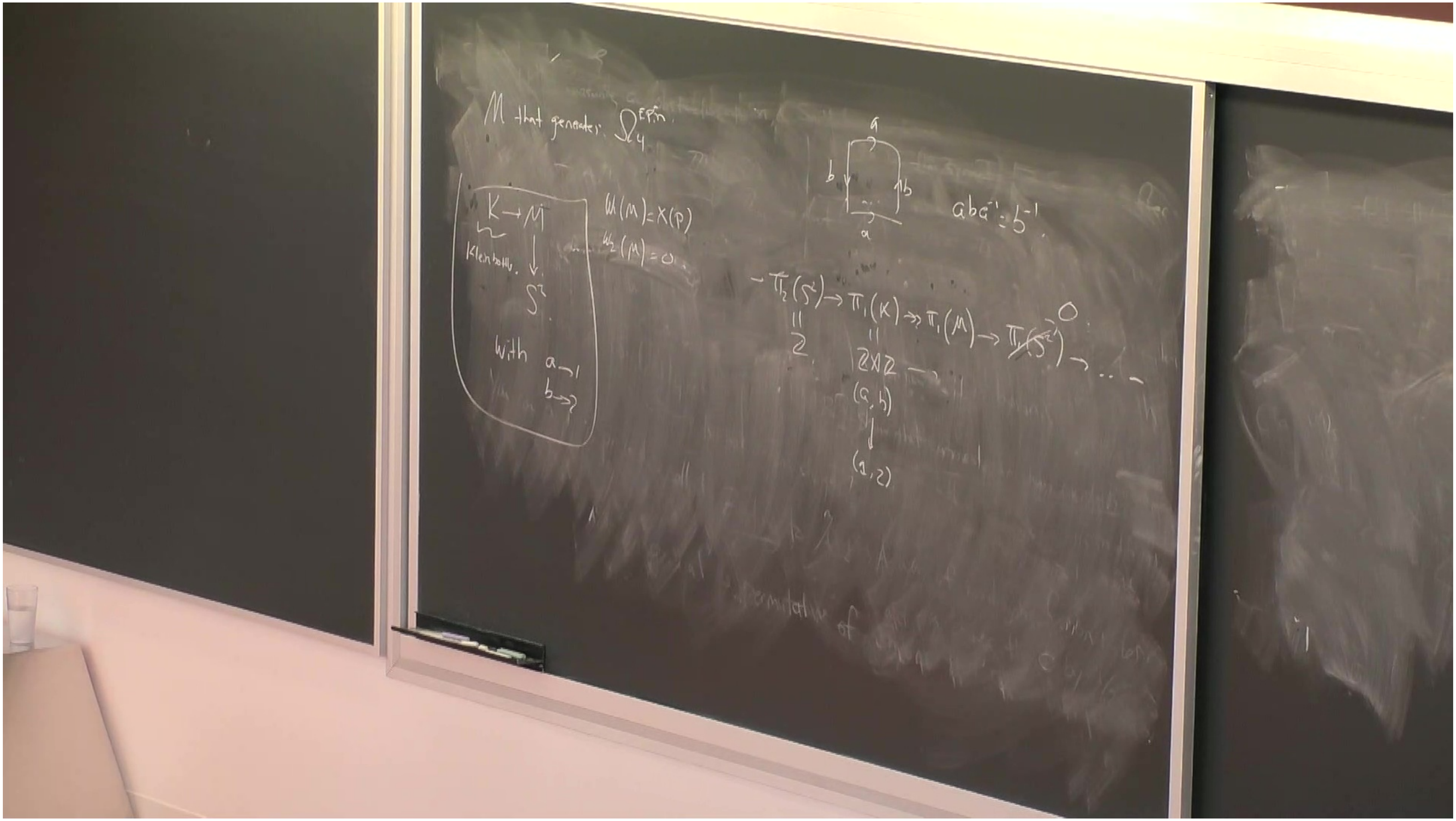


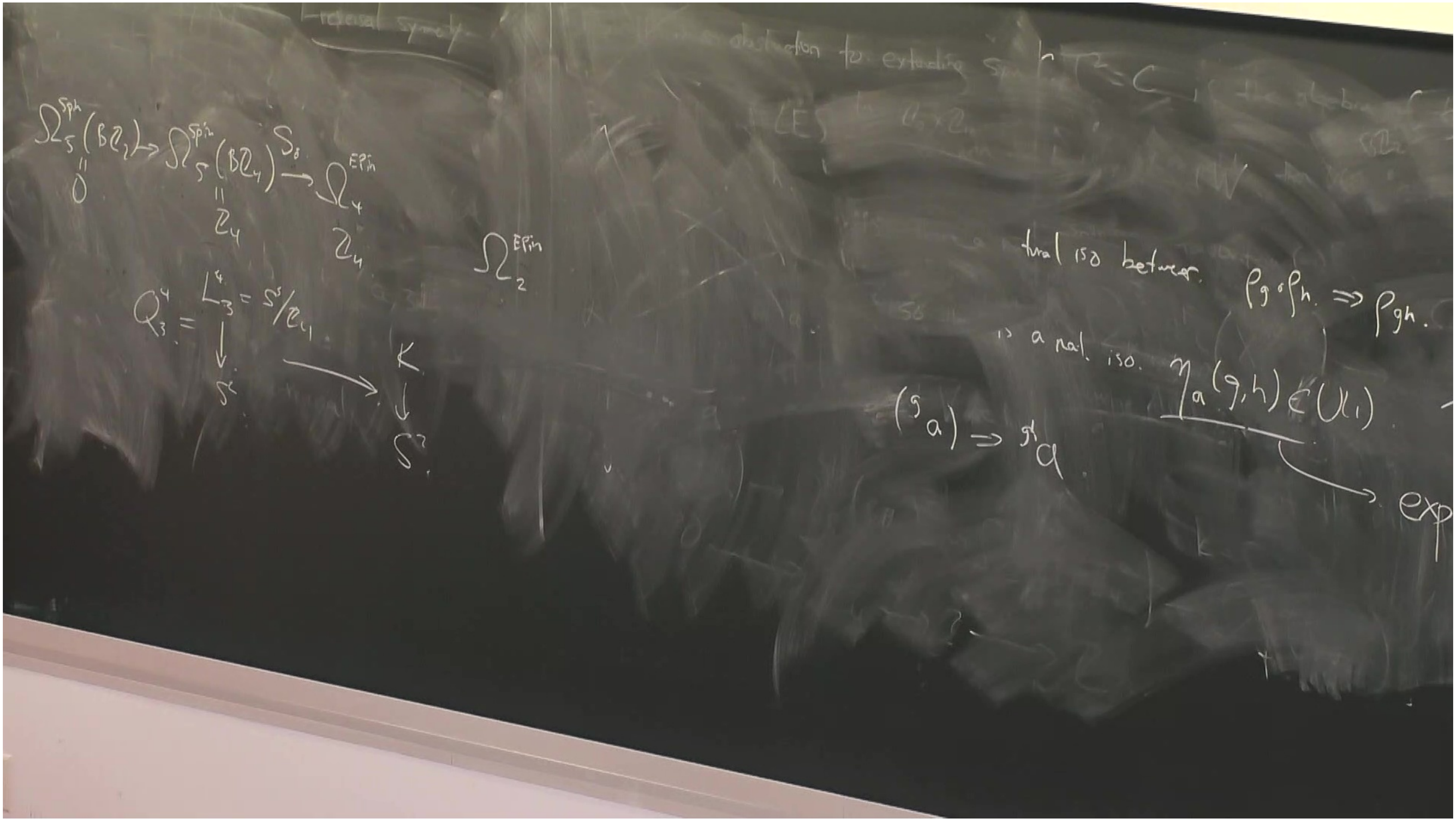
is there a natural iso between $\rho_g \circ \rho_h \Rightarrow \rho_{gh}$.

So there is a nat. iso. $\underline{\eta_a(g,h)} \in U(1)$ $\rightarrow$ You can extend sym. frac to $\mathbb{Z}_2 \times \mathbb{Z}_{10}$

${}^h({}^g a) \Rightarrow {}^{gh}a$.

$\rightarrow \exp\left(\frac{[g]+[h]-[g+h]}{4} * a\right)$.


M that generates Ω
K → M
Klein bottle
S
with a → 1
b → ?
aba⁻¹ = b⁻¹
(a, b)
(1, 2)


Ω_5^Spin(BZ_2) → Ω_5^Spin(BZ_4) → Ω_4^EPin
0
Z_4
Z_4
Ω_2^EPin
Q_3^4 = L_3^4 = S^3/Z_4
K
S^2
tural iso between ρ_g ∘ ρ_h ⇒ ρ_gh
is a nat. iso. η_a(g,h) ∈ U(1)
(^g a) ⇒ ^gh a
exp