Title: Monarch Migration of Carrollian Particles on the Black Hole Horizon

Speakers: David Kubiznak

Series: Strong Gravity

Date: April 13, 2023 - 1:00 PM

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Abstract: After discussing the basics of Carrollian physics, we shall revisit the motion of massless particles with anyonic spin in the black hole horizons. As recently shown, such particles can move within the horizon of the black hole due to the coupling of charges associated with a 2-parametric central extension of the 2-dimensional Carroll group to the magnetic field around the black hole -- the so called "anyonic spin-Hall effect". We shall study several examples of such motions. Of these, the most interesting is the motion in misaligned (asymptotically uniform) magnetic field around Kerr, which results in a time dependent motion of Carrollian particles that is reminiscent of "monarch migration".

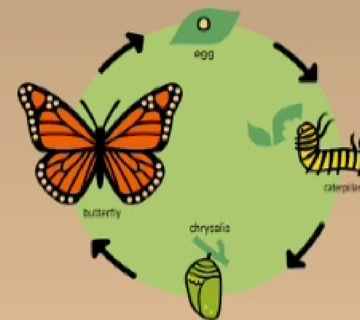
Zoom link: https://pitp.zoom.us/j/97094479078?pwd=SHN6NEU5aE93OXVQYk9aWTVOdzRRUT09

Monarch Migration of Carrollian Particles on the Black Hole Horizon

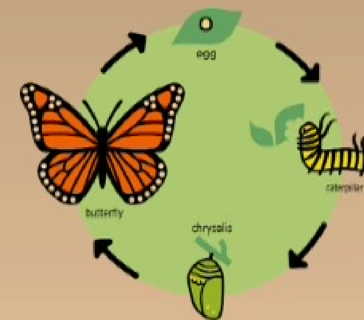
David Kubizňák
(Charles University)

Strong Gravity Seminar
Perimeter Institute, Waterloo, Canada
April 13, 2023

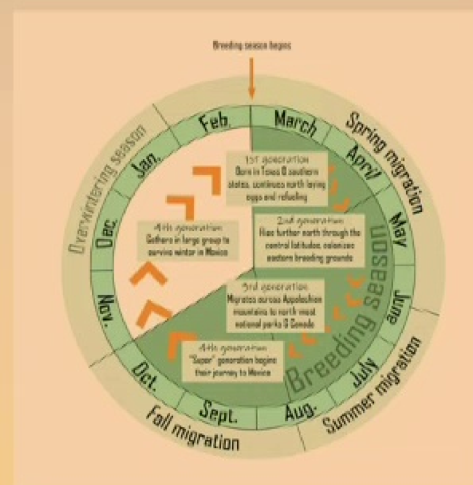
Monarch butterfly



Monarch butterfly



Monarch migration



- Spans generations & thousands of miles

Monarch migration

https://www.nps.gov/subjects/pollinators/migratingmonarchs.htm

Plan of the talk

I. **Lorentz, Galilei, and Carroll**
   a) From Lorentz to Carroll
   b) Carrollian particles and fields

II. **Black holes and test magnetic fields**

III. **Motion of Carrollian particles on BH horizon**
   a) Anyonic spin-Hall effect
   b) Carrollian structure on the horizon
   c) Migrating Carrollian particles ala monarchs

IV. **Summary**

Based on:

- Paper 1: F. Gray, DK, T. Rick Perche, J. Redondo-Yuste, *Carrollian motion in magnetized black hole horizons*, Phys. Rev. D 107 (2023) 064009; Arxiv:2211.13695.
- Paper 2: J. Bicak, DK, T. Rick Perche, *Monarch migration of Carrollian particles on the black hole horizon*, Arxiv:2302.11639.

and

- Paper 3: L. Marsot, P-M Zhang, P. Horvathy, *Anyonic spin-Hall effect on the black hole horizon*, Phys. Rev. D 106 (2022) L121503; Arxiv:2207.06302.
- Paper 4: L. Marsot, P-M Zhang, M. Chernodub, P. Horvathy, *Hall effects in Carroll dynamics*, Arxiv:2212.02360.

I) Lorentz, Galilei, and Carroll

Levy-Leblond (60s):

J.-M. Levy-Leblond, Une nouvelle limite non-relativiste du groupe de poincare, Annales de l'institut Henri Poincare (A) Physique theorique 3 (1965), no. 1 1-12.

From Lorentz to Carroll

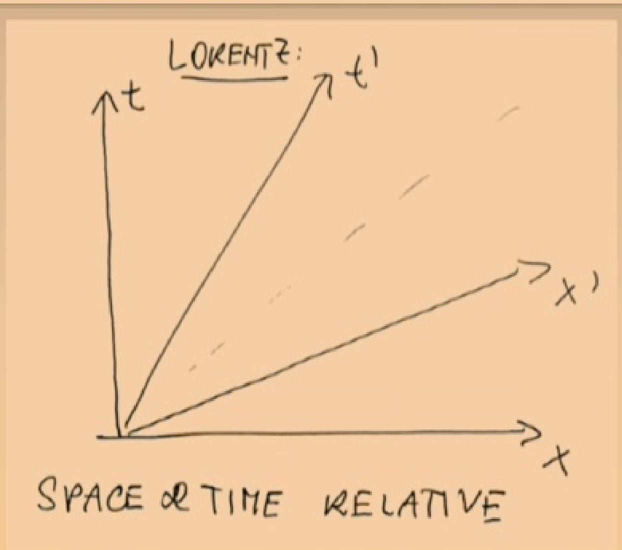
J. De Boer, J. Hartong, NA. Obers, W. Sybesma, S. Vandoren, *Carroll Symmetry, Dark Enery and Inflation*, Front. In Phys. 10 (2022) 810405; Arxiv:2110.02319.

Relativity: Lorentz boost (finite speed of light)

$$ct' = \gamma_\beta(ct - \vec{\beta}\cdot\vec{x}) \; , \qquad \vec{x}_{\parallel}' = \gamma_\beta(\vec{x}_{\parallel} - \vec{\beta}ct)$$

$$\vec{x}_{\perp}' = \vec{x}_{\perp}$$

$$\gamma_\beta = \frac{1}{\sqrt{1-\vec{\beta}^2}}$$



- Relative space and time results in Lorentz contraction, time dilatation,…

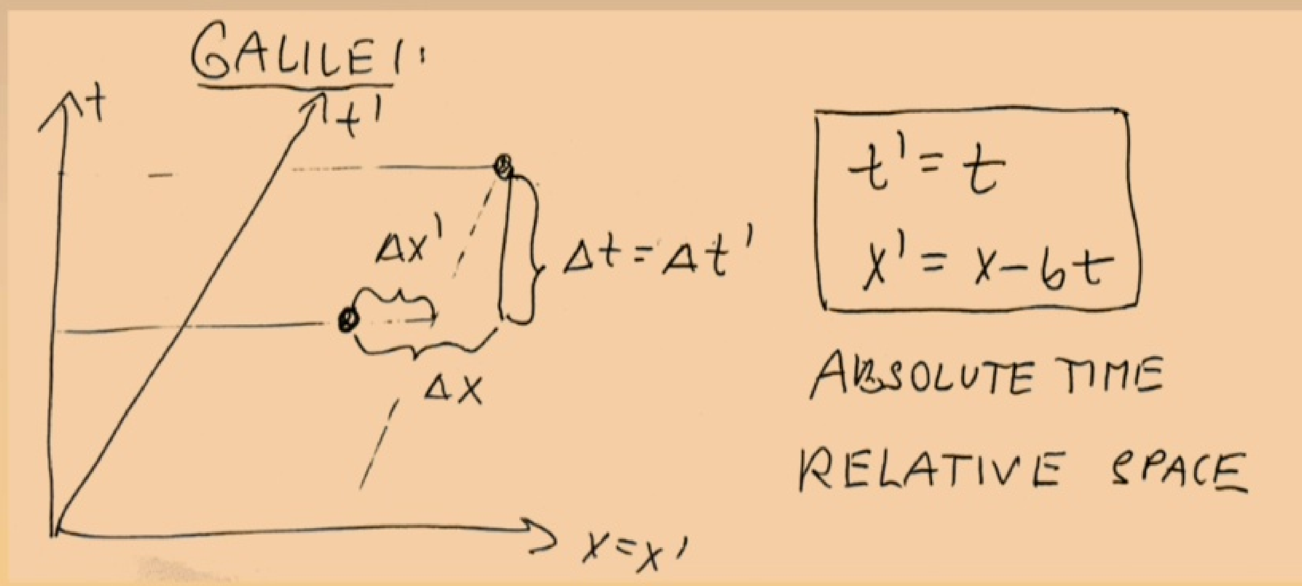
From Lorentz to Carroll

Lorentz boost

$$ct' = \gamma_\beta(ct - \vec{\beta}\cdot\vec{x})\ , \qquad \vec{x}'_{||} = \gamma_\beta(\vec{x}_{||} - \vec{\beta}ct)$$

$$\gamma_\beta = \frac{1}{\sqrt{1-\vec{\beta}^2}}$$

Galilei: Non-relativistic limit $c \to \infty$ $\quad \vec{\beta} = c^{-1}\vec{b}$



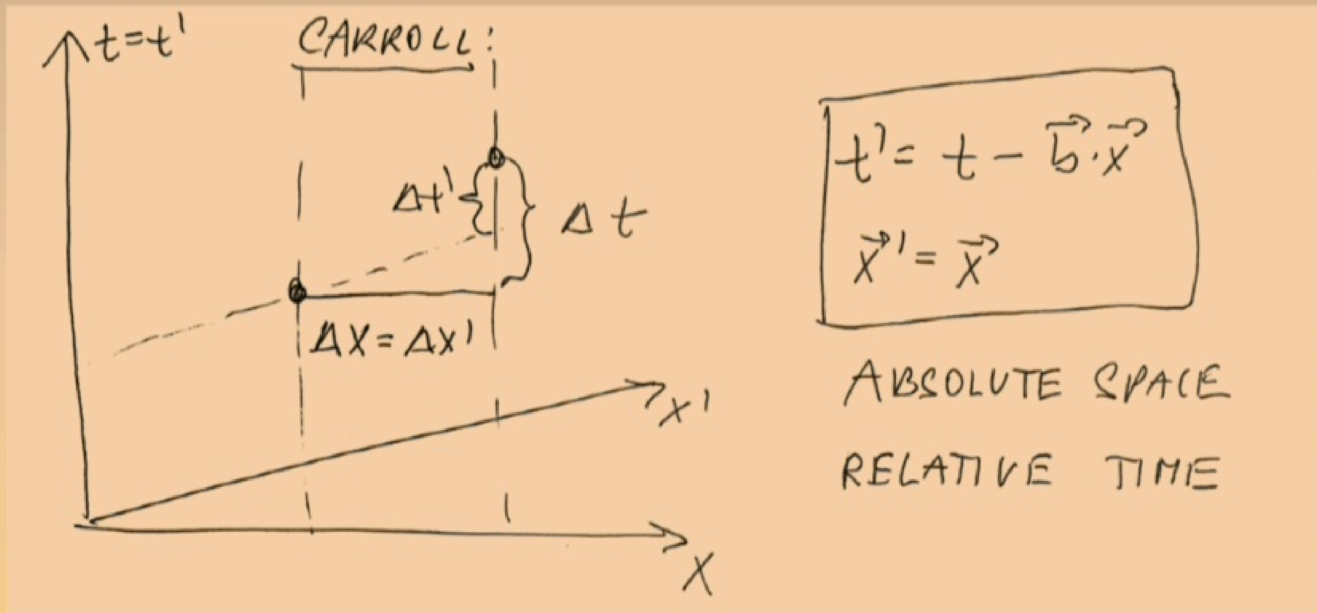
From Lorentz to Carroll

Lorentz boost

$$ct' = \gamma_\beta(ct - \vec{\beta}\cdot\vec{x}) \;, \qquad \vec{x}_{||}' = \gamma_\beta(\vec{x}_{||} - \vec{\beta}ct)$$

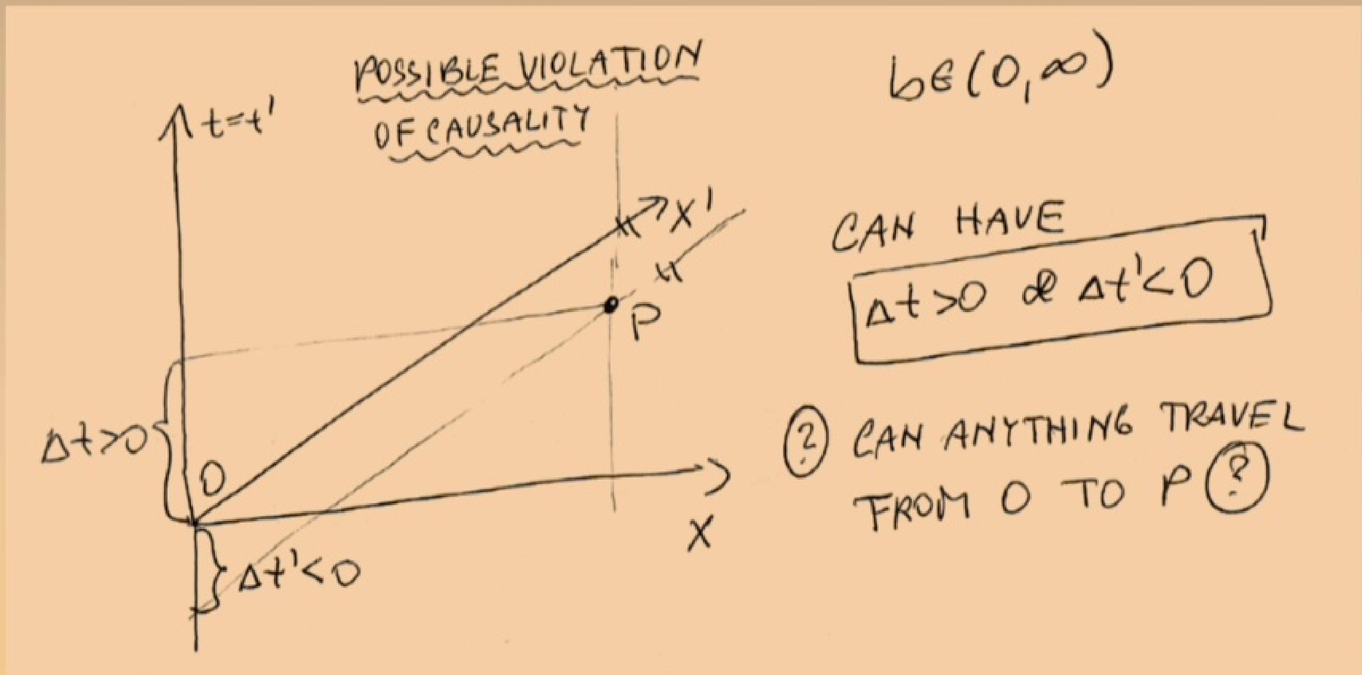
$$\gamma_\beta = \frac{1}{\sqrt{1-\vec{\beta}^2}}$$

Carroll limit: $c \longrightarrow 0$ $\quad \vec{\beta} \equiv c\vec{b} \;, \qquad c \to \epsilon c \;, \qquad \epsilon \to 0$



From Lorentz to Carroll

$$t' = t - \vec{b} \cdot \vec{x} \ , \qquad \vec{x}' = \vec{x}$$



From Lorentz to Carroll

Minkowksi metric: $$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}\vec{x}^{\,2} \to \mathrm{d}\vec{x}^{\,2}$$

Carroll structure:

$$(\xi = \partial_t, dl^2 = d\vec{x}^2, t)$$

CARROLLIAN METRIC $q_{\mu\nu}$: sig $(0,+,+,\dots)$

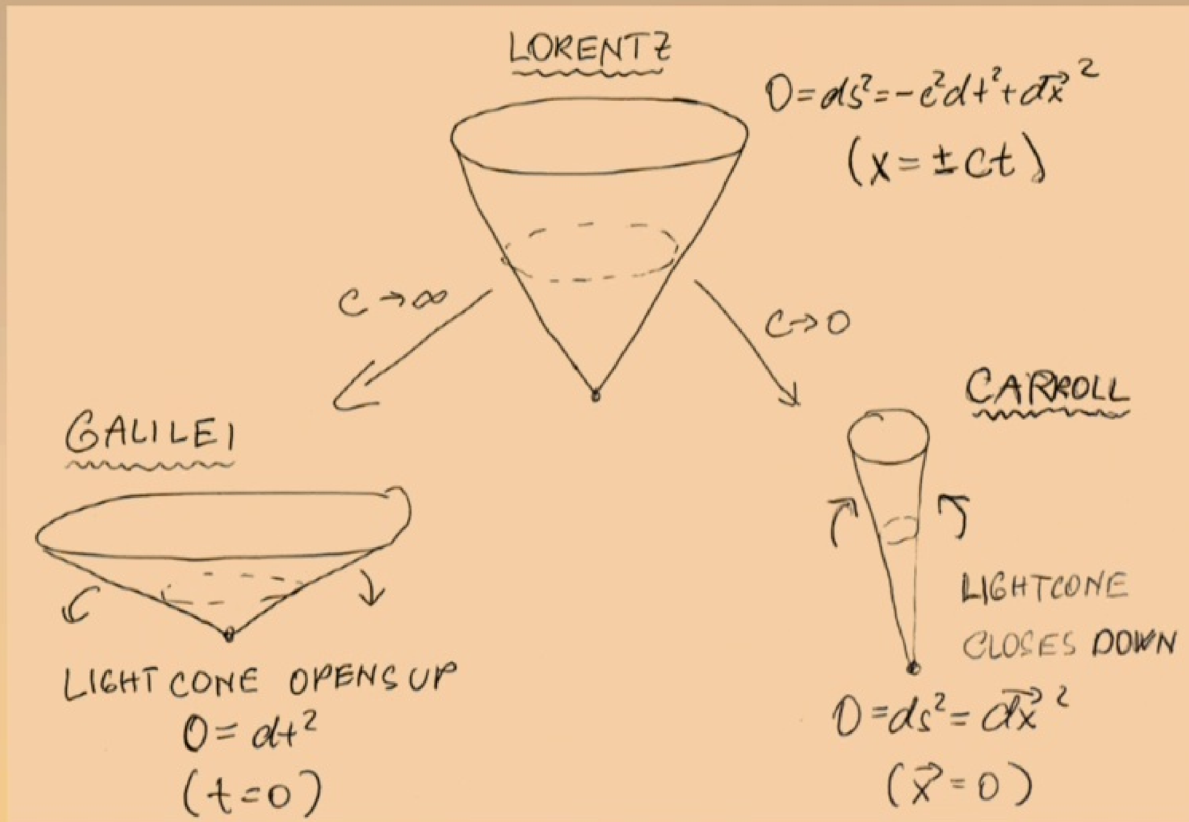
CARROLLIAN VECTOR ξ^μ: $q_{\mu\nu}\xi^\nu = 0$

CARROLL SYMMETRIES

$$\mathcal{L}_\eta q_{\mu\nu} = 0 = \mathcal{L}_\eta \xi^\mu$$

From Lorentz to Carroll

Minkowksi metric: $$ds^2 = -c^2 dt^2 + d\vec{x}^{\,2} \rightarrow d\vec{x}^{\,2}$$



Carrollian particles

- Lorentz boost of momentum:

$$\vec{p}_{\|}{}' = \gamma_\beta \left(\vec{p}_{\|} - \vec{\beta}\,\frac{E}{c} \right) \qquad \frac{E'}{c} = \gamma_\beta \left(\frac{E}{c} - \vec{\beta}\cdot\vec{p} \right)$$

$$\vec{p}\,' = \vec{p} - \vec{b}\,E \ , \qquad E' = E$$

Energy is preserved!

- Lorentz boost of velocity:

$$\vec{u}'_{\|}(t') = \frac{\vec{u}_{\|} - \vec{\beta}c}{\left(1 - \frac{\vec{\beta}\cdot\vec{u}}{c}\right)} \ , \qquad \vec{u}'_{\perp}(t') = \frac{\vec{u}_{\perp}(t)}{\gamma_\beta\left(1 - \frac{\vec{\beta}\cdot\vec{u}}{c}\right)}$$

$$\vec{u}\,'(t') = \frac{\vec{u}(t)}{(1 - \vec{b}\cdot\vec{u})}$$

Discontinuous: zero and non-trivial velocity**!**

Carrollian particles

- Lorentz boost of momentum:

$$\vec{p}_{\|}{}' = \gamma_\beta \left(\vec{p}_{\|} - \vec{\beta}\,\frac{E}{c} \right) \qquad \frac{E'}{c} = \gamma_\beta \left(\frac{E}{c} - \vec{\beta}\cdot\vec{p} \right)$$

$$\vec{p}\,' = \vec{p} - \vec{b}\,E\ , \qquad E' = E$$

Energy is preserved!

- Lorentz boost of velocity:

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$$\vec{u}\,'(t') = \frac{\vec{u}(t)}{(1 - \vec{b}\cdot\vec{u})}$$

Discontinuous: zero and non-trivial velocity**!**

2 options
- Particles **cannot move**
- **Tachyonic** particles $v > c \to 0$.

Carrollian field theory

- Relativistic scalar under Boost:

$$\delta\phi = ct\vec{\beta}\cdot\vec{\partial}\phi + \frac{1}{c}\vec{\beta}\cdot\vec{x}\,\partial_t\phi$$

- Expand around c=0:

$$\phi = c^{\Delta}\left(\phi_0 + c^2\phi_1 + c^4\phi_2 + \cdots\right) = c^{\Delta}\sum_{n=0}^{\infty}\phi_n c^{2n} \qquad \vec{\beta} = c\vec{b},$$

$$\Rightarrow \quad \delta\phi_0 = \vec{b}\cdot\vec{x}\,\partial_t\phi_0\ , \qquad \delta\phi_n = \vec{b}\cdot\vec{x}\,\partial_t\phi_n + t\,\vec{b}\cdot\vec{\partial}\phi_{n-1}$$

- Thus, only ϕ_0 is Carroll scalar ($C_i = x^i\partial_t$ is the boost generator)

Carrollian field theory

- Expanding standard scalar Lagrangian

$$\mathcal{L} = c^{2\Delta-2}\left(\mathcal{L}_0 + c^2\mathcal{L}_1 + O(c^4)\right)$$

$$\mathcal{L}_0 = \frac{1}{2}\dot{\phi}_0^2 - V(\phi_0)\ , \qquad \mathcal{L}_1 = \dot{\phi}_0\dot{\phi}_1 - \frac{1}{2}\partial_i\phi_0\partial_i\phi_0$$

- Concentrating on first **"electric" theory**:

$$T^t{}_t = -\left(\frac{1}{2}\dot{\phi}_0^2 + V\right), \quad T^i{}_t = 0\,,$$
$$T^t{}_i = -\dot{\phi}_0\partial_i\phi_0\,, \quad T^i{}_j = \left(\frac{1}{2}\dot{\phi}_0^2 - V\right)\delta^i{}_j$$

Obeys no flux condition: $T^i{}_t = 0$

Carrollian field theory

- Second term yields

$$\mathcal{L}_1 = \dot{\phi}_0\dot{\phi}_1 - \frac{1}{2}\partial_i\phi_0\partial_i\phi_0$$

$$T^t{}_t = -\dot{\phi}_0\dot{\phi}_1 - \frac{1}{2}(\partial_i\phi_0)^2 \ , \qquad T^i{}_t = \dot{\phi}_0\partial_i\phi_0 \ ,$$

$$T^t{}_i = -\dot{\phi}_1\partial_i\phi_0 - \dot{\phi}_0\partial_i\phi_1 \ , \qquad T^i{}_j = \delta^i{}_j\mathcal{L}_1 + \partial^i\phi_0\partial_j\phi_0$$

- Enforcing $\dot{\phi}_0$ to vanish then yields "**magnetic Carrollian field theory**"

- Similarly, one can formulate electric and magnetic Maxwell theory, and gravity…

  D. Hansem, N.A. Obers, G. Oling, B.T. Sogaard, *Carroll expansion of general relativity*, SciPost Phys. 13 (2022) 3, 055; Arxiv:2112.12684.

Applications to cosmology

J. De Boer, J. Hartong, NA. Obers, W. Sybesma, S. Vandoren, *Carroll Symmetry, Dark Enery and Inflation*, Front. In Phys. 10 (2022) 810405; Arxiv:2110.02319.

- Inflationary model $P = w\mathcal{E}$

$$w = \frac{\frac{1}{2c^2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2c^2}\dot{\phi}^2 + V(\phi)} = -1 + \frac{\pi_\phi^2}{V}c^2 + \mathcal{O}(c^4)$$

- Far outside Hubble sphere $R_H = cH^{-1}$

  recessional velocity is $v \gg c$

In **super-Hubble scales** Carrollian symmetry arises.

Applications to black hole horizons

L. Donnay and C. Marteau, Carrollian physics at the black hole horizon, CQG 36 (2019) 165002; Arxiv:1903.09654.

- Consistent treatment of null surfaces and explicit symmetries of BH horizons
- Approaching horizon (null surface)

$$ds^2 = -\rho\alpha^2 \mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}\rho - 2\rho U_A \mathrm{d}v\mathrm{d}x^A + \\ + (\Omega_{AB} - \rho\lambda_{AB})\mathrm{d}x^A\mathrm{d}x^B + \mathcal{O}(\rho^2),$$

ρ affine distance to the horizon $\rho \to 0 \Leftrightarrow c \to 0$

Carroll structure:

fibre bundle p: $\mathcal{H} \to S \quad S = \mathcal{H}|_{v=v_0}$

S is a Riemannian manifold with topology S^2

Vertical vector field ∂_v

Applications to Bjorken flows

A. Bagchi, K.S. Kolekar, A. Shukla, *Carrollian Origins of Bjorken Flow*, ArxIv:2302.03053.

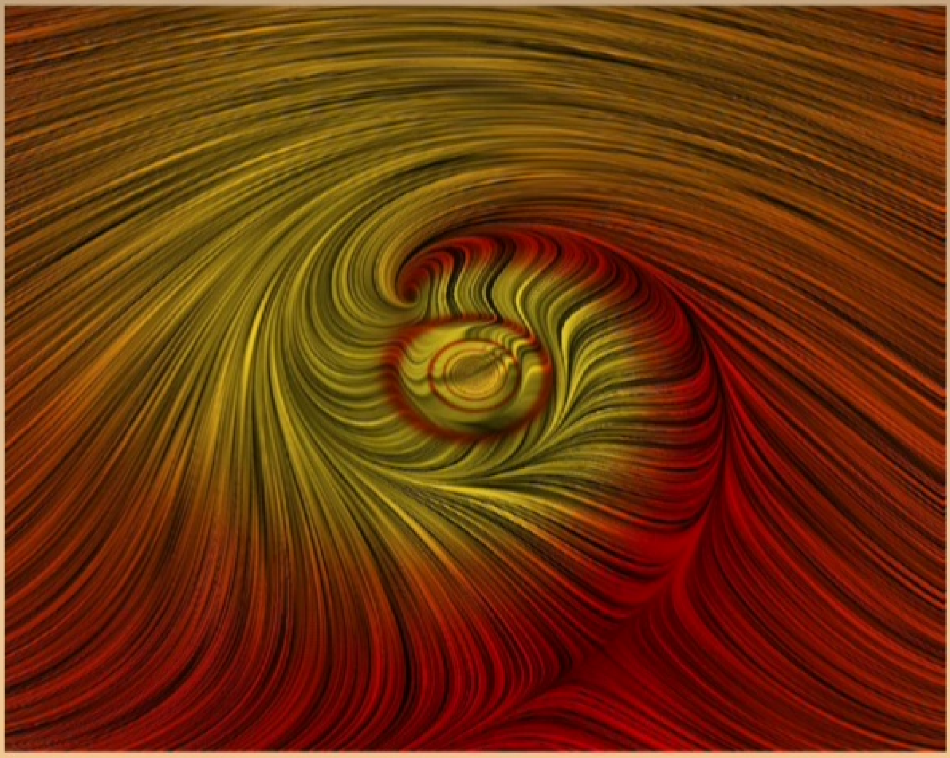
Carrollian hydro describes fluids moving at the speed of light.

- Applications to QGP in Early Universe or LHC

Other applications

- Carrollian algebra isomorphic to BMS group
- Carrollian CFTs should play role in flat space holography
- Fluid/gravity correspondence in Minkowski

II) Black holes and magnetic fields



V. Karas, O. Kopacek, D. Kunneriath, *Influence of frame-dragging on magnetic null points near rotating black hole*, CQG 29 (2012) 035010.

Magnetic fields around Kerr

$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\sin^2\theta}{\Sigma}\left(adt - (r^2+a^2)d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2\,,$$

$$\Delta = r^2 - 2Mr + a^2\,, \quad \Sigma = r^2 + a^2\cos^2\theta$$

Wald's/Papapetrou's trick

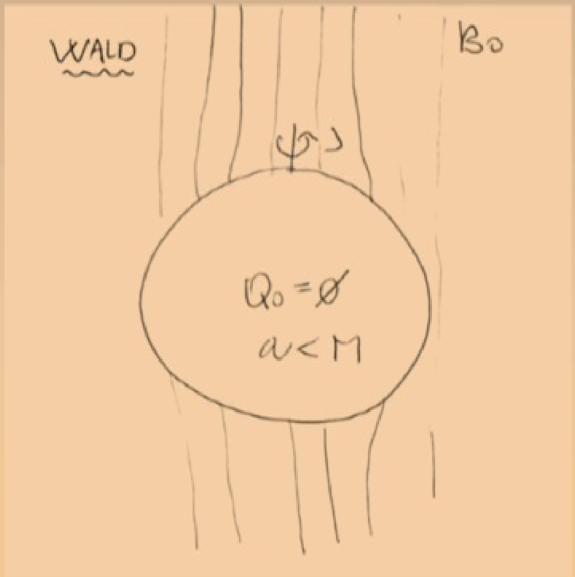
- Killing field: $\nabla_\mu \zeta_\nu + \nabla_\nu \zeta_\mu = 0$

$$\nabla_\mu \nabla_\nu \zeta_\rho = \zeta^\sigma R_{\sigma\mu\rho\nu}$$

$$F_{\mu\nu} \propto \nabla_\mu \zeta_\nu = \nabla_{[\mu}\zeta_{\nu]} \Rightarrow \nabla_\mu F^{\mu\nu} = R_\mu{}^\nu \zeta^\mu$$

Weakly charged Kerr in asymptotically (aligned) uniform magnetic field

$$A = \frac{1}{2} B_0 (\partial_t + 2a\partial_\phi) - \frac{Q_0}{2M}\partial_t$$

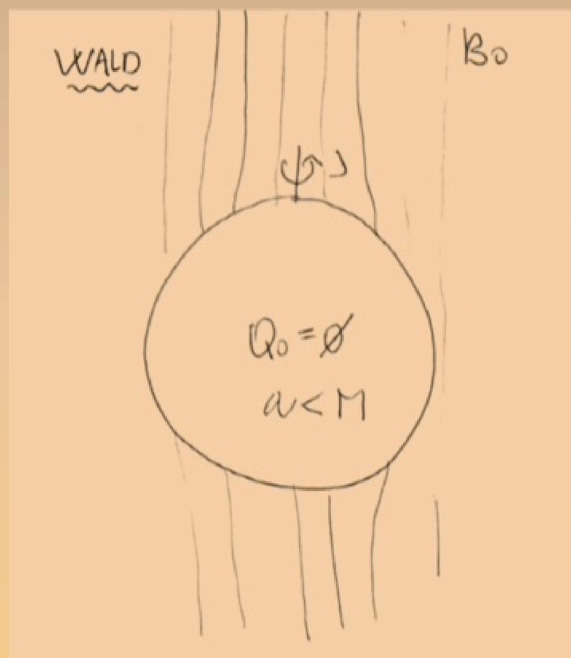


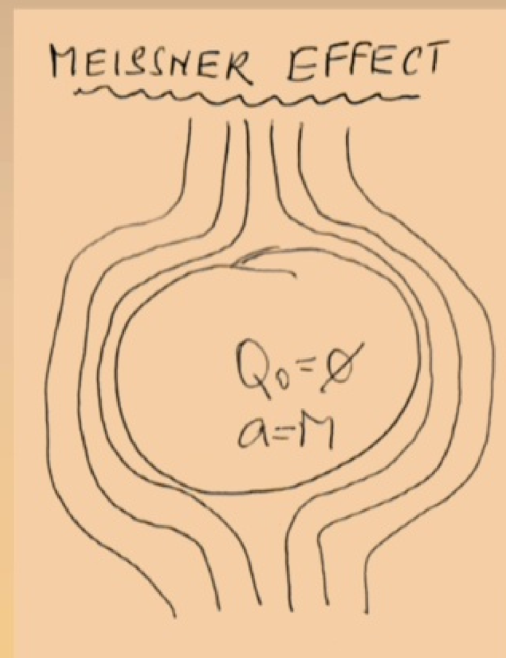
Field produced by aligned **accretion disc**

R.M. Wald, Black hole in a uniform magnetic field, PRD 10, 1680 (1974).

Weakly charged Kerr in asymptotically (aligned) uniform magnetic field

$$A = \frac{1}{2} B_0 (\partial_t + 2a\partial_\phi) - \frac{Q_0}{2M}\partial_t$$

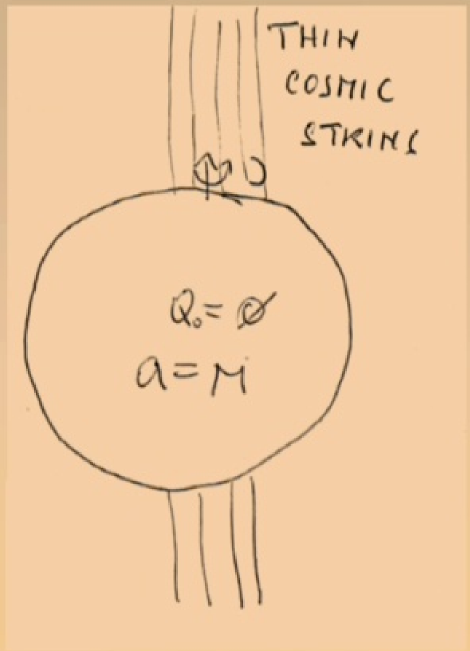


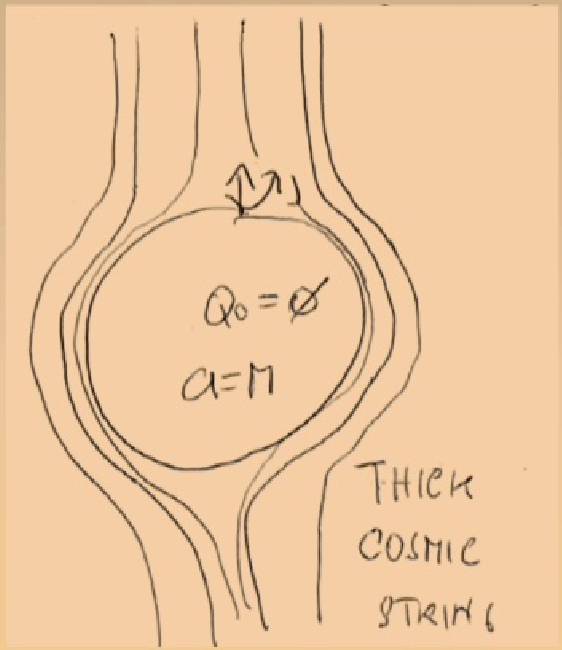


Kerr sporting cosmic string hair

$$S = \int d^4x \sqrt{-g} \left[D_\mu \Phi^\dagger D^\mu \Phi - \tfrac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} - \tfrac{1}{4}\lambda(\Phi^\dagger \Phi - \eta^2)^2 \right]$$

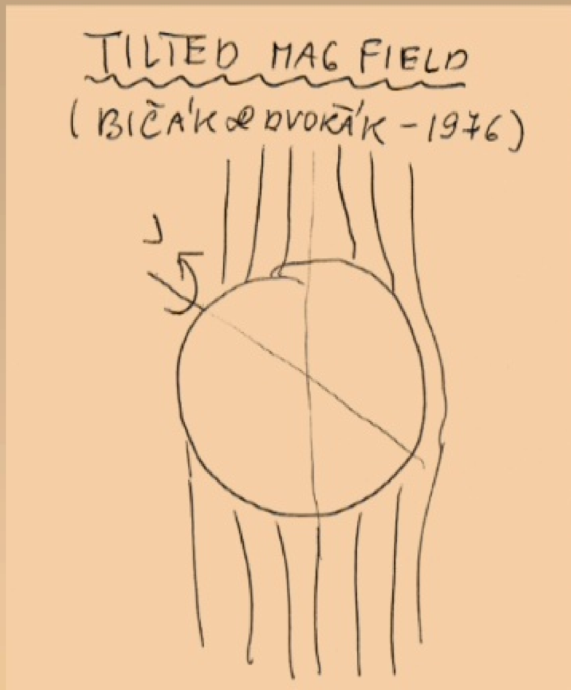
- Meissner effect more complicated





R. Gregory, DK, D. Wills, Rotating black hole hair, JHEP 06 (2013) 023.

Tilted magnetic field in Kerr

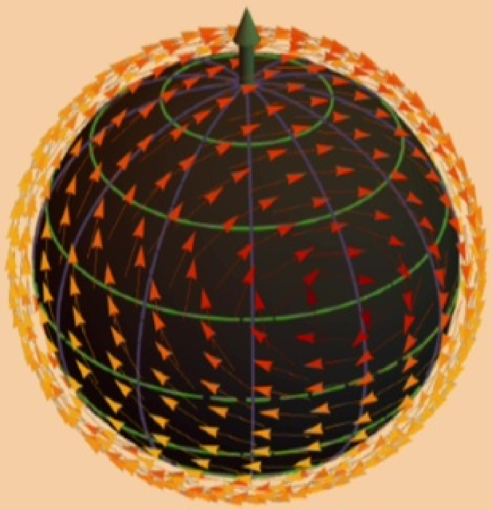


$$
\begin{aligned}
A_t &= \frac{B_1 a M}{\Sigma}\sin\theta\cos\theta\big(r\cos\psi - a\sin\psi\big) \\
&\quad + \frac{B_0 a M r}{\Sigma}(1+\cos^2\theta) - B_0 a\,, \\
A_r &= -B_1(r-M)\cos\theta\sin\theta\sin\psi\,, \\
A_\theta &= -B_1 a(r\sin^2\theta + M\cos^2\theta)\cos\psi \\
&\quad - B_1\Big(r^2\cos^2\theta + (a^2 - Mr)\cos 2\theta\Big)\sin\psi\,, \\
A_\phi &= B_0\sin^2\theta\Big(\frac{r^2+a^2}{2} - \frac{a^2 M r}{\Sigma}(1+\cos^2\theta)\Big) \\
&\quad - \frac{B_1\sin 2\theta}{2}\Big(\Delta\cos\psi + \frac{(r^2+a^2)M}{\Sigma}(r\cos\psi - a\sin\psi)\Big)\,.
\end{aligned}
$$

$$d\psi = d\phi + \frac{a}{\Delta}dr$$

- E-folding time of alignment

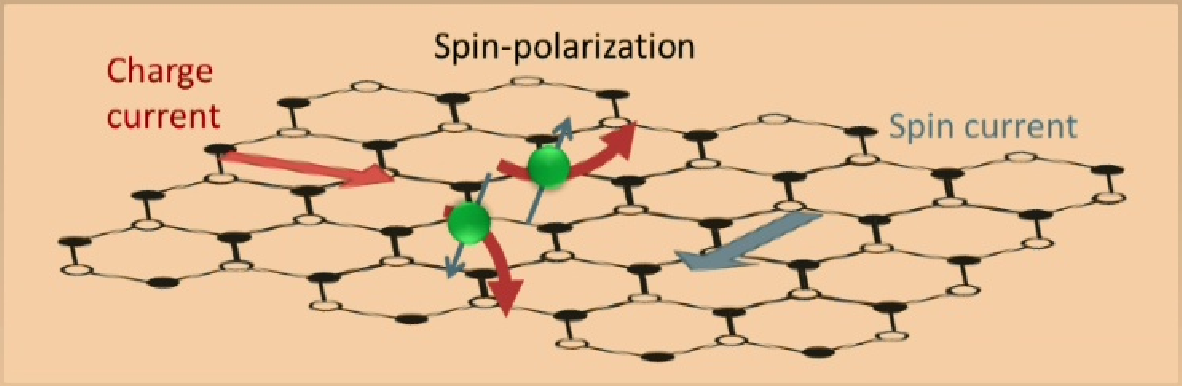
$$10^{10}\,\text{years}\,(10^4\,\text{Gauss}/B)^2\,(10^8 M_\odot/M)$$



III) Motion of Carrollian particles on black hole horizon

Anyonic spin-Hall effect

- **Spin Hall effect:** “x-y-z reaction to external agent due to coupling to some internal dof”



L. Andersson, M.A. Oancea, *Spin Hall effects in the sky*, Arxiv:2302.13634.

- **Anyonic spin Hall effect** (double central extension of Carroll group in 2+1) – paper 3 & 4

$$\frac{dx^A}{dv} = \frac{\mu\chi}{\kappa_{\mathrm{mag}}}\epsilon^{AB}\partial_B B$$

Carroll structure in Kerr BH horizon

$$ds^2 = -\frac{\Delta}{\Sigma}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\sin^2\theta}{\Sigma}\left(adt - (r^2+a^2)d\phi\right)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2\,, \quad (3)$$

$$\Delta = r^2 - 2Mr + a^2\,, \quad \Sigma = r^2 + a^2\cos^2\theta$$

- **Black hole horizon**

$$\Delta(r_+) = 0 \Rightarrow r_+ = M + \sqrt{M^2 - a^2}$$

$$\Omega_+ = \frac{a}{r_+^2 + a^2}$$ … relative rotation w.r.t infinity

Killing horizon of $\xi = \partial_t + \Omega_+ \partial_\phi$

Carroll structure in Kerr BH horizon

- Ingoing co-rotating with horizon coordinates

$$d\phi = d\varphi + \Omega_+ dv - \frac{a}{\Delta} dr .$$
$$dt = dv - \frac{r^2 + a^2}{\Delta} dr .$$

- Then $\xi = \partial_v$ $\quad n = dv - \frac{a(r_+^2 + a^2)}{\Sigma_+} \sin^2\theta d\varphi$ $\quad n_\mu \xi^\mu = 1$

- Projector $q^\mu{}_\nu = \delta^\mu_\nu - \xi^\mu n_\nu - n^\mu \xi_\nu$ $\quad x^A = \{\theta, \varphi\}$

$$q = q_{AB} dx^A dx^B = \Sigma_+ d\theta^2 + \frac{(r_+^2 + a^2)^2 \sin^2\theta}{\Sigma_+} d\varphi^2$$

Carroll structure in Kerr BH horizon

- Ingoing co-rotating with horizon coordinates

$$d\phi = d\varphi + \Omega_+ dv - \frac{a}{\Delta} dr\,.$$
$$dt = dv - \frac{r^2 + a^2}{\Delta} dr\,.$$

- Then $\xi = \partial_v$ $\quad n = dv - \frac{a(r_+^2 + a^2)}{\Sigma_+} \sin^2\theta d\varphi$ $\quad n_\mu \xi^\mu = 1$

- Projector $q^\mu{}_\nu = \delta^\mu_\nu - \xi^\mu n_\nu - n^\mu \xi_\nu$ $\quad x^A = \{\theta, \varphi\}$

$$q = q_{AB} dx^A dx^B = \Sigma_+ d\theta^2 + \frac{(r_+^2 + a^2)^2 \sin^2\theta}{\Sigma_+} d\varphi^2$$

Carroll structure:

$$(\xi = \partial_v, q, v \; - \; Carroll\ time)$$

Magnetic field on the horizon

- Carrollian motion $$\frac{dx^A}{dv} = \frac{\mu\chi}{\kappa_{\rm mag}}\epsilon^{AB}\partial_B B$$

  where $$\hat{F}_{AB} = q^\mu{}_A q^\nu{}_B F_{\mu\nu}$$ $$B = \frac{1}{2}\epsilon^{AB}\hat{F}_{AB}$$

  $$\epsilon^{\theta\varphi} = 1/\sqrt{\det q_{AB}}$$

- Wald solution: $$B = \frac{\left(B_0\left(r_+^4 - a^4\right) + 2aQ_0 r_+\right)\cos\theta}{\left(r_+^2 + a^2\cos^2\theta\right)^2}$$

- Bicak's solution: $$B = \frac{B_1\sin\theta}{2r_+\Sigma_+^2}\Big(2r_+(a^4\cos^2\theta - r_+^4)\cos\psi + a\big[(r_+^2 - a^2)a^2\cos^2\theta + r_+^2(3r_+^2 + a^2)\big]\sin\psi\Big) + \frac{B_0(a^4 - r_+^4)\cos\theta}{\Sigma_+^2}\,.$$

  $$\psi = \varphi + \Omega_+ v$$

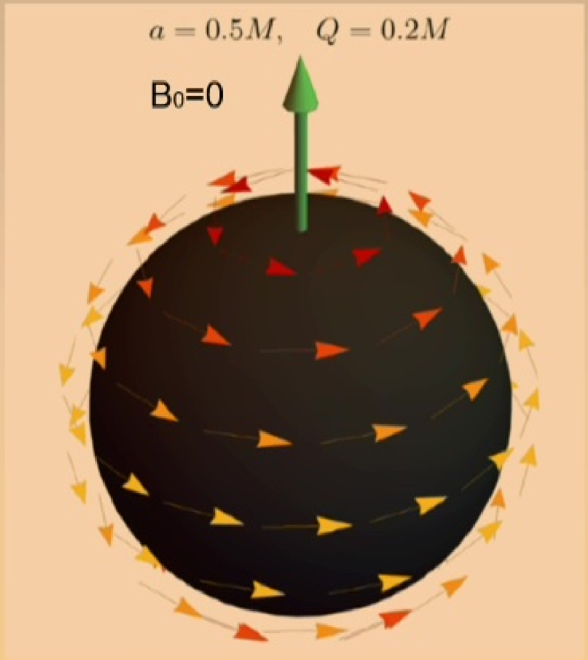
Carroll motion in Wald's field

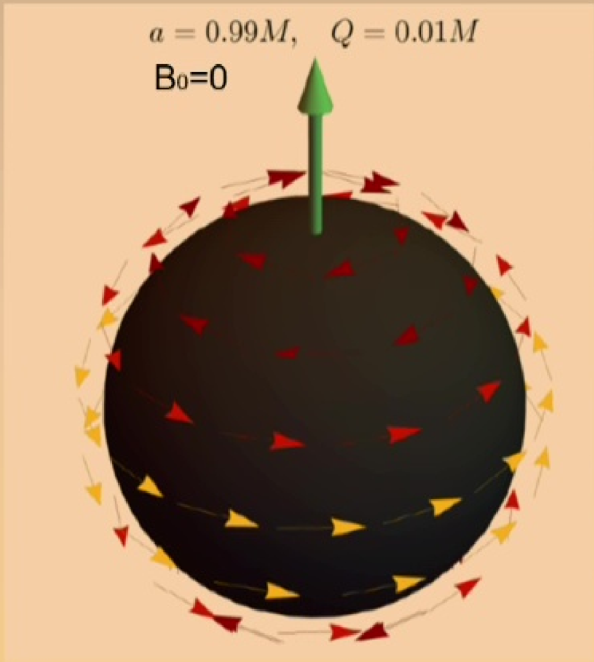
$$\frac{dx^A}{dv} = \frac{\mu\chi}{\kappa_{\text{mag}}}\epsilon^{AB}\partial_B B$$

$$B = \frac{\left(B_0\left(r_+^4 - a^4\right) + 2aQ_0 r_+\right)\cos\theta}{\left(r_+^2 + a^2\cos^2\theta\right)^2}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}v} = 0, \quad \frac{\mathrm{d}\varphi}{\mathrm{d}v} = \frac{2Q\mu\chi}{\kappa_{\text{mag}}}\frac{ar_+}{\Sigma_+^3\left(r_+^2 + a^2\right)}\left(r_+^2 - 3a^2\cos^2\theta\right)$$

- If $Q_0=0$, can see **Meissner effect**





Carroll motion in Kerr-Newmann

$$\frac{dx^A}{dv} = \frac{\mu\chi}{\kappa_{\text{mag}}}\epsilon^{AB}\partial_B B$$

$$B = \frac{(\quad 2aQ\ r_+)\cos\theta}{\left(r_+^2 + a^2\cos^2\theta\right)^2}$$

- Note the counter-rotation near poles for near extremal BHs:

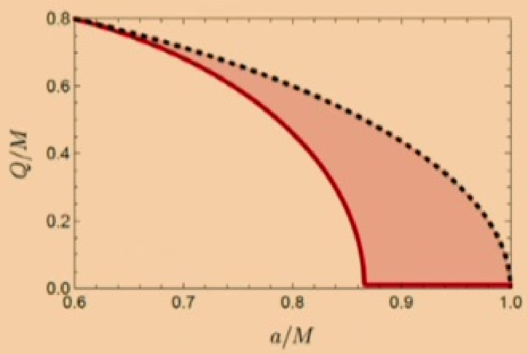
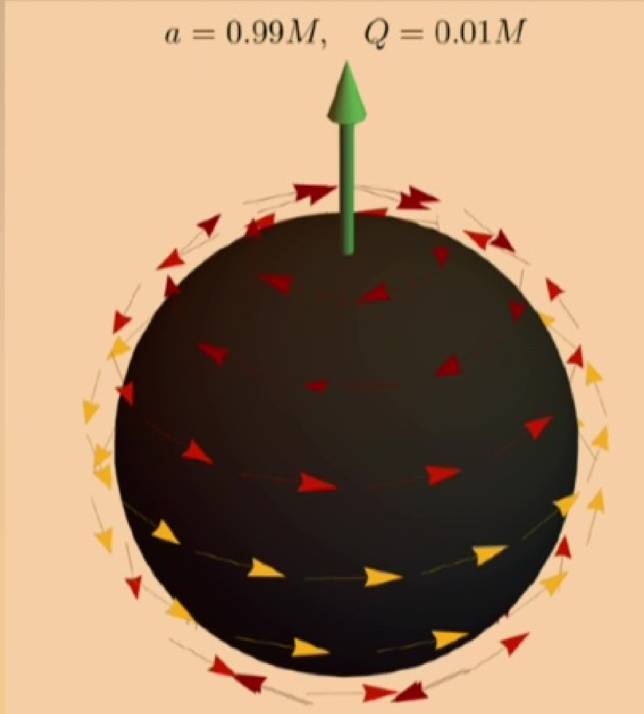


Figure 2. Region in the parameter space $(a/M, Q/M)$ where the exotic particles rotate in opposite directions in different regions of the horizon. The black dashed line represents an extremal black hole.

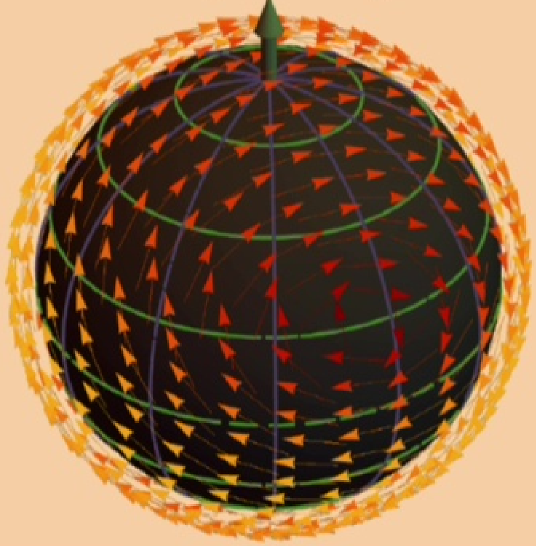


Monarch migration in Bicak's field

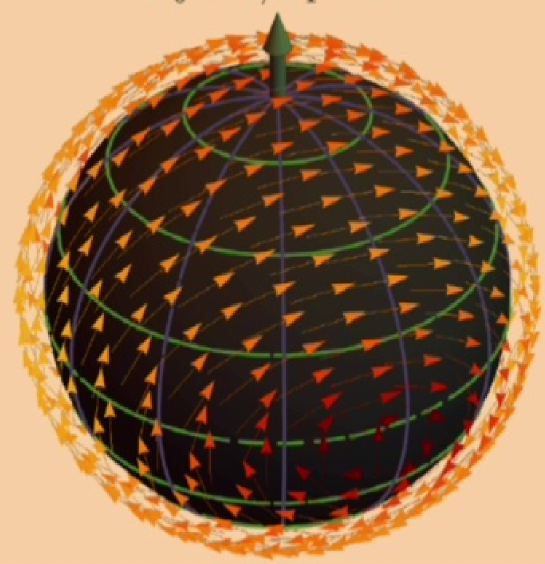
$$B = \frac{B_1 \sin\theta}{2r_+\Sigma_+^2}\Big(2r_+(a^4\cos^2\theta - r_+^4)\cos\psi$$
$$+a\big[(r_+^2 - a^2)a^2\cos^2\theta + r_+^2(3r_+^2 + a^2)\big]\sin\psi\Big)$$
$$+\frac{B_0(a^4 - r_+^4)\cos\theta}{\Sigma_+^2}\,.$$

$$\psi = \varphi + \Omega_+ v$$

$B_0 = \frac{4}{5}M, B_1 = \frac{3}{5}M$



$B_0 = 0, B_1 = M$



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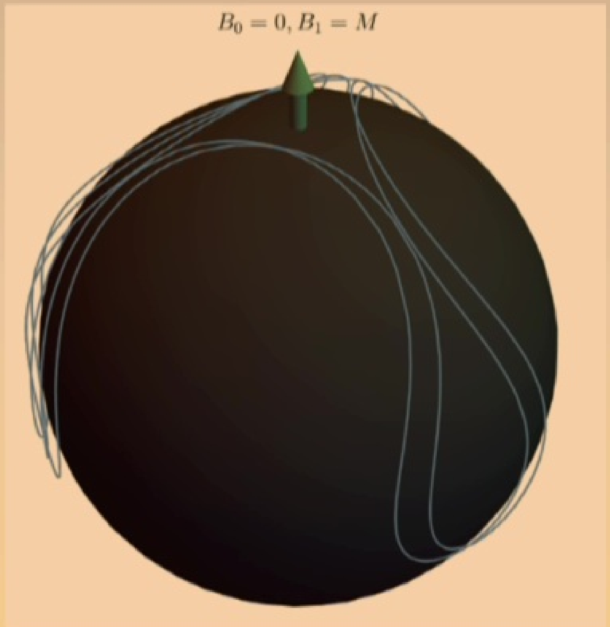

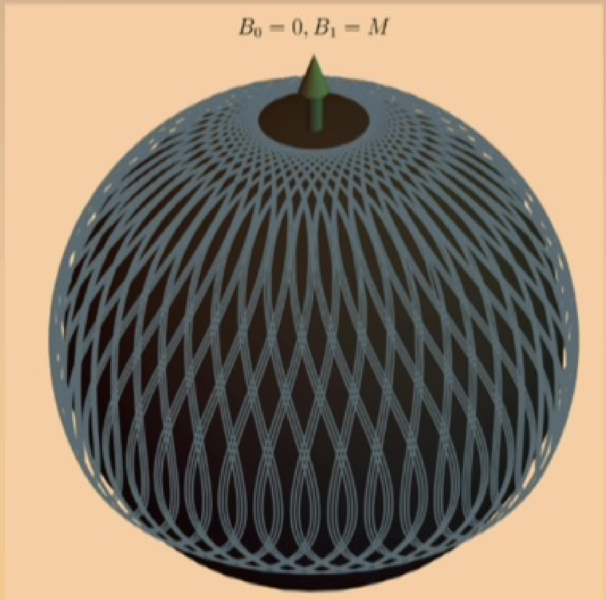

Summary

1) **Carroll physics** (a cousin of Galilie) is quite interesting – obtained by "ultra-relativistic limit"

$$c \to 0$$

In this limit **light cones** completely close down (ultralocal theory)

Carroll structure:

$$(\xi = \partial_v, q, v \; - \; Carroll \; time)$$

Applications to cosmology, ultrarelativistic hydro, flat space holography, black hole horizons, …

Summary

2) **Carrollian** (non-tachyonic) particles cannot move. However, in (2+1) dimensions, one can consider a *double central extension* of Carroll group – equip particle with anyonic spin and couple it to magnetic field:

$$\frac{dx^A}{dv} = \frac{\mu\chi}{\kappa_{\text{mag}}}\epsilon^{AB}\partial_B B$$

Anyonic spin Hall effect on black hole horizon:

… a la **monarch migration**

Summary

3) Do such particles really exist?

… and if so, are there any observable features that could be (at least in principle) detected in future experiments?