Title: Dynamical fixed points in holography: theory and cosmological applications

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Series: Quantum Fields and Strings

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Abstract: Dynamical fixed points are late-time attractors of interactive QFTs that differ from thermal equilibria by a constant entropy production rate. We use holographic framework to analyze such fixed points of strongly coupled gauge theories, driven by homogeneous and isotropic expansion of the background metric - equivalently, a late-time dynamics of a corresponding QFT in Friedmann-Lemaitre-Robertson-Walker Universe. As an application, we discuss gravitational reheating of quark-gluon plasma in the exit from the early-cosmology inflation.

Zoom link: https://pitp.zoom.us/j/96385064629?pwd=NTZrdUdtbW0rZkxPdFZ3ZnZRUzhjZz09

$\Longrightarrow$ Another holographic discovery:

Dynamical fixed points (DFPs)

Outline:

- Hydrodynamics and its breakdown in de Sitter
  - where DFP is hiding?
- A trivial DFP: thermal states of $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) in de Sitter
  - gauge theory perspective
  - holographic picture
  - de Sitter vacuum 'entanglement' entropy
- Nontrivial DFP
  - we focus on QFT_{2+1}
  - $\mathcal{N} = 2^*$ and cascading DFP — see the literature
- Applications: holographic gravitational reheating
  - harvesting the entangelment entropy of de Sitter DFPs
- Conclusions and future directions

$\Longrightarrow$ *Thermodynamic equilibrium* is a late-time attractor of dynamical evolution of isolated interacting quantum system:

$$\lim_{t\to\infty} T_{\mu\nu}(t,\boldsymbol{x}) = \text{diag}\left(\mathcal{E}_{eq}, P_{eq}, \cdots P_{eq}\right)$$

- $T_{\mu\nu}$ are the component of the stress-energy tensor of the system at time t and the spatial location $\boldsymbol{x}$

$\Longrightarrow$ We also have a theory — **the hydrodynamics** — that describes the approach to that equilibrium (assuming we are not-far from it):

- $\nabla_\mu T^{\mu\nu} = 0$
- $T^{\mu\nu} = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + T^{\mu\nu}_{(2)} + \cdots$

- $$T^{\mu\nu} = \underbrace{\mathcal{E}\, u^\mu u^\nu + P\,(g^{\mu\nu} + u^\mu u^\nu)}_{\mathcal{O}(\partial^0 u)} + \underbrace{\left[-\eta\sigma^{\mu\nu} - \zeta(g^{\mu\nu} + u^\mu u^\nu)\nabla\cdot u\right]}_{\mathcal{O}(\partial^1 u):\ \sigma^{\mu\nu}\sim\partial^\mu u^\nu}$$
$$+ \underbrace{[\quad\cdots\quad]}_{\mathcal{O}(\partial^2 u, (\partial u)^2)} + \cdots$$

  - u^μ — local fluid velocity
  - $g^{\mu\nu}$ — background metric
  - η, ζ — shear and bulk viscosities
  - expansion parameter of hydro as EFT:

$$\frac{1}{T}\cdot|\partial u| \ll 1$$

where $T = T(t, \boldsymbol{x})$ is the local temperature

$$\mathcal{E} + P = sT\,, \qquad d\mathcal{E} = T ds$$

- $$\mathcal{S}^\mu = \underbrace{s\ u^\mu}_{\mathcal{O}(\partial^0 u)} + \underbrace{\left[-\frac{1}{T}\ \cdot\ T^{\mu\nu}_{(1)} u_\nu\right]}_{\mathcal{O}(\partial^1 u)} + \underbrace{[\quad \cdots \quad]}_{\mathcal{O}(\partial^2 u, (\partial u)^2)} + \cdots$$

  - from the conservation of the stress-energy tensor,

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \Longrightarrow$$

$$T\ \nabla \cdot \mathcal{S} = \zeta\ (\nabla \cdot u)^2 + \frac{\eta}{2} \sigma_{\mu\nu} \sigma^{\mu\nu}\ \geq\ 0$$

$\Longrightarrow$ As one approaches the equilibrium,

$$\lim_{t\to\infty} u^\mu = u^\mu_{eq} = (1, \mathbf{0}) \qquad \Longrightarrow \qquad \lim_{t\to\infty} T\ \nabla \cdot \mathcal{S} = 0$$

i.e., in the approach to equilibrium the entropy production rate vanishes

We can now provide a formal definition of a dynamical fixed point (DFP):

A *Dynamical Fixed Point* is an internal state of a quantum field theory with spatially homogeneous and time-independent one-point correlation functions of its stress energy tensor $T^{\mu\nu}$, and (possibly additional) set of gauge-invariant local operators $\{\mathcal{O}_i\}$,

and

strictly positive divergence of the entropy current at late-times:

$$\lim_{t\to\infty}\left(\nabla\cdot\mathcal{S}\right) > 0$$

$\Longrightarrow$ Apart from the requirement of the strictly non-zero entropy production rate at late times, characteristics of a DFP coincide with that of the thermodynamic equilibrium.

Why?

$\Longrightarrow$ DFP, *i.e.*, the non-vanishing late-time entropy production in **driven** (open) quantum-mechanical systems/QFT:

- QFTs in cosmological backgrounds,
  asymptotically de Sitter space-times in particular

$\Longrightarrow$ To study DFPs means to classify the end-of-time dynamics of *massive* QFTs, in cosmologies with dark energy

Example: $\mathcal{N}=2^*$ QGP — one of the best understood non-conformal top-down holography

- $\mathcal{N}=2^*$: $\mathcal{N}=4$ SYM with $m_b/m_f \neq 0$ for bosonic/fermionic components of a hypermultiplet
- in Minkowski space time:
  - $g^{\mu\nu}=\eta^{\mu\nu}$
  - $$\mathcal{E}_{eq}=\frac{3}{8}\pi^2N^2T^4\left[1+\left\{\frac{\ln\frac{T}{m_b}}{9\pi^4}\left(\frac{m_b}{T}\right)^4+\cdots\right\}+\left\{-\frac{2\Gamma\left(\frac{3}{4}\right)^4}{3\pi^4}\left(\frac{m_f}{T}\right)^2+\cdots\right\}\right]$$
  - $$\frac{\eta}{s}=\frac{1}{4\pi}\,,\qquad \frac{\zeta}{\eta}=\beta_f^{\Gamma}\cdot\frac{\Gamma\left(\frac{3}{4}\right)^4}{3\pi^3}\left(\frac{m_f}{T}\right)^2+\beta_b^{\Gamma}\cdot\frac{1}{432\pi^2}\left(\frac{m_b}{T}\right)^4+\cdots$$

where

$$\beta_f^{\Gamma}\approx 0.9672\,,\qquad \beta_b^{\Gamma}\approx 8.0000$$

- in FLRW:
  - $$ds_4^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t)^2 \; d\boldsymbol{x}^2$$
  - FLRW cosmology as hydro:

$$\text{local/comoving}: \qquad u^\mu = (1, \mathbf{0})$$

$$\underline{\text{BUT}}: \qquad \nabla \cdot u = 3 \, \frac{\dot{a}}{a} \neq 0$$

  - $$\underbrace{T \; \nabla \cdot \mathcal{S}}_{\frac{T}{a^3} \cdot \frac{d}{dt}[a^3 s]} = \zeta \; \underbrace{(\nabla \cdot u)^2}_{9\left(\frac{\dot{a}}{a}\right)^2} + \frac{\eta}{2} \underbrace{\sigma_{\mu\nu} \sigma^{\mu\nu}}_{=0} + \cdots$$

$\Longrightarrow$ prediction verified in (1603.05344)

$$\underbrace{\frac{d}{dt} \ln[a^3 s]}_{\text{computed from dynamical horizon}} = \underbrace{\frac{1}{T} \cdot (\nabla \cdot u)^2 \cdot \frac{\zeta}{s}}_{\text{agreement from Minkowski } \frac{\zeta}{s}} + \cdots$$

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  - $$\frac{\eta}{s} = \frac{1}{4\pi}\,, \qquad \frac{\zeta}{\eta} = \beta_f^\Gamma \cdot \frac{\Gamma\left(\frac{3}{4}\right)^4}{3\pi^3} \left(\frac{m_f}{T}\right)^2 + \beta_b^\Gamma \cdot \frac{1}{432\pi^2} \left(\frac{m_b}{T}\right)^4 + \cdots$$

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$\Longrightarrow$ so far so good $\cdots$

BUT: validity of 'de Sitter hydro':

$$\frac{\nabla u}{T} \ll 1 \qquad \Longleftrightarrow \qquad \frac{H}{\frac{T_0}{a(t)}} \ll 1$$

$$\lim_{t\to\infty} \frac{H}{T_0 \exp(-Ht)} \longrightarrow +\infty \qquad i.e., \qquad \underline{\text{is always violated!}}$$

$\Longrightarrow$

Hydrodynamics in $R^{3,1}$ $\Longrightarrow$ Thermal equilibrium

Dynamics in dS_4 $\Longrightarrow$ DFP*

*: QFT$\neq$ CFT

$\Longrightarrow \mathcal{N} = 4$ SYM in FLRW CFT perspective

- FLRW is Weyl equivalent to Minkowski:

$$ds_4^2 = -dt^2 + a^2(t)\, d\boldsymbol{x}^2 = a(t)^2 \left(-\frac{dt^2}{a(t)^2} + d\boldsymbol{x}^2 \right) = a^2 \underbrace{\left(-d\tau^2 + d\boldsymbol{x}^2 \right)}_{ds^2_{Minkowski}}$$

- if $\mathcal{O}_\Delta$ is a primary operator of dimension Δ,

$$\langle \mathcal{O}_\Delta \rangle \Big|_{FLRW} = a^{-\Delta} \, \langle \mathcal{O}_\Delta \rangle \Big|_{Minkowski}$$

- stress-energy tensor is not a primary field:

$$\langle T_{\mu\nu} \rangle \Big|_{FLRW} = a^{-4} \, \langle T_{\mu\nu} \rangle \Big|_{Minkowski} + \text{conformal anomaly}$$

$\Longrightarrow$ for a trace of the stress-energy tensor

$$\langle T^\mu_\mu \rangle \Big|_{FLRW} = a^{-4} \underbrace{\langle T^\mu_\mu \rangle \Big|_{Minkowski}}_{=0} + \frac{c}{24\pi^3} \underbrace{\left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right)}_{=-12 \frac{(\dot{a})^2 \ddot{a}}{a^3}}$$

e.g., for $\mathcal{N} = 4$ $SU(N)$ SYM,

$$- \langle T^t_t \rangle \Big|_{FLRW} = \frac{1}{a(t)^4} \mathcal{E} + \frac{3N^2}{32\pi^2} \frac{(\dot{a})^4}{a^4}$$

$$\langle T^{\boldsymbol{x}}_{\boldsymbol{x}} \rangle \Big|_{FLRW} = \frac{1}{a(t)^4} P + \frac{N^2}{8\pi^2} \left\{ \frac{(\dot{a})^4}{4a^4} - \frac{(\dot{a})^2 \ddot{a}}{a^3} \right\}$$

$$\langle T^\mu_\mu \rangle \Big|_{FLRW} = a^{-4} \underbrace{\left(-\mathcal{E} + 3P \right)}_{=0} - \frac{3N^2}{8\pi^2} \frac{(\dot{a})^2 \ddot{a}}{a^3}$$

$\Longrightarrow$ Minkowski space-time thermal equilibrium states of $\mathcal{N} = 4$ SYM (strong coupling) of temperature T_0:

$$\mathcal{E}_0 = \frac{3}{8}\pi^2 N^2 T_0^4\,, \qquad P_0 = \frac{1}{3}\mathcal{E}_0$$

$\Longrightarrow$ in FLRW cosmology,

$$\mathcal{E}(t) = \frac{3}{8}\pi^2 N^2 T(t)^4 + \frac{3N^2}{32\pi^2}\frac{(\dot{a})^4}{a^4}\,, \qquad P(t) = \frac{1}{3}\mathcal{E}(t) - \frac{N^2}{8\pi^2}\frac{(\dot{a})^2\ddot{a}}{a^3}$$

where $T(t)$ is the <u>*effective*</u> temperature

$$T(t) = \frac{T_0}{a(t)}$$

$\Longrightarrow$ Stress-energy tensor in FLRW is covariantly conserved:

$$0 = \langle \nabla^\mu T^\nu_\mu \rangle \qquad \Longleftrightarrow \qquad \frac{d\mathcal{E}(t)}{dt} + 3\frac{\dot{a}}{a}\,\left(\mathcal{E}(t) + P(t)\right) = 0$$

$\Longrightarrow$ entropy density

- In Minkowski space-time:

$$s_0 = \frac{\pi^2}{2} N^2 T_0^3$$

- Assuming the adiabatic expansion in FLRW, the co-moving entropy density, $s_{comoving}$,

$$s_{comoving} \equiv a(t)^3 s(t)$$

is conserved:

$$\frac{d}{dt} s_{comoving} = 0 \qquad \Longrightarrow \qquad s_{comoving} = s_{comoving}\Big|_{t=0} = s_0$$

$\Longrightarrow$

$$s(t) = \frac{\pi^2}{2} N^2 T(t)^3$$

- In expanding FLRW, with $a(t) \to \infty$ as $t \to \infty$,

$$\lim_{t\to\infty} s(t) = 0$$

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$\Longrightarrow$ Let's rephrase the de Sitter entropy discussion in the language of the entropy current $\mathcal{S}^\mu$:

- A locally static observer has $u^\mu = (1, \mathbf{0})$
- The entropy current (in Landau frame $T^{\mu\nu}_{(1)} u_\nu = 0$) is

$$\mathcal{S}^\mu = s \; u^\mu$$

$\Longrightarrow$

$$\nabla \cdot S = \frac{1}{a(t)^3} \, \frac{d}{dt} \left(a(t)^3 s \right) = \frac{1}{a(t)^3} \, \frac{d}{dt} s_{comoving}(t) \; = \; 0$$

That is is why $\mathcal{N} = 4$ SYM (same is true for any conformal theory!) in de Sitter evolved to a trivial DFP

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How would a non-trivial DFP arise?

- Imagine that

$$\lim_{t\to\infty} s(t) = s_{ent} \neq 0$$

This limit is natural to call the *vacuum entanglement entropy* density, hence $_{ent}$

- Then,

$$\lim_{t\to\infty}\left(\nabla\cdot\mathcal{S}\right) = 3\ H\ s_{ent}$$

where

$$H = \lim_{t\to\infty}\frac{d}{dt}\ \ln a(t)$$

$\Longrightarrow$ In strongly coupled non-conformal theories with holographic dual

$$s_{ent}\ >\ 0$$

$\Longrightarrow$ $\mathcal{N} = 4$ SYM in FLRW holographic perspective

$$S_{\mathcal{N}=4} = \frac{1}{16\pi G_5} \int_{\mathcal{M}_5} d^5\xi \sqrt{-g} \left[R + \frac{12}{L^2} \right]$$

$$L^4 = \ell_s^4 \; N g_{YM}^2 \,, \qquad G_5 = \frac{\pi L^3}{2N^2} \,, \qquad 4\pi g_s = g_{YM}^2$$

$\Longrightarrow$ Consider general spatially homogeneous, time-dependent states:

$$ds_5^2 = 2dt\;(dr - Adt) + \Sigma^2 \; d\boldsymbol{x}^2$$

$$A = A(t,r)\,, \qquad \Sigma = \Sigma(t,r)$$

$\Longrightarrow$ We are interested in spatially homogeneous and isotropic states of $\mathcal{N}=4$ SYM in FLRW, so the bulk metric warp approach the AdS boundary $r \to \infty$ as

$$\Sigma = \frac{a(t)r}{L} + \mathcal{O}(r^0)\,, \qquad A = \frac{r^2}{2L^2} + \mathcal{O}(r^1)$$

Indeed, as $r \to \infty$,

$$ds_5^2 = \frac{r^2}{L^2} \underbrace{\left(-dt^2 + a(t)^2 d\boldsymbol{x}^2\right)}_{\text{boundary FLRW}} + \cdots$$

$\Longrightarrow \mathcal{N} = 4$ SYM in FLRW holographic perspective

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$\Longrightarrow$ These equations can be solve in all generality for arbitrary $a(t)$:

$$A = \frac{(r+\lambda)^2}{8} - (r+\lambda)\,\frac{\dot{a}}{a} - \dot{\lambda} - \frac{r_0^4}{8a^4(r+\lambda)^2}\,,$$

$$\Sigma = \frac{(r+\lambda)a}{2}$$

where

- r_0 is a single constant parameter
- $\lambda(t)$ is an arbitrary function - the leftover diffeomorphism of the 5d

gravitational metric reparametrization $r \to \bar{r} = r - \lambda(t)$:

$$A(t,r) \to \bar{A}(t,\bar{r}) = A(t, r+\lambda(r)) - \dot{\lambda}(t)$$

$$\Sigma(t,r) \to \bar{\Sigma}(t,\bar{r}) = \Sigma(t, r+\lambda(t))$$

$\Longrightarrow$

$$ds_5^2 \Longrightarrow d\bar{s}_5^2 \qquad = 2dt\,(d\bar{r} - \bar{A}dt) + \bar{\Sigma}^2\, d\boldsymbol{x}^2$$

$\Longrightarrow$ Identifying

$$\frac{r_0}{2} \equiv T_0$$

$\Longrightarrow$ from holographic computation of the boundary stress energy tensor,

$$\mathcal{E}(t) = \frac{3}{8}\pi^2 N^2 T(t)^4 + \frac{3N^2}{32\pi^2}\frac{(\dot{a})^4}{a^4}\,, \qquad P(t) = \frac{1}{3}\mathcal{E}(t) - \frac{N^2}{8\pi^2}\frac{(\dot{a})^2\ddot{a}}{a^3}$$

$$T(t) = \frac{T_0}{a(t)}$$

Precisely as expected from the Weyl transformation of the thermal state from Minkowski to FLRW!

$\Longrightarrow$ Holography buys us more:

- Chesler-Yaffe pioneered numerical studies of EF metrics:

$$ds_5^2 = 2dt\ (dr - Adt) + \Sigma^2\ d\boldsymbol{x}^2$$

- such metrics has an **apparent horizon** (AH) at r_{AH}

$$d_+\Sigma\bigg|_{r=r_{AH}} = 0 \qquad \Longrightarrow \qquad r_{AH} = \frac{r_0}{a(t)} - \lambda(t)$$

- causal dependence **must** include

$$r \in [r_{AH}, +\infty)$$

- region

$$r < r_{AH}$$

  is causally disconnected from the holographic dynamics and **must be** excised

- AH is a dynamical horizon
-

$$\underbrace{\frac{\Sigma^3}{4G_5}\bigg|_{r=r_{AH}}}_{\text{comoving Bekenstein entropy of the AH}} = \frac{N^2 r_0^3}{128\pi}$$

$$= \underbrace{s_{comoving}}_{\text{SYM comoving entropy density in FLRW}} = a(t)^3 s(t) = \frac{\pi^2}{2} N^2 T_0^3$$

Precisely as expected from the CFT arguments!

$\Longrightarrow$ Nontrivial DFP

- The model:

$$S_4 = \frac{1}{2\kappa^2}\int_{\mathcal{M}_4} dx^4\sqrt{-\gamma}\left[R + 6 - \frac{1}{2}\left(\nabla\phi\right)^2 + \phi^2\right]$$

- ϕ is dual to $\mathcal{O}_\phi$,

$$L^2 m_\phi^2 = -2 \qquad \Longrightarrow \qquad \dim(\mathcal{O}_\phi) = 2$$

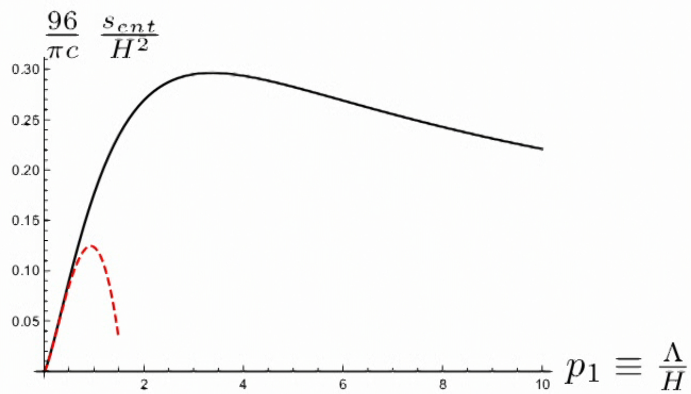
- source therms for the gravitational evolution:
  - the boundary metric is dS_3,

$$ds_3^2 = -dt^2 + e^{2Ht}d\boldsymbol{x}^2$$

  - mass scale Λ of the boundary QFT_3,

$$\phi = \frac{\Lambda}{r} + \mathcal{O}(r^{-2})$$

Recall: $s_{ent} = \lim_{t\to\infty} s(t)$



$$\frac{\kappa^2}{2\pi}\,\frac{s_{ent}}{H^2} = \frac{1}{6}6^{1/3}\,p_1^{4/3} - \frac{1}{12}\,p_1^2 - \frac{5}{216}6^{2/3}\,p_1^{8/3} - \frac{3359}{311040}6^{1/3}\,p_1^{10/3} + \mathcal{O}\left(p_1^4\right)$$

Important:

$$\frac{d(a^2 s)}{dt} = \frac{2\pi}{\kappa^2}\,(\Sigma^2)'\,\frac{(d_+\phi)^2}{\phi^2+6}\bigg|_{r=r_{AH}} \geq 0$$

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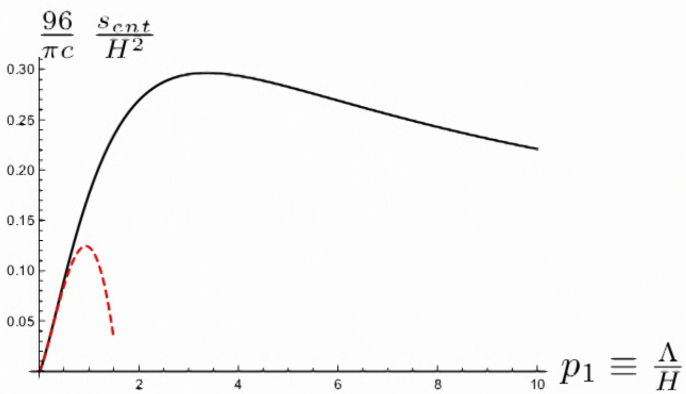

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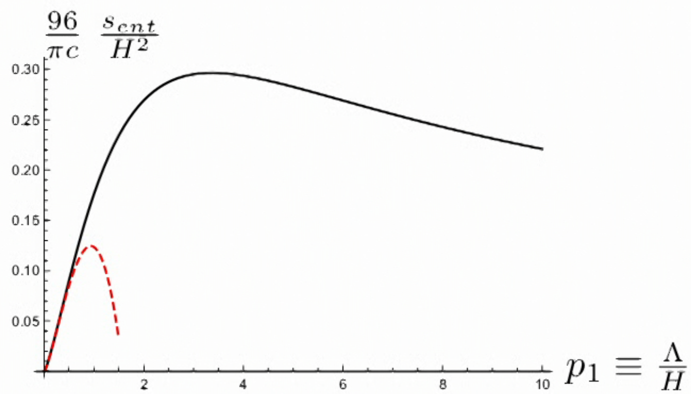

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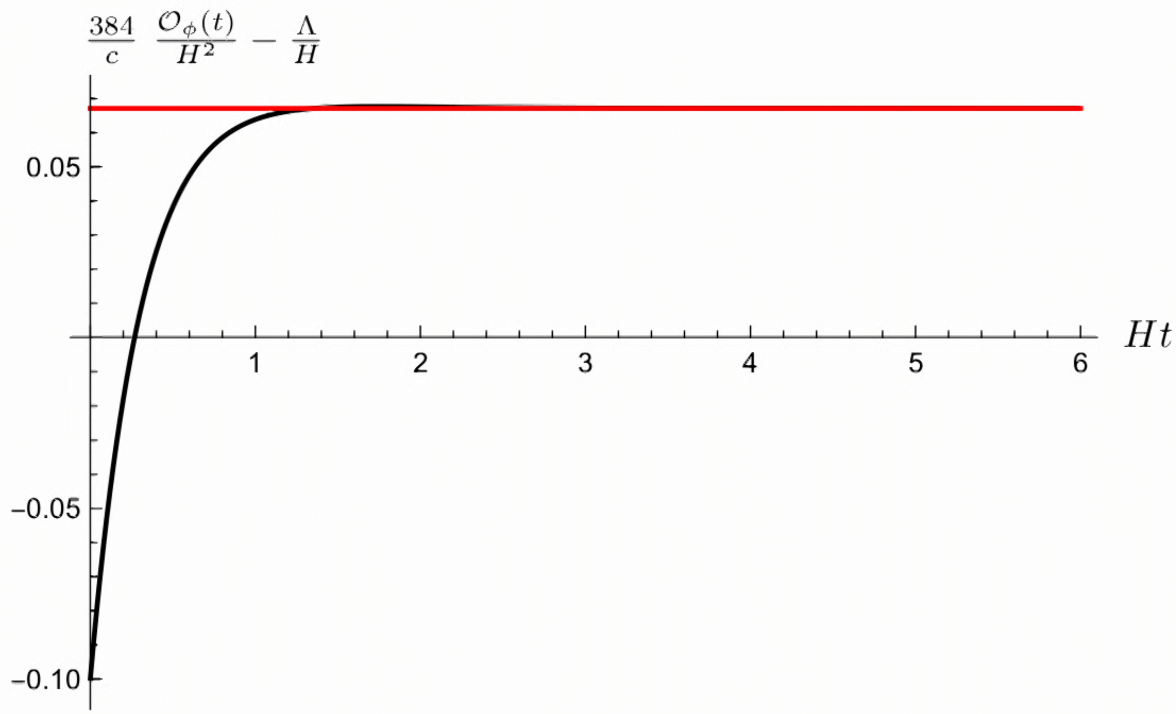
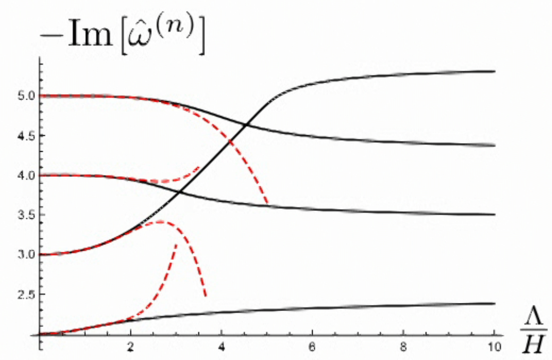
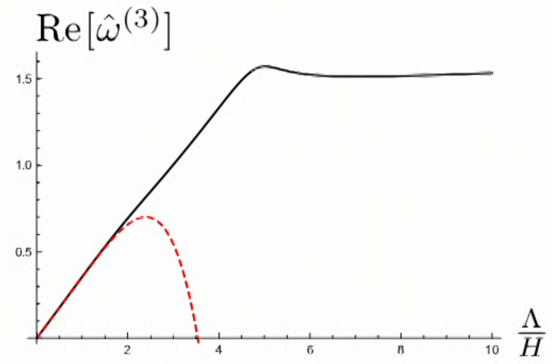
$\Longrightarrow$ DFP as a late-time attractor:



$\Longrightarrow$ Spectrum of DFP fluctuations (*aka* QNMs):

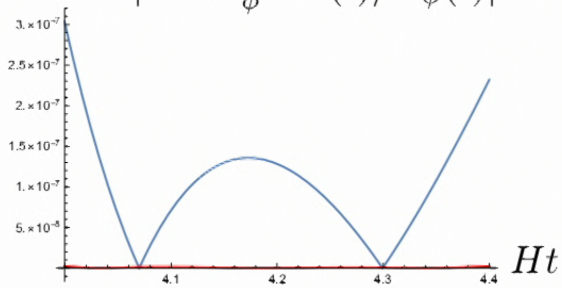




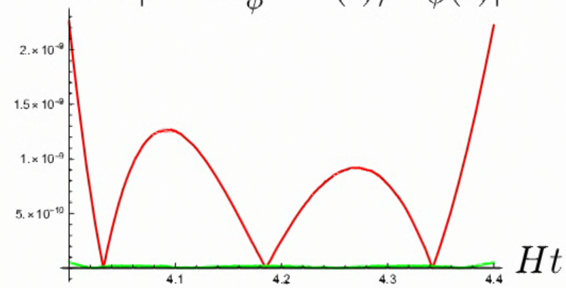
$$\hat{\omega} \equiv \frac{\omega}{H}$$

$\Longrightarrow$ Approach to DFP via 'QNMs':

$\delta = |1 - \mathcal{O}_\phi^{QNM}(t)/\mathcal{O}_\phi(t)|$



$\delta = |1 - \mathcal{O}_\phi^{QNM}(t)/\mathcal{O}_\phi(t)|$



$$\mathcal{O}_\phi^{QNM}(t) = \mathcal{O}_\phi^{DFP} + \sum_{\text{QNM spectrum}} \mathcal{A}e^{-i(\hat{\omega}Ht+\text{phase})}$$

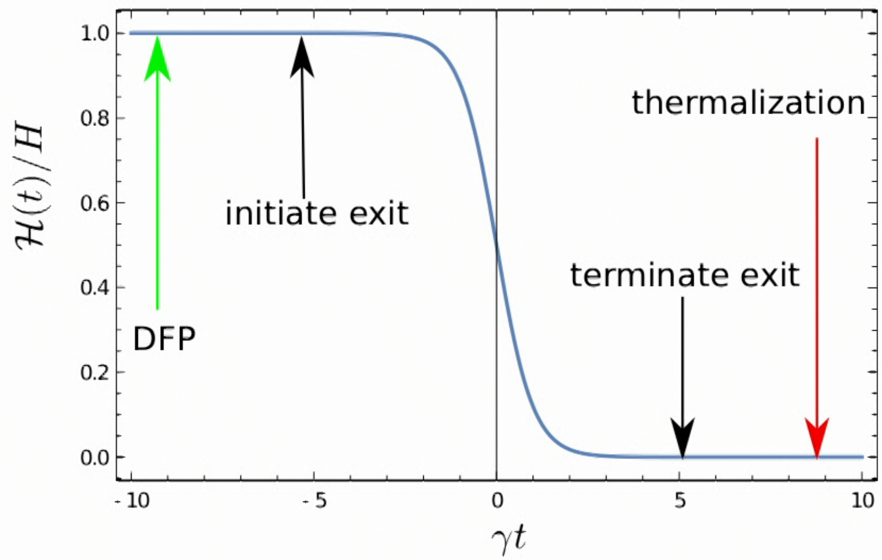
- blue: $n = 2$ QNM only
- red: $n = 2, 3$ QNMs
- green: $n = 2, 3, 4$ QNMs

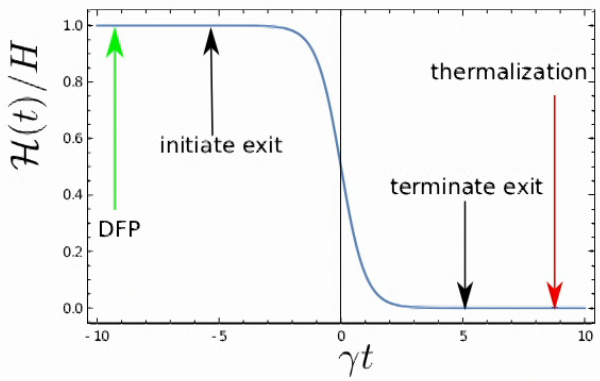
$\Longrightarrow$ Holographic gravitational reheating:

- $$\frac{d(a^2 s)}{dt} = \frac{2\pi}{\kappa^2} \, (\Sigma^2)' \, \frac{(d_+\phi)^2}{\phi^2+6}\bigg|_{r=r_{AH}} \geq 0$$

- consider a scale factor $a(t)$ with a Hubble parameter:

$$\mathcal{H}(t) \equiv \frac{\dot{a}}{a} = \frac{H}{1+\exp(2\gamma t)}$$



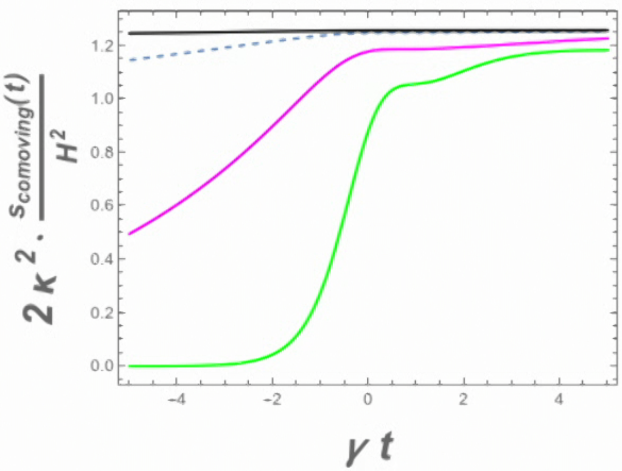


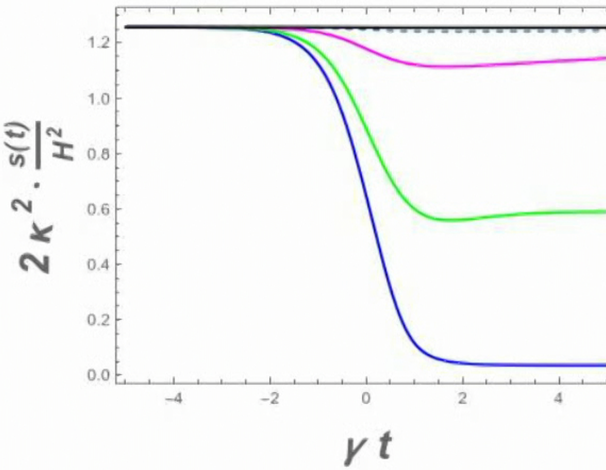
- in the *fast* inflationary exit

$$\ln \frac{a_t}{a_i} = \int_{-5/\gamma}^{5/\gamma} dt\ \mathcal{H}(t) = \frac{5H}{\gamma} \to 0 \qquad \text{as} \qquad \frac{H}{\gamma} \to 0$$

- $$\frac{d(a^2 s)}{dt} \geq 0 \implies s_t a_t^2 \geq s_i a_i^2 \implies s_t \geq s_i \left(\frac{a_i}{a_t}\right)^2 \underbrace{\approx}_{H/\gamma \to 0} s_{ent}^{DFP}$$

- $$s_{ent}^{DFP} \underbrace{\longrightarrow}_{\text{with further thermalization}} s(t)\Big|_{t\to+\infty} = s_{thermal} \geq s_{ent}^{DFP}$$

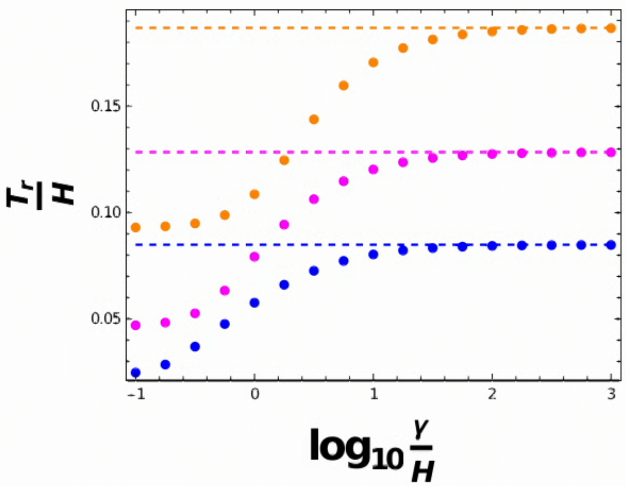


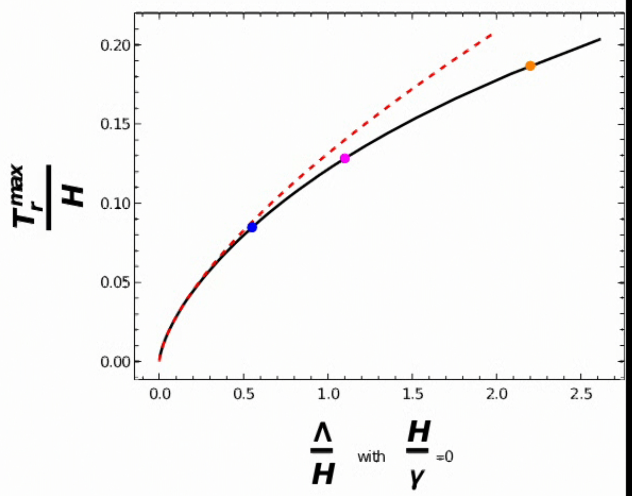


$$\log_{10}\frac{\gamma}{H} = \Bigg\{ \underbrace{-1}_{\text{blue}}, \underbrace{0}_{\text{green}}, \underbrace{1}_{\text{magenta}}, \underbrace{2}_{\text{grey dashed}}, \underbrace{3}_{\text{black}} \Bigg\}$$

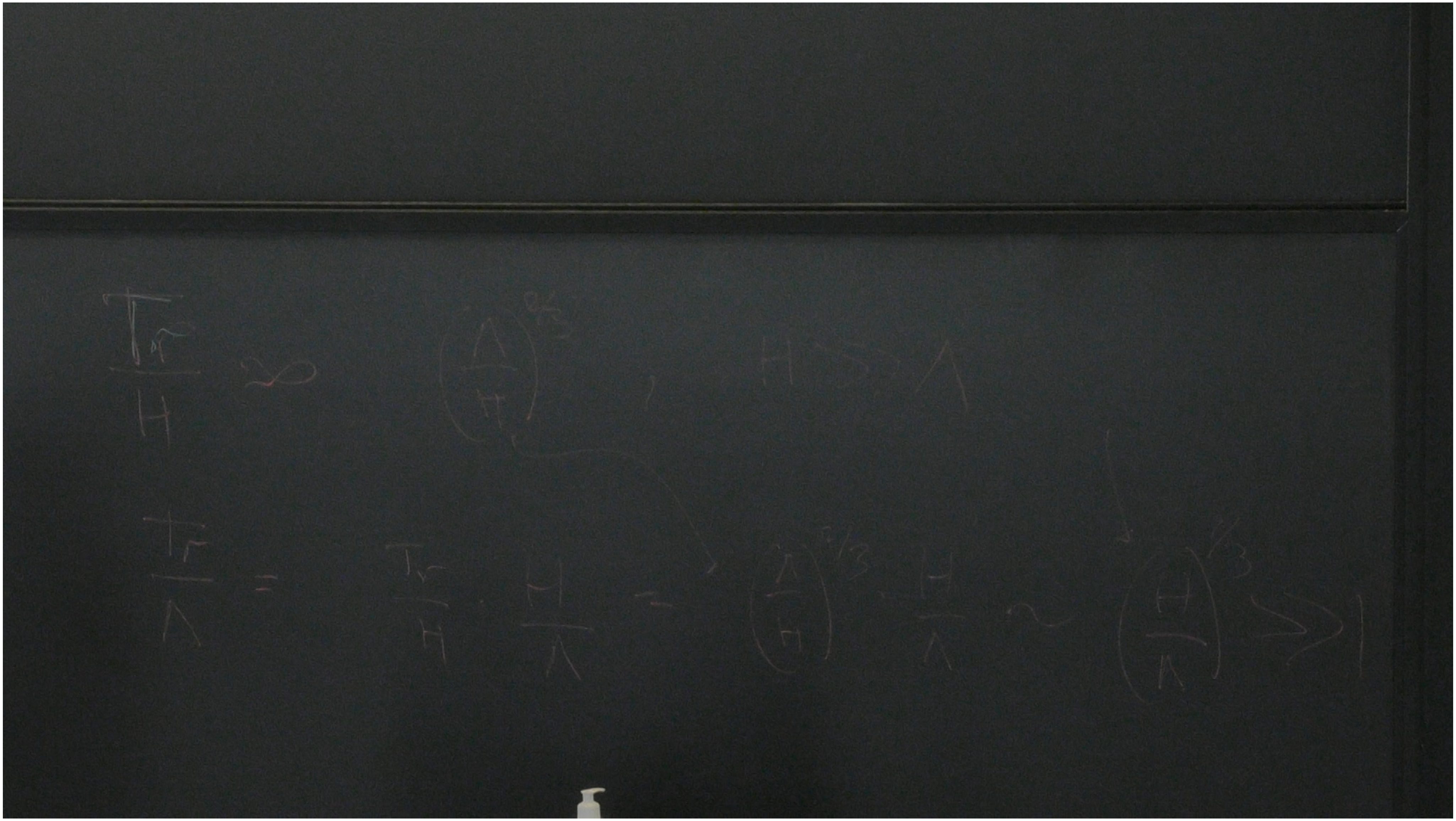
$\Longrightarrow$ evolve until the inflationary exit state thermalized at T_r: $tT_r \sim 1$

$$\frac{\Lambda}{H} = \Bigg\{ \underbrace{0.55}_{\text{orange}} \, , \, \underbrace{1.1}_{\text{magenta}} \, , \, \underbrace{2.2}_{\text{blue}} \Bigg\}$$





$$\frac{T_r^{max}}{H} \approx \frac{3^{2/3}}{2^{7/3}\pi}\left(\frac{\Lambda}{H}\right)^{2/3} , \qquad \text{as} \qquad \frac{\Lambda}{H} \to 0$$



$\frac{T_{rh}}{H} \sim \left(\frac{\Lambda}{H}\right)^{2/3}$, $H \gg \Lambda$
$\frac{T_{rh}}{\Lambda} = \frac{T_{rh}}{H} \cdot \frac{H}{\Lambda} = \left(\frac{\Lambda}{H}\right)^{2/3} \cdot \frac{H}{\Lambda} \sim \left(\frac{H}{\Lambda}\right)^{1/3} \gg 1$

Conclusions:

- A new concept of DFP
- Massive QFT in de Siter has finite physical entropy density s_{ent}
- In the exit from inflation s_{ent} can be harvested — this solves the problem of the initial Hot Big Bang entropy without the inflaton reheating

$\Longrightarrow$ To do:

- compute T_r for $\mathcal{N} = 2^*$
- compute T_r for the cascading theory
- understanding of weakly coupled DFP is missing