Title: Errors from Dynamical Structural Instabilities of Floquet Maps in Quantum Simulation

Speakers: Karthik Chinni

Series: Perimeter Institute Quantum Discussions

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Abstract: We study the behavior of errors in the quantum simulation of spin systems with long-range multibody interactions resulting from the Trotter-Suzuki decomposition of the time evolution operator. We identify a regime where the Floquet operator underlying the Trotter decomposition undergoes sharp changes even for small variations in the simulation step size. This results in a time evolution operator that is very different from the dynamics generated by the targeted Hamiltonian, which leads to a proliferation of errors in the quantum simulation. These regions of sharp change in the Floquet operator, referred to as structural instability regions, appear typically at intermediate Trotter step sizes and in the weakly interacting regime, and are thus complementary to recently revealed quantum chaotic regimes of the Trotterized evolution [L. M. Sieberer et al. npj Quantum Inf. 5, 78 (2019); M. Heyl, P. Hauke, and P. Zoller, Sci. Adv. 5, eaau8342 (2019)]. We characterize these structural instability regimes in p-spin models, transverse-field Ising models with all-to-all p-body interactions, and analytically predict their occurrence based on unitary perturbation theory. We further show that the effective Hamiltonian associated with the Trotter decomposition of the unitary time-evolution operator, when the Trotter step size is chosen to be in the structural instability region, is very different from the target Hamiltonian, which explains the large errors that can occur in the simulation in the regions of instability. These results have implications for the reliability of near-term gate based quantum simulators, and reveal an important interplay between errors and the physical properties of the system being simulated.

Zoom link: https://pitp.zoom.us/j/92045582127?pwd=WDUxcnlIeXdnVWM3WGJoSFVMNDE2dz09

Trotter Errors from Instabilities of Floquet maps in Quantum Simulation

Karthik Chinni, Manuel Muñoz-Arias, Pablo Poggi, Ivan Deutsch
University of New Mexico

This work discussed here: **PRX Quantum 3.1 (2022): 010351**

NISQ-era Quantum Simulation

- Noisy intermediate scale quantum (NISQ) era: ~100 of qubits, no fault-tolerant error correction.

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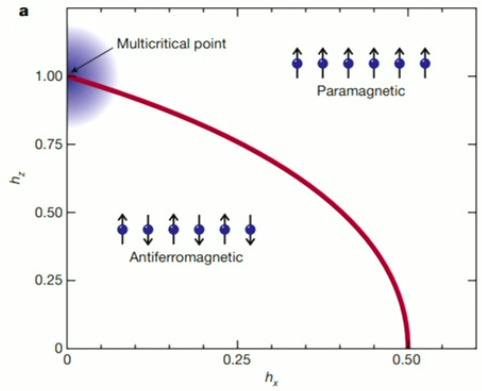
- Noisy intermediate scale quantum (NISQ) era: ~100 of qubits, no fault-tolerant error correction.

- Quantum simulation: less demanding than universal quantum computing. Small errors unlikely to affect macroscopic properties.

- **Can we trust analog quantum simulators** ?*:
  - Interplay between physical properties of the system and how errors accumulate in the simulator?



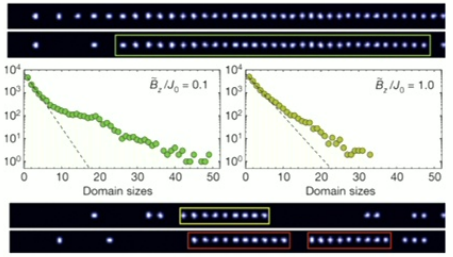
Quantum simulation of phase transition

J. Simon et al, Nature 2011

* Hauke, Philipp, et al. *Reports on Progress in Physics* 75.8 (2012): 082401.

Quantum Simulation

1. Hamiltonian-based simulation (emulation):
   - System in the lab is engineered to obey target Hamiltonian
   - Accessible models platform-dependent: trapped ions ↔ 1D Long-range Ising, etc.



$$H = B_z \sum_i \sigma_z^{(i)} + \sum_{i<j} \frac{J_0}{|i-j|^\alpha} \sigma_x^{(i)} \sigma_x^{(j)}$$

J. Zhang et al, Nature 2017

2. Gate-based simulation:
   - Discrete set of unitaries as building blocks and the target $U_{\text{target}} = \prod_i u_i$

Simulation Using Trotter-Suzuki Decomposition

- $H_{\text{tar}} = \sum_{k=1}^{m} H_k$

Target dynamics:

$$U_{\text{tar}} = e^{i H_{\text{tar}} T}$$

Access to gates $\{e^{iH_1\tau}, e^{iH_2\tau}, \ldots, e^{iH_m\tau}\}$

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Access to gates $\{e^{iH_1\tau}, e^{iH_2\tau}, \dots, e^{iH_m\tau}\}$ → Simulated dynamics:

1st order Trotter-Suzuki

Trotter step size, $\tau = T/n$

$$U_{\text{trot}} = \left(e^{iH_1 \frac{T}{n}} e^{iH_2 \frac{T}{n}} \dots e^{iH_m \frac{T}{n}}\right)^n \equiv \left(U_\delta(\tau)\right)^n$$

- Trotter error scales as

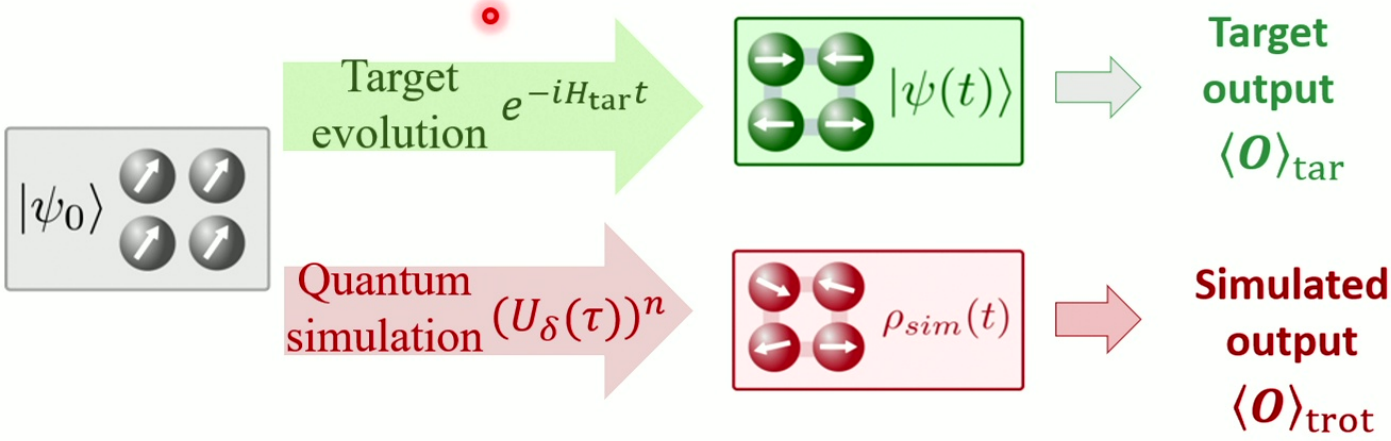
$$\|U_{\text{trot}} - U_{\text{tar}}\| = \mathcal{O}\left(\frac{T^2 \sum_{i,j} \| [H_i, H_j] \|}{n}\right)$$

Trotter Errors

- NISQ-era devices have shallow depth circuits, n is small (or equivalently moderate to large values of τ)

$$U_{\text{trot}} = (U_\delta(\tau))^n \qquad \text{Trotter step size, } \tau = T/n$$

- Errors in simulation from Trotter-Suzuki decomposition:



Error = $\langle O\rangle_{\text{tar}}$ - $\langle O\rangle_{\text{trot}}$

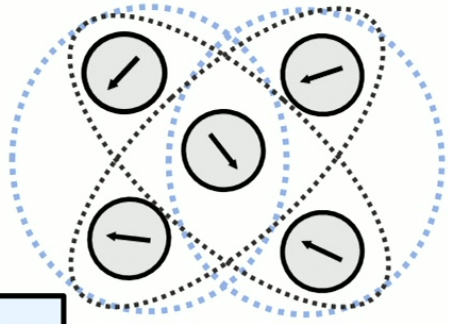
Relate the behavior of these errors to physical properties and get **new insights** beyond the error bound.

5

p-spin Models

- ***p*-spin Hamiltonian**:

$$H(s) = -(1-s)\sum_{i=1}^{N}\frac{\sigma_z^{(i)}}{2} - \frac{s}{p}\sum_{i_1,i_2,\dots i_p=1}^{N}\frac{\sigma_x^{(i_1)}\sigma_x^{(i_2)}\dots\sigma_x^{(i_p)}}{2N^{p-1}}$$



- Completely-connected transverse Ising model with p-body interactions

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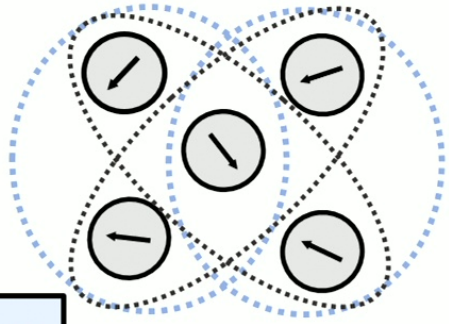

- Completely-connected transverse Ising model with p-body interactions

- Collective spin-operators, $\mathrm{J}_i = \Sigma_k \frac{\sigma_i^{(k)}}{2}$: $H(s) = -(1-s)\mathrm{J}_z - \frac{s}{pJ^{p-1}}\mathrm{J}_x^p$

  control parameter, $0 \le s \le 1$

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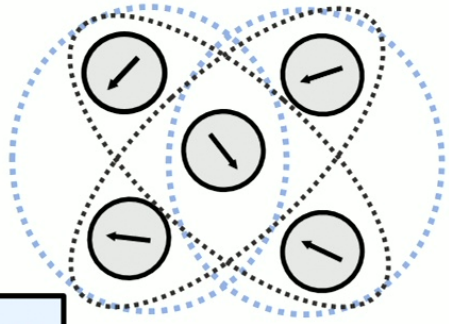

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  control parameter, $0 \le s \le 1$

- Symmetries: dynamics in the symmetric subspace: $[H, \mathrm{J}^2] = 0$. $\mathcal{H}_s = N+1$

  parity operator, $\left[H, e^{i\pi J_z}\right] = 0$ for even p.

Lipkin-Meskov-Glick (LMG) Model

- $\boldsymbol{p = 2}$: Lipkin-Meshkov-Glick (LMG) model

$$H(s) = -(1-s)\mathrm{J}_z - \frac{s}{2J}\mathrm{J}_x^2$$

- "Toy model" for many-body physics: phase transitions of various kinds*.

- Ground-state quantum phase transition (GSQPT):
  - For $s = 0$, the ground state is $|\uparrow\uparrow\uparrow \cdots \uparrow\rangle$
  - For $s = 1$, the ground states are $|\rightarrow\rightarrow\rightarrow \cdots \rightarrow\rangle$ and $|\leftarrow\leftarrow\leftarrow \cdots \leftarrow\rangle$

* Zhang *et al*. Nature **551,** 601-604 (2017)
* P. Jurcevic *et al.* PRL **119**, 080501 (2017)
* M. H. Muñoz Arias, *et al.*, Phys. Rev. A 102, 022610 (2020).

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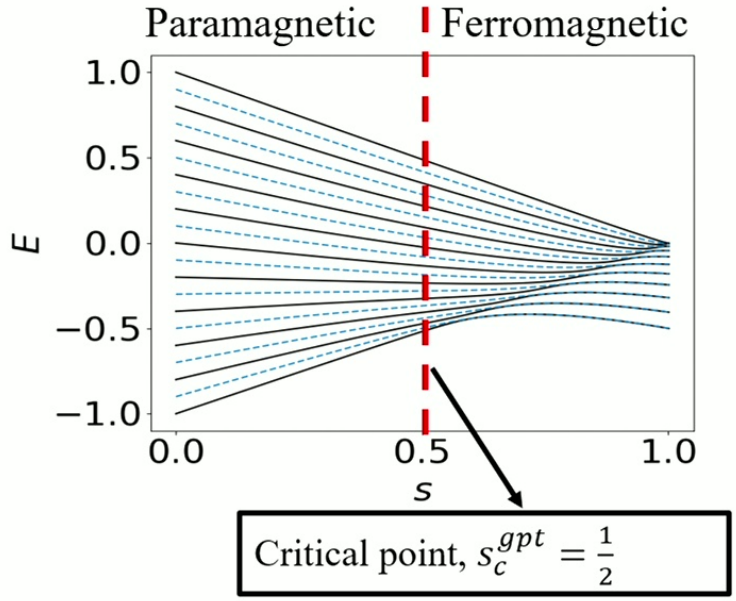

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Mean Field Description

- Mean-field description is exact in the Thermodynamic limit ($N \to \infty$)/ classical limit.

- Mean-field equations of motion: ($\langle AB \rangle = \langle A \rangle \langle B \rangle$)

$$\dot{X} = (1-s)Y$$
$$\dot{Y} = -(1-s)X + \mathrm{s}\, XZ$$
$$\dot{Z} = -\mathrm{s}\, XY$$

$$(X, Y, Z) \equiv \frac{1}{J}\left(\langle J_x \rangle, \langle J_y \rangle, \langle J_z \rangle\right)$$

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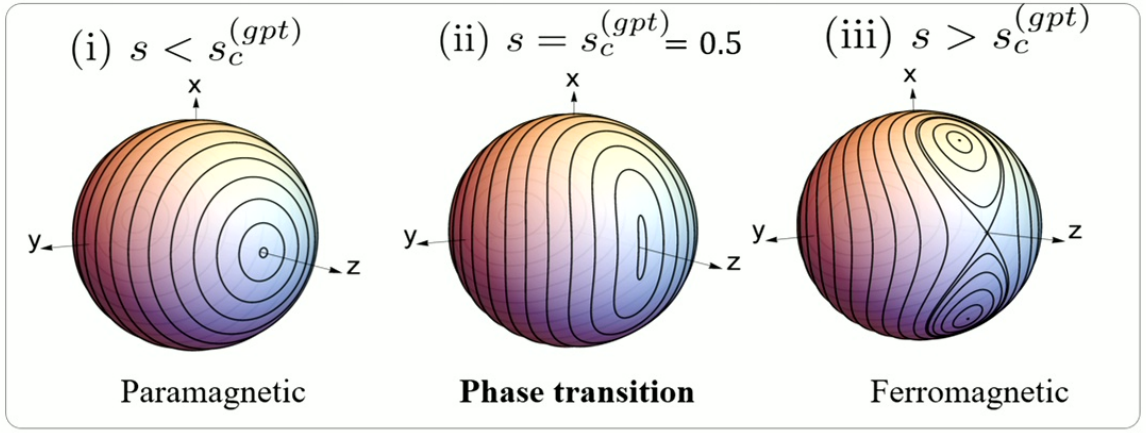

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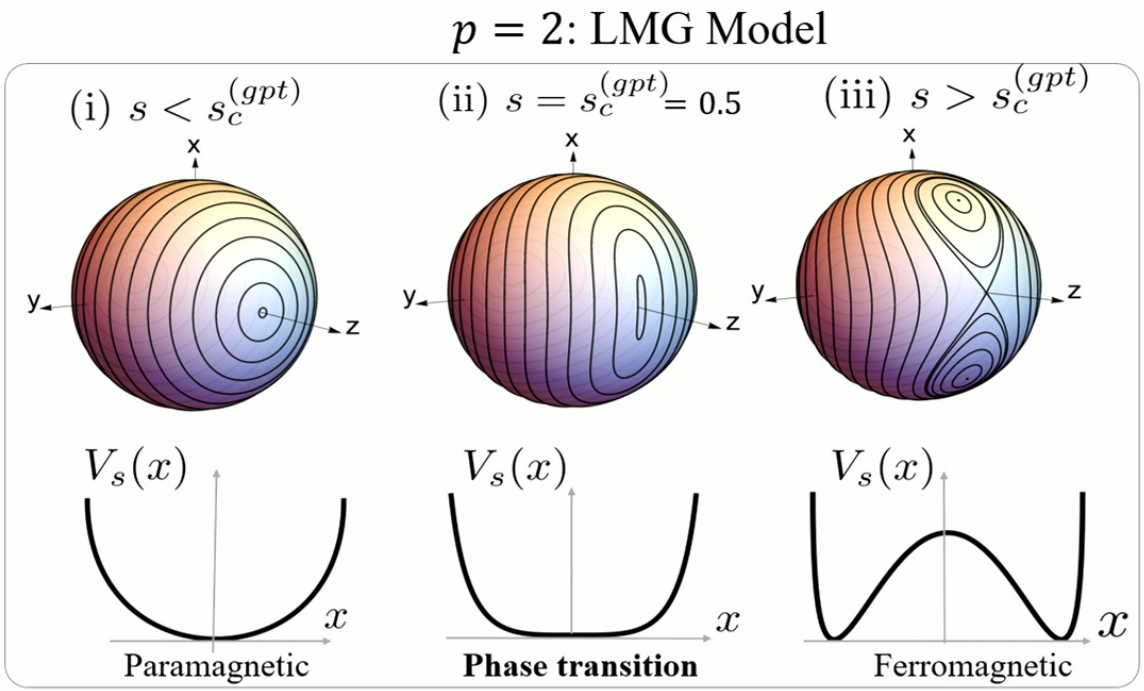
- Connection to double well using Schwinger representation.

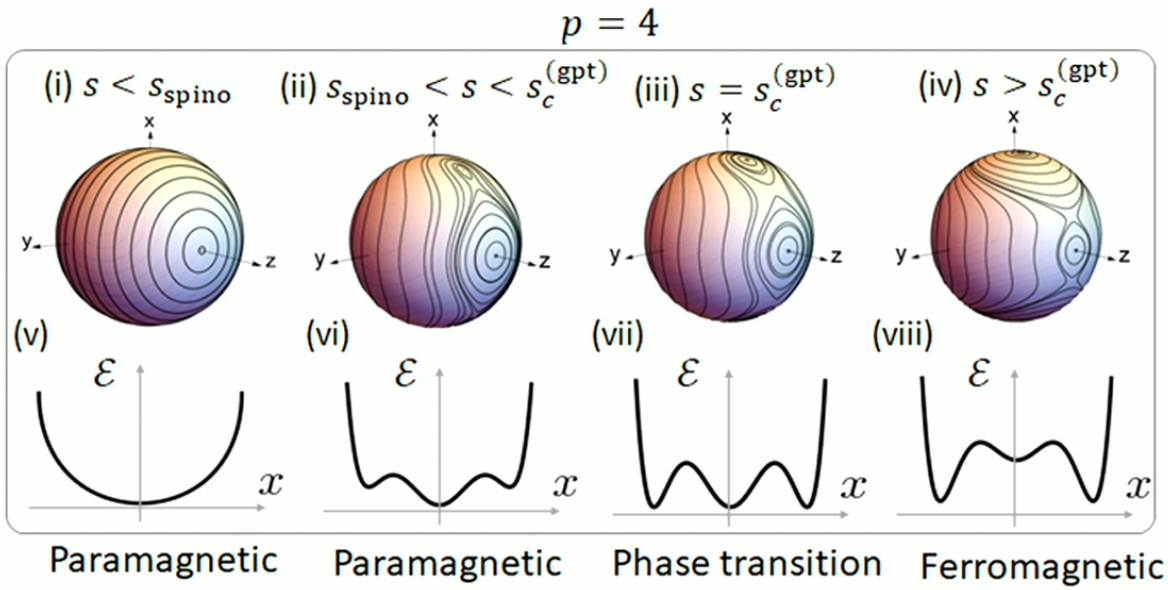


Higher p-spin models

- Mean-field equations of motion:

$$\dot{X} = (1-s)Y$$
$$\dot{Y} = -(1-s)X + sX^{p-1}Z$$
$$\dot{Z} = -sX^{p-1}Y$$

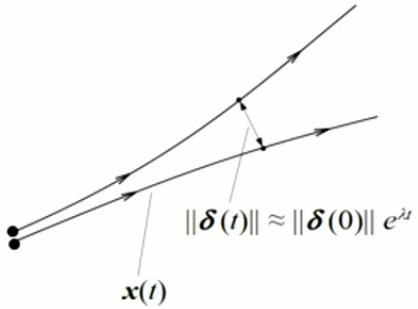
- All models with $p \geq 3$ have first-order GSQPT as opposed to second-order GSQPT for $p = 2$.



Chaos

- In classical mechanics, chaotic systems are characterized by exponential sensitivity to initial conditions.

$$\delta(t) = \delta(0)e^{\lambda t}$$



Trotter-Suzuki Decomposition in p-spin Hamiltonians

Target dynamics

$$H_{\text{tar}} \equiv H_z + H_x = -(1-s)J_z - \frac{s}{pJ^{p-1}}J_x^p$$

$$U_{\text{tar}} = e^{-iT(H_z+H_x)}$$

Simulated dynamics

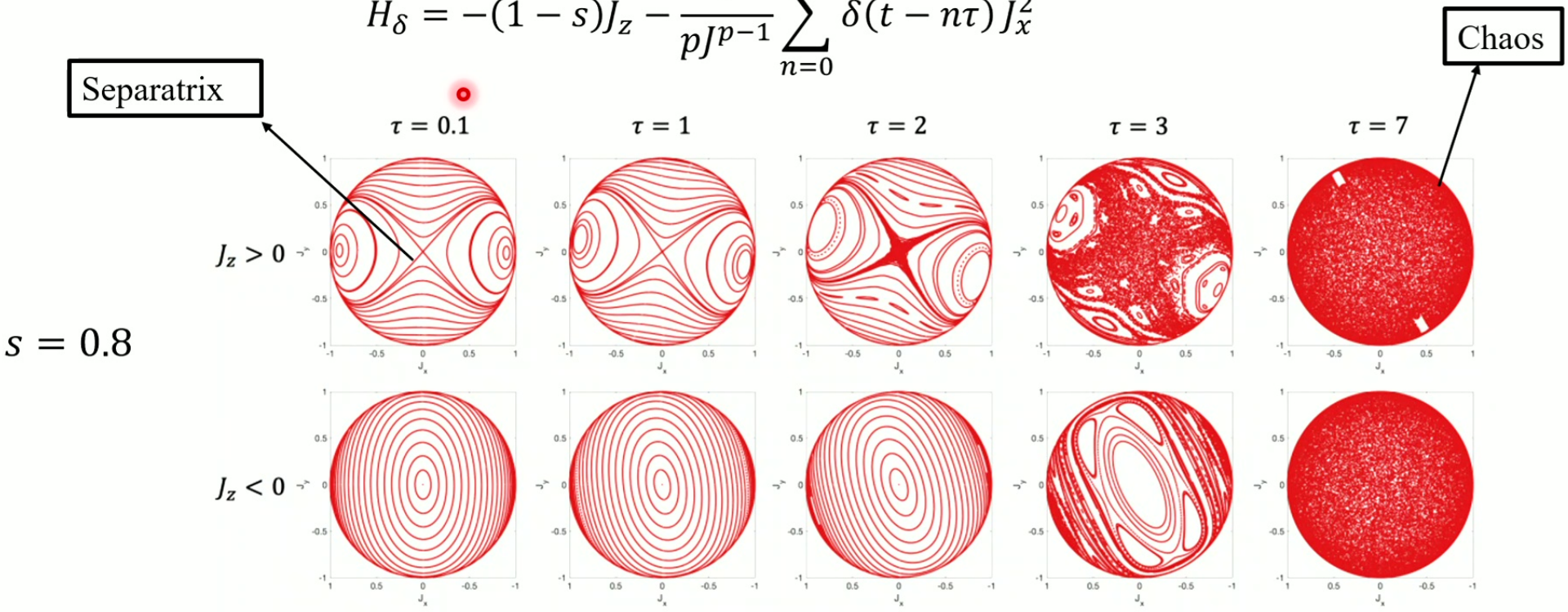
$$U_{\text{trot}} \equiv \left(\mathrm{U}_\delta(\tau)\right)^{\mathrm{n}} = \left(e^{iH_z\tau}\, e^{iH_x\tau} \right)^n$$

$$H_\delta(t) = H_{\text{tar}} + g(t)H_x$$

$$g_\tau(t) = \tau \sum_n \delta(t - n\tau) - 1$$

Poincare Sections

$$H_\delta = -(1-s)J_z - \frac{s}{pJ^{p-1}} \sum_{n=0}^{\infty} \delta(t - n\tau)\, J_x^2$$



Haake, Fritz, M. Kuś, and Rainer Scharf. *Zeitschrift für Physik B Condensed Matter* 65 (1987): 381-395.

Trotter Errors in $p = 2$ Hamiltonian

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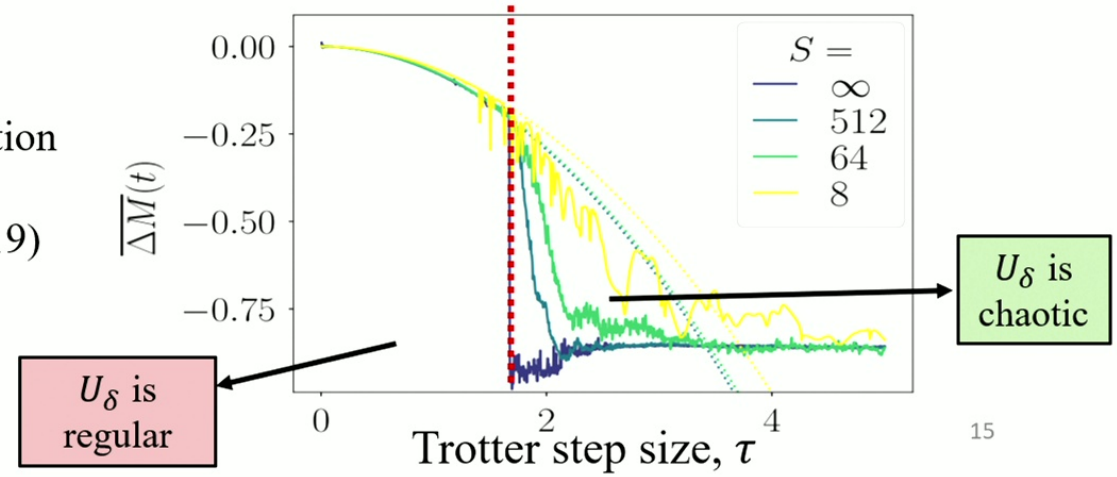
1. L. Sieberer *et al.*, NPJ Quantum Information **5**, 1-11 (2019)
2. M. Heyl *et al.,* Sci. Adv. **5** eaau8342 (2019)
3. C. Kargi *et al.,* arXiv: 2110.11113 (2021)



High Error Regions

$$\mathrm{U_{tar}} = e^{i\left((1-s)J_z + \frac{s}{pJ^{p-1}}J_x^p\right)T} \approx U_{\mathrm{trot}} = (U_\delta(\tau))^n$$

- **High-error regions**: determine the regions where the eigenstates of U_{tar} are very different from those of U_δ.

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- **High-error regions**: determine the regions where the eigenstates of U_{tar} are very different from those of U_δ.

- **Husimi distribution**:

$$Q_j(\theta,\phi) = \frac{2J+1}{4\pi}\left|\langle\, \theta,\phi|\xi_j\rangle\right|^2$$

$$U = \sum_j e^{i\xi_j}|\xi_j\rangle\langle\,\xi_j|$$

Structural Instability Regions

- **Average dissimilarity**: how different are the eigenstates of U_{tar} are from those of U_{δ}.

$$\mathrm{D}(U_{\mathrm{tar}}, U_{\delta}) = \frac{1 - \overline{\mathrm{IPR}}}{1 - \overline{\mathrm{IPR}}_{\mathrm{COE}}}$$

with average inverse participation ratio (IPR),

$$\overline{\mathrm{IPR}} = \frac{1}{d}\sum_{i,j}\left|\left\langle \xi_{\mathrm{tar}}^{(i)} \middle| \xi_{\delta}^{(j)} \right\rangle\right|^4$$

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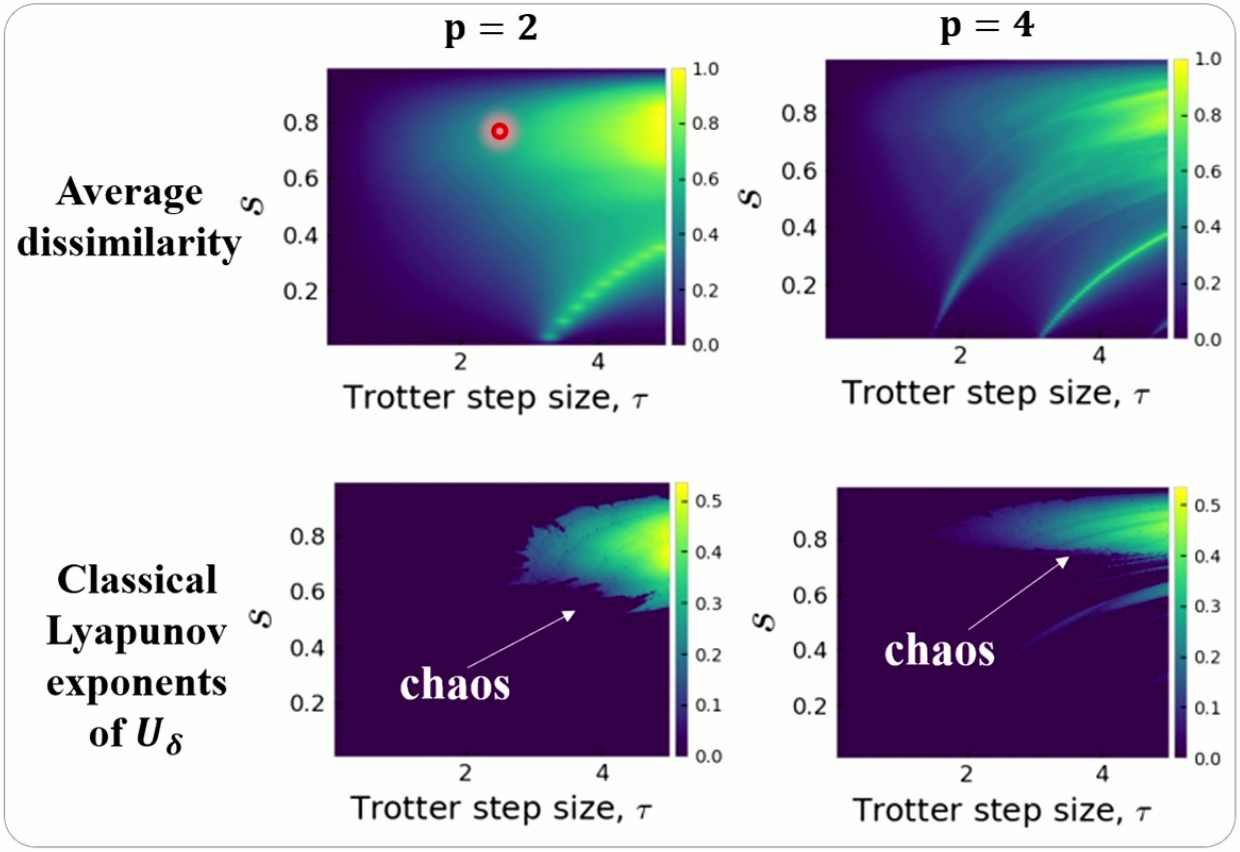

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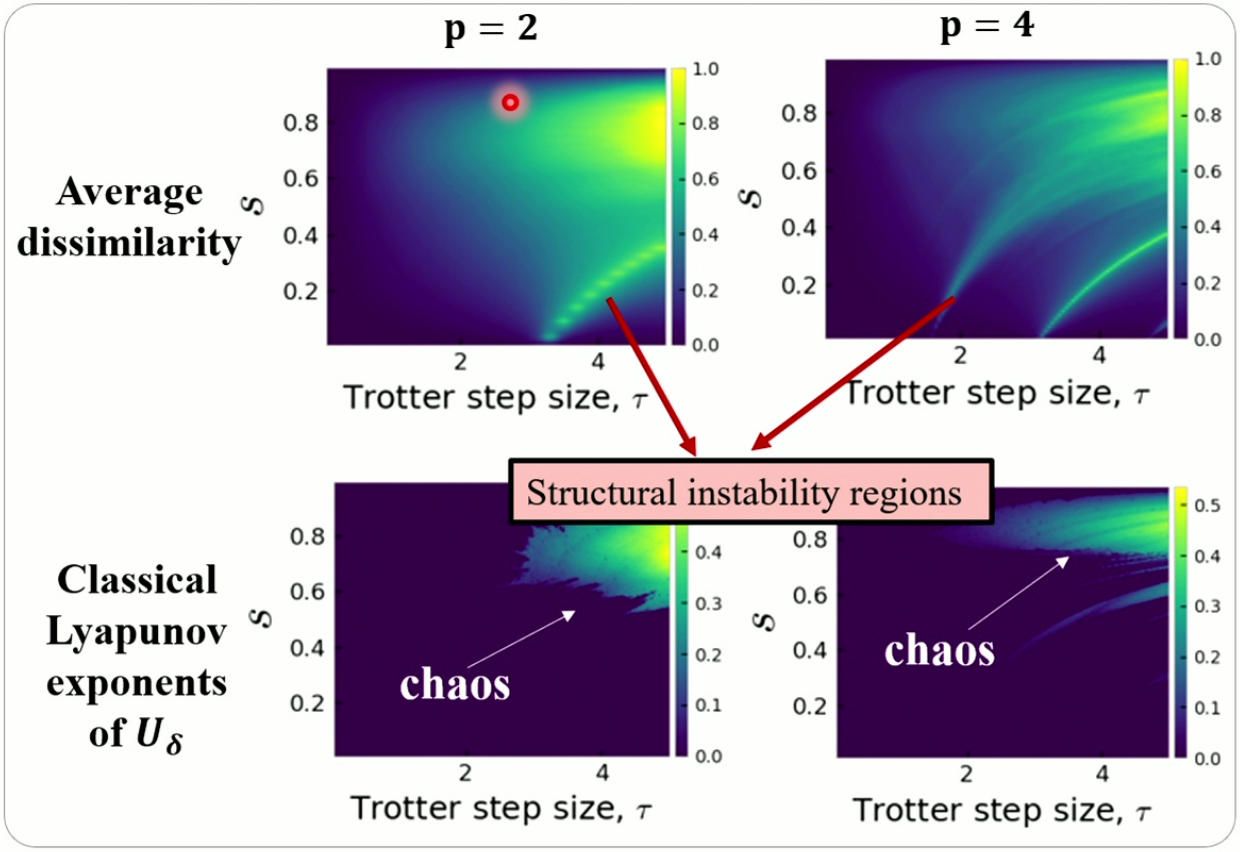

Unitary Perturbation Theory

- **Unitary perturbation theory** to identify points where the eigenvectors change significantly.

  For small s (paramagnetic), $U_\delta = e^{i(1-s)\tau J_z}\, e^{i\frac{s\tau}{pJ^{p-1}}J_x^p}$

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$$\langle J, m_z = -J + m' | \phi_m^{(1)} \rangle = \frac{i}{pJ^{p-1}e^{-i(1-s)\tau m'}} \frac{\left(J_x^p\right)_{m',m}}{\left(e^{i(1-s)\tau m} - e^{i(1-s)\tau m'}\right)}$$

- **Structural instability regions:** $\tau_{p,q}^* = \dfrac{2\pi}{(1-s)q}$ $\quad q = \{p, p-2, \ldots, 2(1)\}$

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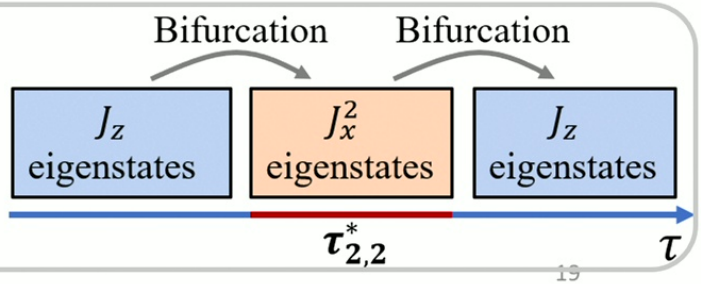
- **Structural instability regions:** $\tau^*_{p,q} = \dfrac{2\pi}{(1-s)q}$ $\quad q = \{p, p-2, \ldots, 2(1)\}$

- $\boldsymbol{p = 2}$: $\tau^*_{2,2} = \dfrac{\pi}{1-s}$ where $U_\delta = e^{i\pi J_z}\, e^{i\frac{s\tau^*}{pJ^{p-1}}J_x^2}$

  eigenstates are $\frac{1}{\sqrt{2}}|J, m_x\rangle \pm \frac{1}{\sqrt{2}}|J, -m_x\rangle$



Magnetization Errors

- **Error in long-time average of J_z** $(s \ll 1)$

$$\mathcal{E}_z^{\infty}(\tau) = \frac{1}{J}\left|\overline{\langle J_z\rangle}_{\mathrm{tar}} - \overline{\langle J_z\rangle}_{\mathrm{trot}}\right|$$

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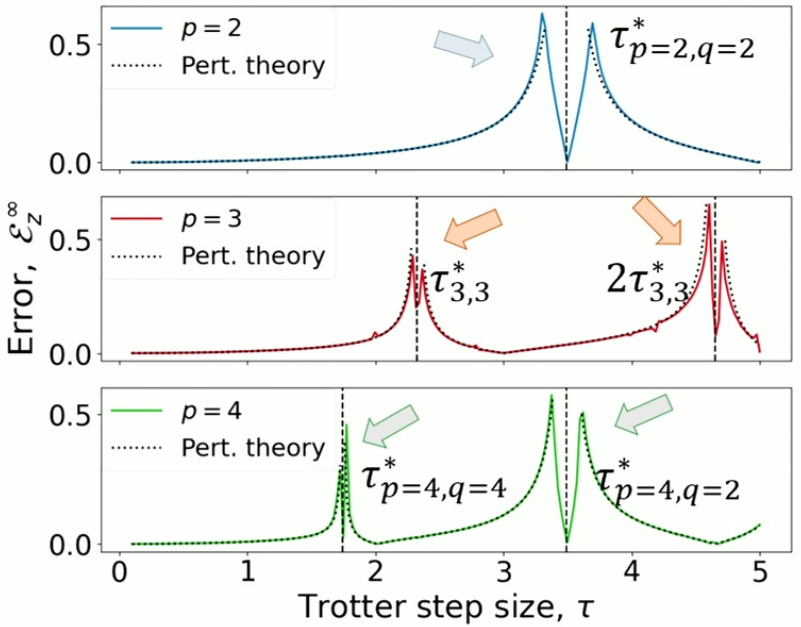
- Perturbation theory expression for error

$$\mathcal{E}_z^{\infty}(\tau) = \left|\sum_{q=p,p-2,\ldots 2(1)} \frac{sq}{p\,J^p}\left[\frac{2}{q(1-s)} - \tau\cot\left(\frac{q(1-s)\tau}{2}\right)\right] f_q(\rho^{(0)}, p)\right|$$



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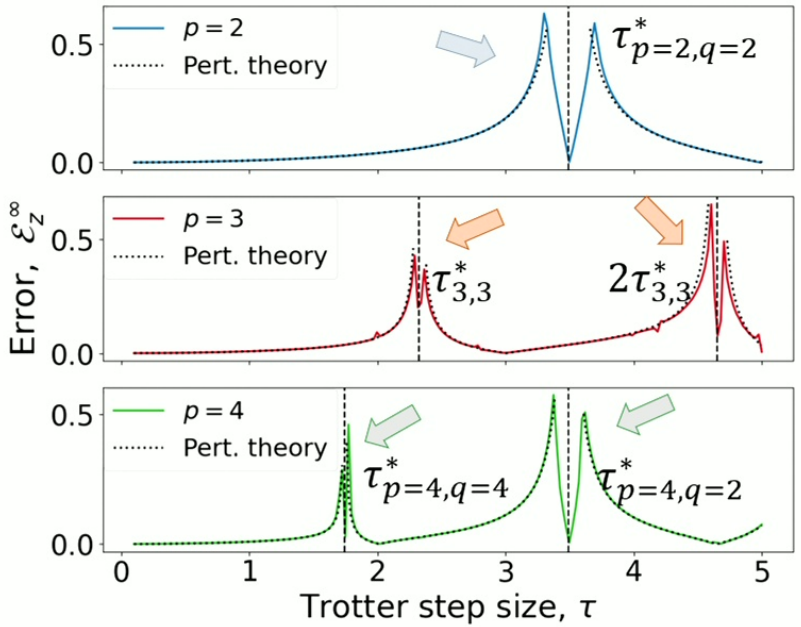
  - Structural instability region → large errors
  - As p is increased, large error regions occur at smaller values of τ.
  - Tradeoff: width of the error region $\propto \frac{1}{\tau^*}$



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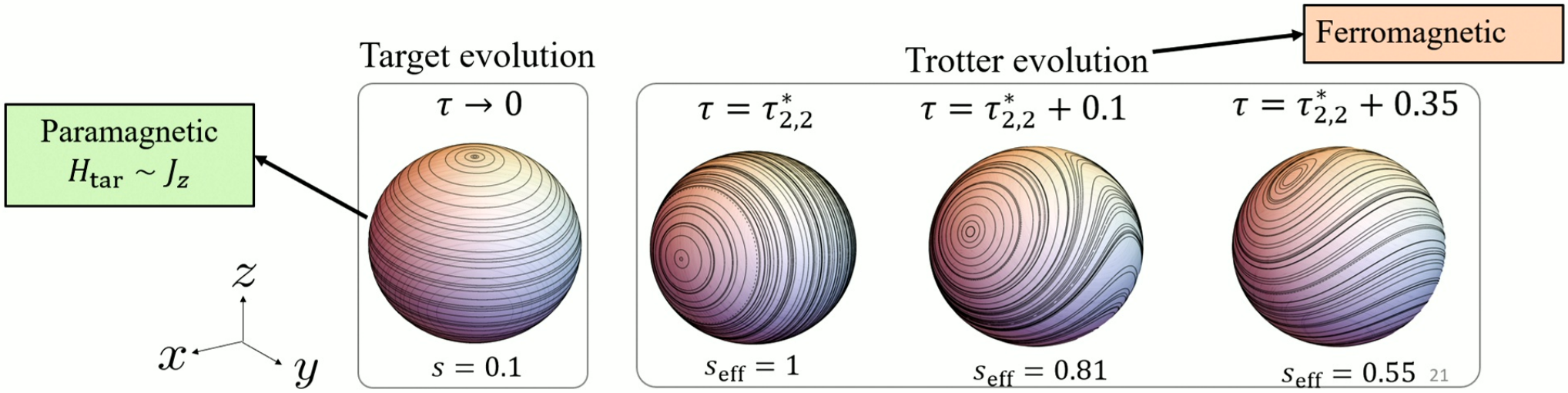
Effective Hamiltonian

- **For small s (paramagnetic)**, around the instability regions ($\tau = \tau^*_{p,q} + \Delta\tau$)

$$(\mathrm{U}_\delta(\tau))^q \equiv e^{-iH_{\mathrm{eff}}^{(p,q)} q\tau}$$

- $\boldsymbol{p = 2}$: $(\mathrm{U}_\delta)^2 = e^{-i2(\tau^*_{2,2}+\Delta\tau)H_{\mathrm{eff}}^{(2,2)}}$ with $H_{\mathrm{eff}}^{(2,2)} = -(1 - s_{\mathrm{eff}})\, J_z - \frac{s_{\mathrm{eff}}}{2J} J_x^2$ where $s_{\mathrm{eff}} > 0.5$



Effective Hamiltonian

$$(\mathrm{U}_\delta(\tau))^q \equiv e^{-iH_{\mathrm{eff}}^{(p,q)} q\tau}$$

- In the general p-spin model and given q,

$$H_{\mathrm{eff}}^{(p,q)} = -(1-s)\frac{\Delta\tau}{\tau_{p,q}^* + \Delta\tau} J_z - \frac{s}{pq\, J^{p-1}} \sum_{n=0}^{q-1} \left(J_x \cos\left(n\frac{2\pi}{q}\right) + J_y \sin\left(n\frac{2\pi}{q}\right)\right)^p$$

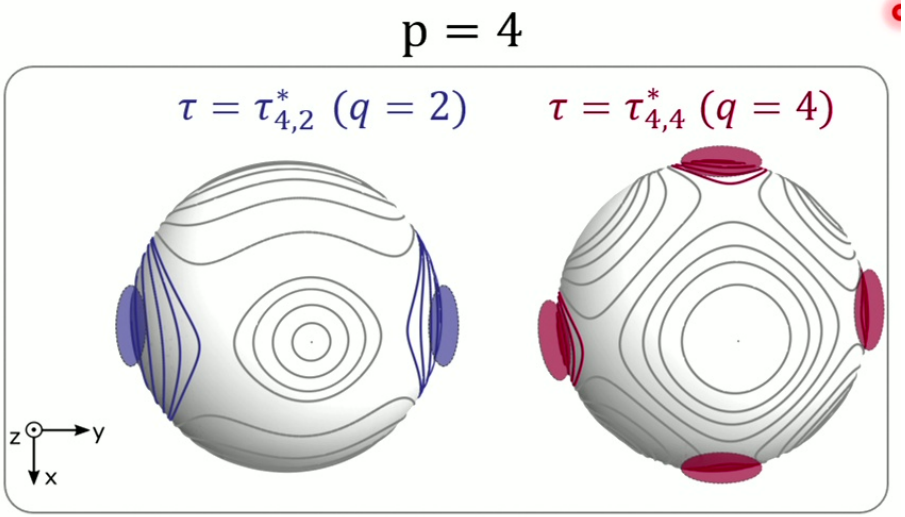
where $\tau_{p,q}^* = \frac{2\pi}{(1-s)q}$, with $q = \{p, p-2, \ldots 2(1)\}$

Subharmonic Response

$$H = -(1-s)J_z - \frac{s\tau}{pJ^{p-1}} \sum_{n=0}^{\infty} \delta(t - n\tau)\, J_x^p$$

- Time translation symmetry: $H(t+\tau) = H(t)$
- Subharmonic response: $f_O(t + m\tau) = f_O(t)$ with $m > 1$

  where $f_O(t) = \langle \psi(t)|O|\psi(t)\rangle$



Muñoz-Arias, Manuel H., Karthik Chinni, and Pablo M. Poggi. *Physical Review Research* 4.2 (2022): 023018.

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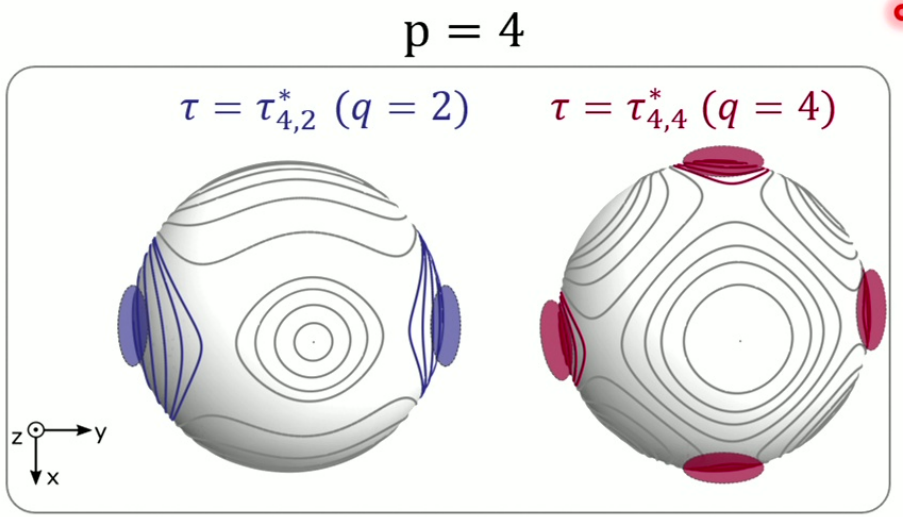
  where $f_O(t) = \langle \psi(t) | O | \psi(t) \rangle$
- Behavior robust to small variations in parameters and the behavior persists for infinitely long times.



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Scrambling

$$H = -(1-s)J_z - \frac{s\tau}{pJ^{p-1}} \sum_{n=0}^{\infty} \delta(t - n\tau)\, J_x^p$$

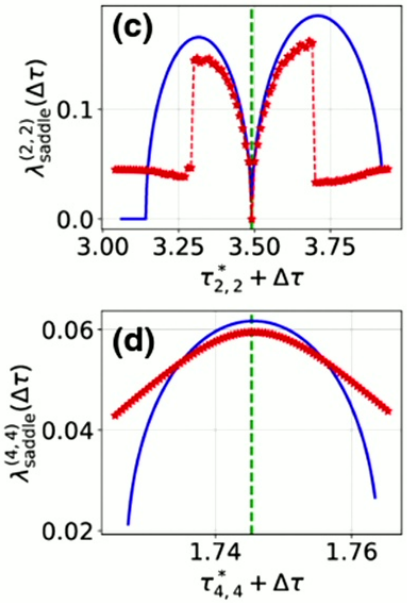
- Infinite temperature square commutator:

$$c(t) = \frac{1}{N+1} \langle |[J_z(t), J_z(0)]|^2 \rangle$$

$\langle . \rangle$ represents infinite-temperature thermal average.

- Saddle point scrambling

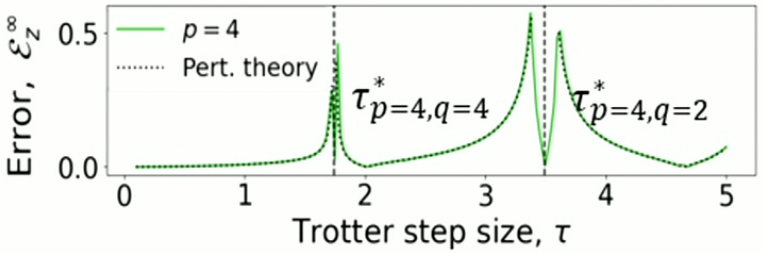
- In chaotic regime and in the presence of saddle points, $c(t) \sim e^{\lambda_{\text{saddle}} t}$.



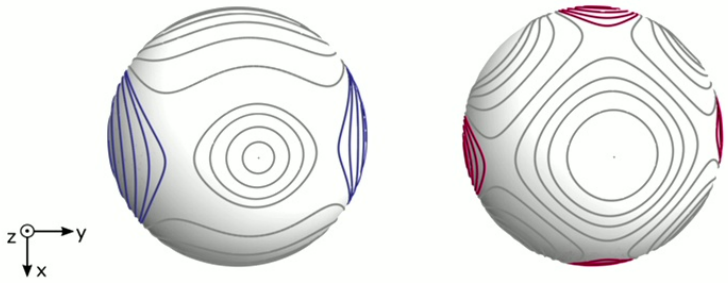
$N = 128$

Summary

- Found a new physical mechanism that leads to Trotter error proliferation.
- In this region, Trotterized dynamics is very different from the target dynamics.
- We characterize structural instability regions using unitary perturbation theory and relate them to degeneracies.



- **Time-crystal like behavior**: in these high error regions system develops a subharmonic response, showing time-crystal like behavior.



Thank you

$$H_{\text{tar}} = \sum_{\gamma=1}^{\Gamma} H_k$$

$$\mathscr{S}_2(t) := e^{-i\frac{t}{2}H_1} \cdots e^{-i\frac{t}{2}H_\Gamma} e^{-i\frac{t}{2}H_\Gamma} \cdots e^{-i\frac{t}{2}H_1}$$

$$H = B_z \sum_i \sigma_z^{(i)} + \sum_{i<j} \frac{J_0}{|i-j|^\alpha} \sigma_x^{(i)} \sigma_x^{(j)}$$

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