Title: Non-vector-bundle Thom spectra and applications to anomalies

Speakers: Arun Debray

Series: Mathematical Physics

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Abstract: There is a by now standard procedure for calculating twisted spin, spin^c, and string bordism groups for applications in physics: realize the twist as arising from a vector bundle, which allows one to split the corresponding Thom spectrum and greatly simplify the Adams spectral sequence computation. Not all twists arise from vector bundles, but Matthew Yu and I noticed that if you ignore this fact and pretend that everything is OK, you still get the right answer! In this talk, I'll discuss a theorem Matthew and I proved explaining this, by calculating the input to Baker-Lazarev's version of the Adams spectral sequence. Then I will discuss applications to anomalies of some quantum field theories and supergravity theories.

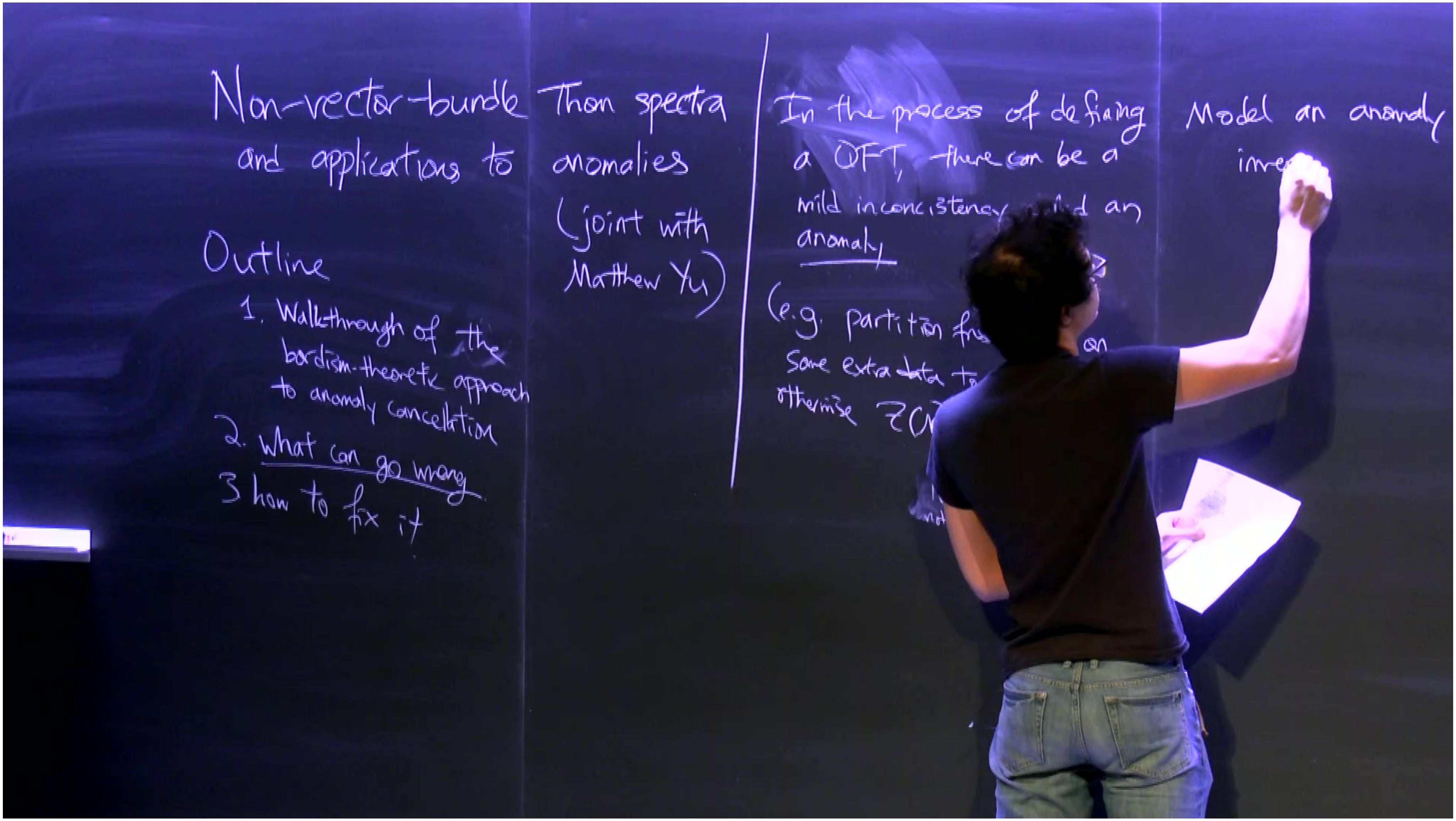
Zoom link: https://pitp.zoom.us/j/93508575689?pwd=YVV6VlRwL1RGSG55V0cwTzdpUWROUT09

Non-vector-bundle Thom spectra
and applications to anomalies

(joint with
Matthew Yu)

Outline

1. Walkthrough of the
bordism-theoretic approach
to anomaly cancellation
2. what can go wrong
3 how to fix it


Non-vector-bundle Thom spectra
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1. Walkthrough of the bordism-theoretic approach to anomaly cancellation
2. What can go wrong
3 how to fix it
In the process of defining a QFT, there can be a
mild inconsistency
anomaly
Model an anomaly

In the process of defining a QFT, there can be a mild inconsistency called an anomaly

(e.g. partition fns depend on some extra data to get a # — otherwise $Z(M) \in \alpha(M)$)

1d cpx vs not trivialized

Model an anomaly of a QFT Z as an invertible field theory in 1 dim higher.

In the process of defining a QFT, there can be a mild inconsistency called an <u>anomaly</u>

(e.g. partition fns depend on some extra data to get a # — otherwise $Z(M) \in \alpha(M)$)

1d cpx vs not trivialized

Model an anomaly of a QFT Z as an <u>invertible field theory</u> α in 1 dim higher.

idea: $\alpha(M)$ is the state space of α on M

<u>Defn</u> (Freed–Moore) A field theory

$$\alpha : \mathrm{Bord}_{n+1} \longrightarrow \mathrm{Vect}$$

is <u>invertible</u> if $\exists\, \alpha^{-1} : \mathrm{Bord}_{n+1} \longrightarrow \mathrm{Vect}$

s.t. $\alpha \otimes \alpha^{-1} \simeq \mathbb{1}$ (trivial theory)

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(stacking)

(almost but not quite trivial)

In many cases, we want the

as an
dim higher.

a on M

→ Vect

theory)

In many cases, we want the anomaly to be trivial.

One approach: invertible field theories in a given dim form an abelian gp (under ⊗)

so: maybe this gp is 0 ⇒ no anomaly

or we can use structure of this gp to assist in anomaly cancellation

→ follows a fairly standard path.

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In many cases, we want the anomaly to be <u>trivial</u>.

One approach: invertible field theories in a given dim form an abelian gp (under $\otimes$)

so: maybe this gp is 0 $\Rightarrow$ no anomaly

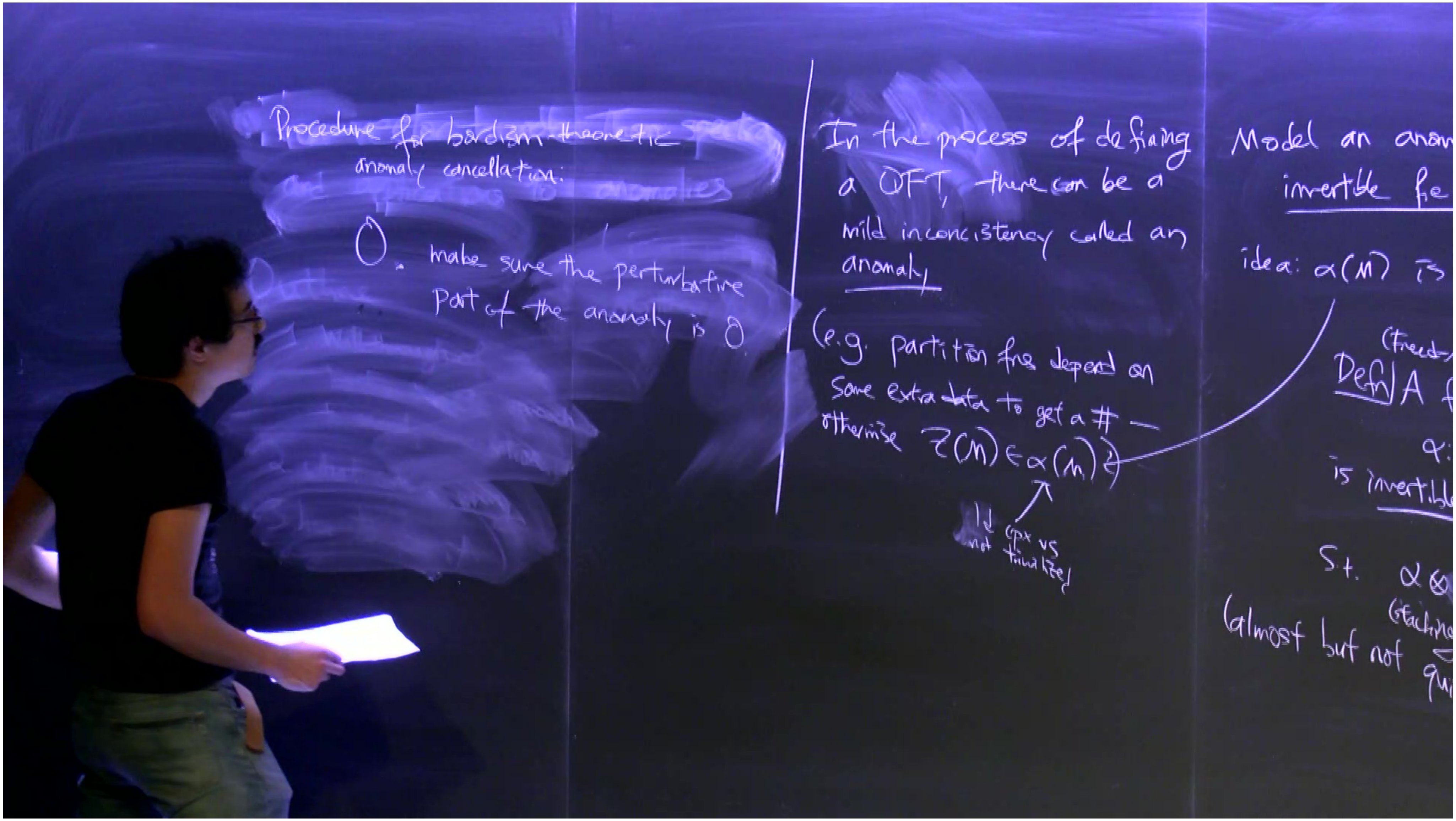
or we can use structure of this gp to assist in anomaly cancellation

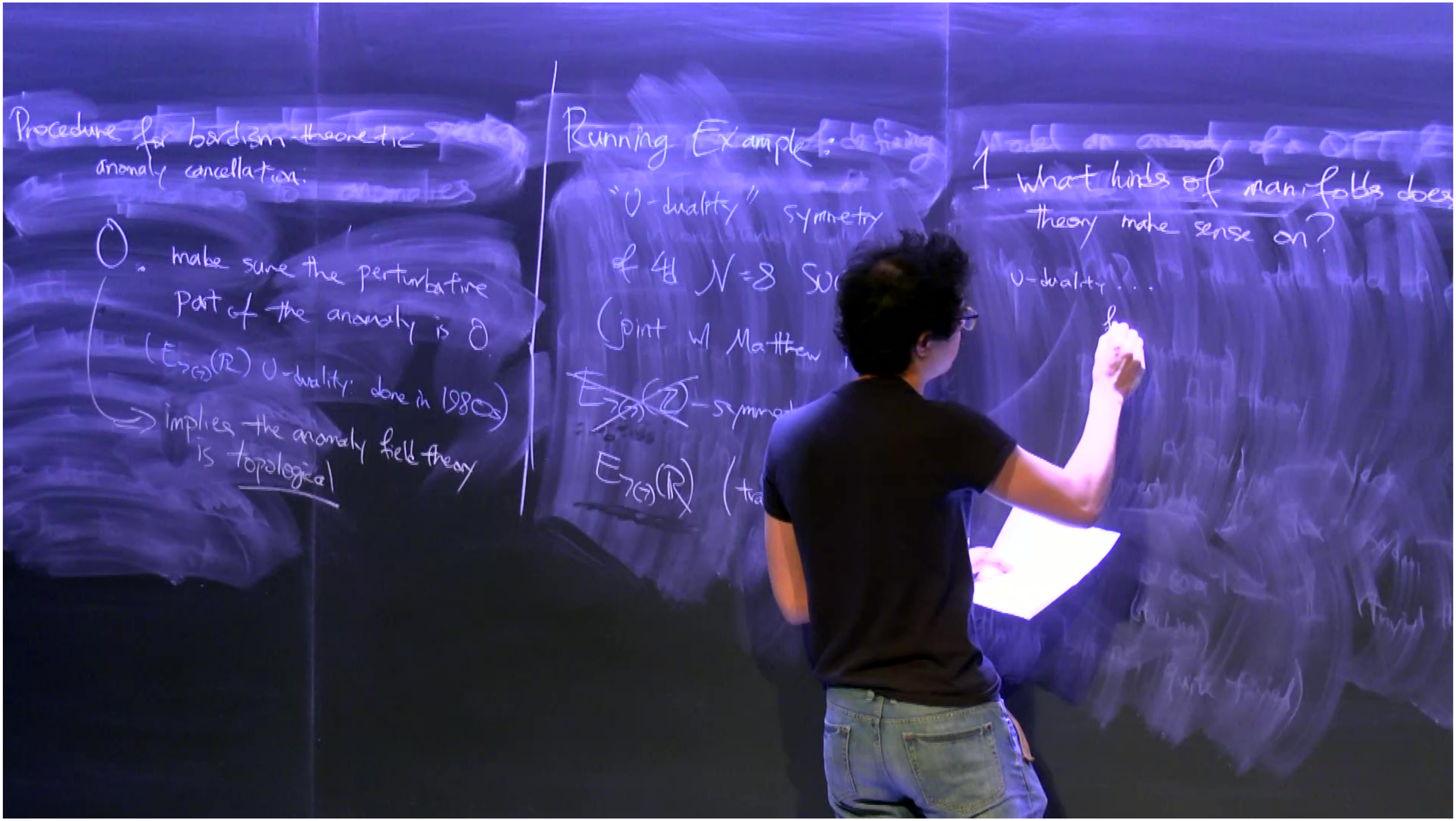
→ follows a fairly standard path.

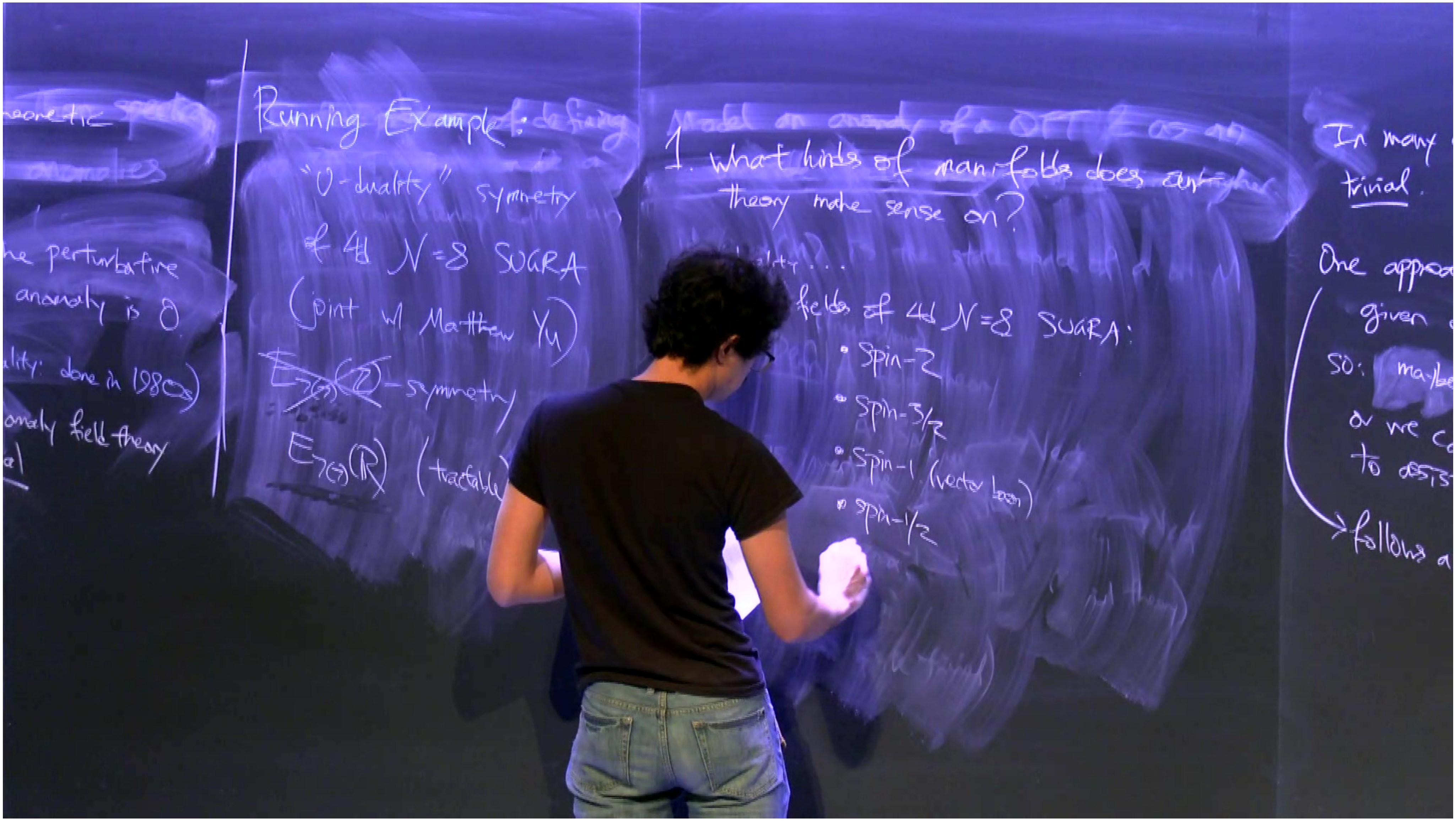
Key fact: <u>Freed-Hopkins-Teleman, Freed-Hopkins</u>

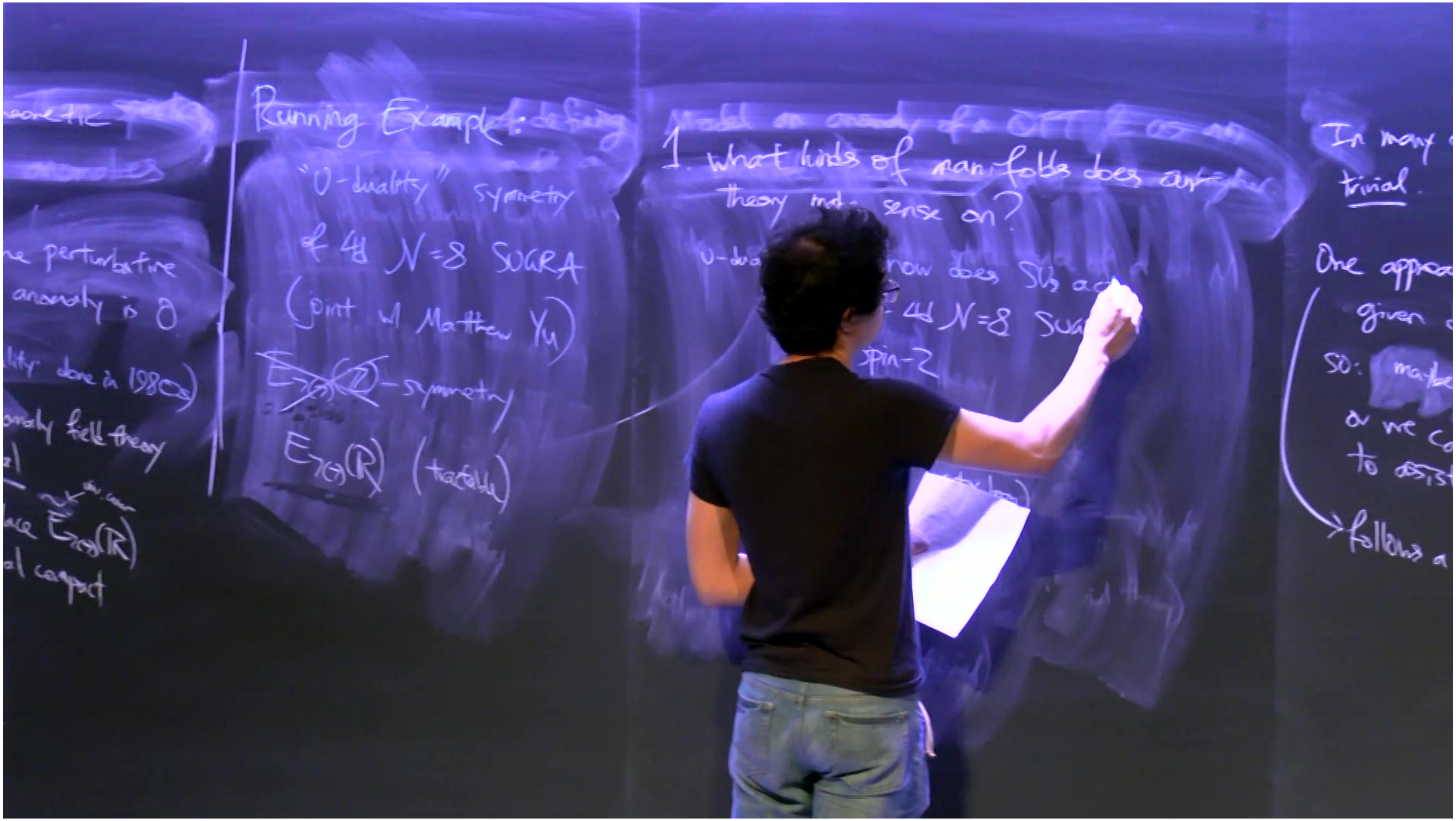
The ab gp of invertible field theories in dim n is isomorphic to a group built from <u>bordism</u> classes of n, $n+1$ -mflds.

(upshot: amenable to standard alg top tools)


Procedure for bordism-theoretic anomaly cancellation:
0. make sure the perturbative part of the anomaly is 0.
In the process of defining a QFT, there can be a mild inconsistency called an anomaly
(e.g. partition fns depend on some extra data to get a # — otherwise Z(M) ∈ α(M))
Cpx vs not trivialized
Model an
invertible
idea: α(M) is
Defn A
is invertible
s.t.
(almost but not


Procedure for bordism-theoretic
anomaly cancellation:
0. make sure the perturbative
part of the anomaly is 0.
(E7(7)(R) U-duality: done in 1980s)
implies the anomaly field theory
is topological
Running Example:
"U-duality" symmetry
of 4d N=8 SU
(joint w/ Matthew
E7(7)(Z)-symm
E7(7)(R) (tra
1. what kinds of manifolds does
theory make sense on?
U-duality?...


Running Example:
"U-duality" symmetry
of 4d N=8 SUGRA
(joint w/ Matthew Yu)
E7(7)(R) (tractable)
1. What kinds of manifolds does a theory make sense on?
fields of 4d N=8 SUGRA:
• Spin-2
• Spin-3/2
• Spin-1 (vector boson)
• Spin-1/2
the perturbative anomaly is 0.
anomaly field theory
In many
trivial.
One approa
given
so:
or we c
to assis
follows a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1. what kinds of manifolds does
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4d N=8 SUGRA
In many
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defining

mmetry

UGRA

Y_4)

1. what kinds of manifolds does any ... theory make sense on?

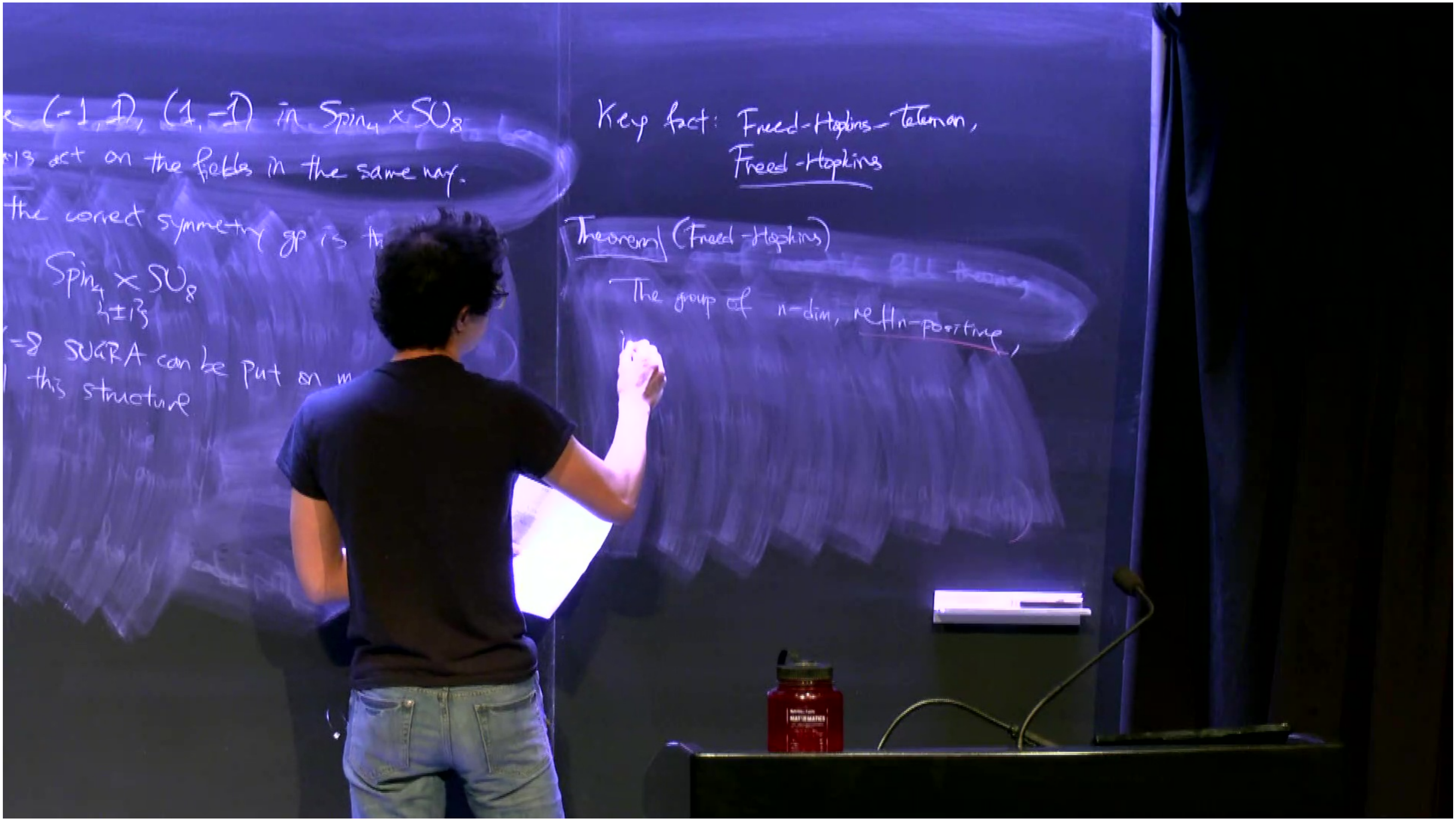
U-duality? ... how does SU_8 act?

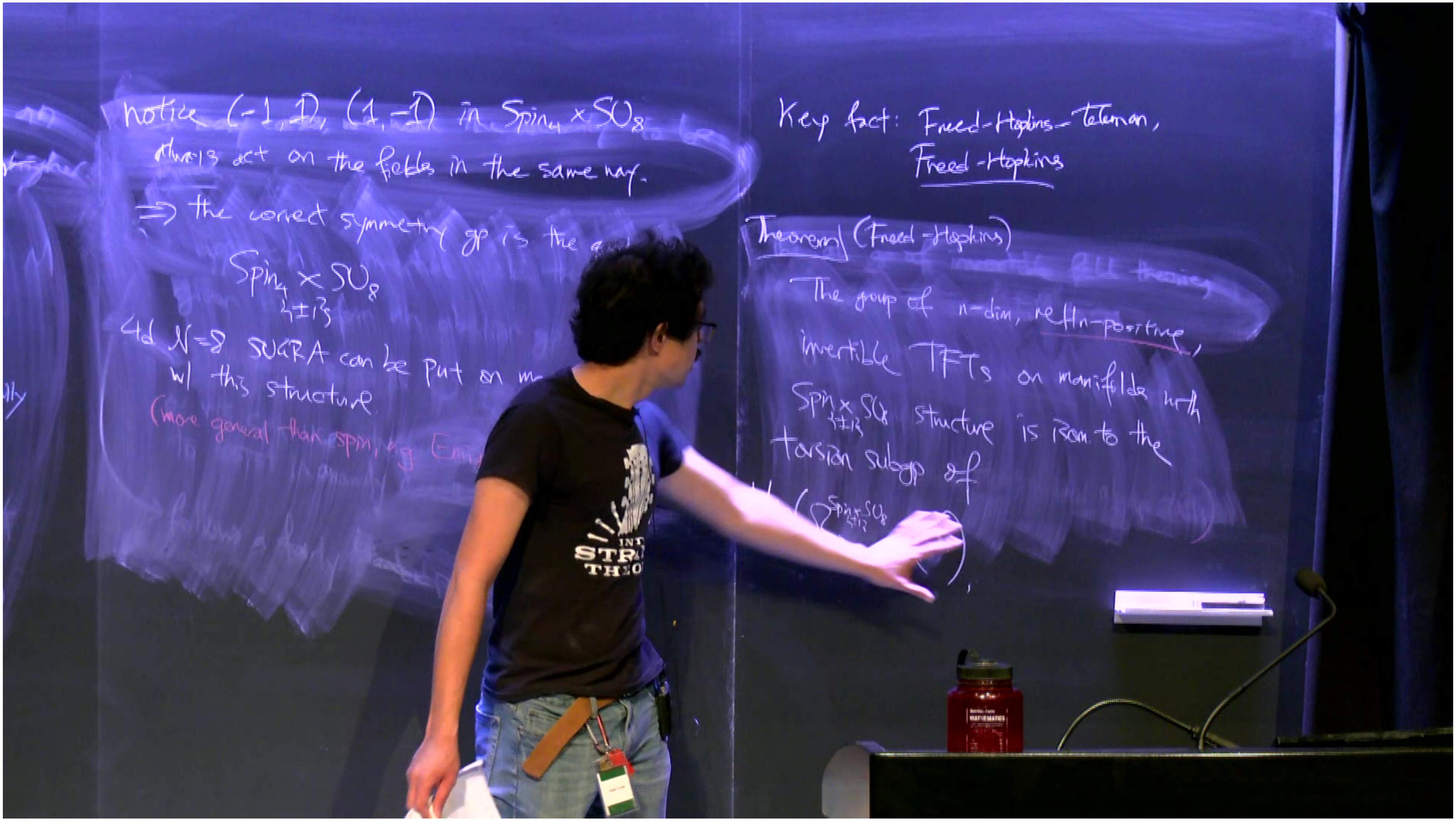
fields of 4d $\mathcal{N}=8$ SUGRA:

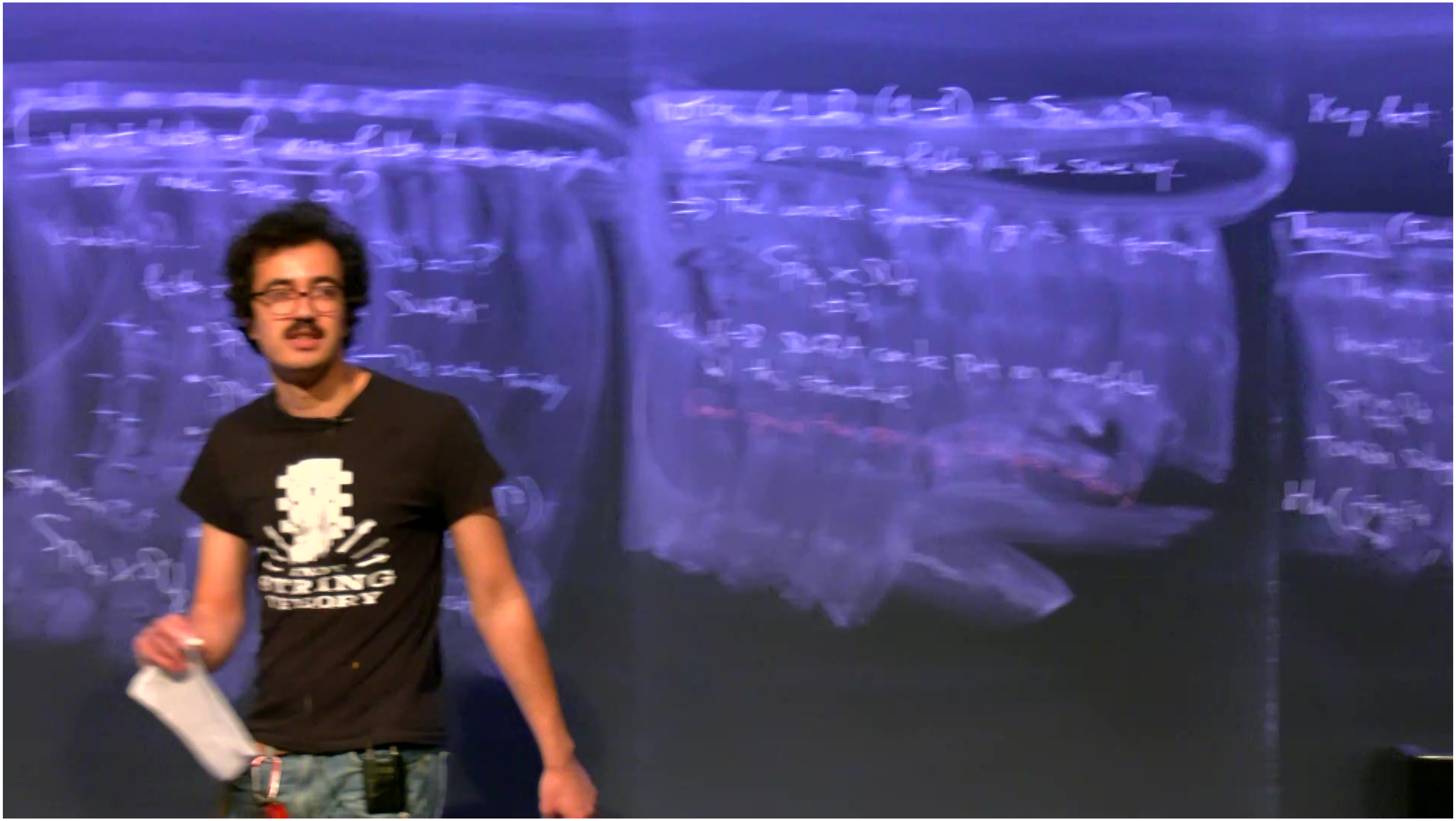
- spin-2 $\longleftarrow$ SU_8 acts trivially
- spin-3/2 $\longleftarrow \Lambda^3(\mathbb{C}^8)$
- spin-1 (vector boson) $\longleftarrow \Lambda^2(\mathbb{C}^8)$
- spin-1/2 $\longleftarrow \mathbb{C}^8$
- scalar field $\longleftarrow$ coset...

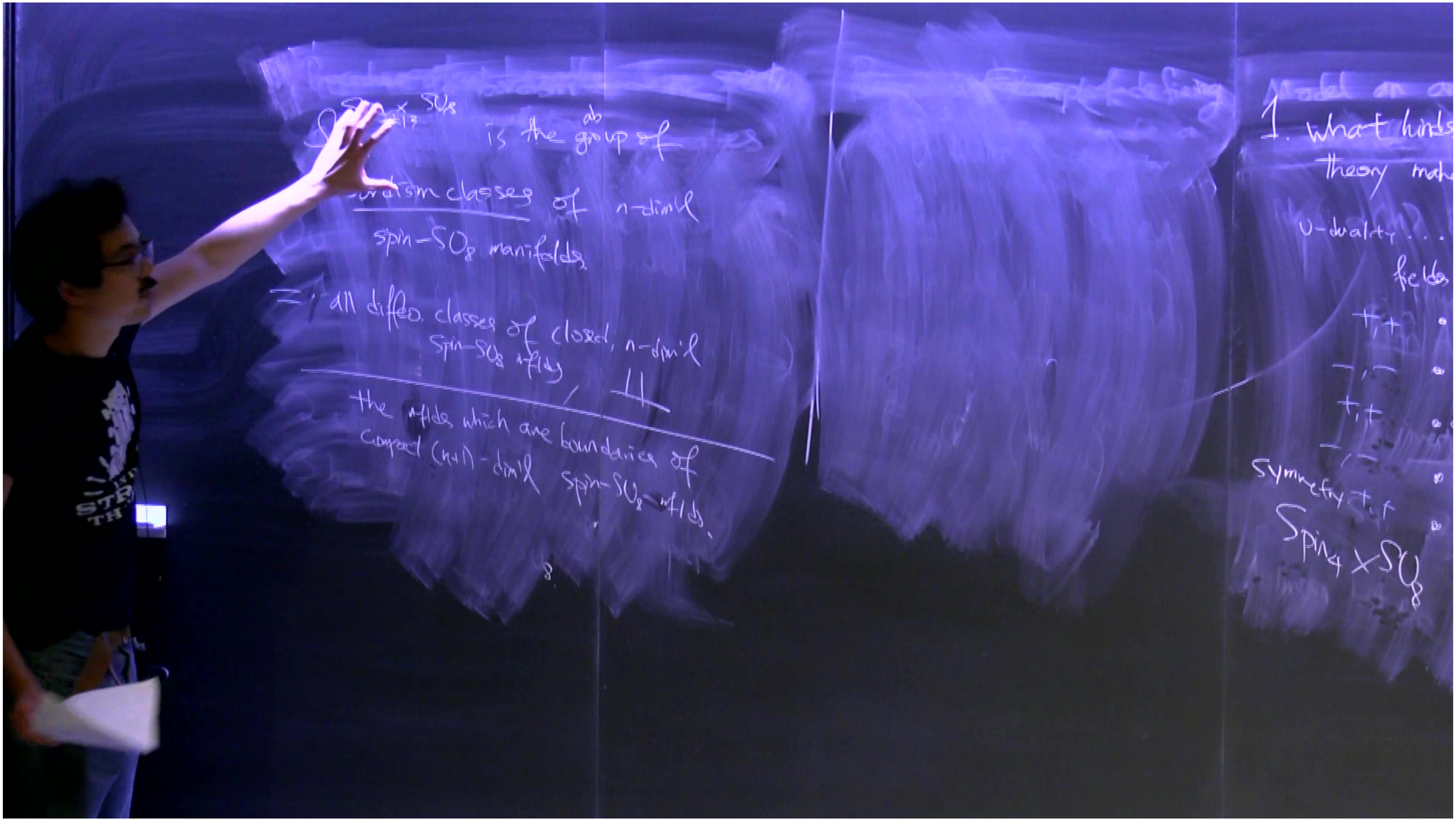
symmetry: $\mathrm{Spin}_4 \times SU_8$

notice $(-1,1)$, $(1,-1)$ in $\mathrm{Spin}_4 \times SU_8$ always act on the fields in the same way.


(-1,1), (1,-1) in $Spin_4 \times SU_8$
act on the fields in the same way.
the correct symmetry gp is
$Spin_4 \times SU_8$
$\{\pm 1\}$
=8 SUGRA can be put on m
this structure
Key fact: Freed-Hopkins-Teleman,
Freed-Hopkins
Theorem (Freed-Hopkins)
The group of n-dim, refl-positive,


notice (-1,1), (1,-1) in Spin4 × SU8
always act on the fields in the same way.
⇒ the correct symmetry gp is the
4d N=8 SUGRA can be put on
w/ this structure
(more general than spin, e.g.
Key fact: Freed-Hopkins-Teleman,
Freed-Hopkins
Theorem (Freed-Hopkins)
The group of n-dim, reflection-positive,
invertible TFTs on manifolds with
structure is iso. to the
torsion subgp of


STRING
THEORY


is the group of
bordism classes of n-dim'l
spin-SU8 manifolds
= all diffeo. classes of closed, n-dim'l
spin-SU8 mflds
the mflds which are boundaries of
compact (n+1)-dim'l spin-SU8 mflds.
1. what hints
theory
U-duality
fields
symmetry
Spin4 × SU8

$[S^1] = 0$

how do we compute $\Omega_5^{\mathrm{Spin}_4 \times_{\{\pm 1\}} SU_8}$?

(shearing)

notice $(-1,1)$, $(1,-1)$

always act on the fields

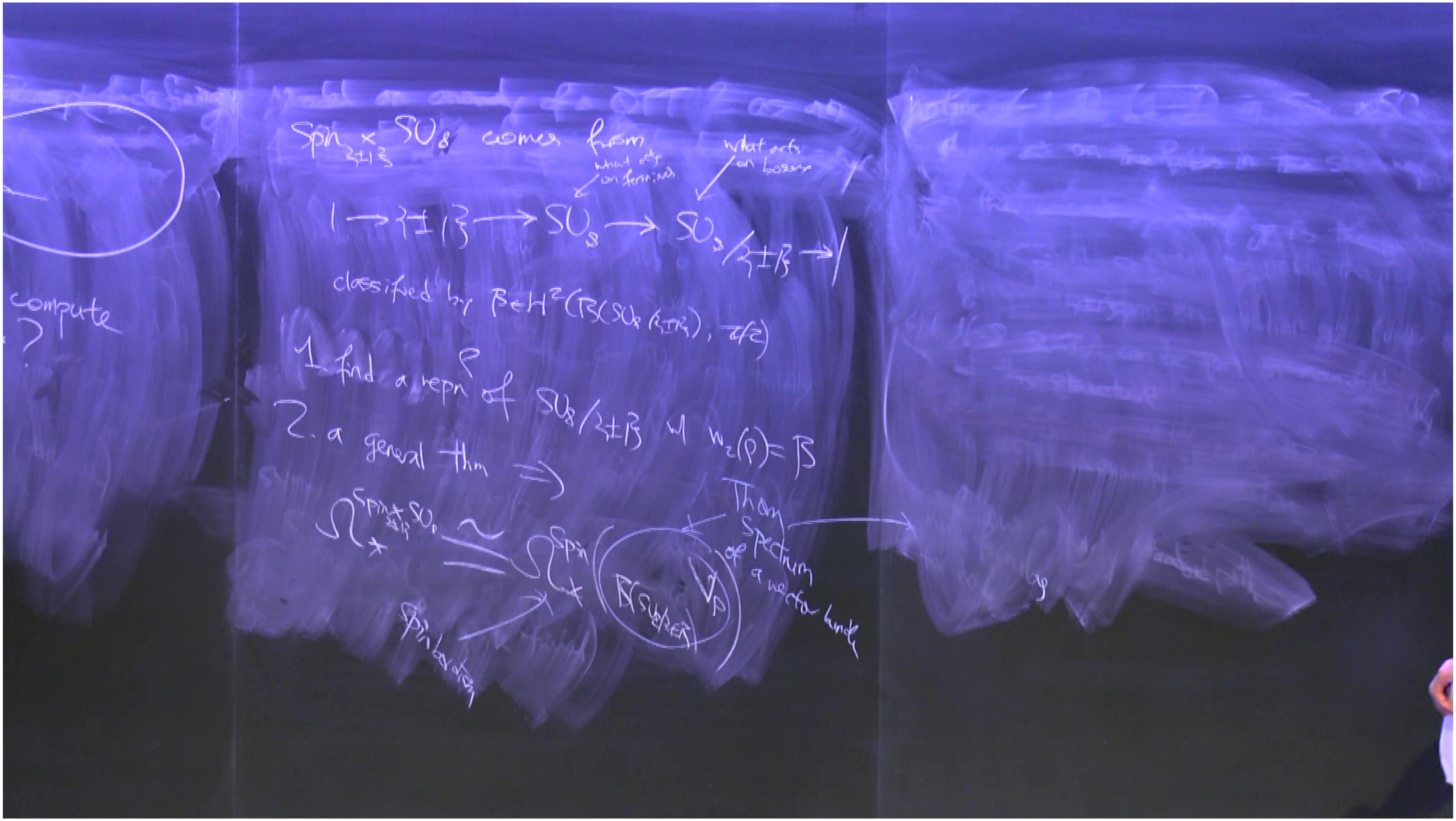
$\Rightarrow$ the correct symmetry

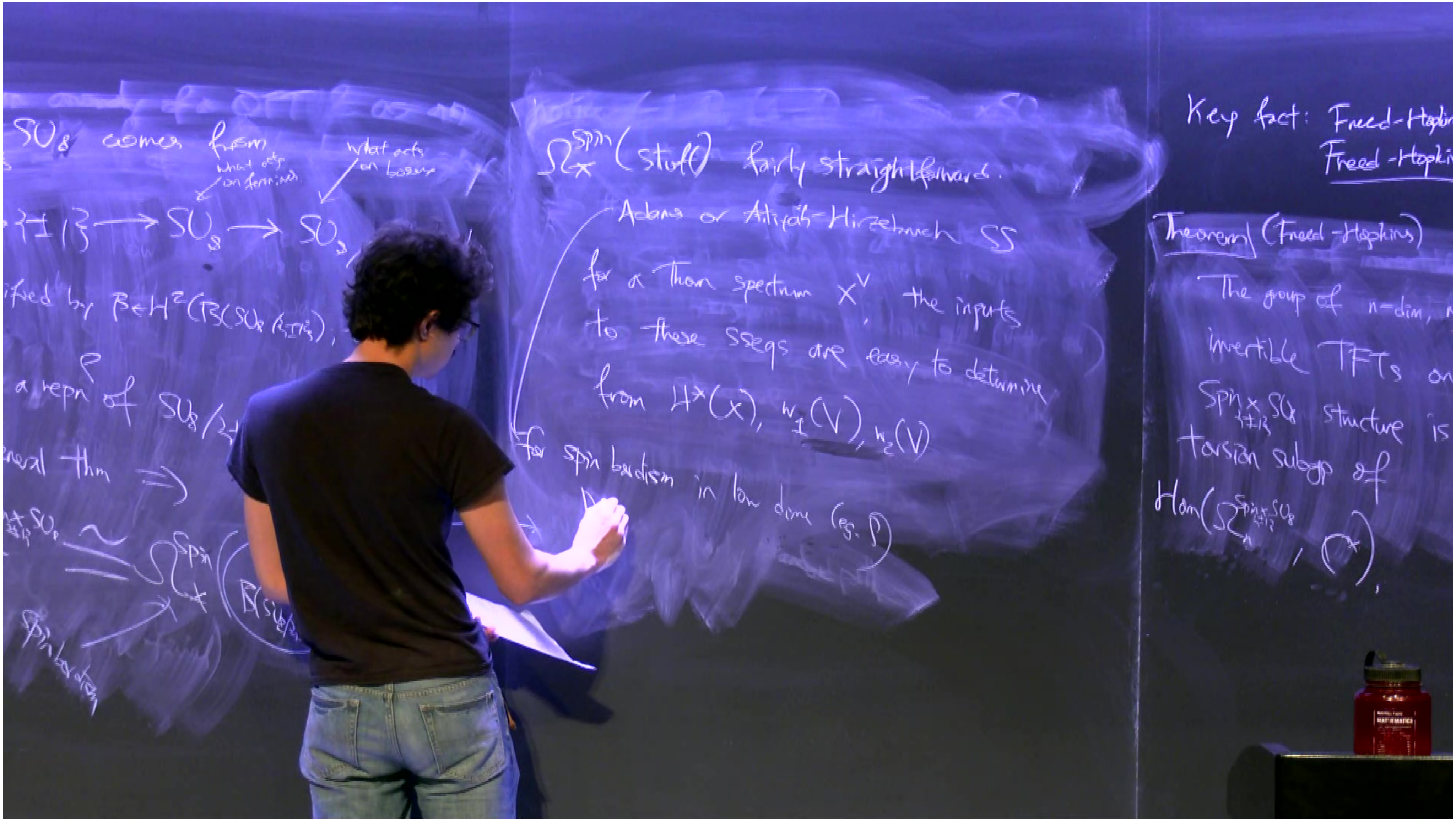
$$\mathrm{Spin}_4 \times_{\{\pm 1\}} SU_8$$

4d $\mathcal{N}=8$ SUGRA can be put

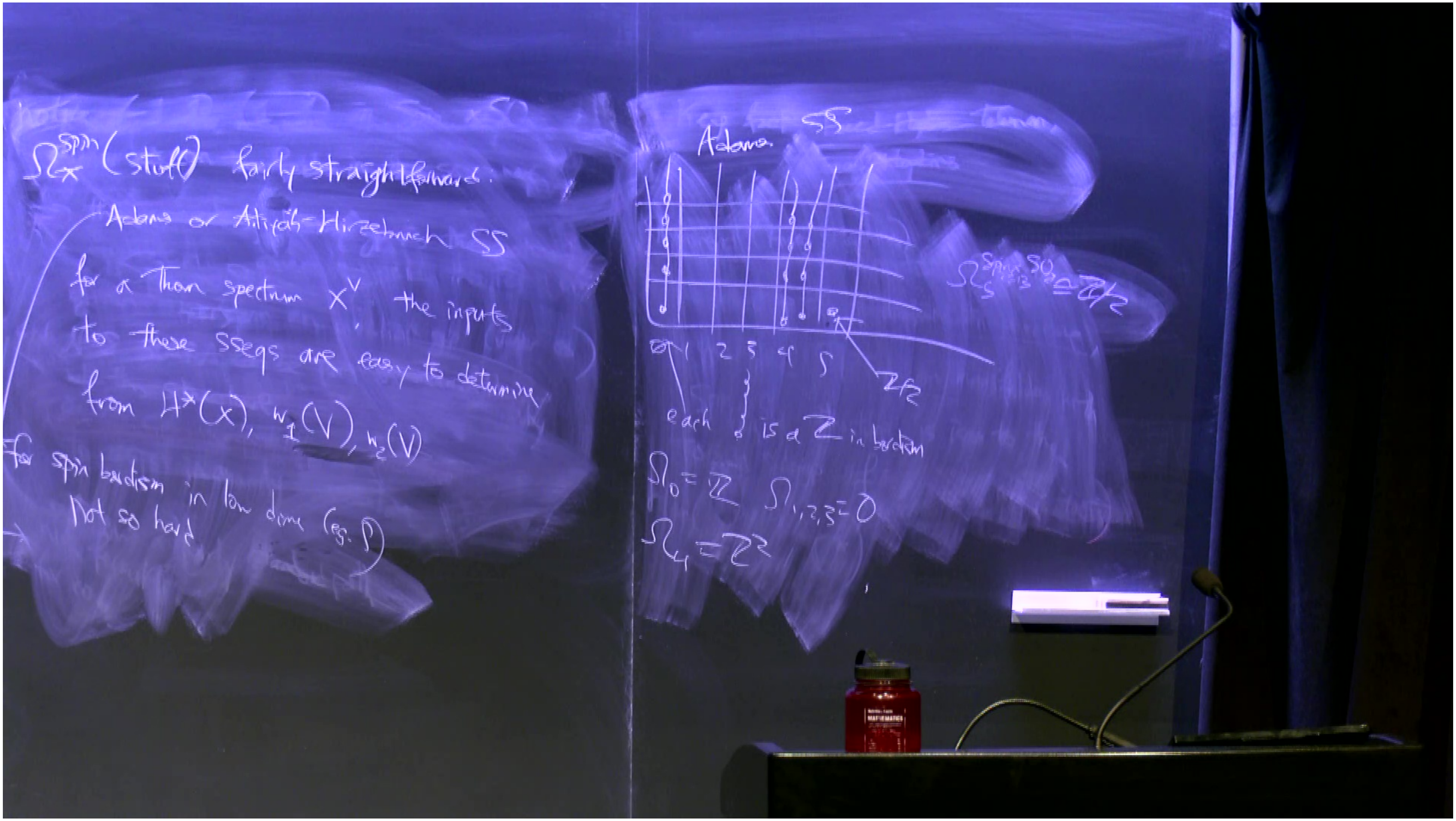
w/ this structure

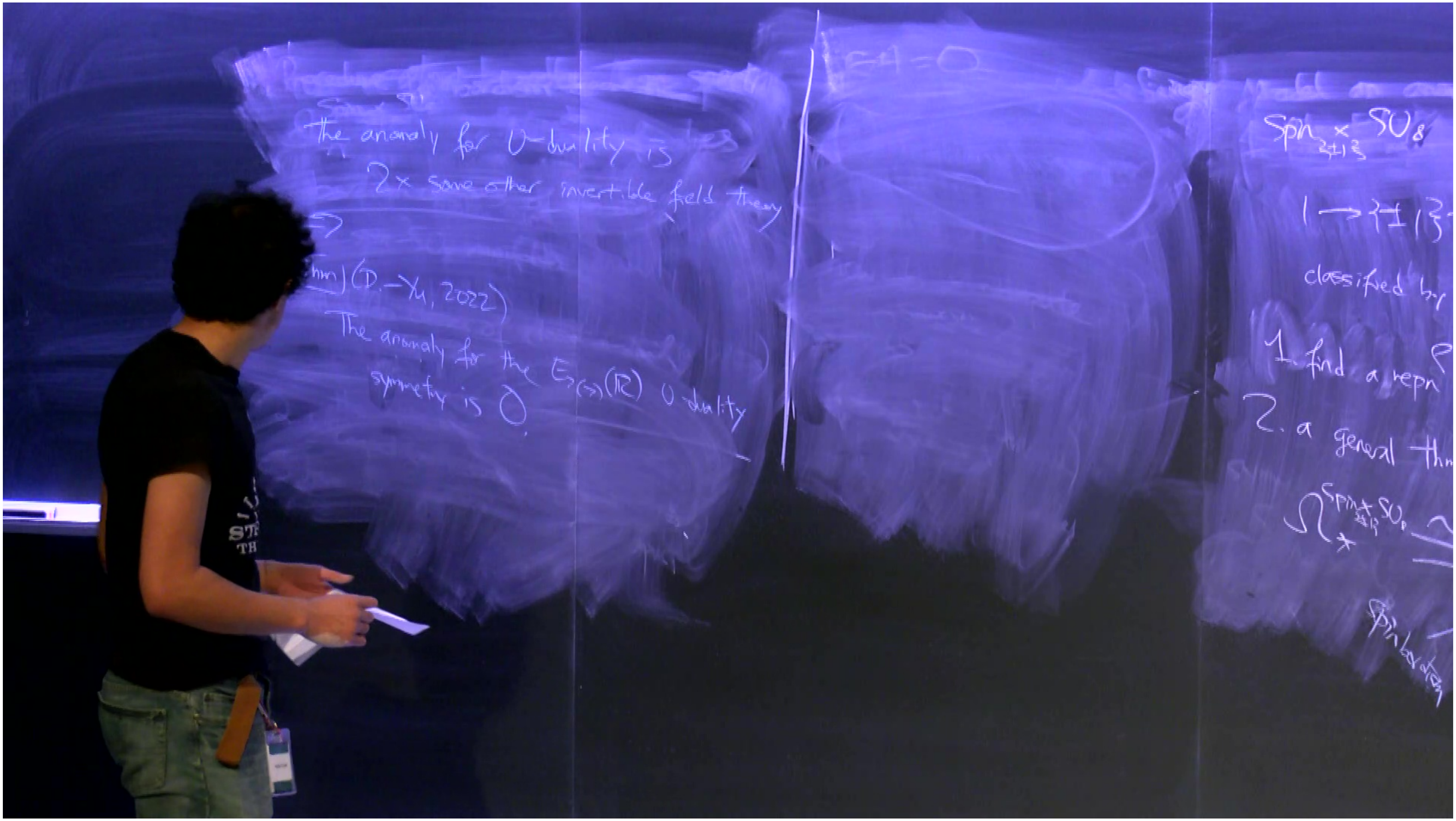
(more general than spin, e.g. E


compute
$\mathrm{Spin} \times_{\{\pm 1\}} SU_8$ comes from
what acts on fermions
what acts on bosons
$1 \to \{\pm 1\} \to SU_8 \to SU_8/\{\pm 1\} \to 1$
classified by $\beta \in H^2(B(SU_8/\{\pm 1\}); \mathbb{Z}/2)$
1. find a repn ρ of $SU_8/\{\pm 1\}$ w/ $w_2(\rho) = \beta$
2. a general thm $\Rightarrow$
$\Omega_*^{\mathrm{Spin} \times_{\{\pm 1\}} SU_8} \cong \Omega_*^{\mathrm{Spin}}(B(SU_8/\{\pm 1\})^{V_\rho})$
Thom spectrum of a vector bundle
spin bordism



SU8 comes from
what acts on fermions
what acts on bosons
→ SU8 → SO8
a repn of SU8
general thm ⟹
spin bordism
Ω^spin_* (stuff) fairly straightforward:
Adams or Atiyah-Hirzebruch SS
for a Thom spectrum X^V, the inputs to these SSeqs are easy to determine from H*(X), w1(V), w2(V)
for spin bordism in low dims (e.g.
Key fact: Freed-Hopkins
Freed-Hopkins
Theorem (Freed-Hopkins)
The group of n-dim,
invertible TFTs on
Spin ×{±1} SU8 structure is
torsion subgp of
Hom(Ω_n^{Spin ×{±1} SU8}, C*),


Ω_X^spin (stuff) fairly straightforward:
Adams or Atiyah-Hirzebruch SS
for a Thom spectrum X^V, the inputs to these sseqs are easy to determine from H*(X), w_1(V), w_2(V)
for spin bordism in low dims (eg. 5)
Not so hard
Adams SS
Z/2
each is a Z in bordism
Ω_0 = Z
Ω_{1,2,3} = 0
Ω_4 = Z^2


The anomaly for U-duality is
2× some other invertible field theory
⟹
Thm] (D.–Yu, 2022)
The anomaly for the $E_{7(7)}(\mathbb{R})$ U-duality
symmetry is 0.
$Spin_{2+1} \times_{\{\pm1\}} SU_8$
$1 \to \{\pm 1\}$
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1. find a repn
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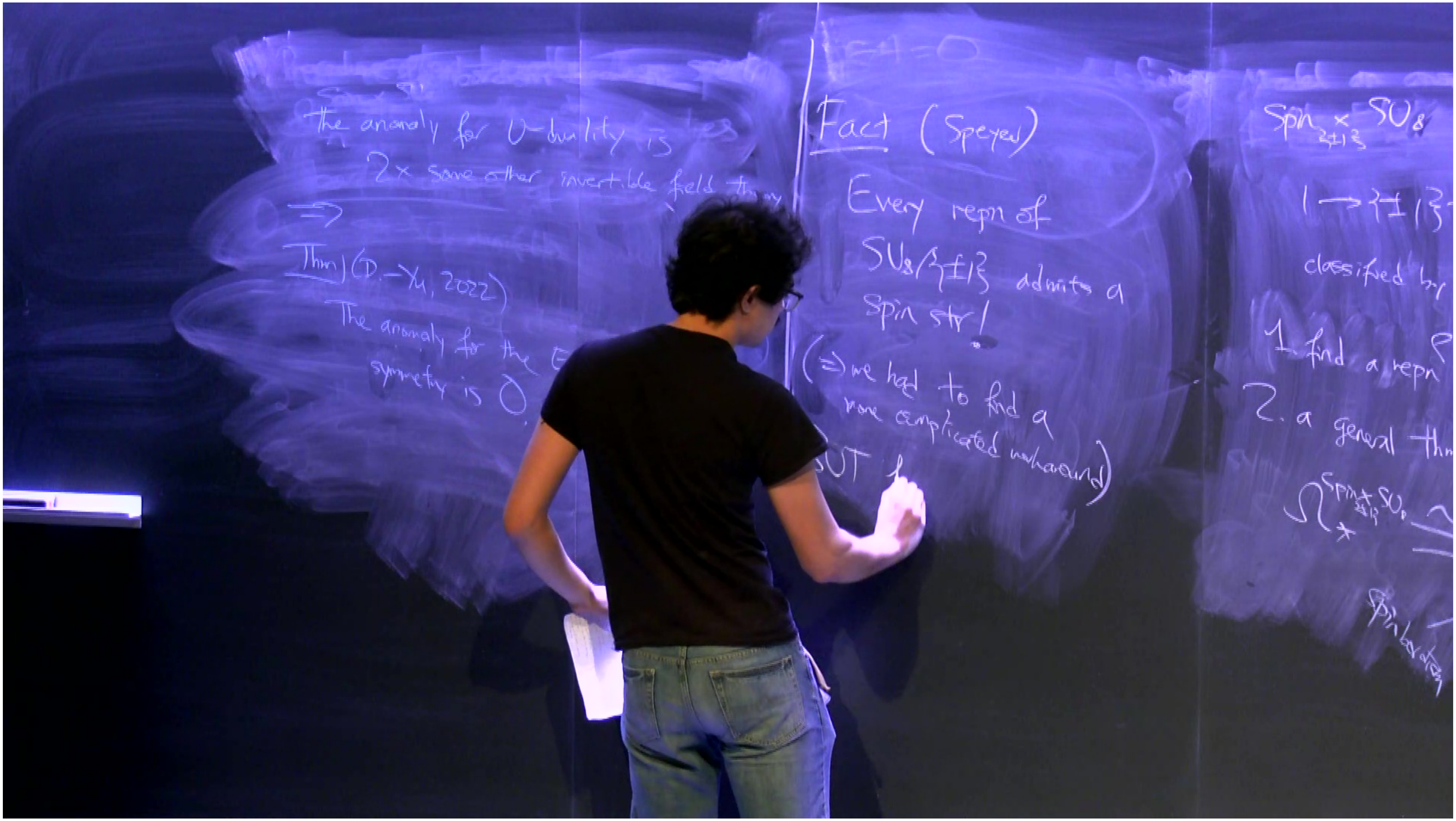
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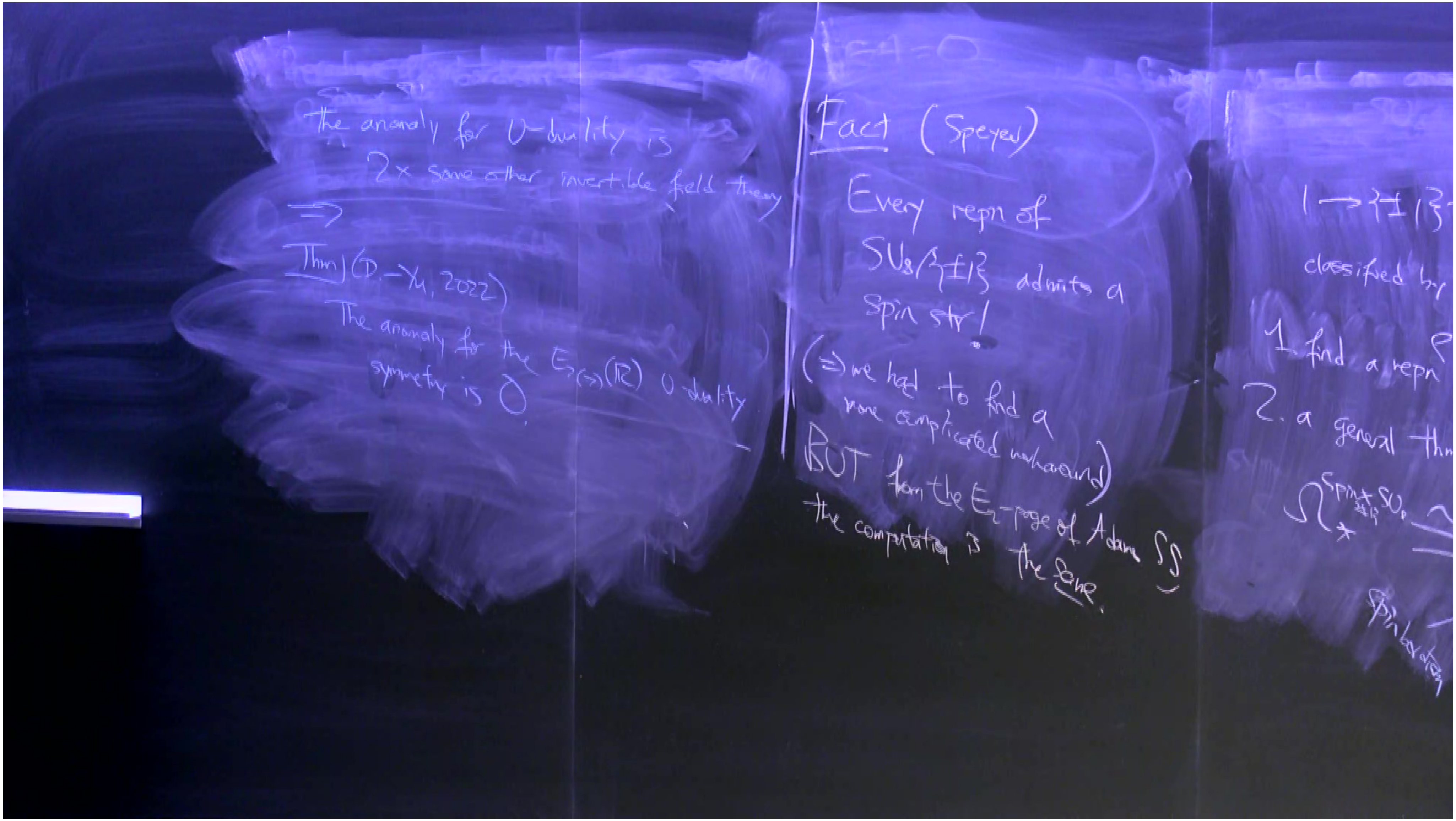
Thm (D.-Yu, 2022)
The anomaly for the E
symmetry is 0.
Fact (Speyer)
Every repn of
SU8/{±1} admits a
spin str!
(⟹ we had to find a
more complicated workaround)
Spin ×{±1} SU8
classified by
1. find a repn
2. a general thm


The anomaly for U-duality is
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Thm (D.–Yu, 2022)
The anomaly for the E7(7)(ℝ) U-duality symmetry is 0.
Fact (Speyer)
Every repn of SU8/{±1} admits a spin str!
(⟹ we had to find a more complicated workaround)
BUT from the E2-page of Adams SS
the computation is the same.
1 → {±1}
classified by
1. find a repn ρ
2. a general thm
Spin bordism

ew)

n of

admits a

find a
(nowhere around)

E₂-page of Adams SS
the same.

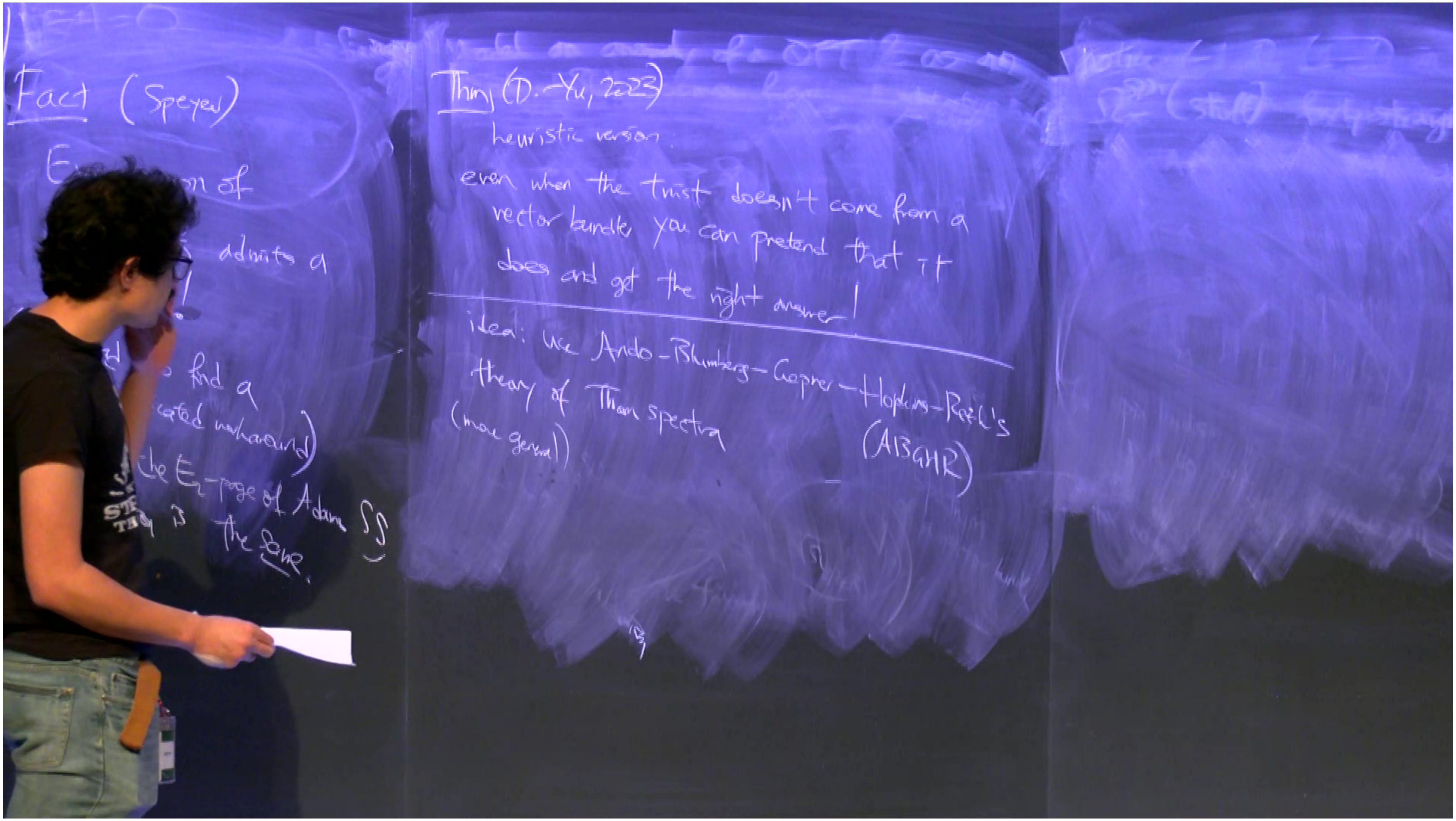
Thm (D.–Yu, 2023)
heuristic version:
even when the twist doesn't come from a vector bundle you can pretend that it does and get the right answer!

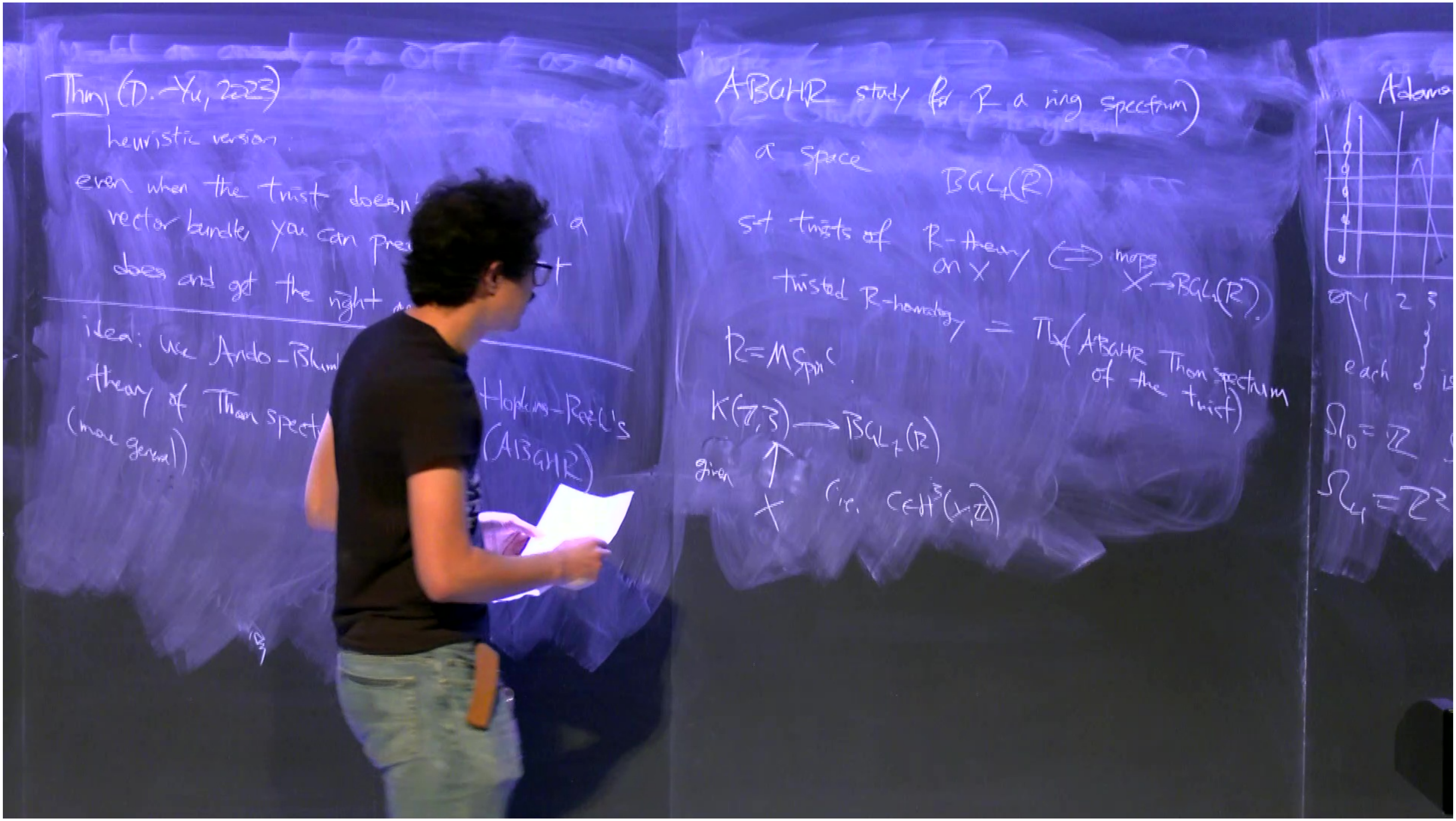
Ω_*^{spin}(stuff) fairly straightforward:

Adams or Atiyah–Hirzebruch SS

for a Thom spectrum X^V, the inputs to these sseqs are easy to determine from $H^*(X)$, $w_1(V)$, $w_2(V)$

for spin bordism in low dim (e.g. $\leq$
Not so hard


Fact (Speyer)
admits a
find a
the E_2-page of Adams SS
is the same.
Thm (D.–Yu, 2023)
heuristic version:
even when the twist doesn't come from a vector bundle you can pretend that it does and get the right answer!
idea: use Ando–Blumberg–Gepner–Hopkins–Rezk's theory of Thom spectra (more general)
(ABGHR)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heuristic version
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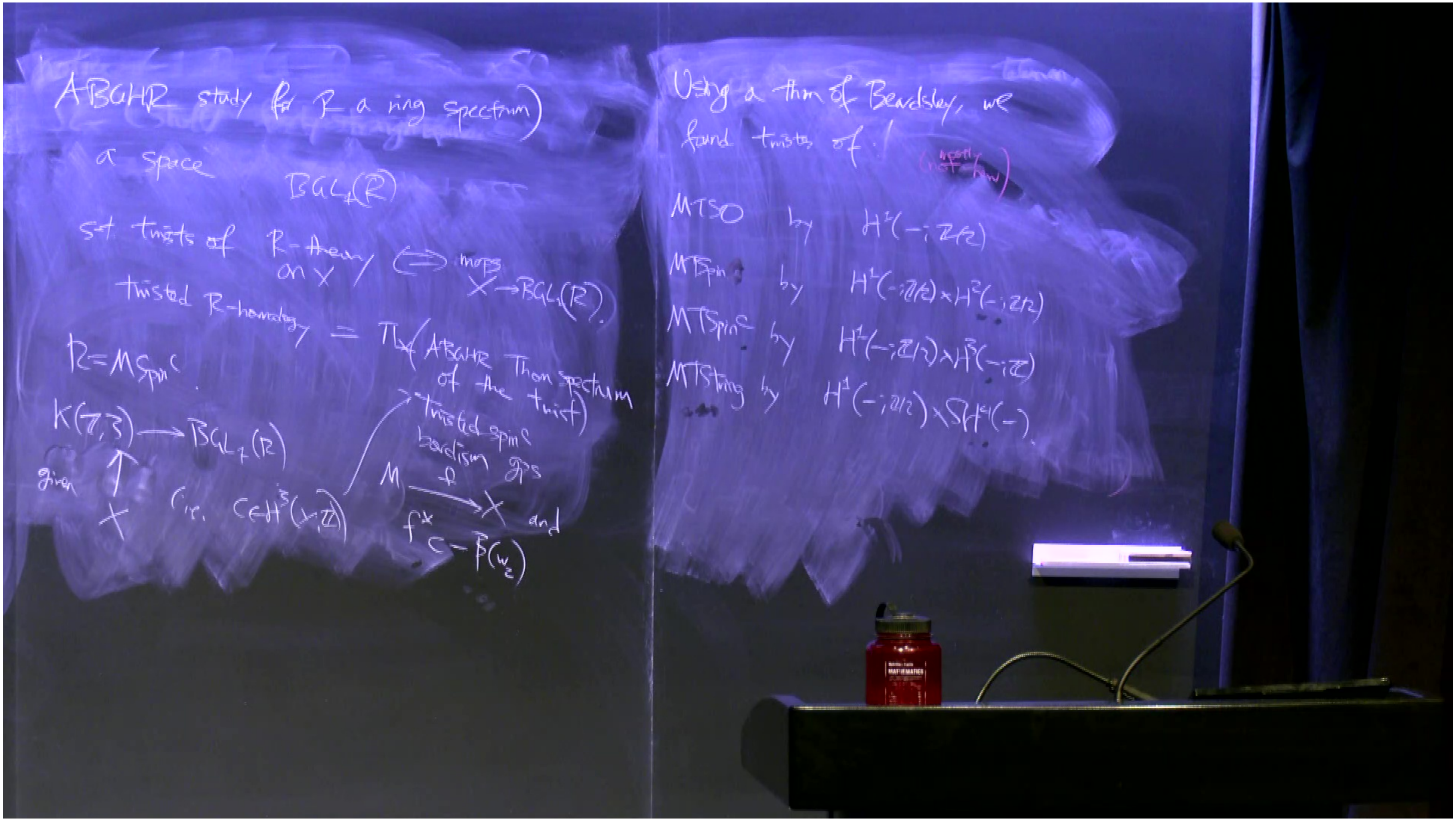
theory of Thom spect
(more general)
Hopkins-Rezk's
(ABGHR)
ABGHR study (for R a ring spectrum)
a space BGL₁(R)
set twists of R-theory on X ⟺ maps X → BGL₁(R)
twisted R-homology = π*(ABGHR Thom spectrum of the twist)
R = MSpin^c
K(Z,3) → BGL₁(R)
given
X
(i.e. c ∈ H³(X;Z))
each
Ω₀ = Z
Ω₄ = Z²


ABGHR study (for R a ring spectrum)
a space BGL1(R)
s.t. twists of R-theory on X ⟺ maps X → BGL1(R)
twisted R-homology = π*(ABGHR Thom spectrum of the twist)
R = MSpin^c
K(Z,3) → BGL1(R)
given X
(i.e. c ∈ H^3(X;Z))
twisted spin^c bordism grps
M → X and f*c = β(w2)
Using a thm of Beardsley, we found twists of (mostly new)
MTSO by H^1(−;Z/2)
MTSpin by H^1(−;Z/2) × H^2(−;Z/2)
MTSpin^c by H^1(−;Z/2) × H^3(−;Z)
MTString by H^1(−;Z/2) × SH^4(−)

Baker–Lazarev studied a variant of Adams SS:

$$E_2^{s,t} = \mathrm{Ext}^{s,t}_{H^*_R H}(H^*_R(M), \mathbb{Z}/2) \Rightarrow \pi_* M^\wedge_2$$

$$H^*_R M = \mathrm{Hom}_R(M, H\mathbb{Z}/2)$$

Thm (D.–Yu, 2023)

For all of the twists on the rightmost board, we compute $H^*_R(M_f)$ and it is $\cong$ to the formula you get for twists coming from a vector bundle.

ABGHR-style Thom spectrum of the twist

Thm (D.–Yu, 2023)

heuristic version:

even when the twist doesn't c vector bundle you can pretend does and get the right answer

idea: use Ando–Blumberg–Gepner– theory of Thom spectra (more general)