

Title: Extended Path Intensity Correlation: Differential Astrometry with Microarcsecond Precision

Speakers: Marios Galanis

Series: Cosmology & Gravitation

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Abstract: The angular resolution of a stellar interferometer, as for a single telescope, becomes better at smaller wavelengths and larger baselines. The goal for ground detectors would then be optical interferometers with baselines as long as the Earth's diameter. The latter goal has been achieved in radio, but it becomes prohibitive in the optical, as the electromagnetic field oscillates too rapidly to record and analyze directly over km-long baselines. Intensity interferometry relying on second-order correlations can make this possible: rather than the amplitude and phase of incoming light, we need only count photons. This technique has a long history and to date the best measurements of nearby stellar radii, dating back to the 1950s. Its main limitations are the need for very bright sources and its narrow field of view, restricting kilometer-long baselines to sources only a few  $\lambda$ s away. In this talk, I will propose an optical-path modification of astronomical intensity interferometers, which introduces an effective time delay in the two-photon interference amplitude, splitting the main intensity correlation fringe into others at finite opening angles, allowing for differential astrometry of multiple compact sources such as stars or quasar images. Together with the exponential progress in the field of single photon detection, such a modification will immensely increase the scope of intensity interferometry beyond measurements of the optical emission region morphology. I will lay out the theory and technical requirements of time-delay intensity interferometry and, time permitting, I will talk about some promising applications, which include astrometric microlensing of stars and quasar images, binary-orbit characterization, exoplanet detection, Galactic acceleration measurements and calibration of the cosmic distance ladder, all at unprecedented relative astrometric precision.

Zoom link: <https://pitp.zoom.us/j/92041231568?pwd=cWo2c0hwTEdmOTRCc042SHNxRWw5UT09>

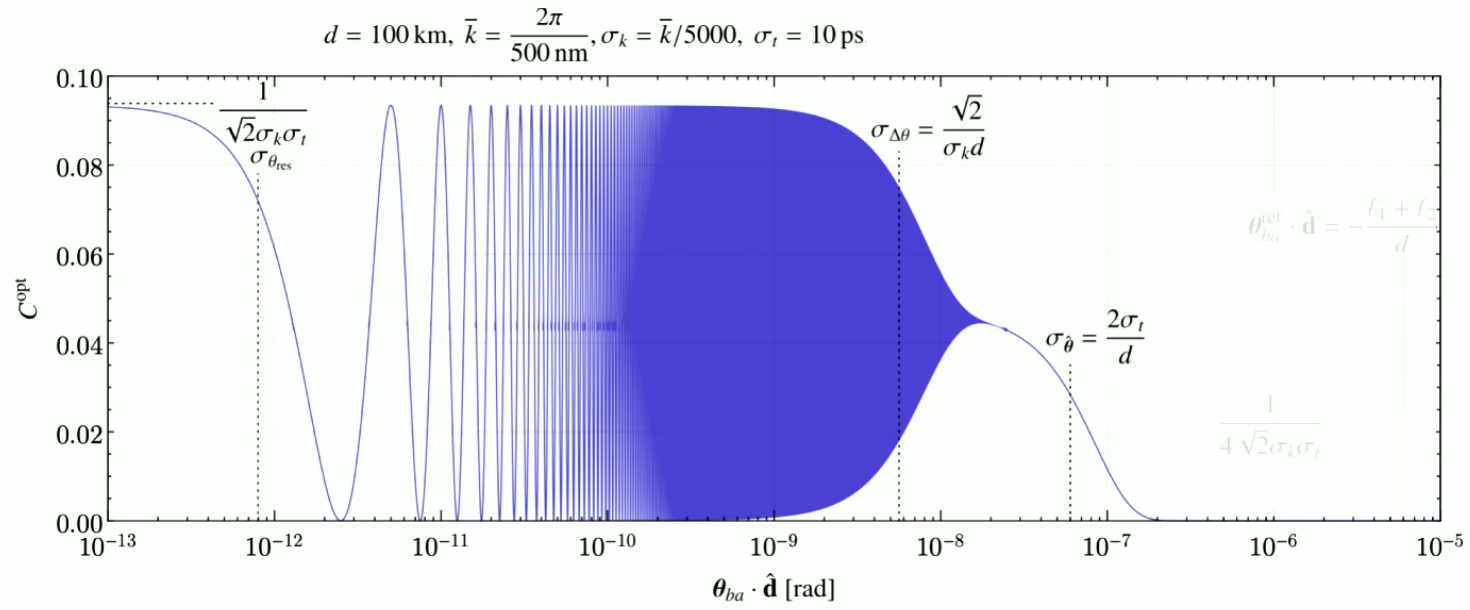
# Extended Path Intensity Correlation: Differential Astrometry with Microarcsecond Precision

Marios Galanis  
Perimeter Institute

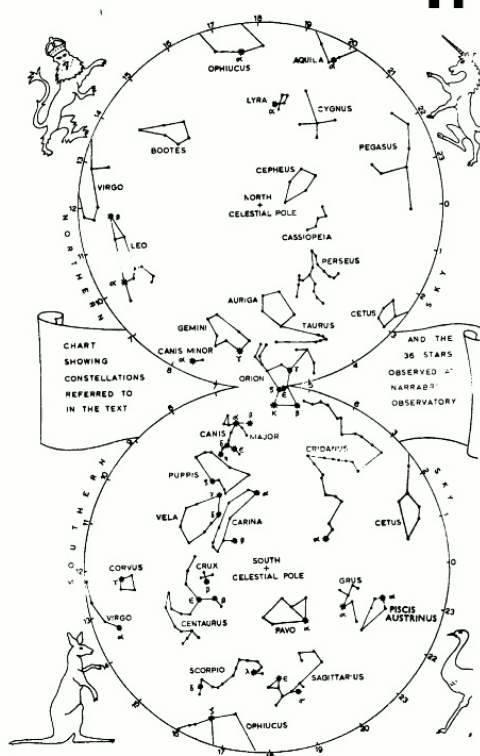
In collaboration with: **Ken Van Tilburg** (NYU & CCA), **Masha Baryakhtar** (UW), **Neal Weiner** (NYU)

arXiv: 2304.xxxxx

# Correlation



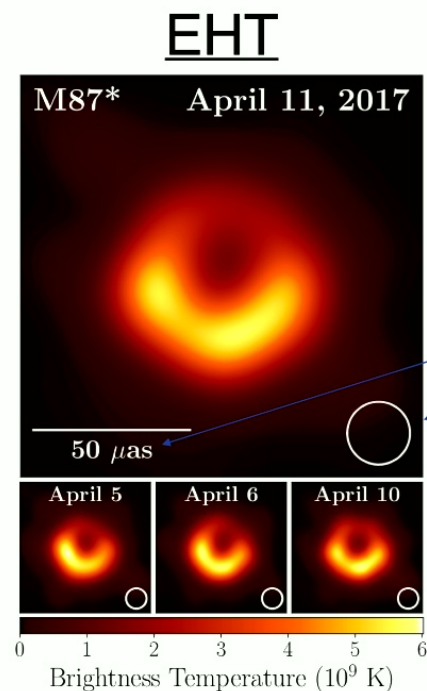
# Narrabri Stellar Intensity Interferometer



Star number	Star name	Type	Zero-baseline correlation $\epsilon_N \pm \sigma$	Angular diameter $\times 10^{-3}$ sec of arc		Temperature [ $T_e(F) \pm \sigma$ ]/K
				$\theta_{UD} \pm \sigma$	$\theta_{LD} \pm \sigma$	
472	$\alpha$ Eri	B 3 (Vp)	$0.98 \pm 0.05$	$1.85 \pm 0.07$	$1.92 \pm 0.07$	$13\,700 \pm 600$
1713	$\beta$ Ori	B 8 (Ia)	$0.98 \pm 0.08$	$2.43 \pm 0.05$	$2.55 \pm 0.05$	$11\,500 \pm 700$
1790	$\gamma$ Ori	B 2 (III)	$1.03 \pm 0.07$	$0.70 \pm 0.04$	$0.72 \pm 0.04$	$20\,800 \pm 1300$
1903	$\epsilon$ Ori	B 0 (Ia)	$0.86 \pm 0.07$	$0.67 \pm 0.04$	$0.69 \pm 0.04$	$24\,500 \pm 2000$
1948	$\zeta$ Ori	O 9.5 (Ib)	$0.60 \pm 0.06$	$0.47 \pm 0.04$	$0.48 \pm 0.04$	$26\,100 \pm 2200$
2004	$\kappa$ Ori	B 0.5 (Ia)	$1.18 \pm 0.09$	$0.44 \pm 0.03$	$0.45 \pm 0.03$	$30\,400 \pm 2000$
2294	$\beta$ CMa	B 1 (II-III)	$1.07 \pm 0.08$	$0.50 \pm 0.03$	$0.52 \pm 0.03$	$25\,300 \pm 1500$
2326	$\alpha$ Car	F 0 (Ib-II)	$0.75 \pm 0.22$	$6.1 \pm 0.7$	$6.6 \pm 0.8$	$7500 \pm 250$
2421	$\gamma$ Gem	A 0 (IV)	$1.17 \pm 0.09$	$1.32 \pm 0.09$	$1.39 \pm 0.09$	$9600 \pm 500$
2491	$\alpha$ CMa	A 1 (V)	$0.91 \pm 0.06$	$5.60 \pm 0.15$	$5.89 \pm 0.16$	$10\,250 \pm 150$
2618	$\epsilon$ CMa	B 2 (II)	$0.89 \pm 0.06$	$0.77 \pm 0.05$	$0.80 \pm 0.05$	$20\,800 \pm 1300$
2693	$\delta$ CMa	F 8 (Ia)	$0.93 \pm 0.18$	$3.29 \pm 0.46$	$3.60 \pm 0.50$	—
2827	$\eta$ CMa	B 5 (Ia)	$0.99 \pm 0.09$	$0.72 \pm 0.06$	$0.75 \pm 0.06$	$14\,200 \pm 1300$
2943	$\alpha$ CMi	F 5 (IV-V)	$0.98 \pm 0.10$	$5.10 \pm 0.16$	$5.50 \pm 0.17$	$6500 \pm 200$
3165	$\zeta$ Pup	O 5 (f)	$1.04 \pm 0.08$	$0.41 \pm 0.03$	$0.42 \pm 0.03$	$30\,700 \pm 2500$
3207	$\gamma^2$ Vel	WC 8 + O 9 (I)	—	$0.43 \pm 0.05$	$0.44 \pm 0.05$	$29\,000 \pm 3000$
3685	$\beta$ Car	A 1 (IV)	$1.01 \pm 0.06$	$1.51 \pm 0.07$	$1.59 \pm 0.07$	$9500 \pm 350$
3982	$\alpha$ Leo	B 7 (V)	$1.12 \pm 0.07$	$1.32 \pm 0.06$	$1.37 \pm 0.06$	$12\,700 \pm 800$
4534	$\beta$ Leo	A 3 (V)	$1.17 \pm 0.10$	$1.25 \pm 0.09$	$1.33 \pm 0.10$	$9050 \pm 450$
4662	$\gamma$ Crv	B 8 (III)	$0.97 \pm 0.10$	$0.72 \pm 0.06$	$0.75 \pm 0.06$	$13\,100 \pm 1200$
4853	$\beta$ Cru	B 0.5 (III)	$0.88 \pm 0.03$	$0.702 \pm 0.022$	$0.722 \pm 0.023$	$27\,900 \pm 1200$
5056	$\alpha$ Vir	B 1 (IV)	—	$0.85 \pm 0.04$	$0.87 \pm 0.04$	$22\,400 \pm 1000$
5132	$\epsilon$ Cen	B 1 (III)	$1.02 \pm 0.07$	$0.47 \pm 0.03$	$0.48 \pm 0.03$	$26\,000 \pm 1800$
5953	$\delta$ Sco	B 0.5 (IV)	$0.75 \pm 0.07$	$0.45 \pm 0.04$	$0.46 \pm 0.04$	—
6175	$\zeta$ Oph	O 9.5 (V)	$1.01 \pm 0.12$	$0.50 \pm 0.05$	$0.51 \pm 0.05$	—
6556	$\alpha$ Oph	A 5 (III)	$0.94 \pm 0.09$	$1.53 \pm 0.12$	$1.63 \pm 0.13$	$8150 \pm 400$
6879	$\epsilon$ Sgr	A 0 (V)	$1.02 \pm 0.06$	$1.37 \pm 0.06$	$1.44 \pm 0.06$	$9650 \pm 400$
7001	$\alpha$ Lyr	A 0 (V)	$0.99 \pm 0.04$	$3.08 \pm 0.07$	$3.24 \pm 0.07$	$9250 \pm 350$
7557	$\alpha$ Aql	A 7 (IV, V)	$0.94 \pm 0.06$	$2.78 \pm 0.13$	$2.98 \pm 0.14$	$8250 \pm 250$
7790	$\alpha$ Pav	B 2.5 (V)	$1.01 \pm 0.07$	$0.77 \pm 0.05$	$0.80 \pm 0.05$	$17\,100 \pm 1400$
8425	$\alpha$ Gru	B 7 (IV)	$1.11 \pm 0.08$	$0.98 \pm 0.07$	$1.02 \pm 0.07$	$14\,800 \pm 1200$
8728	$\alpha$ PsA	A 3 (V)	$1.02 \pm 0.08$	$1.98 \pm 0.13$	$2.10 \pm 0.14$	$9200 \pm 500$

1965–1974

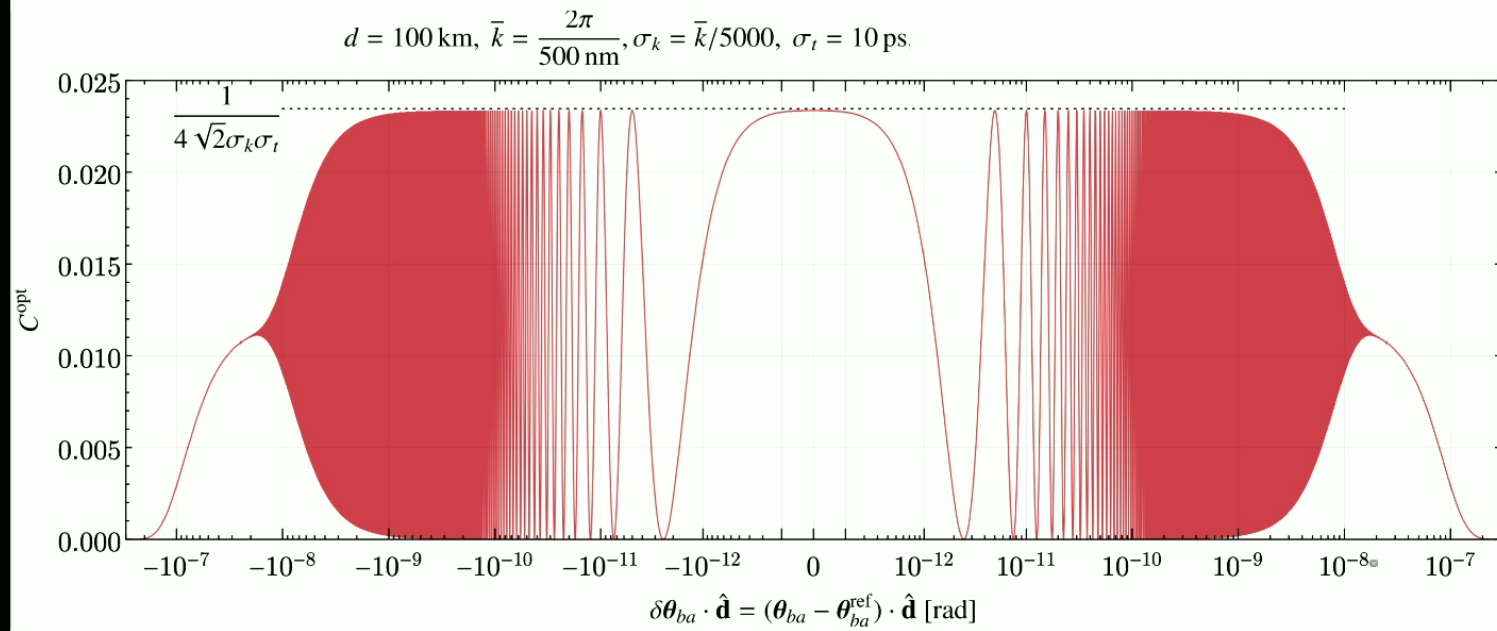
# Narrabri Stellar Intensity Interferometer



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1965–1974

# Correlation - Extended FOV





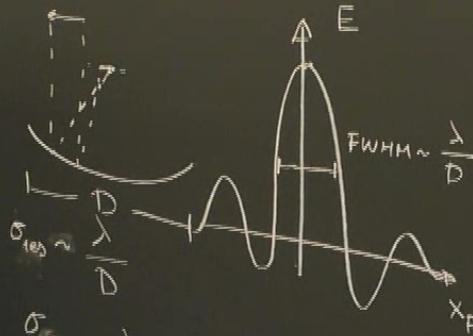
Correlation:  
Microarcsecond Precision



# Optical Telescopes

Imagers

Keck  
HST  
JWST  
Gaia



$$\lambda = 4 \cdot 10^{-6} \text{ rad}$$

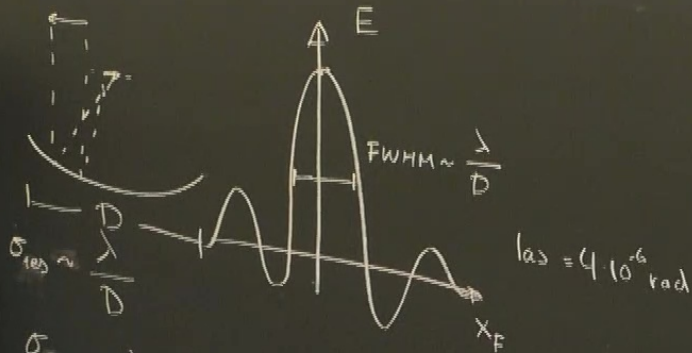
$$\sigma_{\text{prec}} \sim \frac{\lambda}{D} \left( \frac{1}{\text{SNR}}, \text{PSF} \right) \left\{ \begin{array}{l} 10^4 \\ 10^{-2} \end{array} \right\}$$

$$\frac{400 \text{ nm}}{10 \text{ m}} \sim 4 \cdot 10^{-8} \text{ rad} \sim 0.01 \text{ mas} = 10 \text{ mas}$$

# Optical Telescopes

Imagers

Keck  
HST  
JWST  
Gaia



$$\lambda_0 = 4 \cdot 10^{-6} \text{ rad}$$

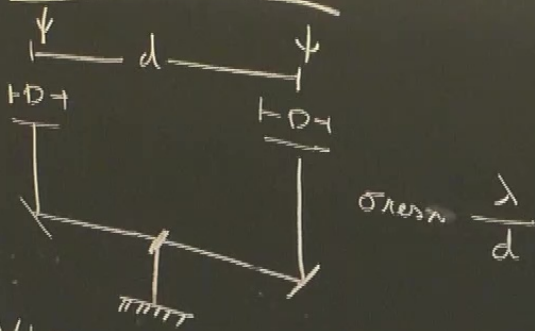
$$\sigma_{\text{prec}} \sim \frac{\lambda}{D} \min \left\{ \frac{1}{\text{SNR}}, \text{PSF} \right\}$$

$\downarrow$   
 $10^{-4}, 10^{-3}$

$$\frac{400 \text{ nm}}{10 \text{ m}} \sim 4 \cdot 10^{-8} \text{ rad} \sim 0.01 \text{ arc} = 10 \text{ mas}$$



## Interferometers

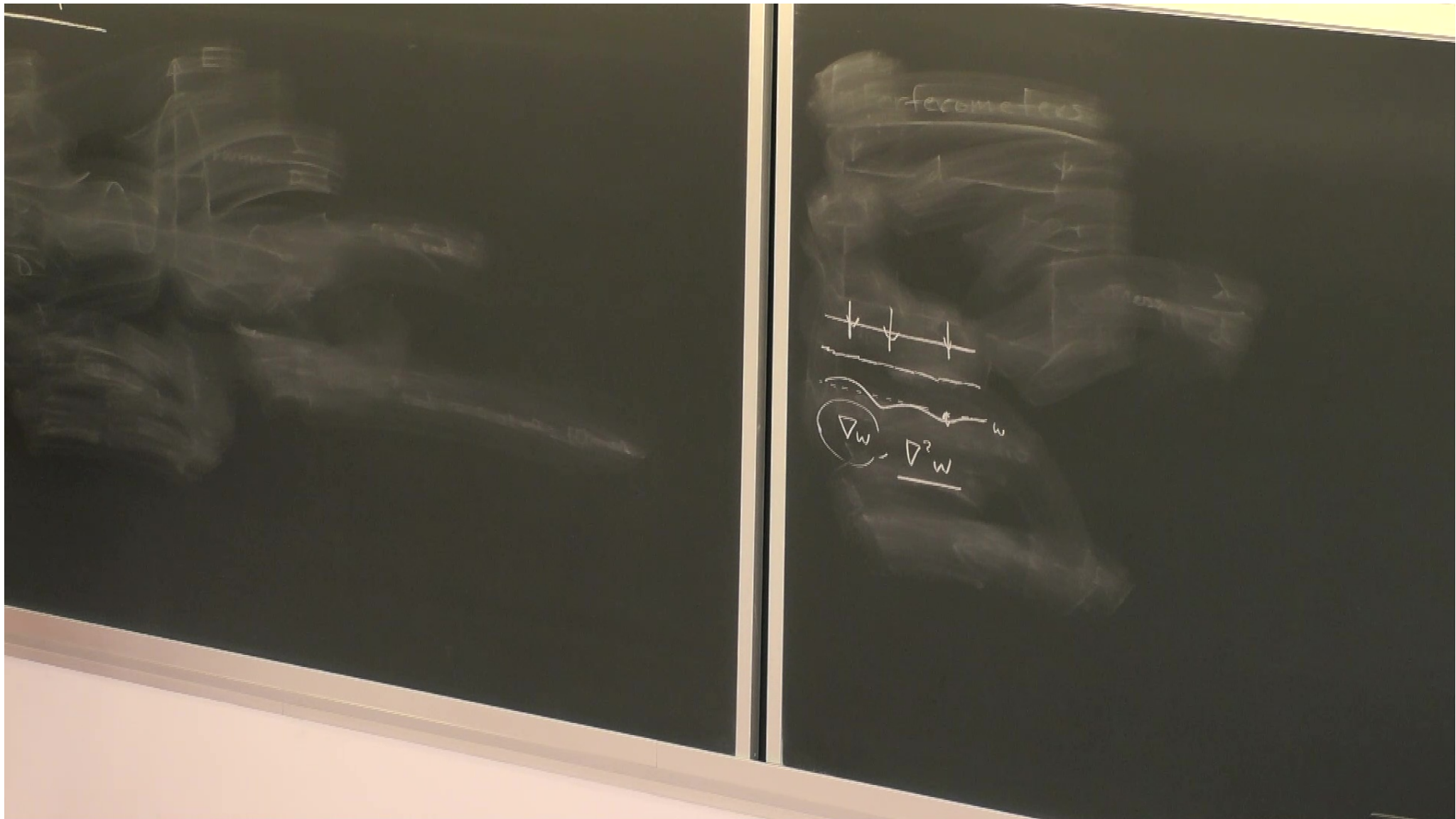


$$\sigma_{\text{res}} \sim \frac{\lambda}{d}$$

VLBI Radio

$$\lambda \sim 1 \text{ mm}$$

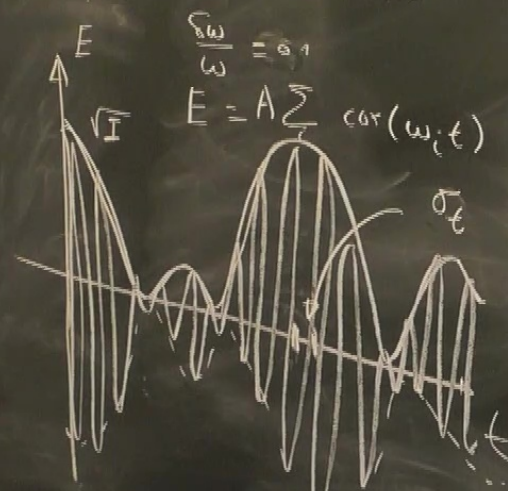
$$\lambda_{\text{opt}} \sim 500 \text{ nm} \sim \text{fs}$$



lation:  
cond Precision

# Intensity Interferometry

Hanbury-Brown & Twiss 1950s



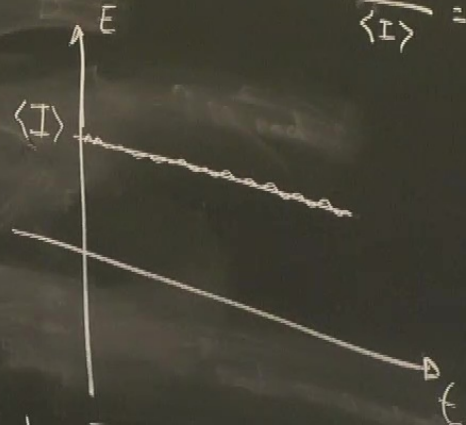
$$\frac{\delta \omega}{\omega} = \frac{1}{N}$$

$$E = A \sum \cos(\omega_i t)$$

Hg

$$\sigma_n^2 = \langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle$$

$$\frac{\sigma_{\langle I \rangle}}{\langle I \rangle} = \frac{\sqrt{\langle n \rangle}}{\langle n \rangle} \rightarrow 0 \quad \langle n \rangle \rightarrow \infty$$



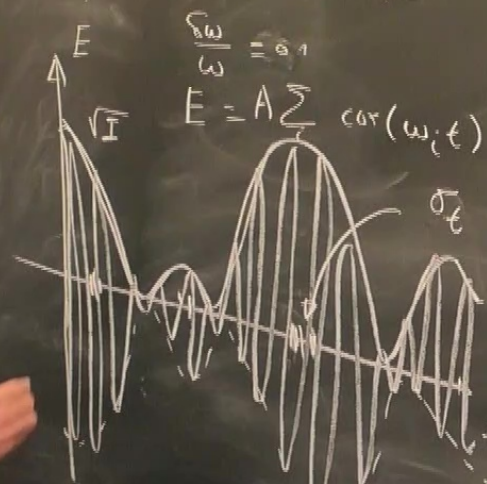
$$\Delta t_c = \delta \omega^{-1}$$

$$I \sim N^2 \alpha^2 + \sum_{i,j} \cos((\omega_i - \omega_j) t)$$



# Intensity Interferometry

Hanbury-Brown & Twiss 1950s



$$\frac{\delta\omega}{\omega} \approx 1$$

$$E = A \sum \cos(\omega_i t)$$

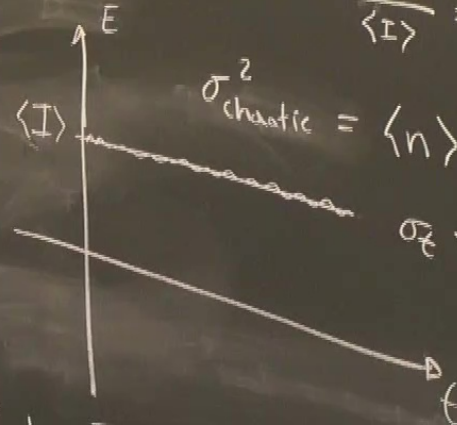
Hg

$$\sigma_w^2 = \langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle$$

$$\frac{\sigma_{\langle I \rangle}}{\langle I \rangle} = \frac{\sqrt{\langle n \rangle}}{\langle n \rangle} \rightarrow 0 \quad \langle n \rangle \rightarrow \infty$$

$$\sigma_{\text{chaotic}}^2 = \langle n \rangle + \langle n \rangle^2$$

$$\sigma_E \gg \tau_c$$



$$\tau_c = \delta\omega^{-1}$$

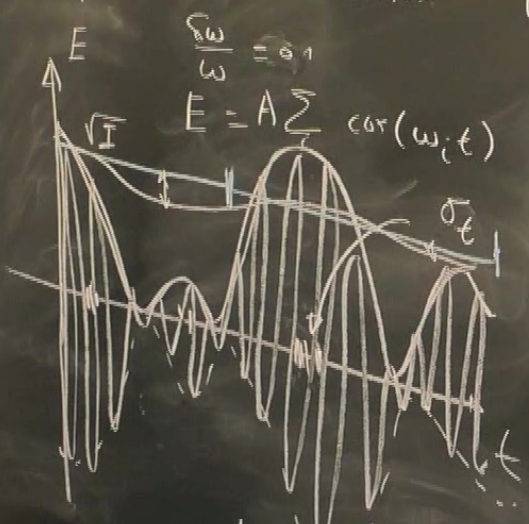
$$I \sim N^2 \alpha^2 + \sum_{i,j} \cos((\omega_i - \omega_j)t)$$

on:  
nd Precision



# Intensity Interferometry

Hanbury-Brown & Twiss 1950s



$$\frac{\delta \omega}{\omega} \approx 1$$

$$E = A \sum \cos(\omega_i t)$$

Hg

$$\tau_c = \delta \omega^{-1}$$

$$I \sim N^2 \langle a^2 \rangle + \sum_{i \neq j} \cos((\omega_i - \omega_j)t)$$

$$\sigma_{\omega}^2 = \langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle$$

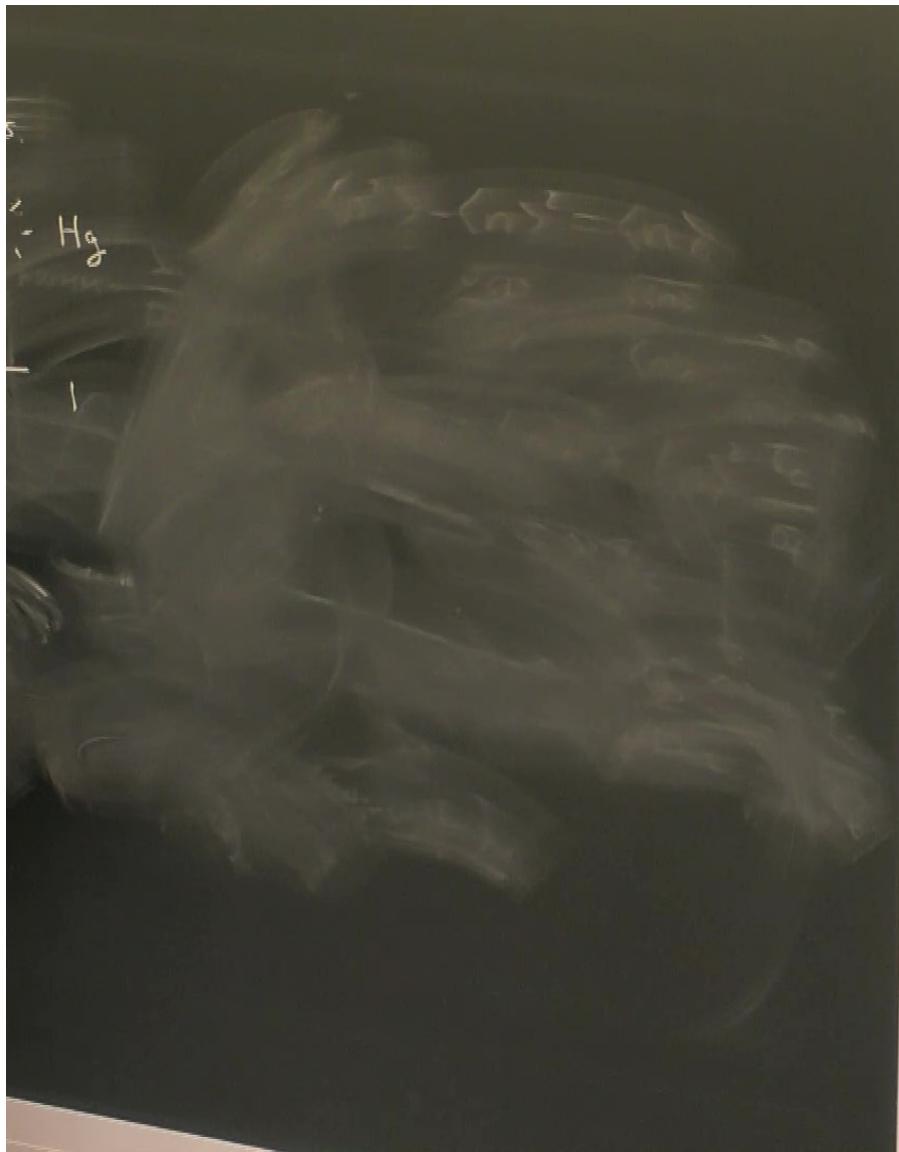
$$\frac{\sigma_{\langle I \rangle}}{\langle I \rangle} = \frac{\sqrt{\langle n \rangle}}{\langle n \rangle} \rightarrow 0 \quad \langle n \rangle \rightarrow \infty$$

$$\sigma_{\text{chaotic}}^2 = \langle n \rangle + \langle n \rangle^2 \frac{\tau_c}{\sigma_t}$$

$$\sigma_t \gg \tau_c$$

$$\frac{\sigma_{\langle I \rangle}^{\text{ch}}}{\langle I \rangle} \rightarrow \frac{\tau_c}{\sigma_t}$$





Interferometers

$$T_c \sim \lambda \sim fs \quad T_c \sim \Delta\lambda$$
$$\sigma_t \sim ns \quad 1950s \quad PMTs$$
$$\sim 10ps \quad SN\ SPDs$$
$$\quad \quad \quad SPADs$$
$$E = A \sum_i e^{i\omega_i t + \phi_i}$$
$$\langle |E|^2 \rangle = \left\langle \sum_i \sum_j A_i A_j^* e^{i(\omega_i - \omega_j)t + (\phi_i - \phi_j)} \right\rangle$$

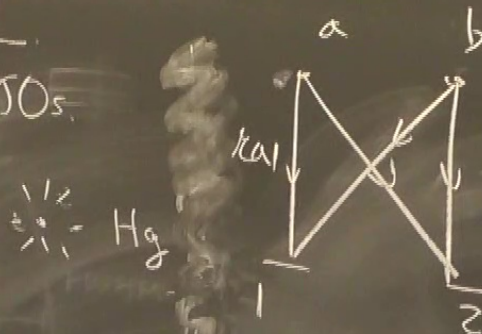
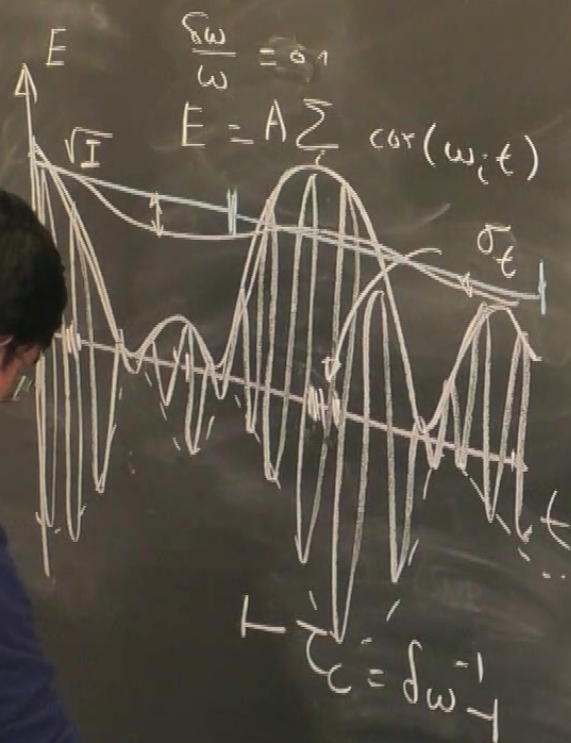
$\delta_{ij}$   $\downarrow$

$$= \sum_i |A_i|^2 \rightarrow \int dk f(k)$$



# Intensity Interferometry

Hanbury-Brown & Twiss 1950s



$$E_j = A e^{i\omega t - ik\tau_{a1}} + i\phi_a(t - \tau_{a1}) + B e^{i\omega t - ik\tau_{b1} + i\phi_b(t - \tau_{b1})}$$

$$E_{a,1} = \tau_{a1}$$

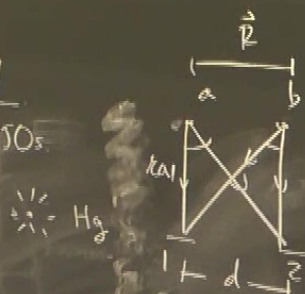
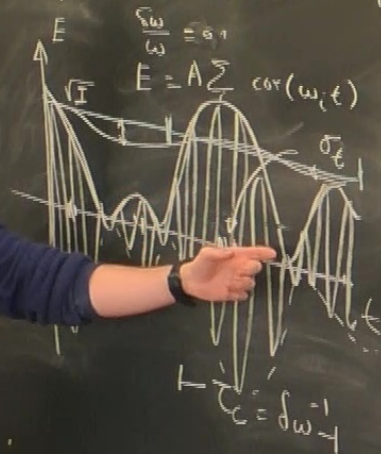
$\langle I_1 I_2 \rangle = E_1 E_1^* E_2 E_2^*$

$\geq |A|^2 |B|^2 e^{ik(\tau_{a1} - \tau_{a2} + \tau_{b2} - \tau_{b1})}$

$e^{i(\phi_a(t - \tau_{a1}) - \phi_a(t - \tau_{a2}) + \phi_b(t - \tau_{b2}) - \phi_b(t - \tau_{b1}))}$

# Intensity Interferometry

Hanbury-Brown & Twiss 1950s



$$E_j = A e^{i\omega t - ik\ell_{a1}} + B e^{i\omega t - ik\ell_{b1} + i\phi_b(t - \ell_{b1})}$$

$$E_{a,1} = \ell_{a1}$$

$$\langle I_1 I_2 \rangle = \langle E_1 E_1^* E_2 E_2^* \rangle$$

$$= |A|^2 |B|^2 e^{ik(\ell_{a1} - \ell_{a2} + \ell_{b2} - \ell_{b1})}$$

$$e^{i(\phi_a(t - \ell_{a1}) - \phi_a(t - \ell_{a2} + \tau) + \phi_b(t - \ell_{b2}) - \phi_b(t - \ell_{b1}))}$$

$$T = \ell_{a1} - \ell_{a2} = \ell_{b1} - \ell_{b2}$$

$$C = \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} - 1 = \# \cos(k \cdot d \cdot \vec{\Theta}_{rel}) \frac{\tau_c}{\sigma_t}$$



$$C \equiv \frac{\langle I_1, I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} - 1 = \# \cos(k \vec{d} \cdot \vec{\Theta}_{rel}) \frac{\tau_c}{\sigma_t} e^{-\frac{1}{\sigma_t^2} (\tau - \vec{d} \cdot \vec{\Theta}_{rel})^2}$$

$$\frac{\lambda}{d} e^{-\left(\frac{1}{\Delta \lambda}\right)^2 (k \vec{d} \cdot \vec{\Theta}_{rel})^2}$$

$$\tau = t_{a2} - t_{a1}$$

$$\tau = t_{b2} - t_{b1}$$

$$\begin{aligned}
 & e^{i\omega t - ik r_{b,j} + i\phi_b(t - r_{b,j})} \\
 & - r_{a,1} \\
 & E_z^* \\
 & - r_{a,2} + r_{b,2} - r_{b,1} \\
 & - r_{a,1} - \phi_a(t - r_{a,2} + \tau) \\
 & (t - r_{b,2} + \tau) \phi_b(t - r_{b,1}) \\
 & r_{a,2} = r_{b,1} - r_{b,2}
 \end{aligned}$$

$$C = \frac{\langle I_1, I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} - 1 = \# \cos(k \vec{d} \cdot \vec{\Theta}_{rel}) \frac{\tau_c}{\sigma_t} e^{-\frac{1}{\sigma_t^2} (\tau - \vec{d} \cdot \vec{\Theta}_{rel})^2}$$

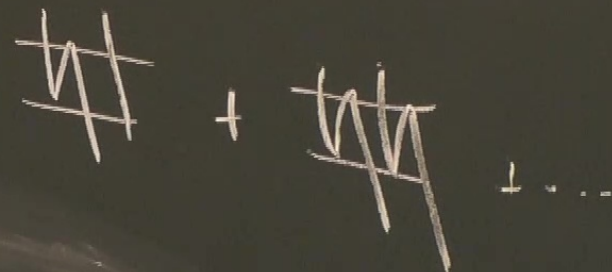
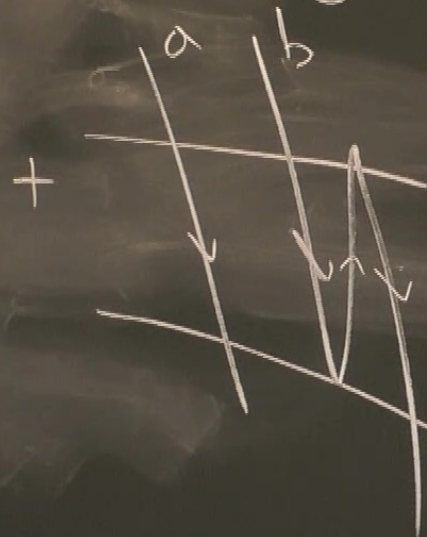
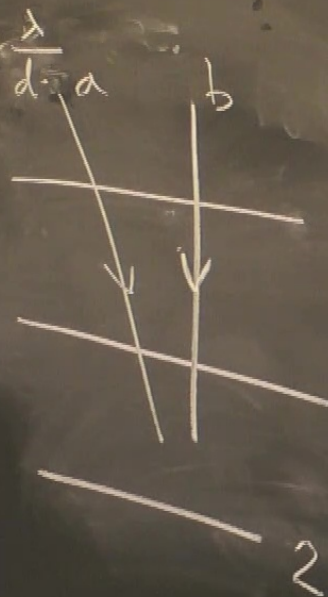
$$\frac{\lambda}{d} e^{-\left(\frac{1}{\Delta\lambda}\right)^2 (\vec{d} \cdot \vec{\Theta}_{rel})^2}$$

$$\tau = r_{a,2} - r_{a,1} \quad \sigma_{\Theta_{rel}} \sim \frac{\lambda}{d}$$

$$\tau = r_{b,2} - r_{b,1} \quad FOV \sim \frac{\Delta\lambda}{d}$$

$$C \equiv \frac{\langle I_1, I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} - 1 = \# \cos(k \vec{d} \cdot \vec{\Theta}_{\text{rel}}) \frac{\tau_c}{\sigma_t} e^{-\frac{1}{\sigma_t^2} (\tau - \vec{d} \cdot \vec{\Theta}_{\text{rel}})^2}$$

$$e^{-\left(\frac{1}{\Delta x}\right)^2 (\vec{d} \cdot \vec{\Theta}_{\text{rel}})^2}$$





$C = \frac{\langle I_1, I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle} - 1 = \#$

$\frac{\lambda}{d \cdot \dots}$

$\cos(k(\vec{d} \cdot \vec{\Theta}_{rel} - z_l)) \frac{\tau_c}{\sigma_t} e^{-\frac{1}{\sigma_t^2} (\tau - \vec{d} \cdot \vec{\Theta}_{rel})^2}$

$e^{-\left(\frac{1}{\Delta\lambda}\right)^2 (\vec{d} \cdot \vec{\Theta}_{rel} - z_l)^2}$   
 $z_l \approx \vec{d} \cdot \vec{\Theta}_{rel}$

$+ \text{[Diagram: A series of parallel lines with arrows pointing downwards, representing a diffraction pattern or wave interference.]}$

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$+ \dots$