Title: Smoothly Splitting Amplitudes and Semi-Locality

Speakers: Bruno Gimenez Umbert
Date: April 03, 2023 - 2:00 PM

URL: https://pirsa.org/23040080

Abstract: I will present a novel behavior developed by scattering amplitudes in certain regions of the kinematic space, in which amplitudes factorize into 3 pieces without becoming singular. This is opposed to what happens in standard factorization or soft limits. We call this behavior a 3-split, and it is "semi-local" in nature. As 3-splits cannot be obtained from standard unitarity arguments, they represent a new phenomenon in QFT. I will introduce 3-splits in their simplest version, i.e. for the biadjoint scalar theory (which I will also introduce). If time allows, I will also comment on how 3-splits arise in other QFTs and how they lead to the discovery of more general theories.

Zoom link: https://pitp.zoom.us/j/95575818902?pwd=V1RKYW9WMmZQNElVL2VyUGFGMFgxQT09

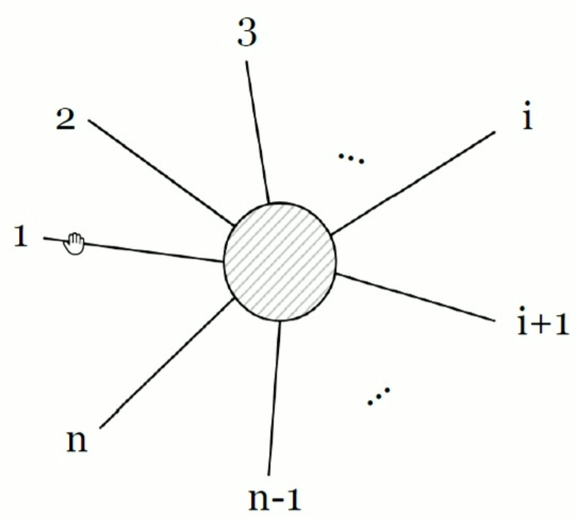
Smoothly Splitting Amplitudes and Semi-Locality

PI Grad Students Seminar Series

Based on: Freddy Cachazo, Nick Early, BGU
JHEP 08 (2022) 252

Bruno Giménez Umbert

Interaction of elementary particles: scattering amplitudes determine the probability of a scattering process to happen.



Construction of amplitudes using recursion relations allowed by unitarity/locality constraints on S-matrix.

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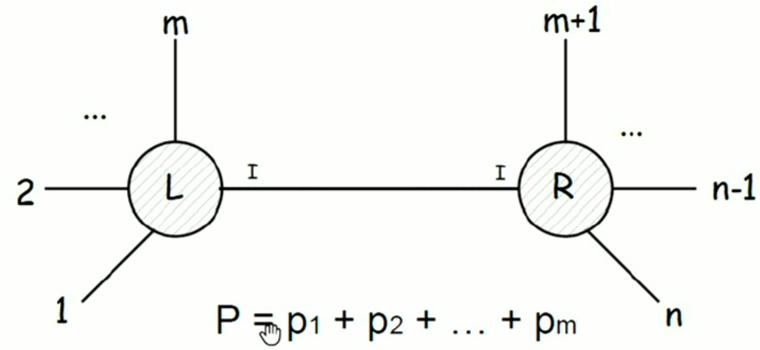
At (massless) tree level:

- amplitudes have simple poles ~ $1/P^2$

- residues of the form $A_L \times A_R$

(factorization)



$P^2 = 0$

$$\mathcal{A} \to \frac{\mathcal{A}_L \times \mathcal{A}_R}{P^2}$$

Scattering amplitudes with color/flavor structure admit decomposition into partial amplitudes.

Important for this talk: biadjoint scalar theory

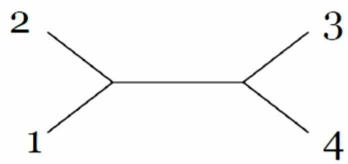
$$\mathcal{M}_{\mathrm{BA}}^{\phi^3} = \sum_{\alpha,\tilde{\alpha}\in S_n/\mathbb{Z}_n} \overbrace{\mathrm{Tr}[T^{a_{\alpha_1}}\cdots T^{a_{\alpha_n}}]\mathrm{Tr}[\tilde{T}^{\tilde{a}_{\tilde{\alpha}_1}}\cdots \tilde{T}^{\tilde{a}_{\tilde{\alpha}_n}}]}^{\text{color}}\overbrace{m(\alpha|\tilde{\alpha})}^{\text{poles/kinematics}}$$

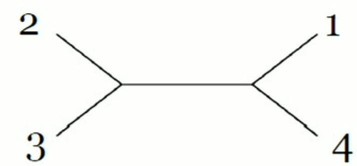
partial amplitude

$$\boxed{\mathcal{L}_{\mathrm{BA}}^{\phi^3} = \partial_\mu \phi^{a\tilde{a}} \partial^\mu \phi_{a\tilde{a}} + f^{abc} f^{\tilde{a}\tilde{b}\tilde{c}} \phi_{a\tilde{a}} \phi_{b\tilde{b}} \phi_{c\tilde{c}}}$$

Partial amplitude (biadjoint scalar theory) for ordering $\mathbb{I} = (1, 2, \ldots, n)$:

n = 4





Partial amplitude (biadjoint scalar theory) for ordering $\mathbb{I} = (1, 2, \ldots, n)$:

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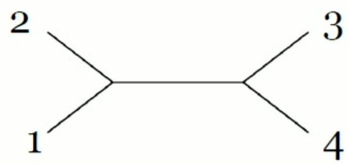
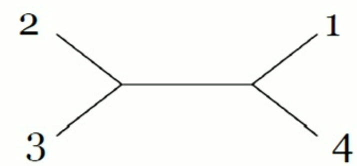


$$\frac{1}{s_{12}} = \frac{1}{s_{34}}$$



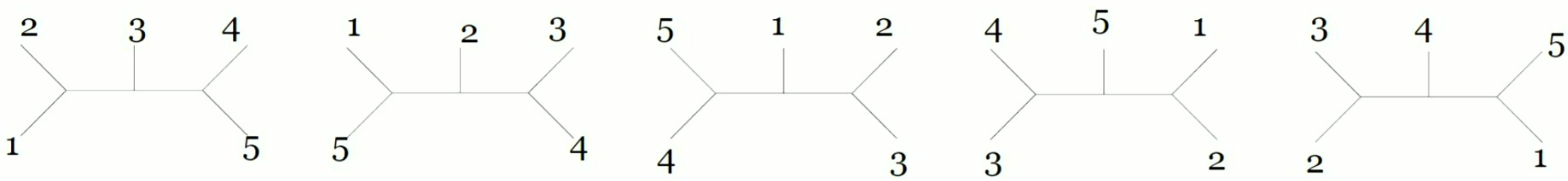
$$\frac{1}{s_{23}} = \frac{1}{s_{14}}$$

$$m_4(\mathbb{I}, \mathbb{I}) = \frac{1}{s_{12}} + \frac{1}{s_{23}}$$

$$s_{ab} = (p_a + p_b)^2$$

Partial amplitude (biadjoint scalar theory) for ordering $\mathbb{I} = (1, 2, \ldots, n)$:

n = 5



$$m_5(\mathbb{I}, \mathbb{I}) = \frac{1}{s_{12}s_{45}} + \frac{1}{s_{34}s_{51}} + \frac{1}{s_{23}s_{45}} + \frac{1}{s_{12}s_{34}} + \frac{1}{s_{23}s_{51}}$$

Scattering amplitudes with color/flavor structure admit decomposition into partial amplitudes.

$\downarrow$

Planar poles of the form $$\frac{1}{(p_i + p_{i+1} + \cdots + p_{i+m})^2}$$ (for canonical ordering $\mathbb{I} = (1, 2, \ldots, n)$)

Scattering amplitudes with color/flavor structure admit decomposition into partial amplitudes.

Planar poles of the form $$\frac{1}{(p_i + p_{i+1} + \cdots + p_{i+m})^2}$$ (for canonical ordering $\mathbb{I} = (1, 2, \ldots, n)$)

vanishing $s_{ij} := (p_i + p_j)^2$ do not seem interesting...

with i and j **non-consecutive**

... but $\exists$ regions where some non-planar invariants vanish such that

$$\mathcal{A} = \mathcal{J}_1 \times \mathcal{J}_2 \times \mathcal{J}_3$$

without becoming singular: Smooth Splitting

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$$\mathcal{A} = \mathcal{J}_1 \times \mathcal{J}_2 \times \mathcal{J}_3$$

without becoming singular: Smooth Splitting

Amputated current

LSZ formalism:

$$G(x_1, x_2, \ldots, x_n) \xrightarrow{\text{Fourier transform}} \tilde{G}(p_1, p_2, \ldots, p_n)$$

$$A(p_1, p_2, \ldots, p_n) = \left(\prod_{i=1}^{n} \lim_{p_i^2 \to 0} p_i^2 \right) \tilde{G}(p_1, p_2, \ldots, p_n)$$

(amplitude)

Amputated current

$$\mathcal{J}(p_1, p_2, \ldots, p_{n-1}) := p_n^2 J(p_1, p_2, \ldots, p_{n-1})$$

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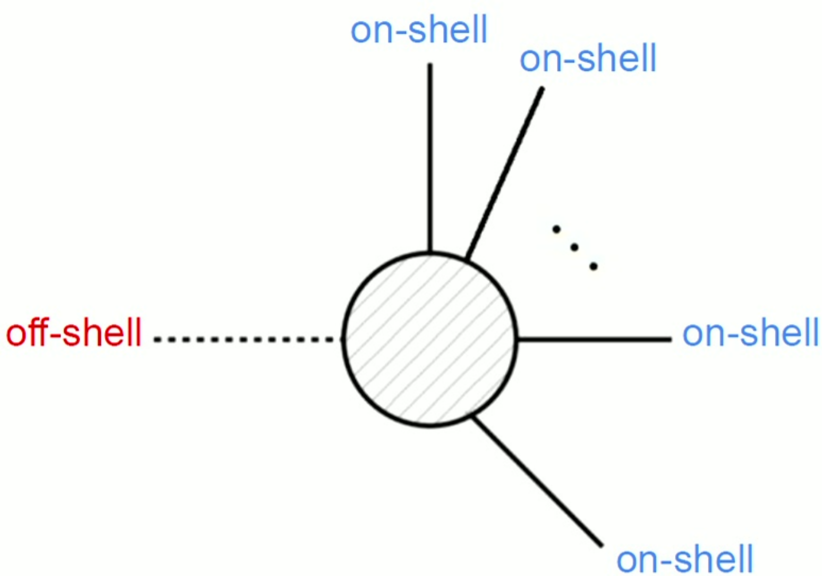
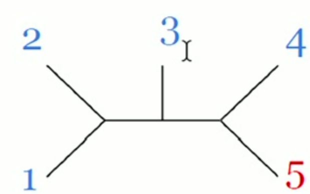
$$\mathcal{J}(p_1, p_2, \ldots, p_{n-1}) := p_n^2 J(p_1, p_2, \ldots, p_{n-1})$$

In terms of Feynman diagrams:





$$\sim \frac{1}{s_{12}s_{123}}$$

Split Kinematics

$$s_{ab} = (p_a + p_b)^2$$

Split kinematics (i, j, k) with $i < j < k$ (can set i=1 WLG):

$$(s_{ab}) = \begin{bmatrix} 0 & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} & s_{17} & s_{18} & s_{19} \\ s_{12} & 0 & s_{23} & s_{24} & s_{25} & s_{26} & s_{27} & s_{28} & s_{29} \\ s_{13} & s_{23} & 0 & s_{34} & s_{35} & s_{36} & s_{37} & s_{38} & s_{39} \\ s_{14} & s_{24} & s_{34} & 0 & s_{45} & s_{46} & s_{47} & s_{48} & s_{49} \\ s_{15} & s_{25} & s_{35} & s_{45} & 0 & s_{56} & s_{57} & s_{58} & s_{59} \\ s_{16} & s_{26} & s_{36} & s_{46} & s_{56} & 0 & s_{67} & s_{68} & s_{69} \\ s_{17} & s_{27} & s_{37} & s_{47} & s_{57} & s_{67} & 0 & s_{78} & s_{79} \\ s_{18} & s_{28} & s_{38} & s_{48} & s_{58} & s_{68} & s_{78} & 0 & s_{89} \\ s_{19} & s_{29} & s_{39} & s_{49} & s_{59} & s_{69} & s_{79} & s_{89} & 0 \end{bmatrix}$$

example n=9

$$s_{ab} = (p_a + p_b)^2$$

Split kinematics (i, j, k) with $i < j < k$ (can set i=1 WLG):

$$(s_{ab}) = \begin{bmatrix} 0 & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} & s_{17} & s_{18} & s_{19} \\ s_{12} & 0 & s_{23} & s_{24} & s_{25} & s_{26} & s_{27} & s_{28} & s_{29} \\ s_{13} & s_{23} & 0 & s_{34} & s_{35} & s_{36} & s_{37} & s_{38} & s_{39} \\ s_{14} & s_{24} & s_{34} & 0 & s_{45} & s_{46} & s_{47} & s_{48} & s_{49} \\ s_{15} & s_{25} & s_{35} & s_{45} & 0 & s_{56} & s_{57} & s_{58} & s_{59} \\ s_{16} & s_{26} & s_{36} & s_{46} & s_{56} & 0 & s_{67} & s_{68} & s_{69} \\ s_{17} & s_{27} & s_{37} & s_{47} & s_{57} & s_{67} & 0 & s_{78} & s_{79} \\ s_{18} & s_{28} & s_{38} & s_{48} & s_{58} & s_{68} & s_{78} & 0 & s_{89} \\ s_{19} & s_{29} & s_{39} & s_{49} & s_{59} & s_{69} & s_{79} & s_{89} & 0 \end{bmatrix} \qquad (s_{ab}) = \begin{bmatrix} 0 & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} & s_{17} & s_{18} & s_{19} \\ s_{12} & 0 & s_{23} & s_{24} & 0 & 0 & s_{27} & 0 & 0 \\ s_{13} & s_{23} & 0 & s_{34} & 0 & 0 & s_{37} & 0 & 0 \\ s_{14} & s_{24} & s_{34} & 0 & s_{45} & s_{46} & s_{47} & s_{48} & s_{49} \\ s_{15} & 0 & 0 & s_{45} & 0 & s_{56} & s_{57} & 0 & 0 \\ s_{16} & 0 & 0 & s_{46} & s_{56} & 0 & s_{67} & 0 & 0 \\ s_{17} & s_{27} & s_{37} & s_{47} & s_{57} & s_{67} & 0 & s_{78} & s_{79} \\ s_{18} & 0 & 0 & s_{48} & 0 & 0 & s_{78} & 0 & s_{89} \\ s_{19} & 0 & 0 & s_{49} & 0 & 0 & s_{79} & s_{89} & 0 \end{bmatrix}$$

example n=9 and split kin. (1,4,7)

Examples

3-split in n=6 for (1,3,5) split kinematics:

$$m_6(\mathbb{I},\mathbb{I}) = \frac{1}{s_{12}s_{34}s_{56}} + \frac{1}{s_{23}s_{45}s_{61}} + \frac{1}{s_{12}s_{45}s_{123}} + \frac{1}{s_{23}s_{45}s_{123}} + \frac{1}{s_{12}s_{56}s_{123}} + \frac{1}{s_{23}s_{56}s_{123}} + \frac{1}{s_{23}s_{56}s_{234}}$$
$$+ \frac{1}{s_{34}s_{56}s_{234}} + \frac{1}{s_{23}s_{61}s_{234}} + \frac{1}{s_{34}s_{61}s_{234}} + \frac{1}{s_{12}s_{34}s_{345}} + \frac{1}{s_{12}s_{45}s_{345}} + \frac{1}{s_{34}s_{61}s_{345}} + \frac{1}{s_{45}s_{61}s_{345}}$$

$$s_{abc} = (p_a + p_b + p_c)^2$$

Examples

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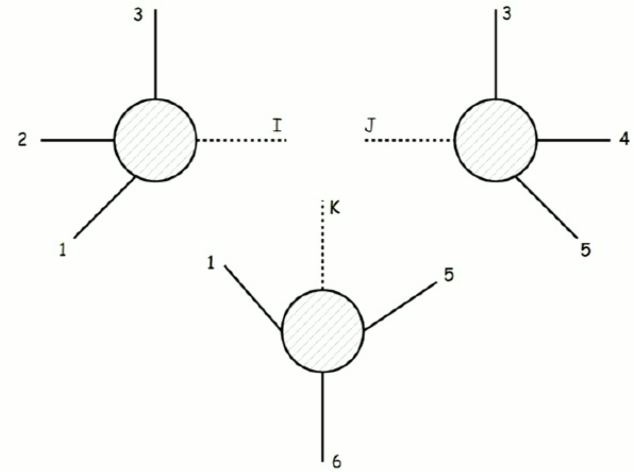

$$m_6(\mathbb{I},\mathbb{I})\big|_{\text{split kin.}} = \left(\frac{1}{s_{12}}+\frac{1}{s_{23}}\right)\left(\frac{1}{s_{34}}+\frac{1}{s_{45}}\right)\left(\frac{1}{s_{56}}+\frac{1}{s_{61}}\right)$$

Do 3-splits happen in other QFTs?

U(N) non-linear sigma model

For split kin. (1,j,k) with j odd and k even:

$$A_n^{\text{NLSM}}(\mathbb{I})\big|_{\text{split kin.}} = \mathcal{J}^{\text{NLSM}}(\mathbb{I})\;\mathcal{J}^{\text{NLSM}\oplus\phi^3}(\mathbb{I}_1|\beta_1)\;\mathcal{J}^{\text{NLSM}\oplus\phi^3}(\mathbb{I}_2|\beta_2)$$

$$\beta_1 = \{I, j, k\}$$
$$\beta_2 = \{1, J, k\}$$

Example: n=8 for (1,3,6) split kinematics

$$A_8^{\text{NLSM}}(\mathbb{I})|_{(1,3,6)} = (s_{12}+s_{23})\left(\frac{s_{34}+s_{45}}{s_{345}} + \frac{s_{45}+s_{56}}{s_{456}} - 1\right)\left(\frac{s_{67}+s_{78}}{s_{678}} + \frac{s_{78}+s_{81}}{s_{781}} - 1\right)$$

Special Galileon

For split kin. (1,j,k) with j odd and k even:

$$A_n^{\mathrm{sGal}}\Big|_{\text{split kin.}} = \mathcal{J}^{\mathrm{sGal}}\ \mathcal{J}^{\mathrm{sGal}\oplus\phi^3}(\beta_1)\ \mathcal{J}^{\mathrm{sGal}\oplus\phi^3}(\beta_2)$$

$$\beta_1 = \{I, j, k\}$$
$$\beta_2 = \{1, J, k\}$$

Discovery of extended theories (same as seen in [Cachazo-Cha-Mizera 2016])

Summary

We found a subspace of the kinematic space in which massless tree-level scattering amplitudes either vanish or "semi-locally" factorize —without becoming singular— into three amputated currents.

3-splits allow to discover extended theories from simpler ones by just exploring regions of the kinematic space!

This is a new phenomenon observed in QFT, and its physical meaning is still unclear.

Final comments

Recent developments in the understanding of scattering amplitudes has lead to build profound bridges between physics and pure mathematics.

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Example: CHY formula

$$\mathcal{A}_n = \int d\mu_n \, \mathcal{I}_L \, \mathcal{I}_R$$

	$\mathcal{I}_L$	$\mathcal{I}_R$
bi-adjoint scalar	$\mathcal{C}_n(\omega)$	$\mathcal{C}_n(\tilde{\omega})$
Yang-Mills	$\mathcal{C}_n(\omega)$	$\mathrm{Pf}'\Psi_n$
Einstein gravity	$\mathrm{Pf}'\Psi_n$	$\mathrm{Pf}'\tilde{\Psi}_n$
Born-Infeld	$(\mathrm{Pf}'\mathsf{A}_n)^2$	$\mathrm{Pf}'\Psi_n$
Non-linear sigma model	$\mathcal{C}_n(\omega)$	$(\mathrm{Pf}'\mathsf{A}_n)^2$
Yang-Mills-scalar	$\mathcal{C}_n(\omega)$	$\mathrm{Pf}\,\mathsf{X}_n\,\mathrm{Pf}'\mathsf{A}_n$
Einstein-Maxwell-scalar	$\mathrm{Pf}\,\mathsf{X}_n\,\mathrm{Pf}'\mathsf{A}_n$	$\mathrm{Pf}\,\mathsf{X}_n\,\mathrm{Pf}'\mathsf{A}_n$
Dirac-Born-Infeld (scalar)	$(\mathrm{Pf}'\mathsf{A}_n)^2$	$\mathrm{Pf}\,\mathsf{X}_n\,\mathrm{Pf}'\mathsf{A}_n$
special Galileon	$(\mathrm{Pf}'\mathsf{A}_n)^2$	$(\mathrm{Pf}'\mathsf{A}_n)^2$

[Cachazo-He-Yuan 2013]

Final comments

Recent developments in the understanding of scattering amplitudes has lead to build profound bridges between physics and pure mathematics.

We used CHY to prove 3-splits in all the theories we studied (useful to use it instead of FDs due to semi-locality): looks like other theories like Yang-Mills or gravity allow for 3-splits.

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We showed how 3-splits can be used to reconstruct NLSM amplitudes from recursion relations.

Where are 3-splits coming from? They seem analogous to factorizations in higher-k amplitudes (natural generalization of QFT).

Thank you!