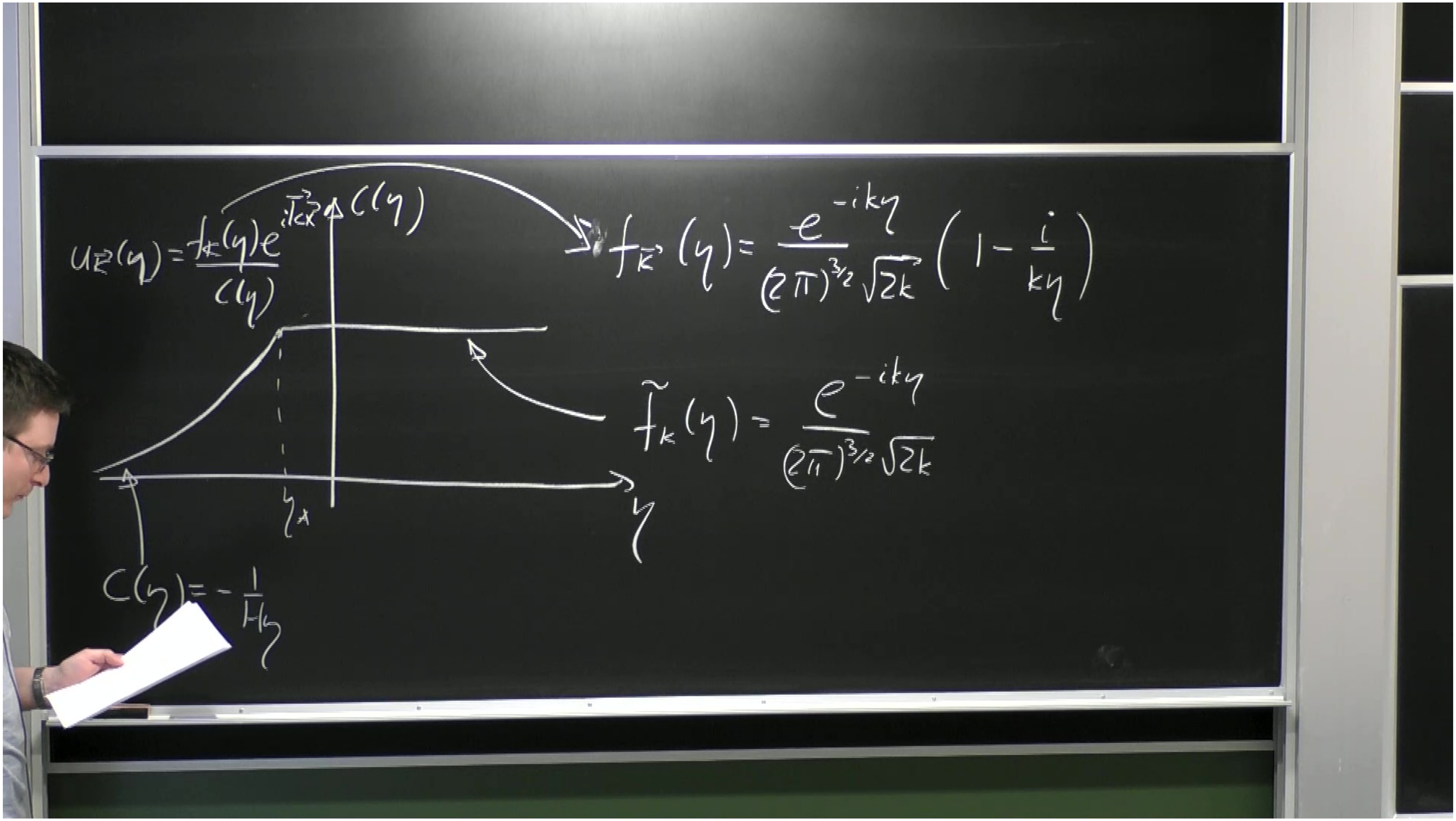
Title: Quantum Field Theory in Curved Spacetime (PM) - 2023-04-14

Speakers:

Collection: Quantum Field Theory in Curved Spacetime

Date: April 14, 2023 - 2:00 PM

URL: https://pirsa.org/23040074



$u_{\vec{k}}(\eta) = \frac{f_{k}(\eta) e^{i\vec{k}\cdot\vec{x}}}{C(\eta)}$
$C(\eta)$
$f_{\vec{k}}(\eta) = \frac{e^{-ik\eta}}{(2\pi)^{3/2}\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)$
$\tilde{f}_{k}(\eta) = \frac{e^{-ik\eta}}{(2\pi)^{3/2}\sqrt{2k}}$
η_*
η
$C(\eta) = -\frac{1}{H\eta}$

$$\varphi(x,\eta_*) = \int \frac{d^3k}{(2\pi)^{3/2}} \underbrace{\left(\frac{e^{-ik\eta_*}\, b_{\vec k} + e^{ik\eta_*}\, b^+_{-\vec k}}{C_* \sqrt{2k}}\right)}_{\varphi_{\vec k} \sim \text{coordinate}} e^{i\vec k \vec x}$$

$$\Pi_{\vec k} = -i\, C_* \sqrt{\frac{k}{2}} \left(b_{-\vec k}\, e^{-ik\eta_*} - b^+_{\vec k}\, e^{ik\eta_*}\right) \sim \text{momentum}$$

$$[\Pi_{\vec k}, \varphi_{\vec k'}] = -i\,\delta(\vec k - \vec k')$$

$$a_{\vec{k}}|0\rangle_{BD} = 0 \Rightarrow \text{eq. } \Psi_{BD}[\varphi_{\vec{k}}]$$

$$\Psi_{BD} = N \exp\left\{-\frac{1}{2}\int d^3k\, |\varphi_{\vec{k}}|^2 \cdot D(k,\eta)\right\}$$

$$D(k,\eta) \overset{k\eta_* \ll 1}{\approx} C_*^2 k \frac{(k\eta_*)^2}{\cos^2 k(\eta_*-\eta)} + i C_*^2 k \frac{\sin k(\eta_*-\eta)}{\cos k(\eta_*-\eta)}$$

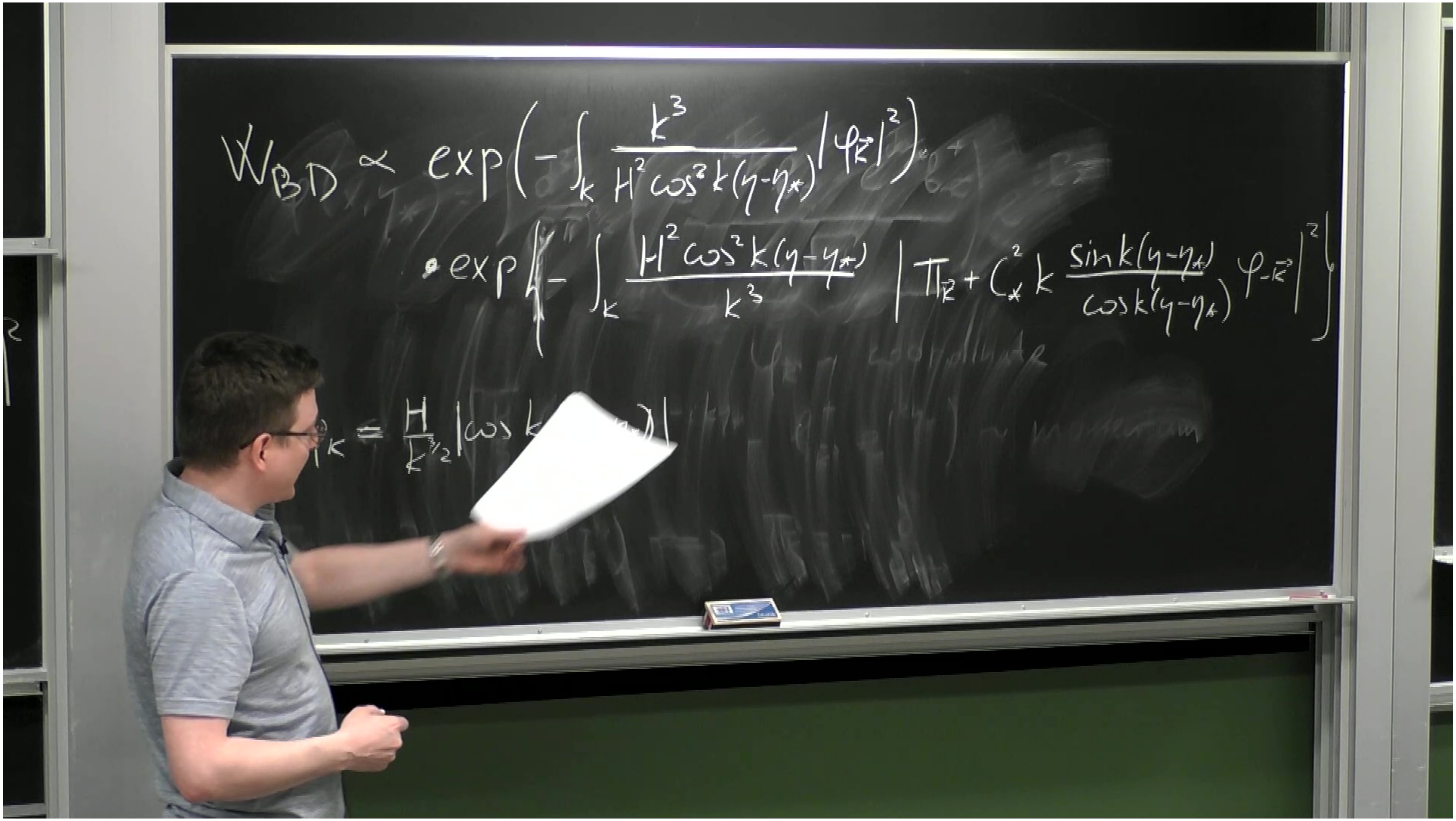
$(2\pi)^{3/2}\sqrt{2k}$

$$C(\eta) = -\frac{1}{H\eta}$$

$$= \int [d\zeta_k]\, e^{i\int_k \pi_k \zeta_k}\, e^{-\frac{1}{2}\int_k D(k,\eta)\left|\varphi_k - \frac{\zeta_k}{2}\right|^2 - \frac{1}{2}\int_k D^*(k,\eta)\left|\varphi_k + \frac{\zeta_k}{2}\right|^2}$$

$$W_{BD} \propto \exp\left(-\int_k \mathrm{Re}\, D(k,\eta)\,|\varphi_k|^2\right)$$

$$\cdot \exp\left(-\int_k \frac{|\pi_k - \mathrm{Im}\, D(k,\eta)\, \varphi_{-k}|^2}{\mathrm{Re}\, D}\right)$$



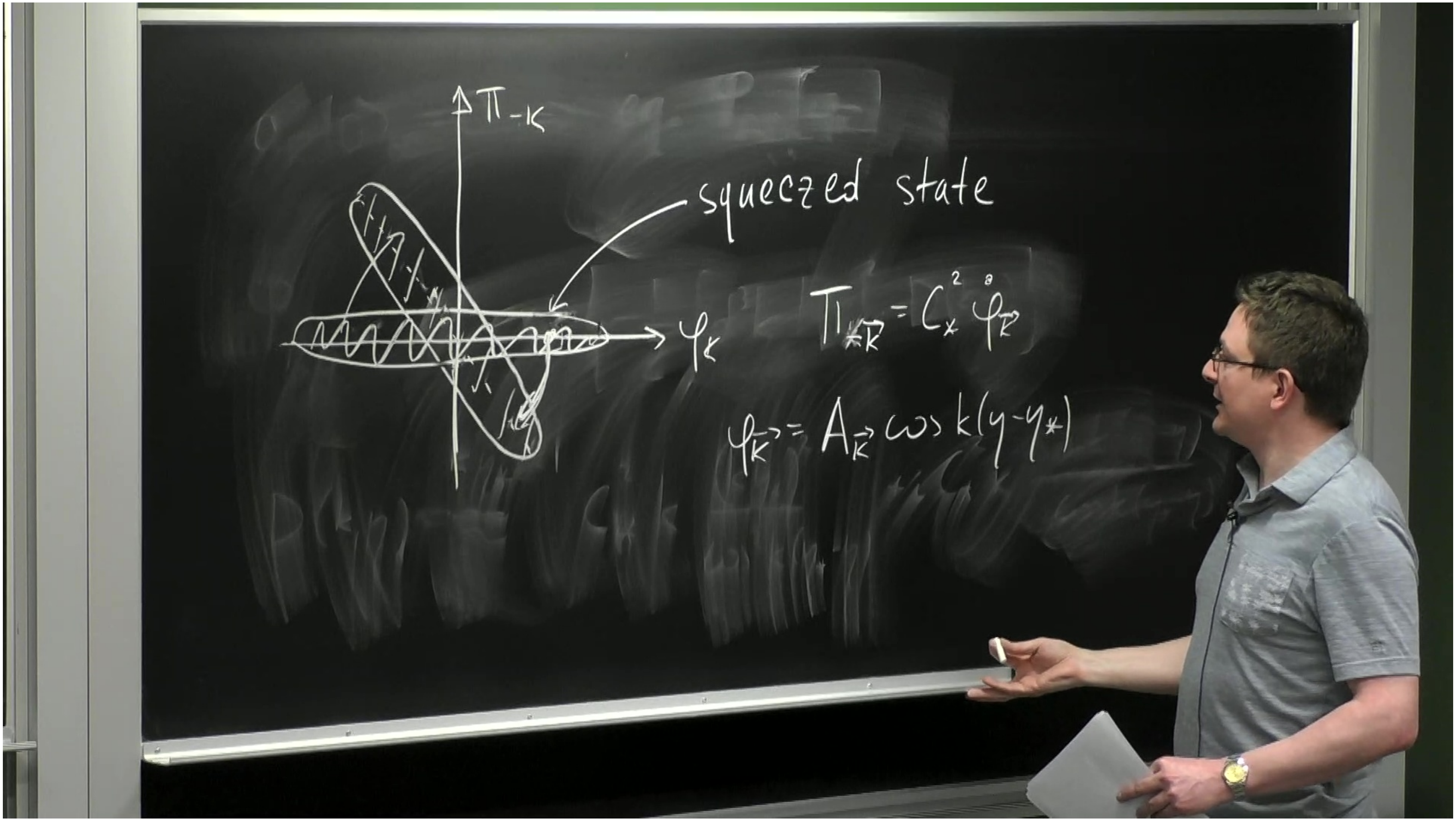
$\Psi_{BD} \propto \exp\left(-\int_k \frac{k^3}{H^2\cos^2 k(\eta-\eta_*)}|\varphi_{\vec{k}}|^2\right)$
$\cdot \exp\left\{-\int_k \frac{H^2\cos^2 k(\eta-\eta_*)}{k^3}\left|\Pi_{\vec{k}} + C_*^2 k \frac{\sin k(\eta-\eta_*)}{\cos k(\eta-\eta_*)}\varphi_{-\vec{k}}\right|^2\right\}$

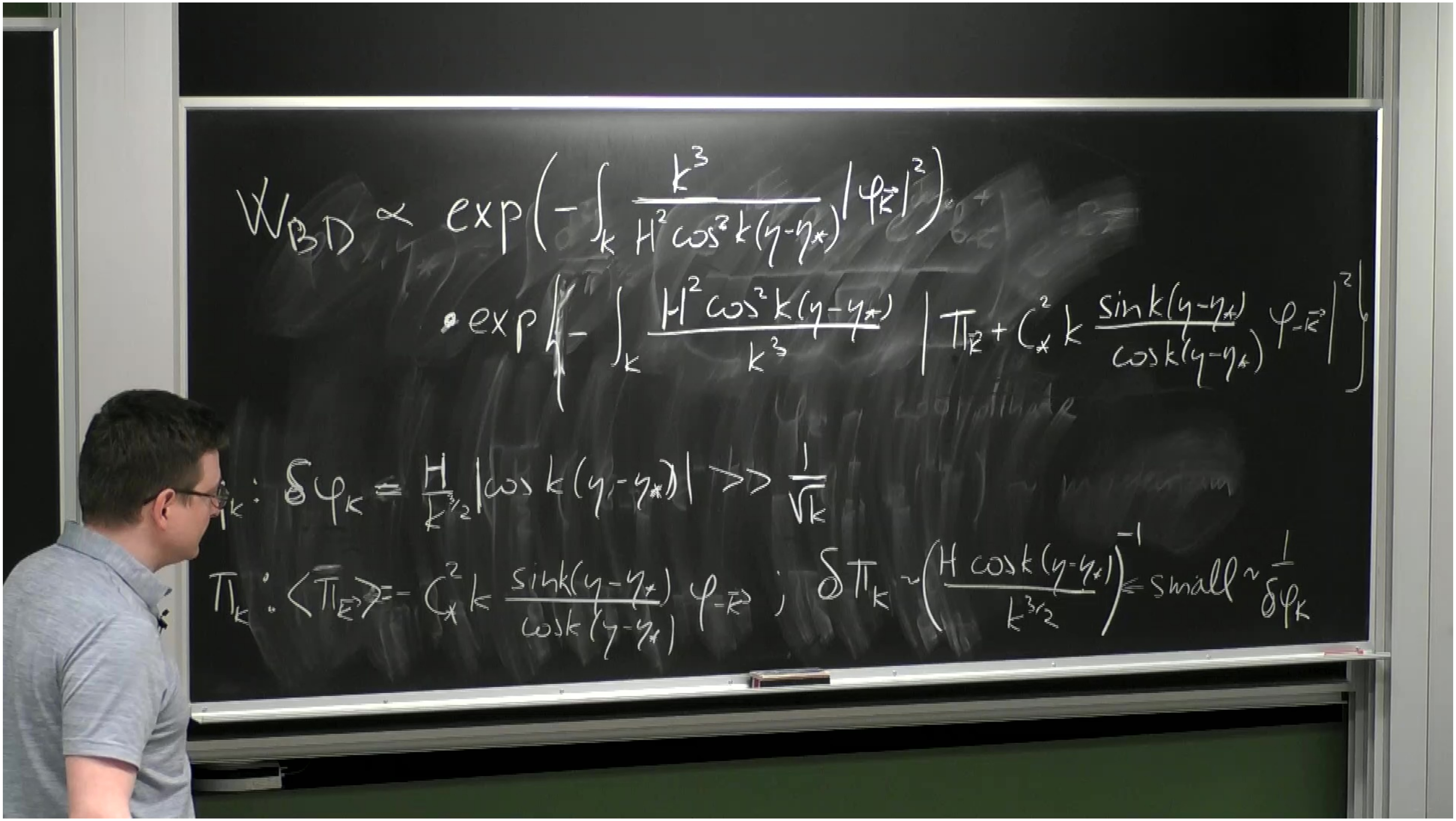
$$\Psi_{BD} \propto \exp\left(-\int_k \frac{k^3}{H^2\cos^2 k(\eta-\eta_*)}|\varphi_{\vec{k}}|^2\right)$$

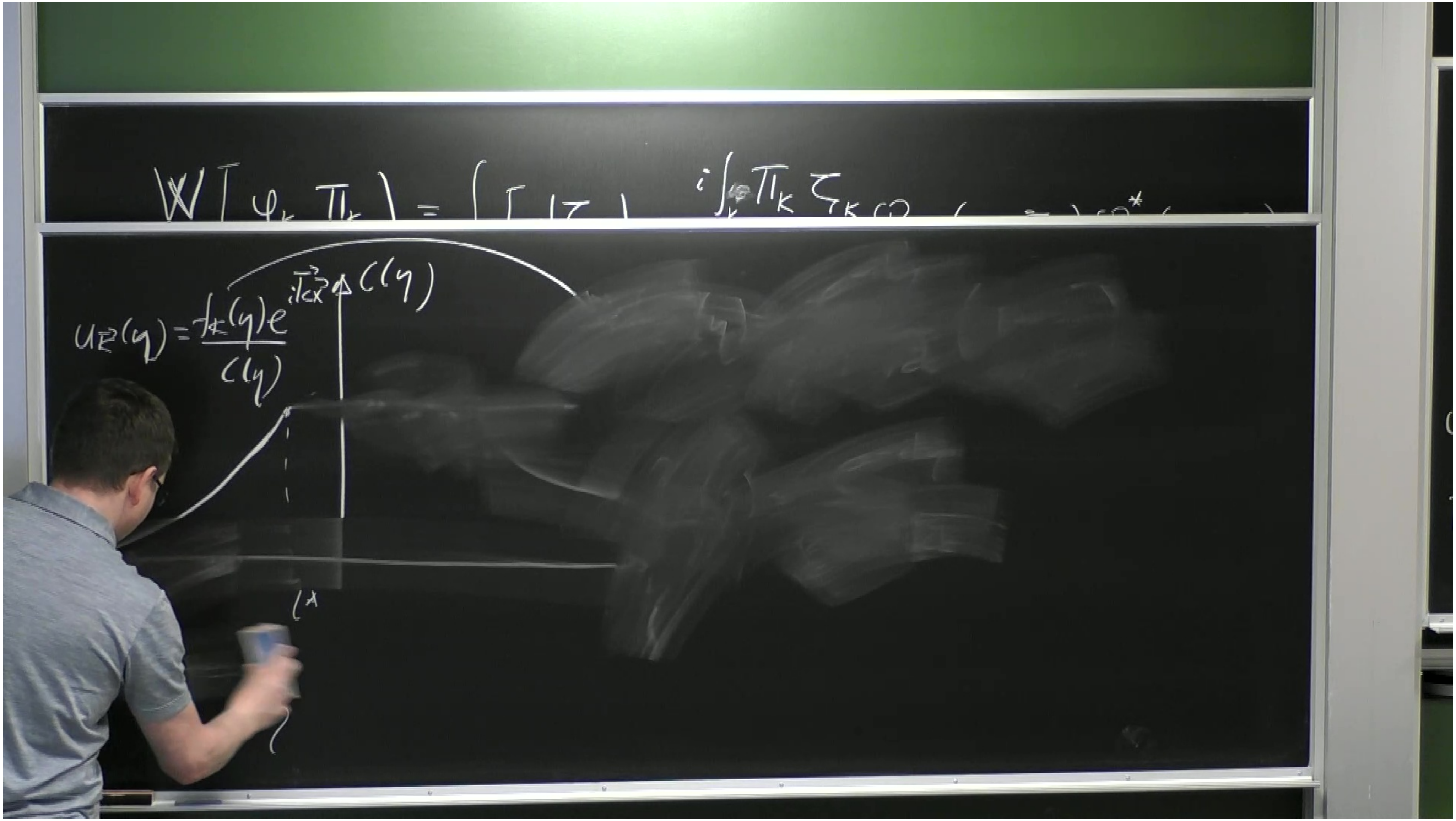
$$\cdot \exp\left(-\int_k \frac{H^2\cos^2 k(\eta-\eta_*)}{k^3}\left|\Pi_{\vec{k}} + c_*^2 k \frac{\sin k(\eta-\eta_*)}{\cos k(\eta-\eta_*)}\varphi_{-\vec{k}}\right|^2\right)$$

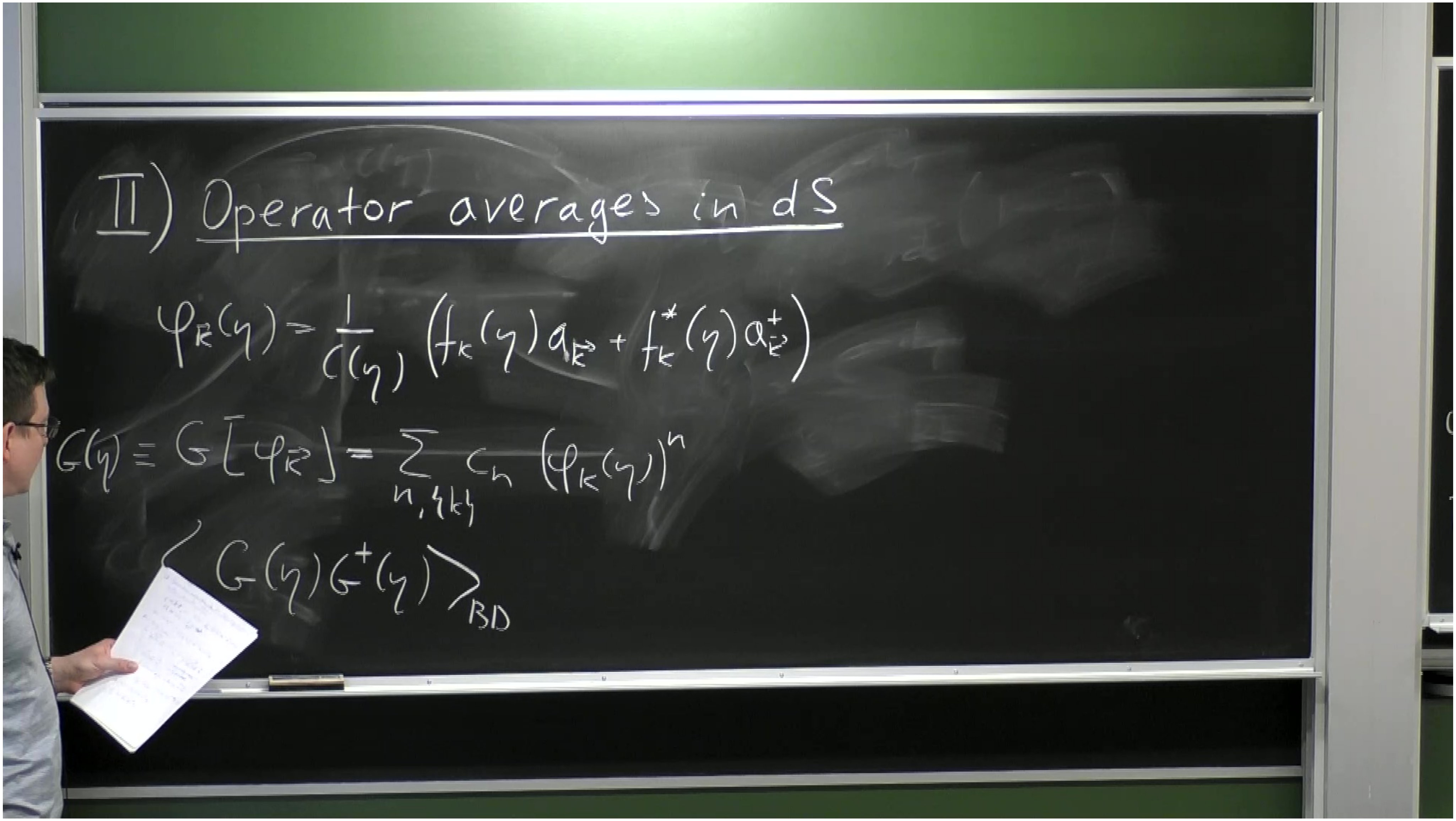
φ_k : $\delta\varphi_k = \frac{H}{k^{3/2}}|\cos k(\eta-\eta_*)| \gg \frac{1}{\sqrt{k}}$

Π_k : $\bar{\Pi}_{\vec{k}} = -c_*^2 k \frac{\sin(\eta-\eta_*)}{\cos k(\eta-\eta_*)}\varphi_{-\vec{k}}$; $\delta\Pi_k \sim H\cos k(\eta-\eta_*)$


Π_{-k}
φ_k
squeezed state
$\Pi_{*\vec{k}} = C_*^2 \dot{\varphi}_{\vec{k}}$
$\varphi_{\vec{k}} = A_{\vec{k}} \cos k(y-y_*)$


$\Psi_{BD} \propto \exp\left(-\int_k \frac{k^3}{H^2\cos^2 k(\eta-\eta_*)}|\varphi_{\vec{k}}|^2\right)$
$\cdot \exp\left\{-\int_k \frac{H^2\cos^2 k(\eta-\eta_*)}{k^3}\left|\Pi_{\vec{k}} + C_*^2 k \frac{\sin k(\eta-\eta_*)}{\cos k(\eta-\eta_*)}\varphi_{-\vec{k}}\right|^2\right\}$
$\varphi_k : \delta\varphi_k = \frac{H}{k^{3/2}}|\cos k(\eta-\eta_*)| \gg \frac{1}{\sqrt{k}}$
$\Pi_k : \langle \Pi_{\vec{k}} \rangle = -C_*^2 k \frac{\sin k(\eta-\eta_*)}{\cos k(\eta-\eta_*)}\varphi_{-\vec{k}}$; $\delta\Pi_k \sim \left(\frac{H\cos k(\eta-\eta_*)}{k^{3/2}}\right)^{-1} \leftarrow$ small $\sim \frac{1}{\delta\varphi_k}$


C(η)

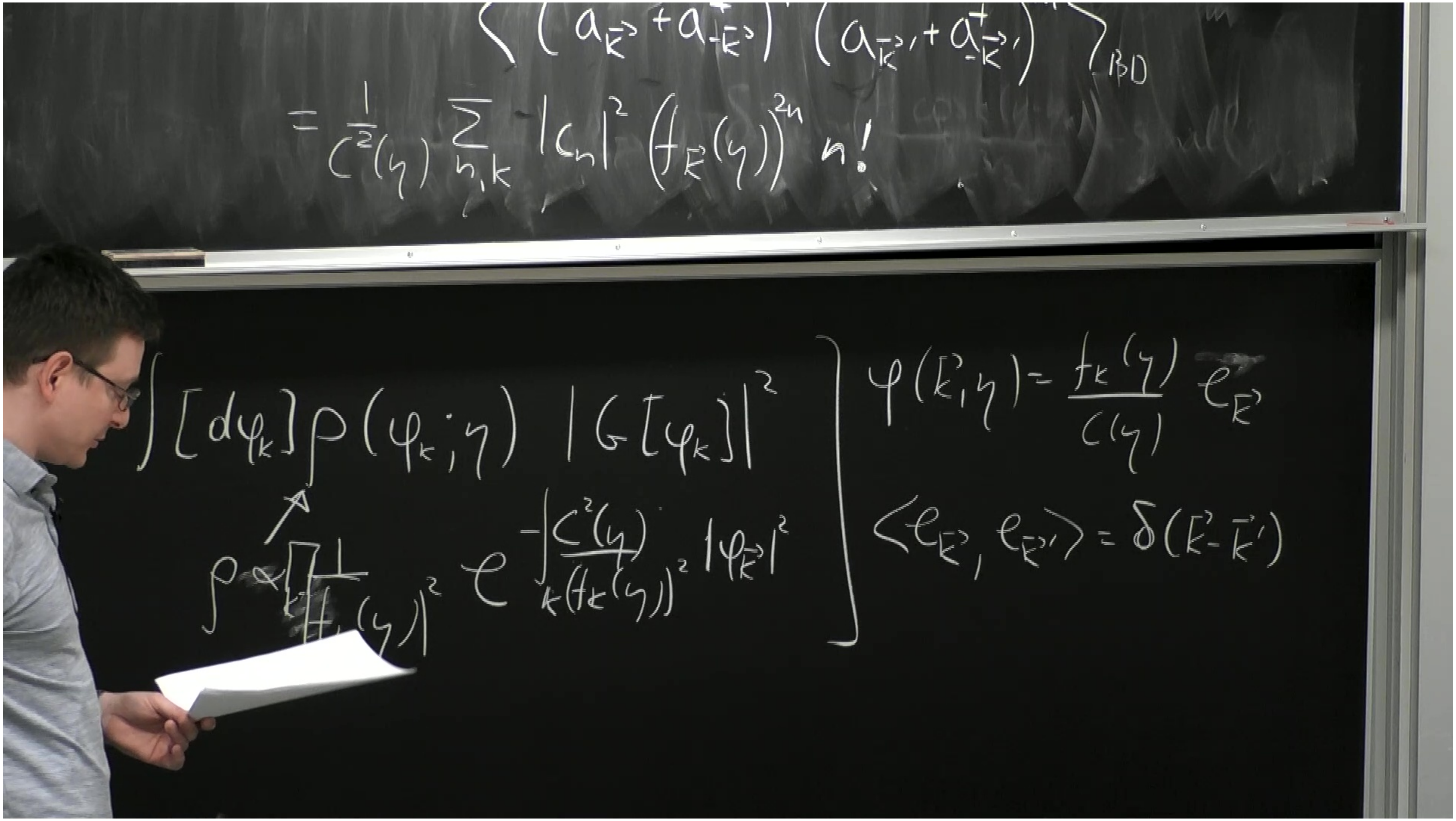


II) Operator averages in dS
$\varphi_{\vec{k}}(\eta) = \frac{1}{C(\eta)} \left(f_k(\eta) a_{\vec{k}} + f_k^*(\eta) a^+_{-\vec{k}} \right)$
$G(\eta) \equiv G[\varphi_{\vec{k}}] = \sum_{n,\{k\}} c_n \left(\varphi_k(\eta)\right)^n$
$\langle G(\eta) G^+(\eta) \rangle_{BD}$

$$f_k = \frac{i e^{-ik\eta}}{(2\pi)^{3/2}\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right) \approx \underbrace{\frac{1}{k\eta}\,\frac{1}{(2\pi)^{3/2}\sqrt{2k}}}\left(1 + O((k\eta)^2)\right)$$

$$\langle G^\dagger G \rangle_{BD} = \frac{1}{C^2(\eta)} \sum_{n,m,k,k'} c_n C_m^* \left(f_k(\eta)\right)^n \left(f_{k'}(\eta)\right)^m$$

$$\left\langle \left(a_{\vec{k}} + a^\dagger_{-\vec{k}}\right)^n \left(a_{\vec{k}'} + a^\dagger_{-\vec{k}'}\right)^m \right\rangle_{BD}$$

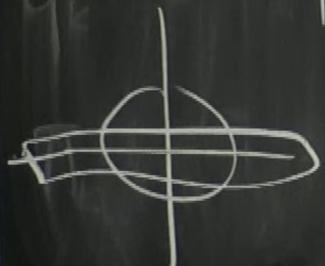

$\langle (a_{\vec k} + a^\dagger_{-\vec k}) \cdots (a_{\vec k'} + a^\dagger_{-\vec k'}) \cdots \rangle_{BD}$
$= \frac{1}{C^2(\eta)} \sum_{n,k} |c_n|^2 (f_{\vec k}(\eta))^{2n} n!$
$\int [d\varphi_k] \rho(\varphi_k;\eta) |G[\varphi_k]|^2$
$\rho \propto \prod_k \frac{1}{|f_k(\eta)|^2} e^{-\int_k \frac{C^2(\eta)}{|f_k(\eta)|^2} |\varphi_{\vec k}|^2}$
$\varphi(\vec k,\eta) = \frac{f_k(\eta)}{C(\eta)} e_{\vec k}$
$\langle e_{\vec k}, e_{\vec k'} \rangle = \delta(\vec k - \vec k')$

$$f_k = \frac{i e^{-ik\eta}}{(2\pi)^{3/2}\sqrt{2k}}\left(1-\frac{i}{k\eta}\right) \approx \frac{1}{k\eta}\,\frac{1}{(2\pi)^{3/2}\sqrt{2k}}\left(1+O\left((k\eta)^2\right)\right)$$

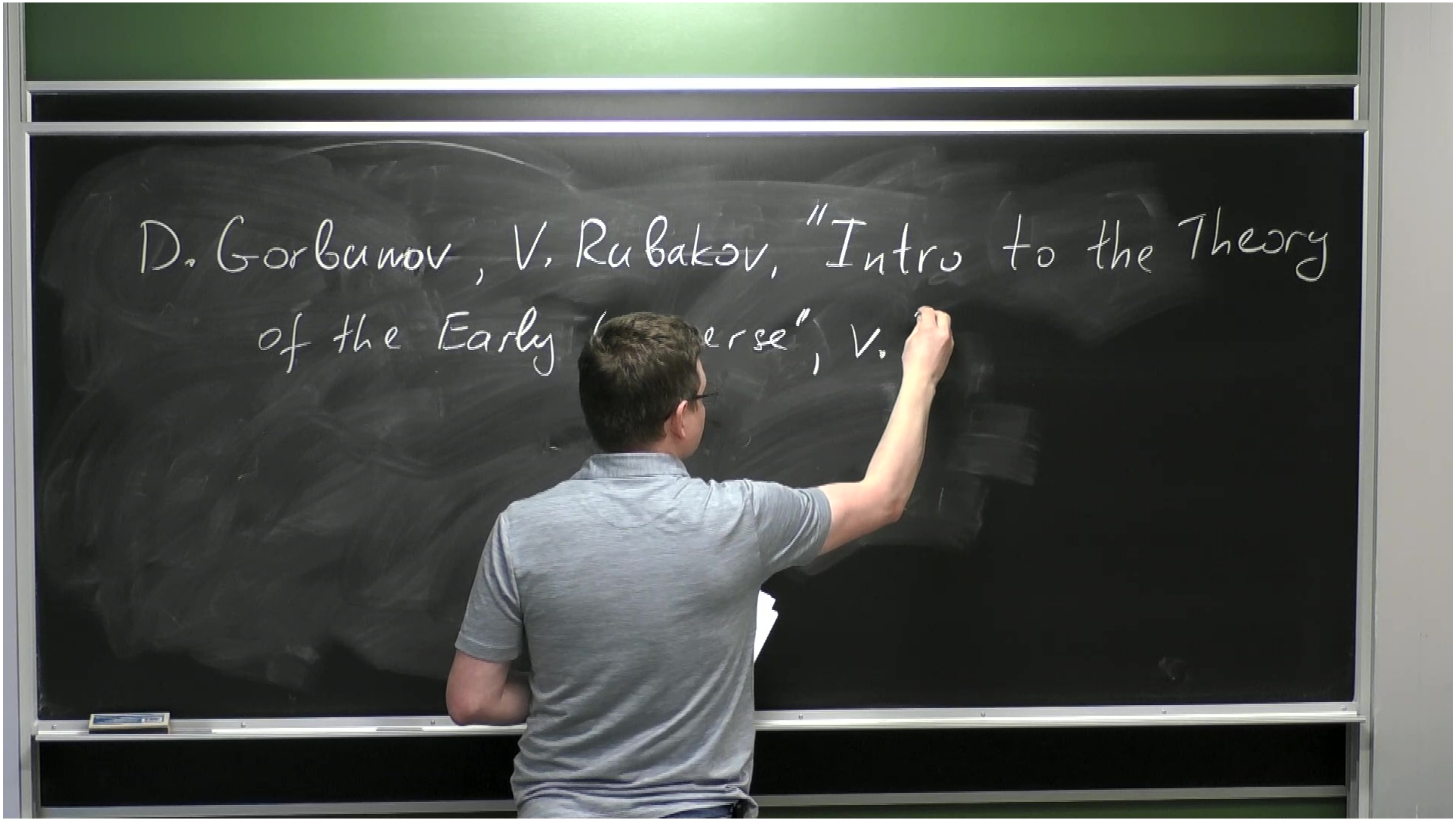
$$\langle G^\dagger G\rangle_{BD} = \frac{1}{C^2(\eta)}\sum_{n,m,k,k'} c_n c_m^* \left(f_k(\eta)\right)^n \left(f_{k'}(\eta)\right)^m$$

$$\delta\pi \sim \delta\varphi$$

$$\left\langle \left(a_{\vec k} + a^\dagger_{-\vec k}\right)^n \left(a_{\vec k'} + a^\dagger_{-\vec k'}\right)^m \right\rangle_{BD}$$

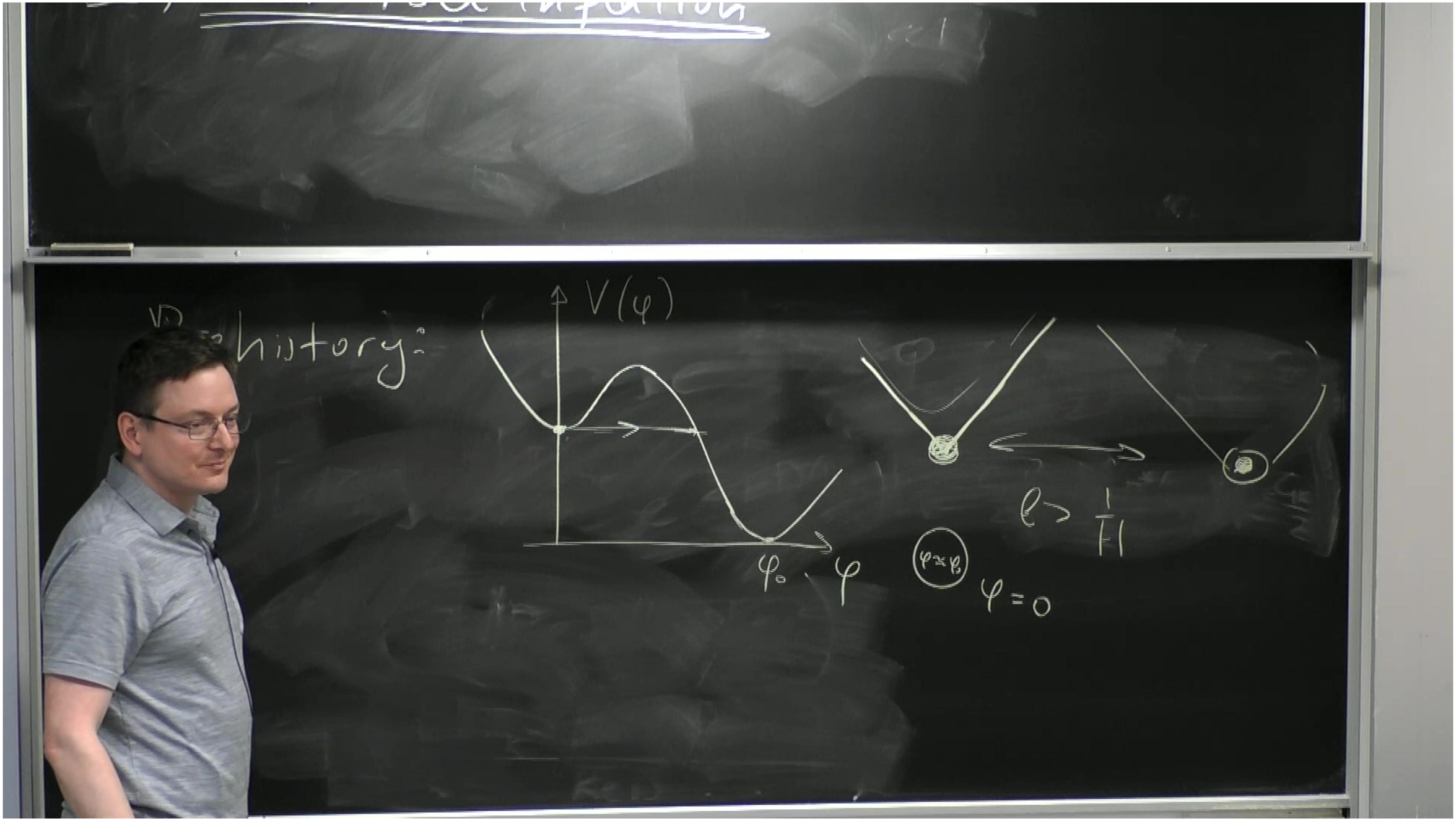


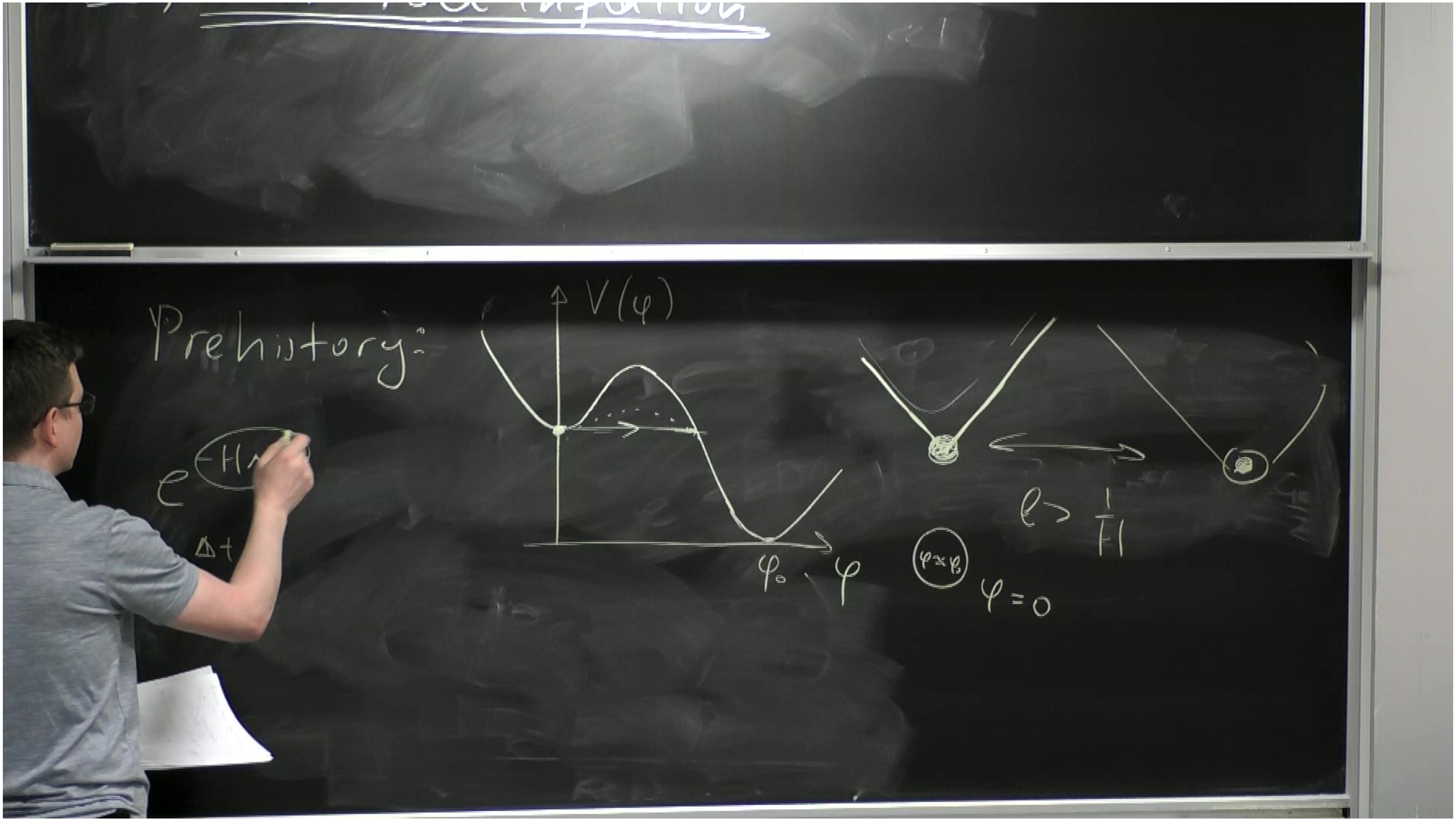
$$= \frac{1}{C^2(\eta)}\sum_{n,k}|c_n|^2 \left(f_{\vec k}(\eta)\right)^{2n} n!$$

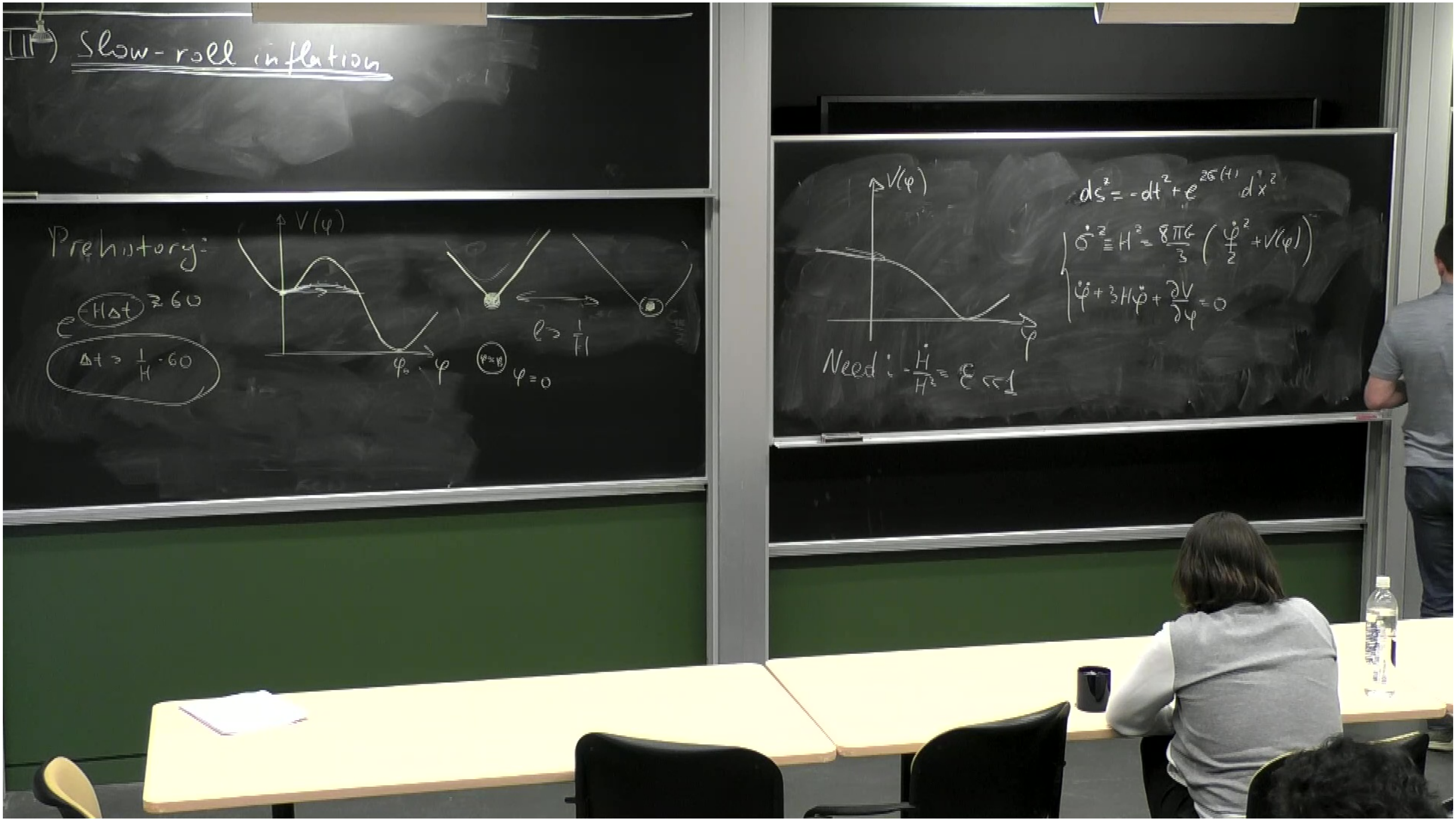

D. Gorbunov, V. Rubakov, "Intro to the Theory
of the Early
erse", V.

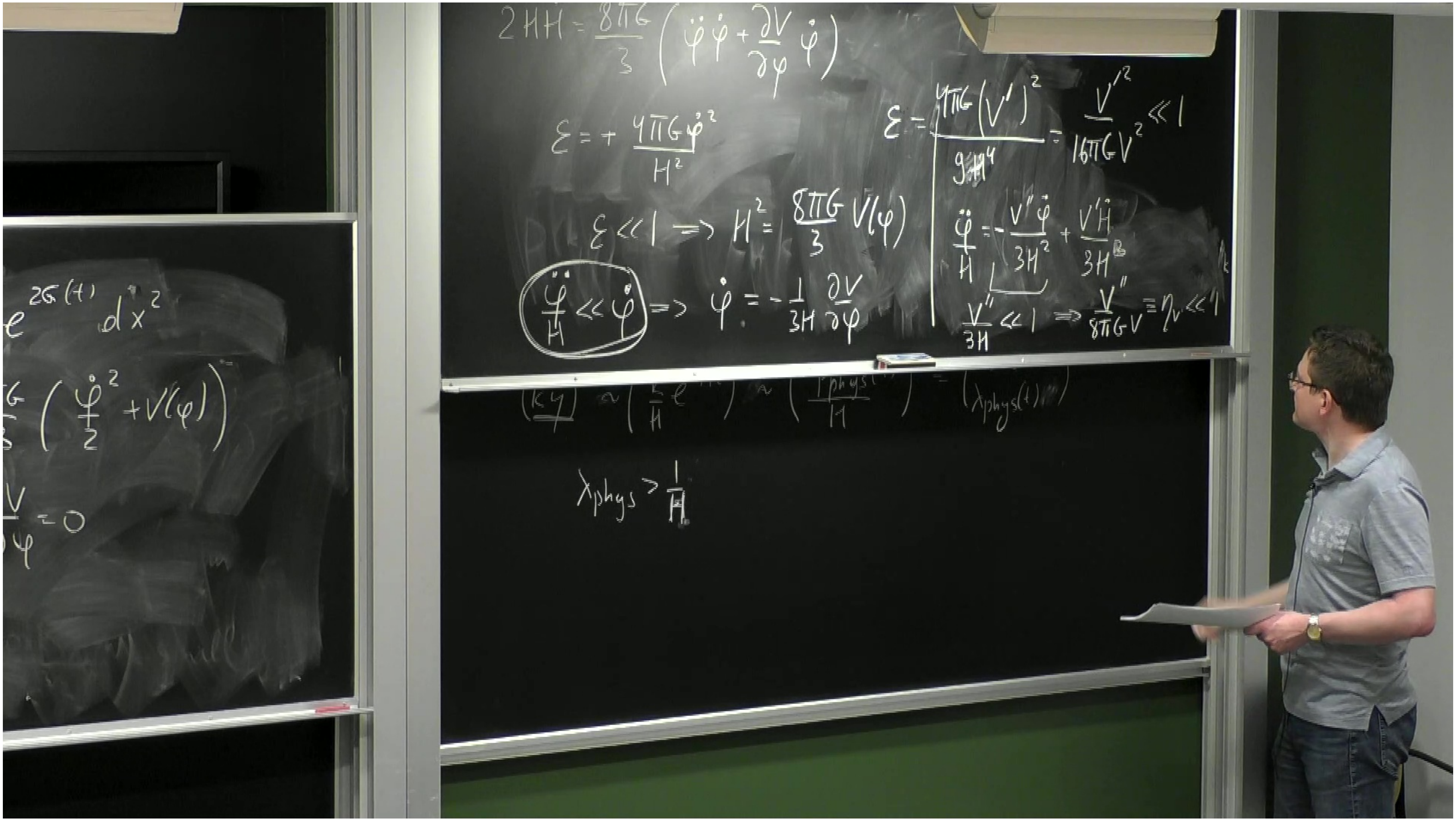
D. Gorbunov, V. Rubakov, "Intro to the Theory of the Early Universe", v. 2

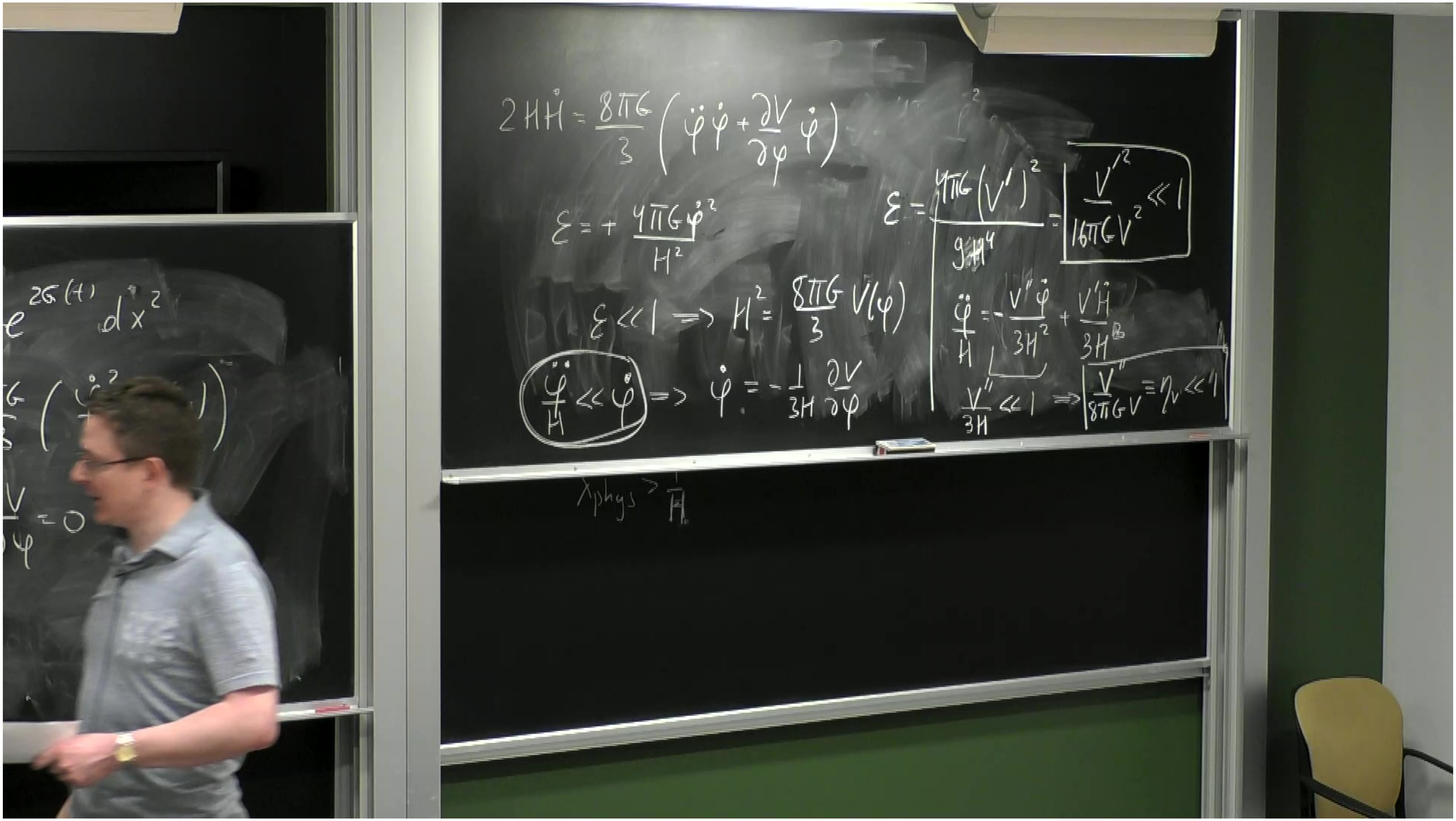
III) Slow-roll inflation

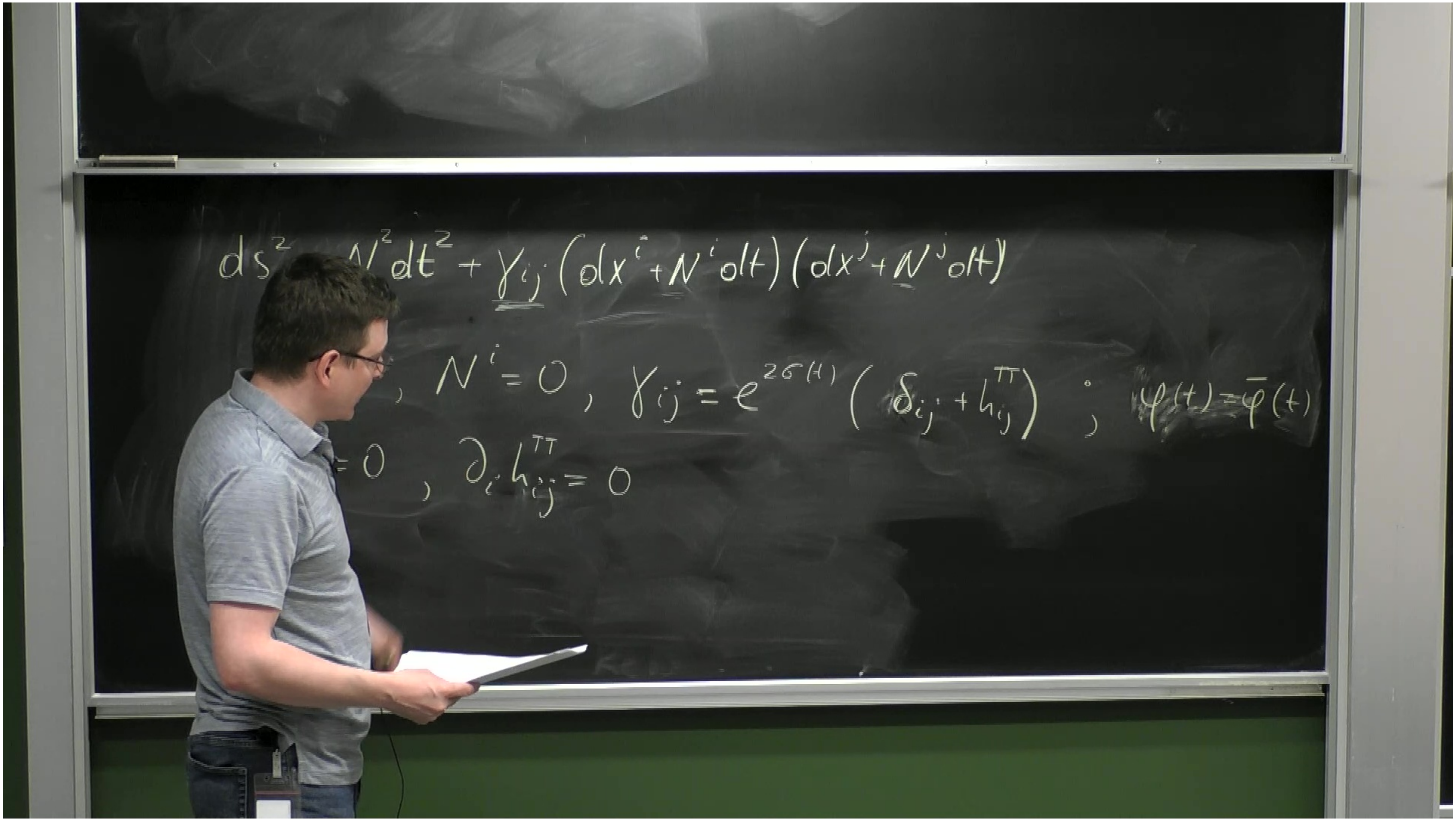

history:
V(φ)
φ₀
φ
φ ≈ φ₀
φ = 0
ℓ > 1/H


Prehistory:
V(φ)
φ₀
φ
φ≈φ₀
φ = 0
ℓ > 1/H
Δt

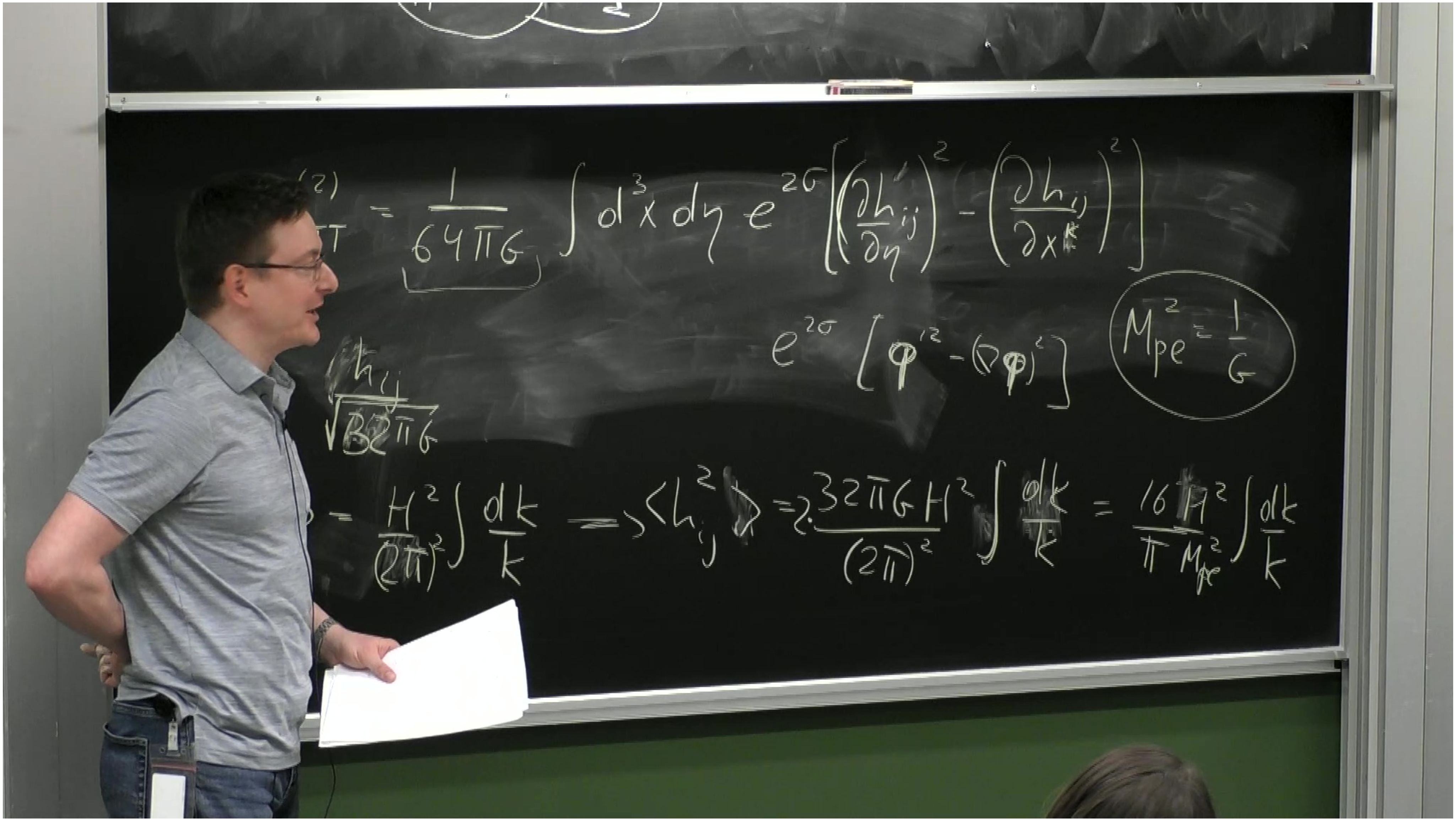

III) Slow-roll inflation
Prehistory:
$e^{H\Delta t} \gtrsim 60$
$\Delta t \gtrsim \frac{1}{H} \cdot 60$
$V(\varphi)$
φ_0 φ
$\ell > \frac{1}{H}$
$\varphi = 0$
$ds^2 = -dt^2 + e^{2\sigma(t)} d\vec{x}^2$
$\dot{\sigma}^2 \equiv H^2 = \frac{8\pi G}{3}\left(\frac{\dot{\varphi}^2}{2} + V(\varphi)\right)$
$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V}{\partial \varphi} = 0$
Need: $-\frac{\dot{H}}{H^2} = \epsilon \ll 1$

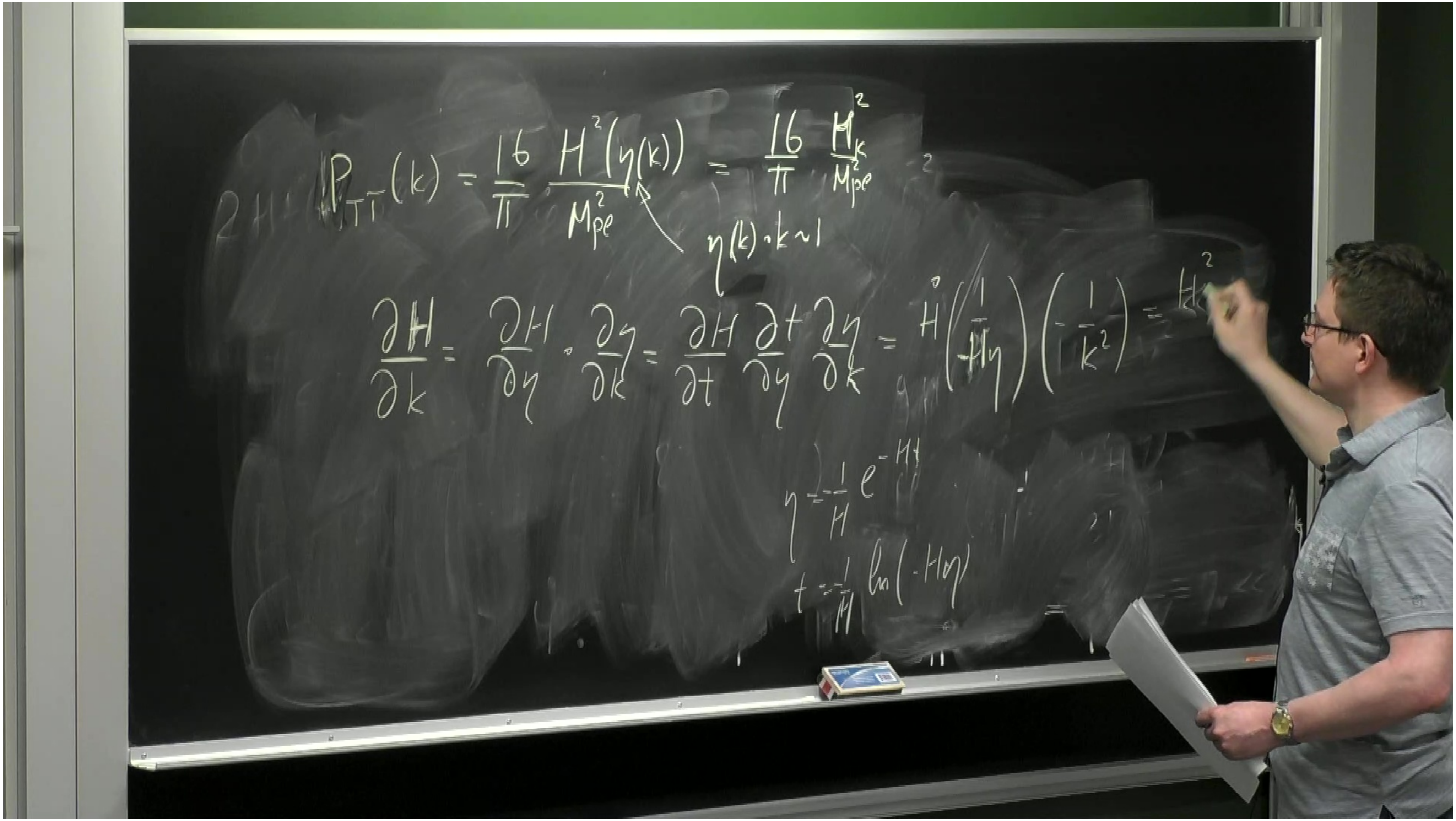

2HH = 8πG/3 (φ̈φ̇ + ∂V/∂φ φ̇)
ε = + 4πGφ̇²/H²
ε ≪ 1 ⟹ H² = 8πG/3 V(φ)
φ̈/H ≪ φ̇ ⟹ φ̇ = − 1/3H ∂V/∂φ
ε = 4πG(V′)²/9H⁴ = V′²/16πGV² ≪ 1
V″/3H ≪ 1 ⟹ V″/8πGV ≡ ηV ≪ 1
λphys > 1/H


$2H\dot{H} = \frac{8\pi G}{3}\left(\ddot{\varphi}\dot{\varphi} + \frac{\partial V}{\partial \varphi}\dot{\varphi}\right)$
$\varepsilon = + \frac{4\pi G \dot{\varphi}^2}{H^2}$
$\varepsilon \ll 1 \Rightarrow H^2 = \frac{8\pi G}{3} V(\varphi)$
$\frac{\ddot{\varphi}}{H} \ll \dot{\varphi} \Rightarrow \dot{\varphi} = -\frac{1}{3H}\frac{\partial V}{\partial \varphi}$
$\lambda_{phys} > \frac{1}{H}$



$ds^2 \quad N^2 dt^2 + \gamma_{ij} (dx^i + N^i dt)(dx^j + N^j dt)$
$, \; N^i = 0 \,, \; \gamma_{ij} = e^{2\sigma(t)} (\delta_{ij} + h^{TT}_{ij}) \;; \; \varphi(t) = \bar{\varphi}(t)$
$= 0 \,, \; \partial_i h^{TT}_{ij} = 0$


$^{(2)}{}_{T} = \frac{1}{64\pi G}\int d^3x\, d\eta\, e^{2\sigma}\left[\left(\frac{\partial h_{ij}}{\partial \eta}\right)^2 - \left(\frac{\partial h_{ij}}{\partial x^k}\right)^2\right]$
$e^{2\sigma}\left[\varphi'^2 - (\nabla\varphi)^2\right]$
$M_{pl}^2 = \frac{1}{G}$
$\frac{h_{ij}}{\sqrt{32\pi G}}$
$= \frac{H^2}{(2\pi)^2}\int \frac{dk}{k} \Rightarrow \langle h_{ij}^2\rangle = 2\cdot\frac{32\pi G H^2}{(2\pi)^2}\int\frac{dk}{k} = \frac{16 H^2}{\pi M_{pl}^2}\int\frac{dk}{k}$

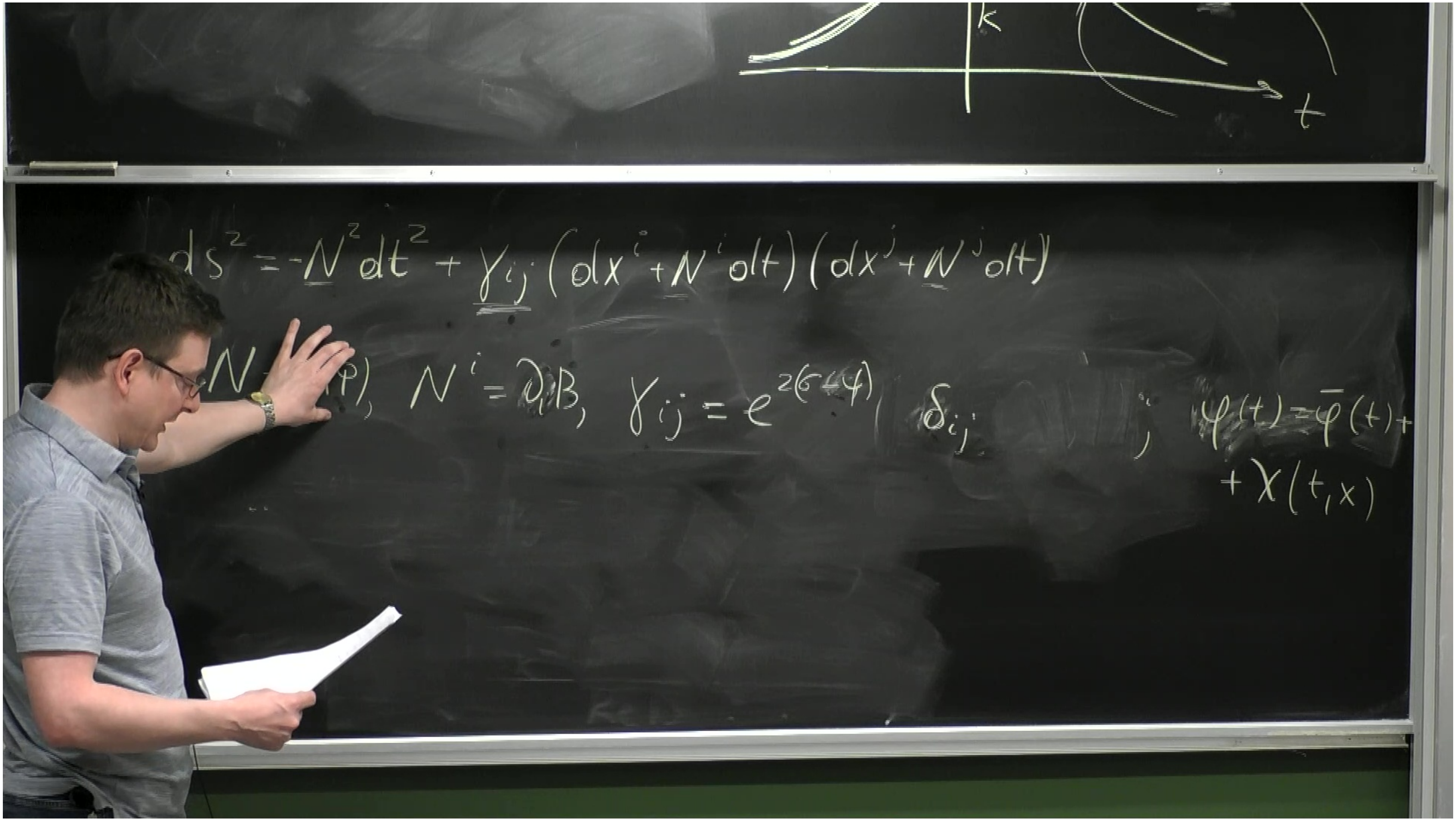


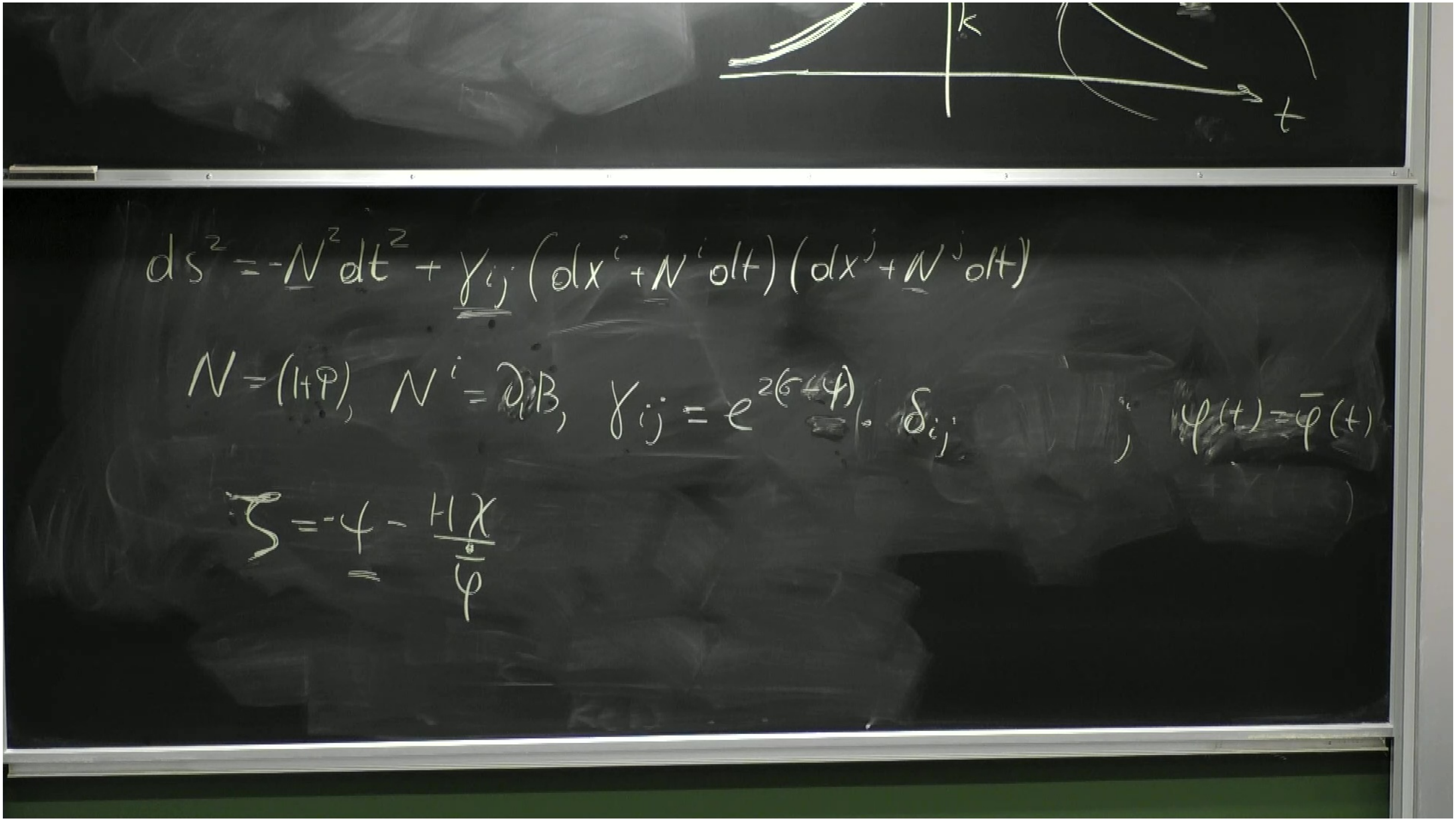
$P_{TT}(k) = \frac{16}{\pi}\frac{H^2(\eta(k))}{M_{pl}^2} = \frac{16}{\pi}\frac{H_k^2}{M_{pl}^2}$
$\eta(k)\cdot k \sim 1$
$\frac{\partial H}{\partial k} = \frac{\partial H}{\partial \eta}\cdot\frac{\partial \eta}{\partial k} = \frac{\partial H}{\partial t}\frac{\partial t}{\partial \eta}\frac{\partial \eta}{\partial k} = \dot{H}\left(\frac{1}{-H\eta}\right)\left(-\frac{1}{k^2}\right) =$
$\eta = -\frac{1}{H}e^{-Ht}$
$t = -\frac{1}{H}\ln(-H\eta)$

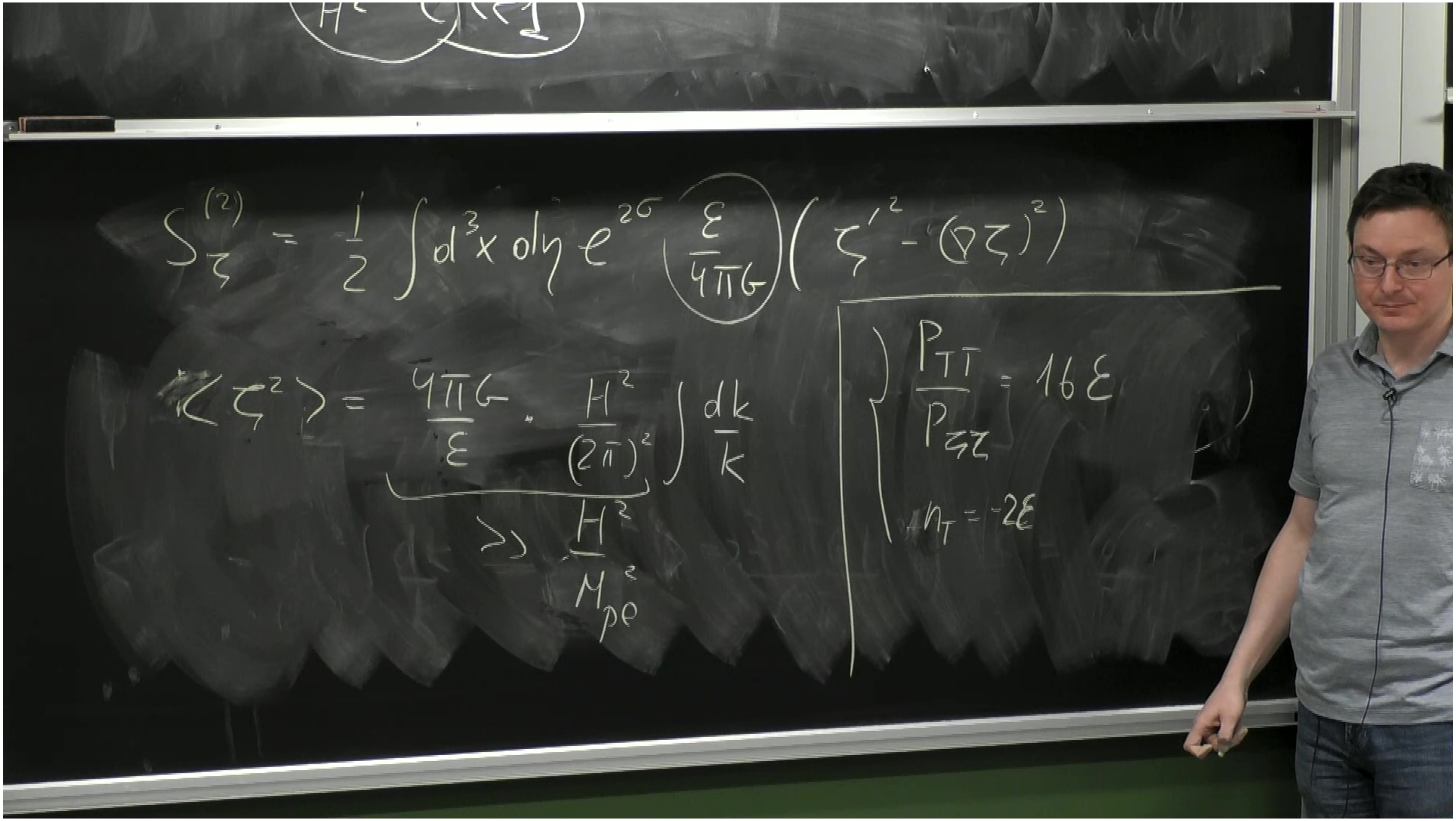
$$t = -\frac{1}{H}\ln(-H\eta)$$

$$\frac{\partial \log P_{TT}}{\partial \log k} = 2\,\frac{\partial \log H}{\partial \log k} = 2\,\frac{\dot{H}}{H^2} = \boxed{-2\varepsilon}$$

$$P_{TT} \propto (k)^{-2\varepsilon}$$


$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$N^i = \partial_i B, \quad \gamma_{ij} = e^{2(\zeta+\psi)} \delta_{ij}$
$\varphi(t) = \bar{\varphi}(t) + \chi(t,x)$
t
k


k
t
$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
$N = (1+\varphi),\ N^i = \partial_i B,\ \gamma_{ij} = e^{2(\zeta - \psi)}\delta_{ij}$; $\varphi(t) = \bar{\varphi}(t)$
$\zeta = -\psi - \frac{H\chi}{\dot{\bar{\varphi}}}$



$S^{(2)}_{\zeta} = \frac{1}{2}\int d^3x\, d\eta\, e^{2\sigma}\, \frac{\varepsilon}{4\pi G}\left(\zeta'^2 - (\nabla\zeta)^2\right)$
$\langle \zeta^2 \rangle = \underbrace{\frac{4\pi G}{\varepsilon} \cdot \frac{H^2}{(2\pi)^2}}_{\gg \frac{H^2}{M_{pl}^2}} \int \frac{dk}{k}$
$\frac{P_{TT}}{P_{\zeta\zeta}} = 16\,\varepsilon$
$n_T = -2\varepsilon$

$$\frac{\partial \log P_{\zeta\zeta}}{\partial \log k} = \frac{2\, d\log H}{d \log k} - \frac{d \log \varepsilon}{d \log k} = -2\varepsilon + 2\eta_V - 4\varepsilon = 2\eta_V - 6\varepsilon$$

$$\boxed{n_s - 1 = 2\eta_V - 6\varepsilon}$$

$$H^2 \sim V(\varphi)$$

$$n_s = 0{,}965 \pm 0{,}004$$