Title: Cosmology Lecture (230424)

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Collection: Cosmology (2022/2023)

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Perturbed Boltzmann eqn:

unpert. $f = f_0(E) = \frac{1}{e^{E/T} \pm 1}$

$f = f_0 + f_1(\vec{x}, \vec{p})$

$\frac{df}{dt} = C[f] = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{dx^i}{dt} + \frac{\partial f}{\partial E}\frac{dE}{dt} + \frac{\partial f}{\partial \hat{p}^i}\frac{d\hat{p}^i}{dt}$

$\frac{dx^i}{dt} = \frac{p\hat{p}^i}{aE}(1 - \Phi + \Psi)$

$\frac{dE}{dt} = -\frac{p\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i} - \frac{p^2}{E}\left(H + \frac{\partial \Phi}{\partial t}\right)$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{p\hat{p}^i}{aE} - \frac{\partial f}{\partial E}\left[\frac{p\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i} + \frac{p^2}{E}\left(H + \frac{\partial \Phi}{\partial t}\right)\right] = 0$$

$$\int \frac{\partial f}{\partial t}\frac{d^3p}{(2\pi)^3} = \frac{\partial n}{\partial t}$$

$$\int \frac{\partial f}{\partial x^i}\frac{p\hat{p}^i}{aE}\frac{d^3p}{(2\pi)^3} = \frac{1}{a}\frac{\partial}{\partial x}\underbrace{\int f\frac{p\hat{p}^i}{E}\frac{d^3p}{(2\pi)^3}}_{\equiv n\vec{v}} = \frac{1}{a}\nabla\cdot(n_0\vec{v})$$

$$-\frac{\partial \Psi}{\partial x^i}\int \frac{\partial f_0}{\partial E}\frac{p\hat{p}^i}{a}\frac{d^3p}{(2\pi)^3}$$

$$\left(H+\frac{\partial\Phi}{\partial t}\right)\Bigg] = 0$$

$$\nabla\cdot(n_0\vec{v})$$

$$-\frac{\partial\Psi}{\partial x^i}\int\frac{\partial f_0}{\partial E}\,\frac{p\hat{p}^i}{a}\,\frac{p^2\,dp\,d^2\hat{p}}{(2\pi)^3} = 0$$

$$-\int\frac{\partial f}{\partial E}\,\frac{p^2}{E}\left(H+\frac{\partial\Phi}{\partial t}\right)\frac{d^3p}{(2\pi)^3} = \left(H+\frac{\partial\Phi}{\partial t}\right)\int f\;3\;\frac{p^2\,dp\,d^2\hat{p}}{(2\pi)^3}$$

$$\frac{\partial f}{\partial E} = \frac{\partial f}{\partial p}\cdot\frac{dp}{dE} = \frac{E}{p}\frac{\partial f}{\partial p}$$

$$\left(H + \frac{\partial\Phi}{\partial t}\right)\Big] = 0$$

$$\nabla\cdot(n_0 \vec{v})$$

$$-\frac{\partial\Psi}{\partial x^i}\int \frac{\partial f_0}{\partial E}\,\frac{p\hat{p}^i}{a}\,\frac{p^2\, dp\, d^2\hat{p}}{(2\pi)^3} = 0$$

$$-\int \frac{\partial f}{\partial E}\,\frac{p^2}{E}\left(H+\frac{\partial\Phi}{\partial t}\right)\frac{d^3p}{(2\pi)^3} = 3\left(H+\frac{\partial\Phi}{\partial t}\right)\int f\, \frac{d^3p}{(2\pi)^3}$$

$$\frac{\partial f}{\partial E} = \frac{\partial f}{\partial p}\cdot\frac{dp}{dE} = \frac{E}{p}\,\frac{\partial f}{\partial p}$$

$$\left(H + \frac{\partial\Phi}{\partial t}\right)\Big] = 0$$

$$\nabla\cdot(n_0\vec{v})$$

$$-\frac{\partial\Psi}{\partial x^i}\int \frac{\partial f_0}{\partial E}\,\frac{p\hat{p}^i}{a}\,\frac{p^2\,dp\,d\hat{p}}{(2\pi)^3} = 0$$

$$-\int \frac{\partial f}{\partial E}\,\frac{p^2}{E}\left(H + \frac{\partial\Phi}{\partial t}\right)\frac{d^3p}{(2\pi)^3} = 3\left(H + \frac{\partial\Phi}{\partial t}\right) n$$

$$\frac{\partial f}{\partial E} = \frac{\partial f}{\partial p}\cdot\frac{dp}{dE} = \frac{E}{p}\,\frac{\partial f}{\partial p}$$

$$\boxed{\frac{\partial n}{\partial t} + \frac{1}{a}\nabla\cdot(n_0\vec{v}) + 3\left(H + \frac{\partial\Phi}{\partial t}\right)n = 0}$$

$$n \equiv n_0(1+\delta)$$

$$\frac{\partial n_0}{\partial t} + 3Hn_0 = 0 \Rightarrow n_0 \propto a^{-3}$$

$$\frac{\partial\delta}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{v} + 3\frac{\partial\Phi}{\partial t} = 0$$

$$a^2(1+2\Phi)\delta_{ij}\,dx^i dx^j$$

$= 0$

$$\frac{\partial \rho}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{q} + 3\left(H + \frac{\partial \Phi}{\partial t}\right)(\rho + P) = 0$$

$$\rho = \rho_0 (1+\delta)$$

$$\frac{\partial \delta}{\partial t} + (1+w)\left[\frac{1}{a}\vec{\nabla}\cdot\vec{v} + 3\frac{\partial \Phi}{\partial t}\right] + 3H\left(c_s^2 - w\right)\delta = 0$$

$$w \equiv P_0/\rho_0 \qquad c_s^2 = \frac{\partial P}{\partial \rho}$$

$= 0$

$$\frac{\partial \rho}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{q} + 3\left(H + \frac{\partial \Phi}{\partial t}\right)(\rho + P) = 0$$

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$$w \equiv P_0/\rho_0 \qquad c_s^2 = \frac{\delta P}{\delta \rho}$$

NR: $\rho = m\,n$

$$\delta_\rho = \delta_n$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{p\hat{p}^i}{aE} - \frac{\partial f}{\partial E}\left[\frac{p\hat{p}^i}{a}\frac{\partial \Phi}{\partial x^i} + \frac{p^2}{E}\left(H + \frac{\partial \Phi}{\partial t}\right)\right] = 0$$

$$\int f \frac{d^3p}{(2\pi)^3} = n$$

$$\rho = \int f E \frac{d^3p}{(2\pi)^3}$$

$$\int \frac{\partial f}{\partial x^i}\frac{p\hat{p}^i}{aE}\frac{d^3p}{(2\pi)^3} = \frac{1}{a}\frac{\partial}{\partial x^i}\underbrace{\int f \frac{p\hat{p}^i}{E}\frac{d^3p}{(2\pi)^3}}_{\equiv n\vec{v}} = \frac{1}{a}\nabla\cdot(n_0\vec{v})$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{q} + 3(H +$$

$$\rho = \rho_0(1+\delta)$$

$$\frac{\partial \delta}{\partial t} + (1+w)\left[\frac{1}{a}\vec{\nabla}\cdot\vec{v} + 3\right.$$

$$w \equiv P_0/\rho_0$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{q} + 3\left(H + \frac{\partial \Phi}{\partial t}\right)(\rho + P) = 0$$

$$\rho = \rho_0 (1+\delta)$$

$$\frac{\partial \delta}{\partial t} + (1+w)\left[\frac{1}{a}\vec{\nabla}\cdot\vec{v} + 3\frac{\partial \Phi}{\partial t}\right] + 3H\left(c_s^2 - w\right)\delta = 0$$

$$w \equiv P_0/\rho_0 \qquad c_s^2 = \frac{\partial P}{\partial \rho}$$

NR: $\rho = m\, n$

$$\delta_\rho = \delta_n$$

$$\vec{q} = (\rho_0 + P_0)\,\vec{v}$$

$$\frac{\partial \vec{q}}{\partial t}$$

$$\Phi)$$

$$\left(H + \frac{\partial \Phi}{\partial t}\right)$$

$$= 0$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{p\hat{p}^i}{aE} - \frac{\partial f}{\partial E}\left[\frac{p\hat{p}^i}{a}\frac{\partial \Phi}{\partial x^i} + \frac{p^2}{E}\left(H + \frac{\partial \Phi}{\partial t}\right)\right] = 0$$

$$\int f \frac{d^3p}{(2\pi)^3} = n$$

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$$\frac{\partial \rho}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{q} +$$

$$\rho = \rho_0(1+\delta)$$

$$\frac{\partial \delta}{\partial t} + (1+w)\left[\frac{1}{a}\right.$$

$$w \equiv P_0/$$

$$\Big] = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{q} + 3\left(H + \frac{\partial \Phi}{\partial t}\right)(\rho + P) = 0$$

$$\rho = \rho_0 (1+\delta)$$

$$\frac{\partial \delta}{\partial t} + (1+w)\left[\frac{1}{a}\vec{\nabla}\cdot\vec{v} + 3\frac{\partial \Phi}{\partial t}\right] + 3H\left(c_s^2 - w\right)\delta = 0$$

$$w \equiv P_0/\rho_0 \qquad c_s^2 = \frac{\partial P}{\partial \rho}$$

NR: $\rho = m\,n$

$$\delta_\rho = \delta_n$$

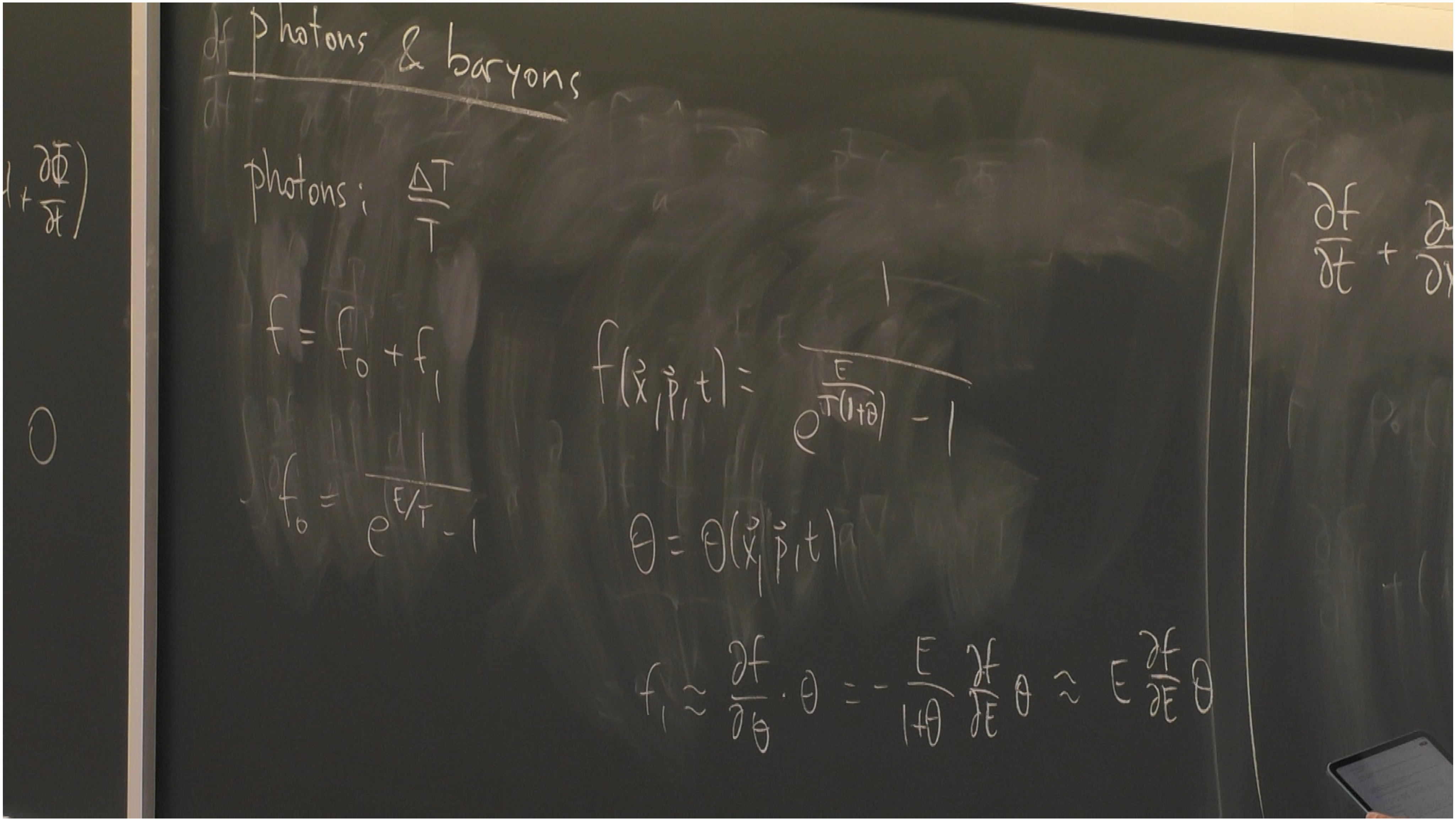
$$\vec{q} = (\rho_0 + P_0)\,\vec{v}$$

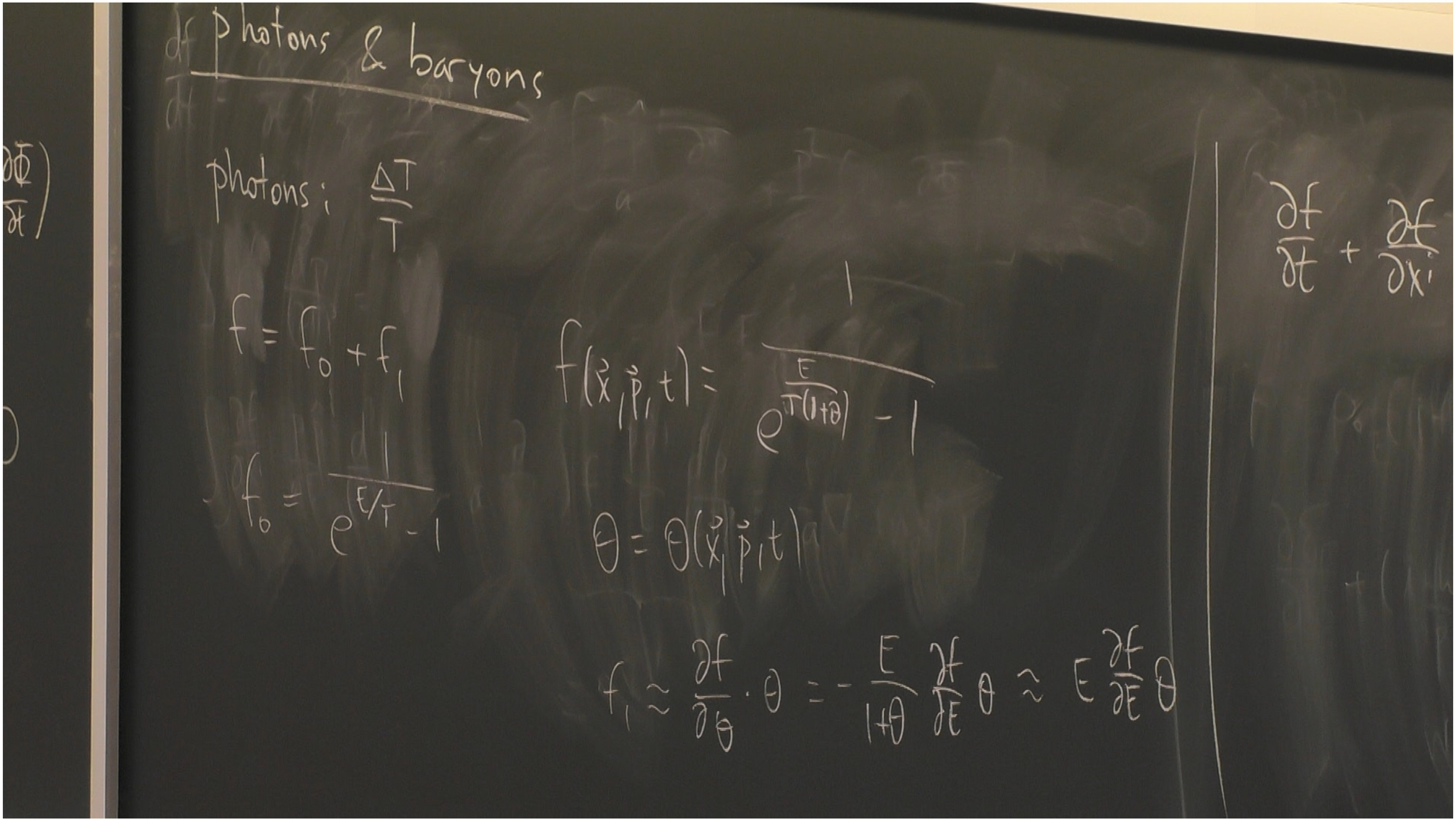
$$\frac{\partial \vec{q}}{\partial t}$$

$$\frac{\partial q^j}{\partial t} + 4 H q^j + \frac{1}{a}\frac{\partial}{\partial x^i} T^{ij} + \frac{1}{a}\frac{\partial \Psi}{\partial x^i}\left(T^{ij} + \rho\,\delta^{ij}\right) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + H\vec{v} + \frac{1}{a}\vec{\nabla}\Psi = 0$$

Cold DM : $T^{ij} = 0$ $\quad P = 0$


Photons & baryons
photons: $\frac{\Delta T}{T}$
$f = f_0 + f_1$
$f_0 = \frac{1}{e^{E/T} - 1}$
$f(\vec{x}, \vec{p}, t) = \frac{1}{e^{\frac{E}{T(1+\Theta)}} - 1}$
$\Theta = \Theta(\vec{x}, \vec{p}, t)$
$f_1 \simeq \frac{\partial f}{\partial \Theta} \cdot \Theta = -\frac{E}{1+\Theta} \frac{\partial f}{\partial E} \Theta \simeq E \frac{\partial f}{\partial E} \Theta$
$\frac{\partial f}{\partial t} +$


Photons & baryons
photons: $\frac{\Delta T}{T}$
$f = f_0 + f_1$
$f_0 = \frac{1}{e^{E/T} - 1}$
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$\theta = \theta(\vec{x}, \vec{p}, t)$
$f_1 \simeq \frac{\partial f}{\partial \theta} \cdot \theta = -\frac{E}{1+\theta} \frac{\partial f}{\partial E} \theta \simeq E \frac{\partial f}{\partial E} \theta$
$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}$

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{\hat{p}^i}{a} - E\frac{\partial f}{\partial E}\left[\frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x} + \left(H + \frac{\partial \Phi}{\partial t}\right)\right]$$

$$f \to f_0 - E\,\frac{\partial f_0}{\partial E}\,\Theta$$

$$E\frac{\partial f}{\partial E}\Theta$$

$$\frac{\partial}{\partial t}\left(f_0 - E\frac{\partial f}{\partial E}\Theta\right) + \frac{\partial}{\partial x}\left(f_0 - E\frac{\partial f_0}{\partial E}\Theta\right)\frac{\hat{p}^i}{a} - \left[\frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x} + \left(H + \frac{\partial \Phi}{\partial t}\right)\right] E\frac{\partial}{\partial E}\left(f_0 - E\frac{\partial f}{\partial E}\,\Theta\right)$$

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{\hat{p}^i}{a} - E\frac{\partial f}{\partial E}\left[\frac{\hat{p}^i}{a}\frac{\partial \Phi}{\partial x} + \left(H + \frac{\partial \Phi}{\partial t}\right)\right]$$

$$f \to f_0 - E\frac{\partial f_0}{\partial E}\Theta$$

$$E\frac{\partial f}{\partial E}\Theta$$

$$\frac{\partial}{\partial t}\left(f_0 - E\frac{\partial f}{\partial E}\Theta\right) + \frac{\partial}{\partial x}\left(f_0 - E\frac{\partial f_0}{\partial E}\Theta\right)\frac{\hat{p}^i}{a} - \left[\frac{\hat{p}^i}{a}\frac{\partial \Phi}{\partial x} + \left(H + \frac{\partial \Phi}{\partial t}\right)\right]E\frac{\partial}{\partial E}\left(f_0 - E\frac{\partial f}{\partial E}\Theta\right)$$

$$\frac{\partial f_0}{\partial t} - HE\frac{\partial f_0}{\partial E} = 0$$

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i}\frac{\hat{p}^i}{a} - E\frac{\partial f}{\partial E}\left[\frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x} + \left(H + \frac{\partial \Phi}{\partial t}\right)\right]$$

$$f \to f_0 - E\frac{\partial f_0}{\partial E}\Theta$$

$$E\frac{\partial f}{\partial E}\Theta$$

$$\frac{\partial}{\partial t}\left(f_0 - E\frac{\partial f}{\partial E}\Theta\right) + \frac{\partial}{\partial x}\left(f_0 - E\frac{\partial f_0}{\partial E}\Theta\right)\frac{\hat{p}^i}{a} - \left[\frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x} + \left(H + \frac{\partial \Phi}{\partial t}\right)\right]E\frac{\partial}{\partial E}\left(f_0 - E\frac{\partial f}{\partial E}\Theta\right)$$

$$\frac{\partial f_0}{\partial t} - HE\frac{\partial f_0}{\partial E} = 0$$

$$-E\frac{\partial f_0}{\partial E}\left[\frac{\partial\Theta}{\partial t}+\frac{\hat{p}^i}{a}\frac{\partial\Theta}{\partial x^i}-HE\frac{\partial\Theta}{\partial E}+\left(\frac{\hat{p}^i}{a}\frac{\partial\Psi}{\partial x^i}+\frac{\partial\Phi}{\partial t}\right)\right]=$$