Title: Cosmology Lecture (230403)

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Collection: Cosmology (2022/2023)

Date: April 03, 2023 - 3:15 PM

URL: https://pirsa.org/23040062

Cosmology

me: Neal Dalal

book: Dodelson & Schmidt

online: Baumann notes

Review of classical cosmology

Facts:

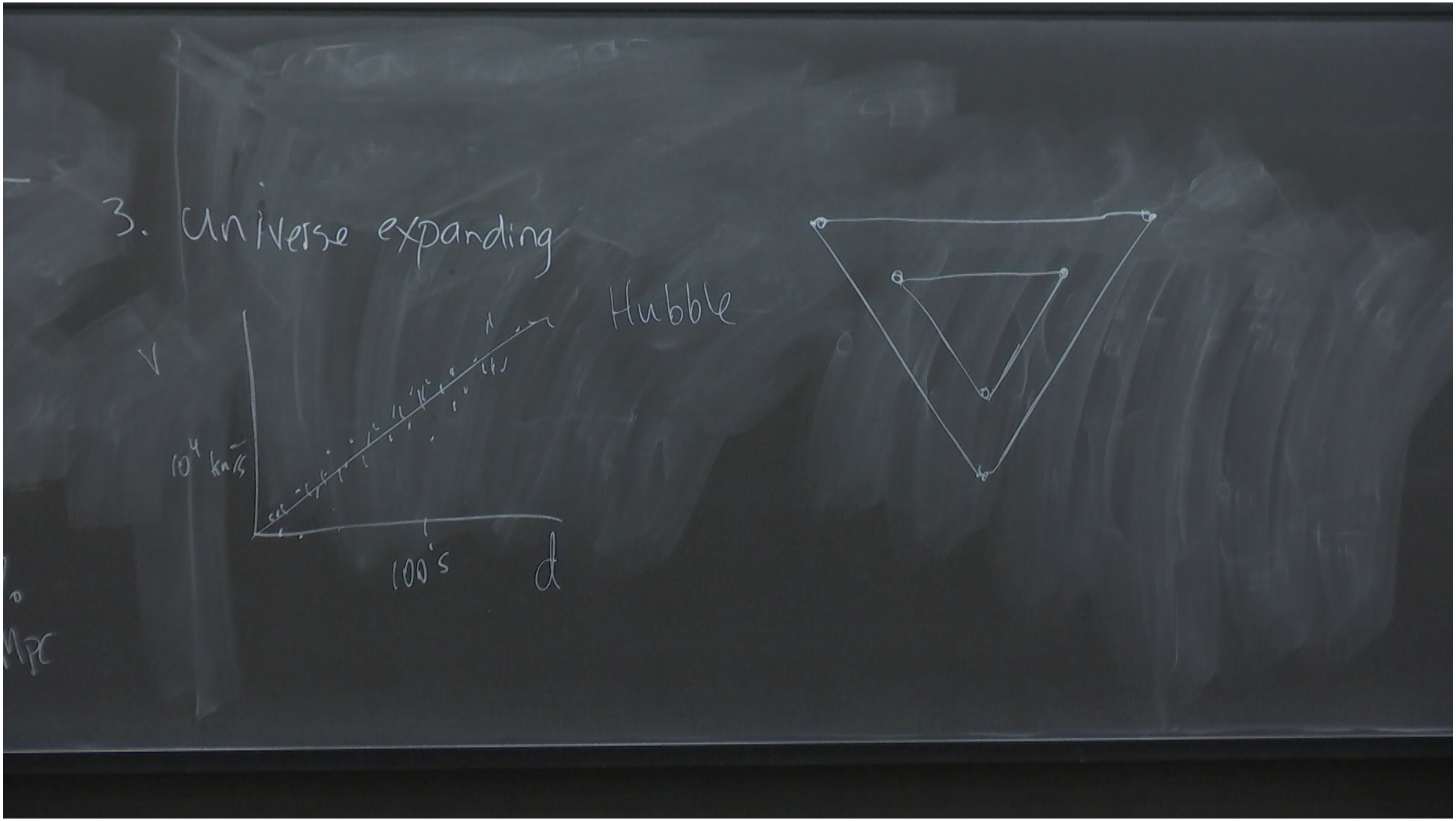
1. gravity = GR

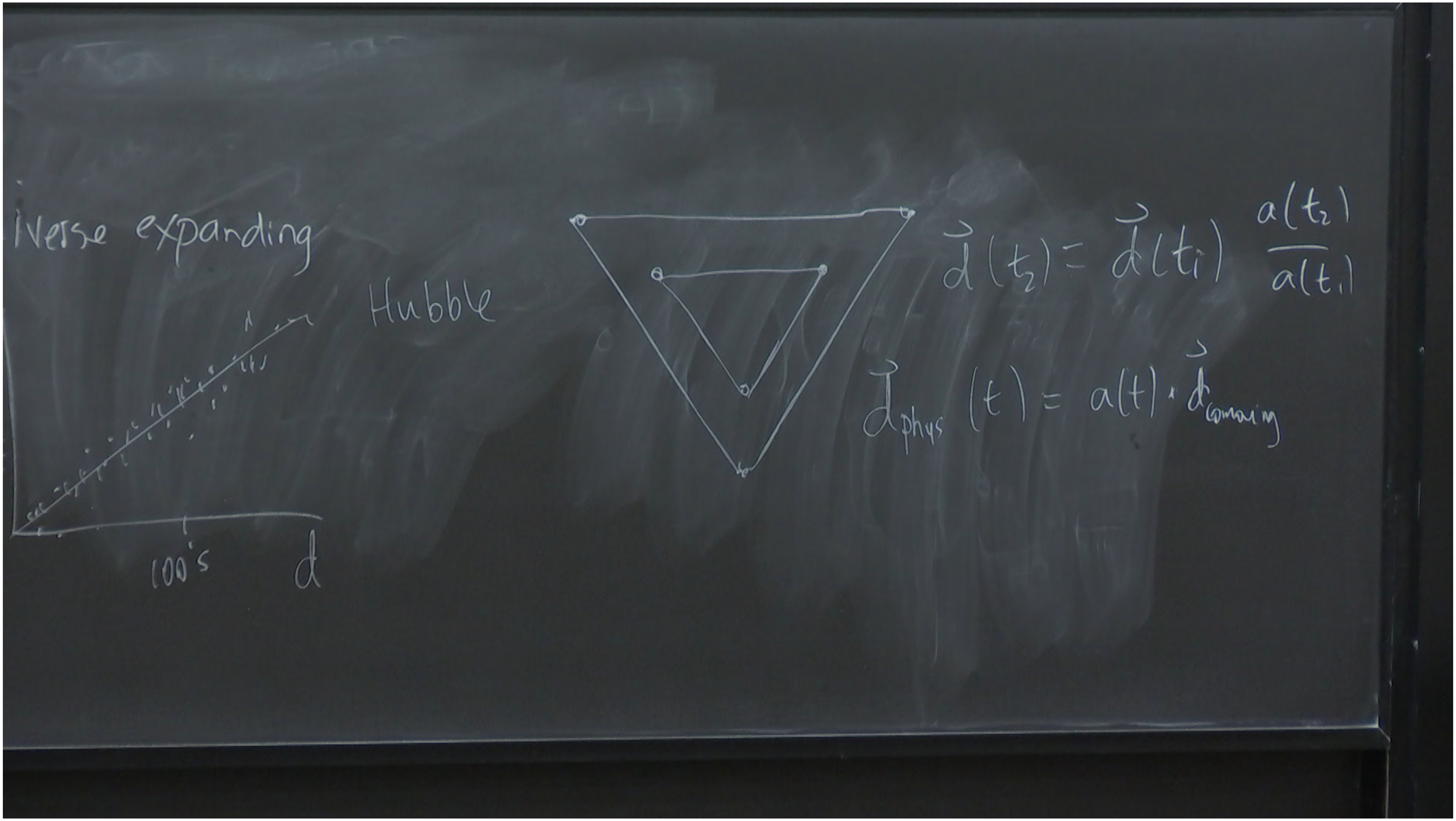
2. homog & isotropy

CMB: 10^{-5} isotropic

galaxies uniform $\lesssim 10\%$

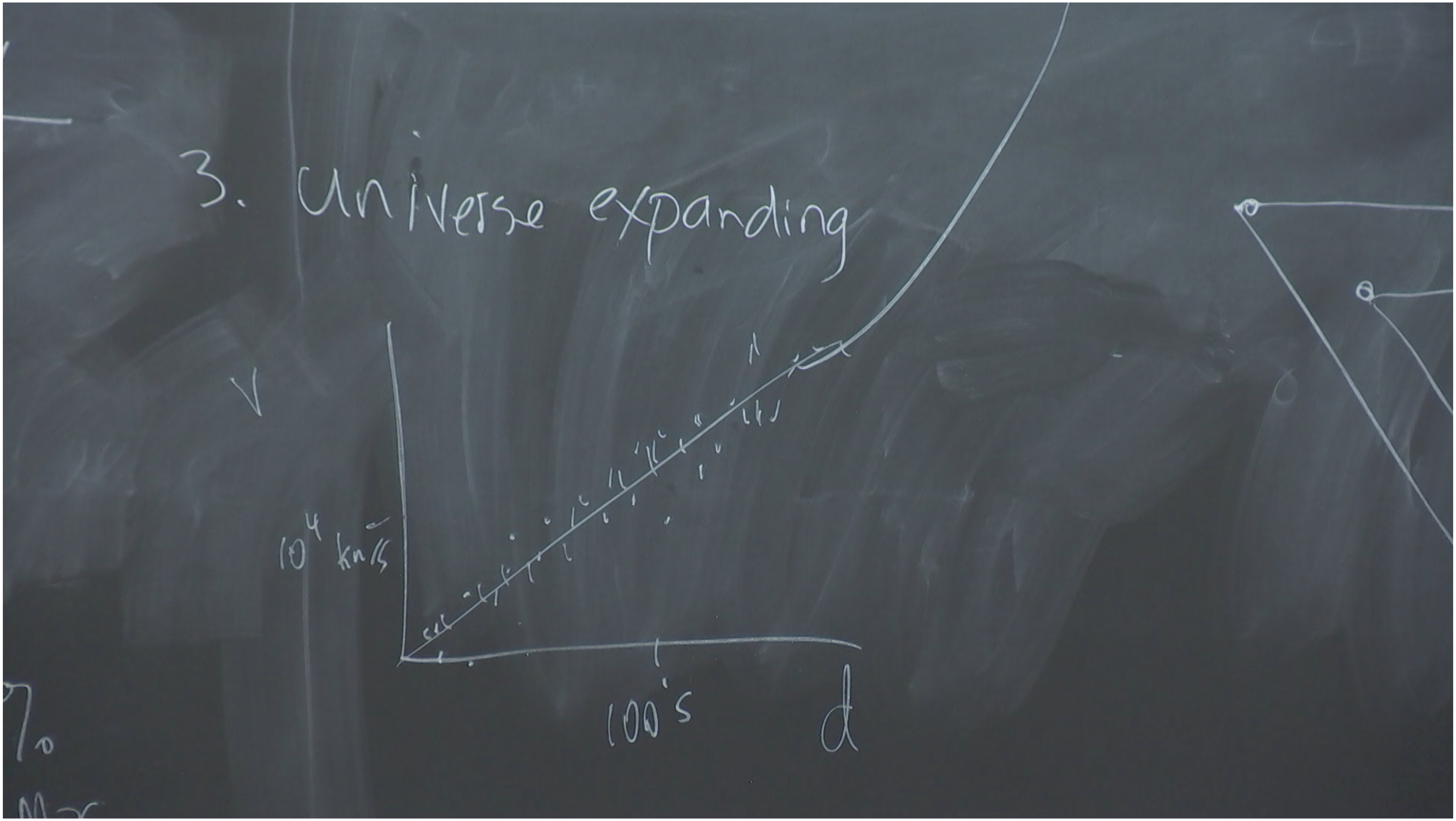
100's of Mpc

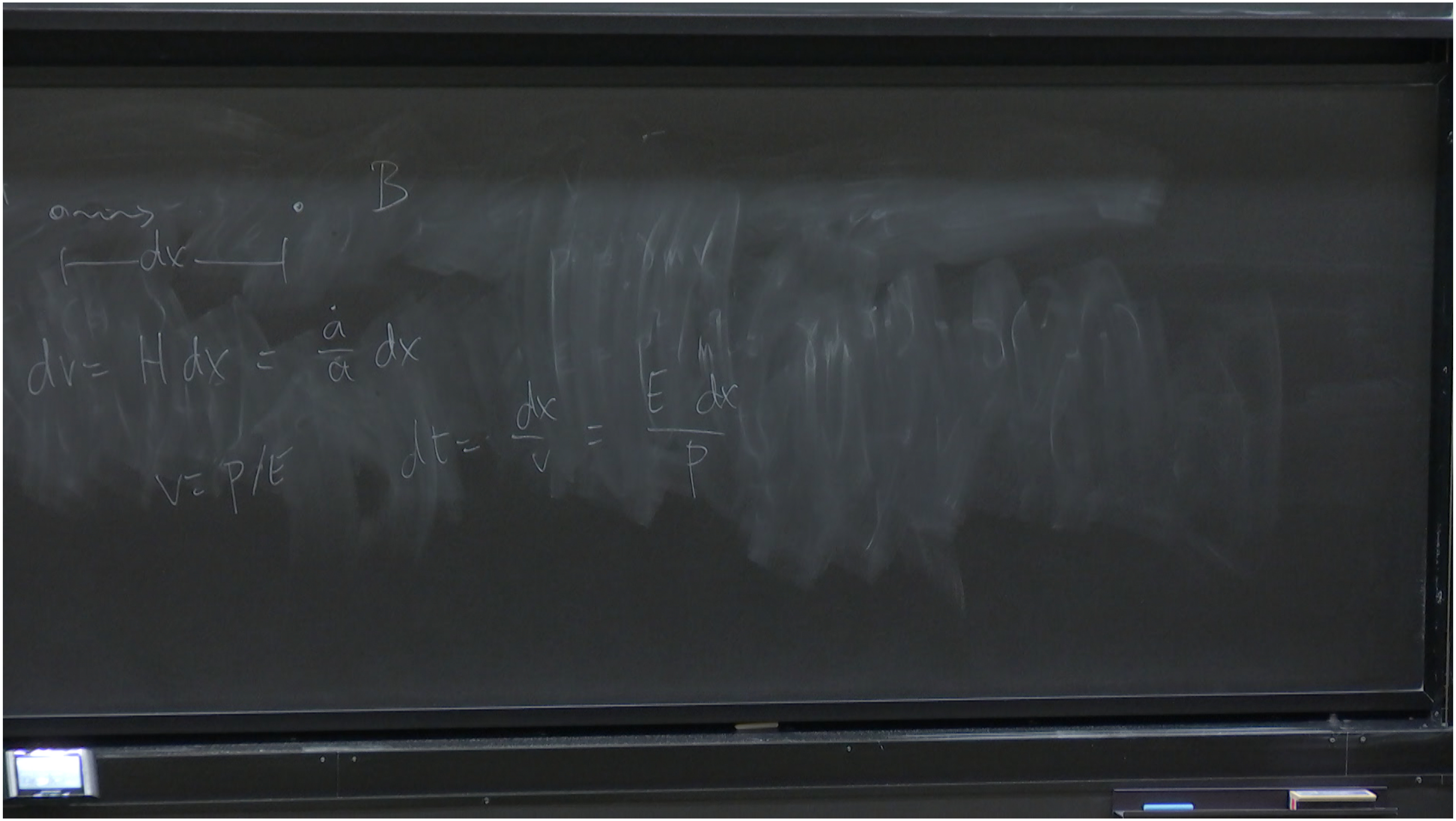

3. Universe expanding
Hubble
v
10^4 km/s
100's
d
Mpc


iverse expanding
Hubble
100's
d
$\vec{d}(t_2) = \vec{d}(t_1) \frac{a(t_2)}{a(t_1)}$
$\vec{d}_{phys}(t) = a(t) \cdot \vec{d}_{comoving}$

$$\vec{v}(t) = \frac{d}{dt}\left(\vec{d}\right) = \frac{da}{dt}\,\vec{d}_{\text{comoving}} = \frac{1}{a}\frac{da}{dt}\,\vec{d}_p$$

$$H(t) \equiv \frac{\dot{a}}{a}$$


3. Universe expanding
v
10⁴ km/s
100's
d


B
dx
$dv = H\,dx = \frac{\dot{a}}{a}\,dx$
$v = p/E$
$dt = \frac{dx}{v} = \frac{E\,dx}{p}$

B

$$\dot{p} = \frac{dp}{dt} = -Hp$$

$$= -\frac{\dot{a}}{a}\,p$$

dx

$$dt = \frac{dx}{v} = \frac{E\,dx}{p} \qquad \frac{dp}{p} = -\frac{da}{a} \Rightarrow p \propto \frac{1}{a}$$

$$dv = -EH\,dx$$

cosmological redshifts

$$p_{obs} = p_{emit} \times \frac{a_{emit}}{a_{obs}} \qquad a(\text{today}) = 1$$

$$p = h\nu = h/\lambda$$

$$\lambda_{obs} = \lambda_{emit} \times \frac{a(t_{obs})}{a(t_{emit})} = \frac{\lambda_{emit}}{a_{emit}} = \lambda_{emit}(1+z)$$

$= 1$

$$a = \frac{1}{1+z}$$

$\lambda_{emit}(1+z)$

FRW spacetime

coords : t, x, y, z / r, θ, ϕ

$$ds^2 = -dt^2 + a^2(t)\,|dx|^2$$

$$|dx|^2 = g_{ij}\,dx^i dx^j$$

$$|dx|^2 = dr^2 + D^2(r)\, d\Omega^2$$

$$D(r) = \begin{cases} R \sin(r/R) & \text{positive curv.} \\ r & \text{flat} \\ R \sinh(r/R) & \text{negative curv.} \end{cases}$$

$$|dx|^2 = dr^2 + D^2(r)\, d\Omega^2$$

$$D(r) = \begin{cases} R \sin(r/R) & \text{positive curv.} \\ r & \text{flat} \\ R \sinh(r/R) & \text{negative curv.} \end{cases}$$

$$= R\, k^{-1/2} \sin\left(k^{1/2} \frac{r}{R}\right)$$

$$k^{-1/2} \sin\left(k^{1/2} \frac{r}{R}\right)$$

positive : $r = R \sin^{-1}(D/R)$

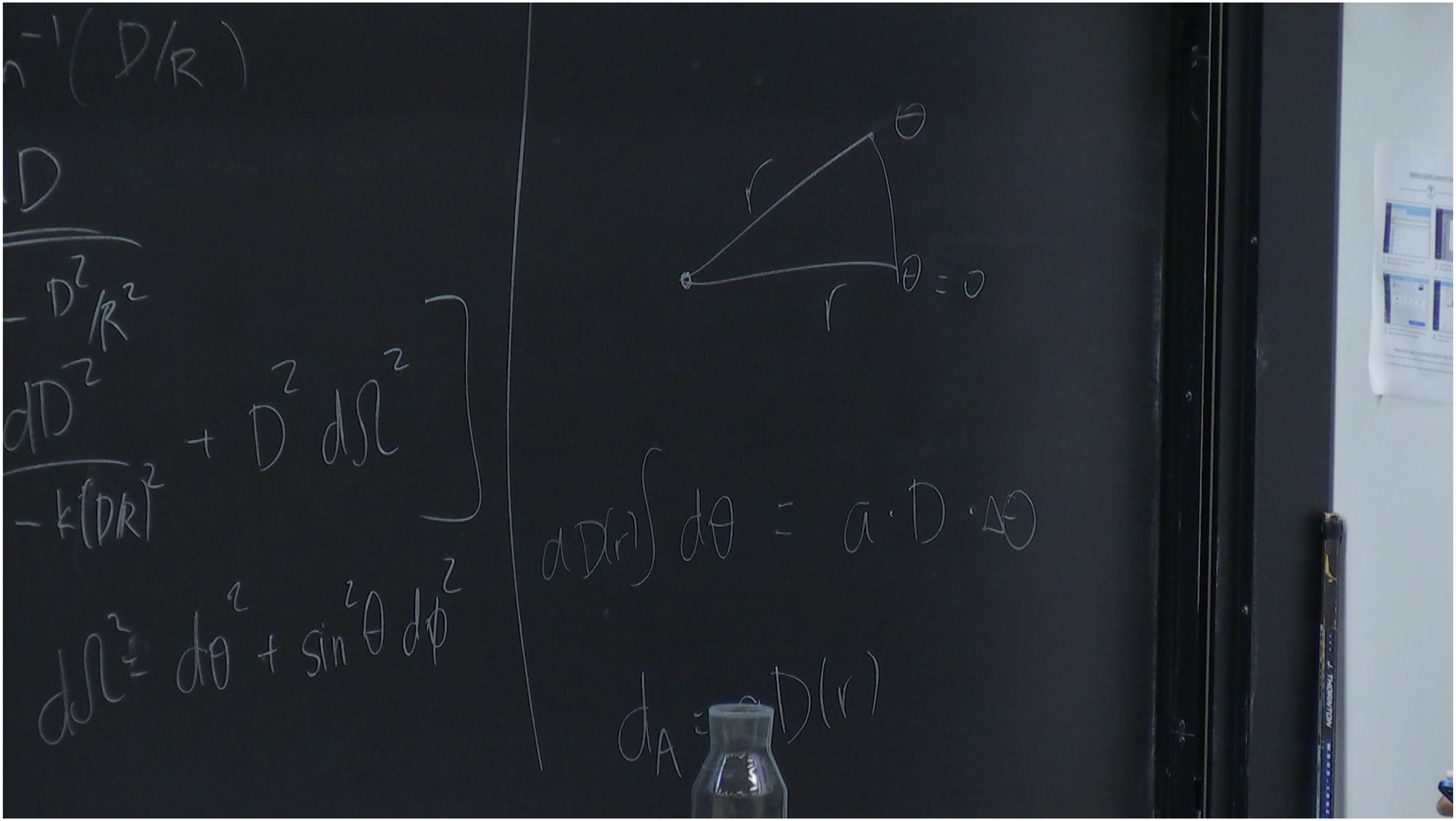
$$dr = \frac{dD}{\sqrt{1 - D^2/R^2}}$$

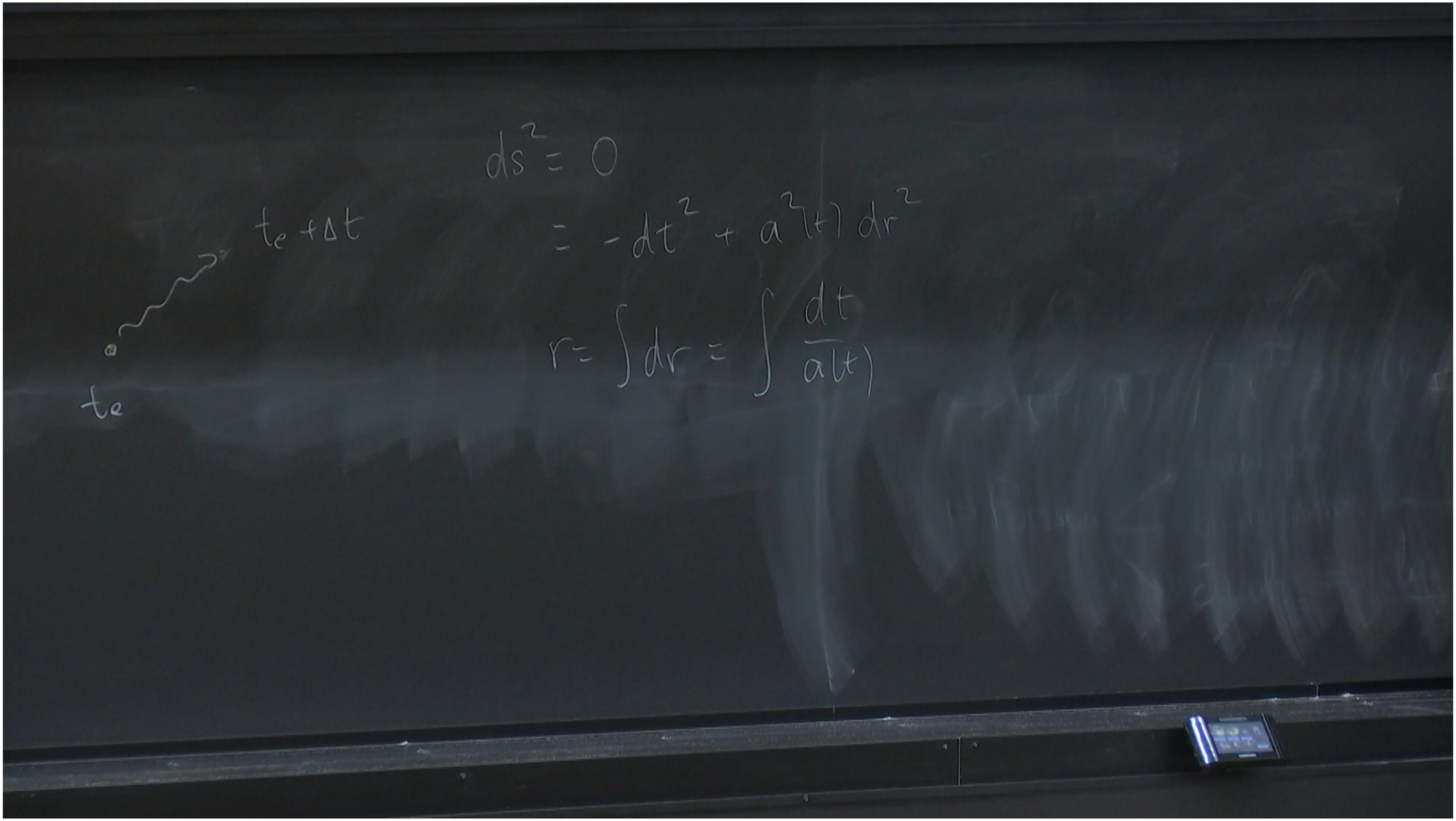
$$ds^2 = -dt^2 + a^2\left[\frac{dD^2}{1 - k(D/R)^2} + D^2 d\Omega^2\right]$$

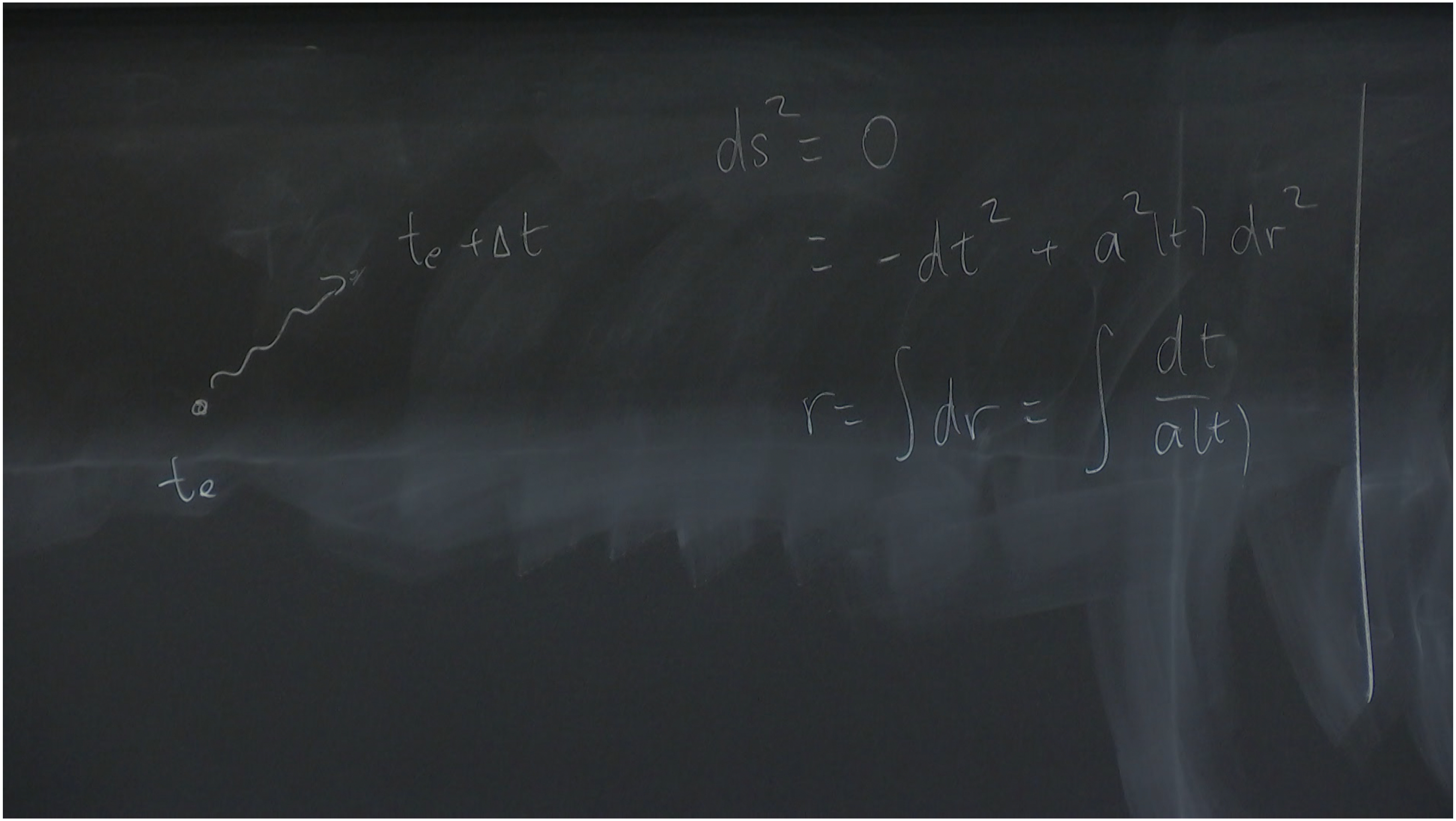
$t, D=0, \theta, \phi$

t, D, θ, ϕ

$$ds = a(t) \frac{dD}{\sqrt{1-k\left(\frac{D}{R}\right)^2}}$$


$\sin^{-1}(D/R)$
dD^2
D^2/R^2
$-k(D/R)^2$
$+ D^2 d\Omega^2$
$d\Omega^2 = d\theta^2 + \sin^2\theta \, d\phi^2$
r
θ
$\theta = 0$
$a D(r) \int d\theta = a \cdot D \cdot \Delta\theta$
$d_A = a D(r)$


$t_e + \Delta t$
t_e
$ds^2 = 0$
$= -dt^2 + a^2(t)\, dr^2$
$r = \int dr = \int \frac{dt}{a(t)}$


$t_e + \Delta t$
t_e
$ds^2 = 0$
$= -dt^2 + a^2(t)\,dr^2$
$r = \int dr = \int \frac{dt}{a(t)}$

$\dot{p}^\mu = 0$

$\frac{dp^\mu}{dt} = \frac{\partial p^\mu}{\partial t} + \Gamma^\mu_{t\alpha} p^\alpha = 0$

$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(\partial_\beta g_{\nu\alpha} + \partial_\alpha g_{\nu\beta} - \partial_\nu g_{\alpha\beta} \right)$

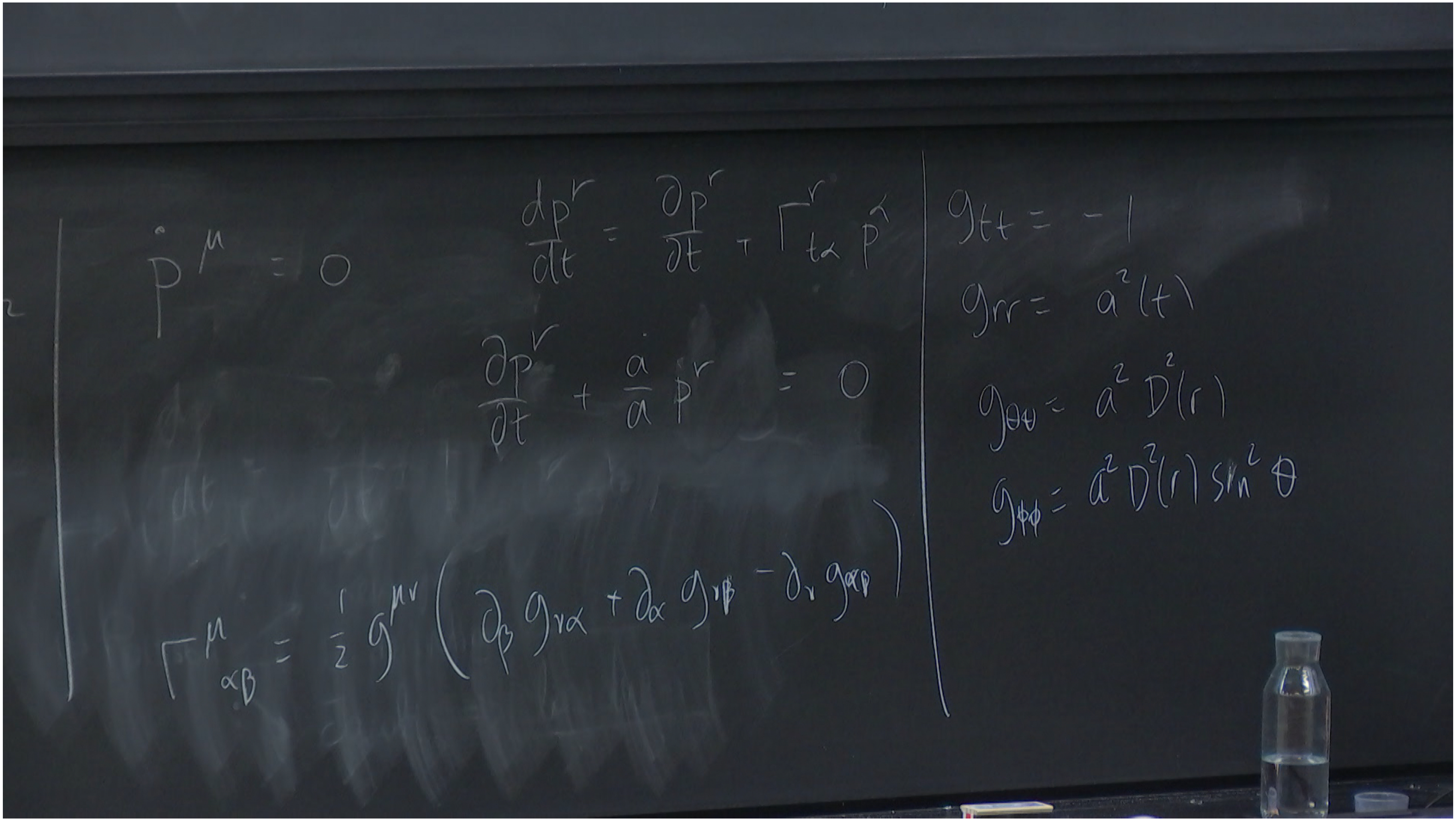
dr^2

t

(t)

g_{tt}

g_{rr}



$\dot{p}^\mu = 0$
$\frac{dp^r}{dt} = \frac{\partial p^r}{\partial t} + \Gamma^r_{t\alpha} p^\alpha$
$\frac{\partial p^r}{\partial t} + \frac{\dot{a}}{a} p^r = 0$
$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(\partial_\beta g_{\nu\alpha} + \partial_\alpha g_{\nu\beta} - \partial_\nu g_{\alpha\beta} \right)$
$g_{tt} = -1$
$g_{rr} = a^2(t)$
$g_{\theta\theta} = a^2 D^2(r)$
$g_{\phi\phi} = a^2 D^2(r) \sin^2\theta$