Title: Cosmology Lecture (230428)

Speakers: Neal Dalal

Collection: Cosmology (2022/2023)

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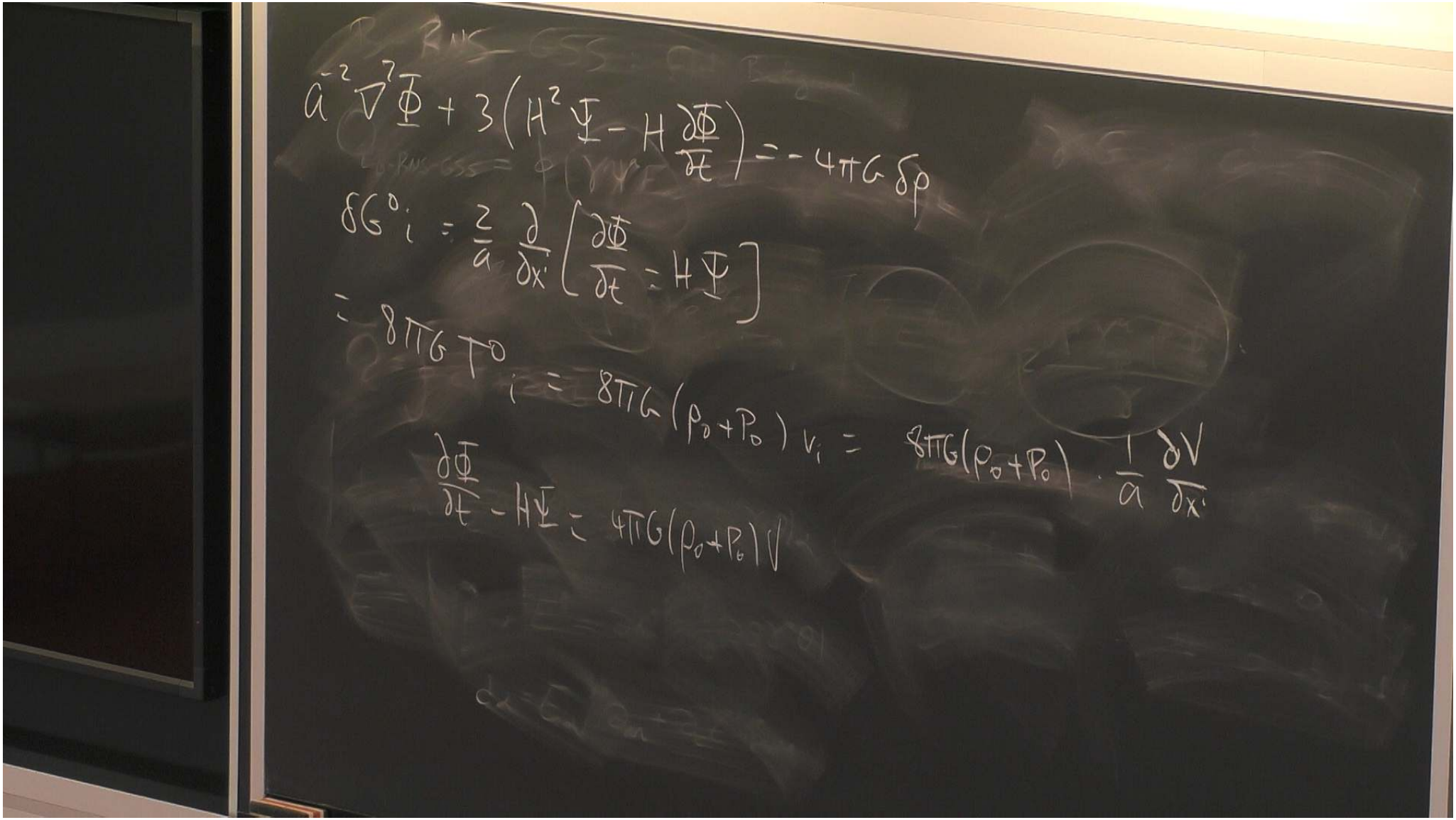
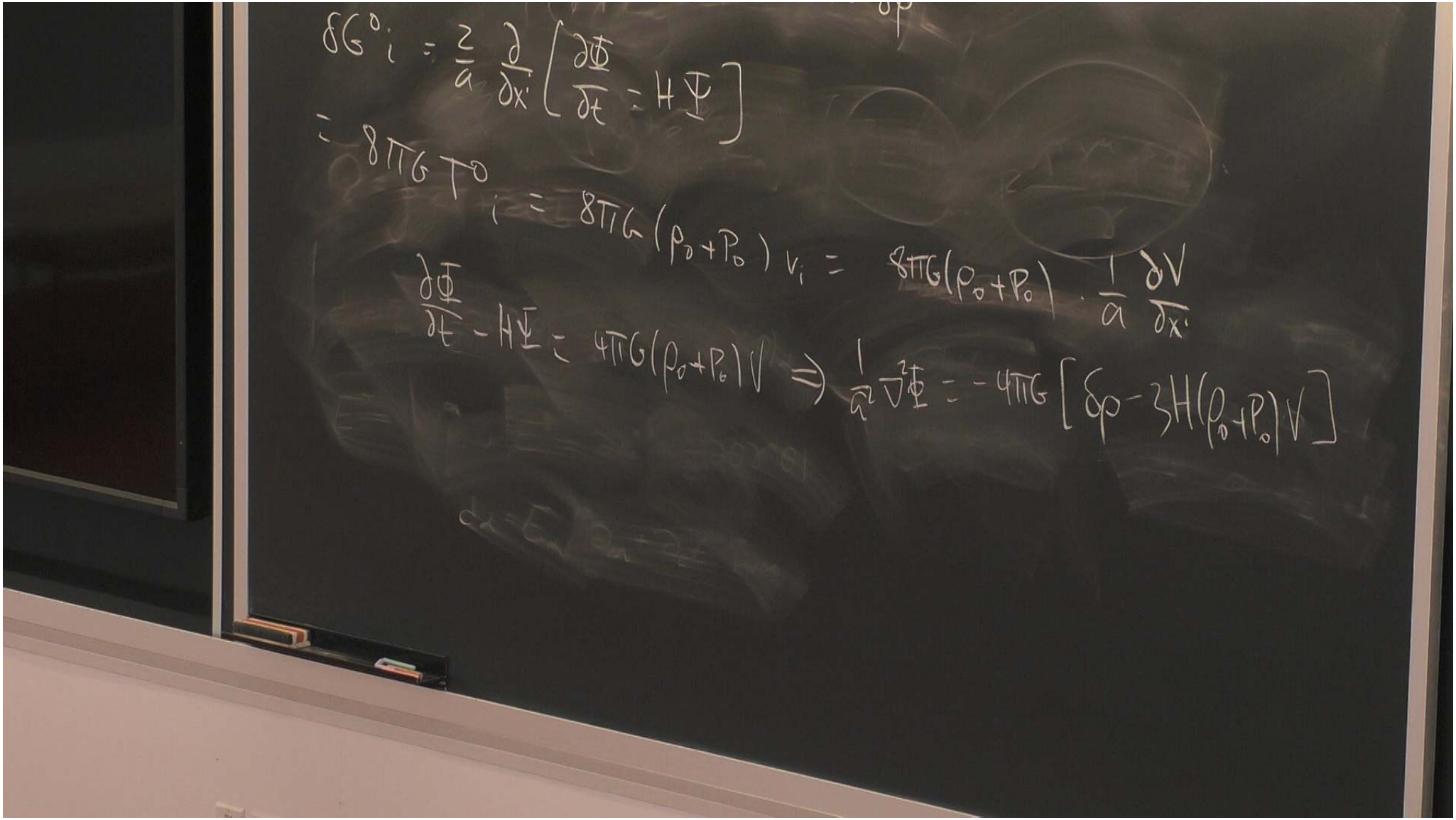
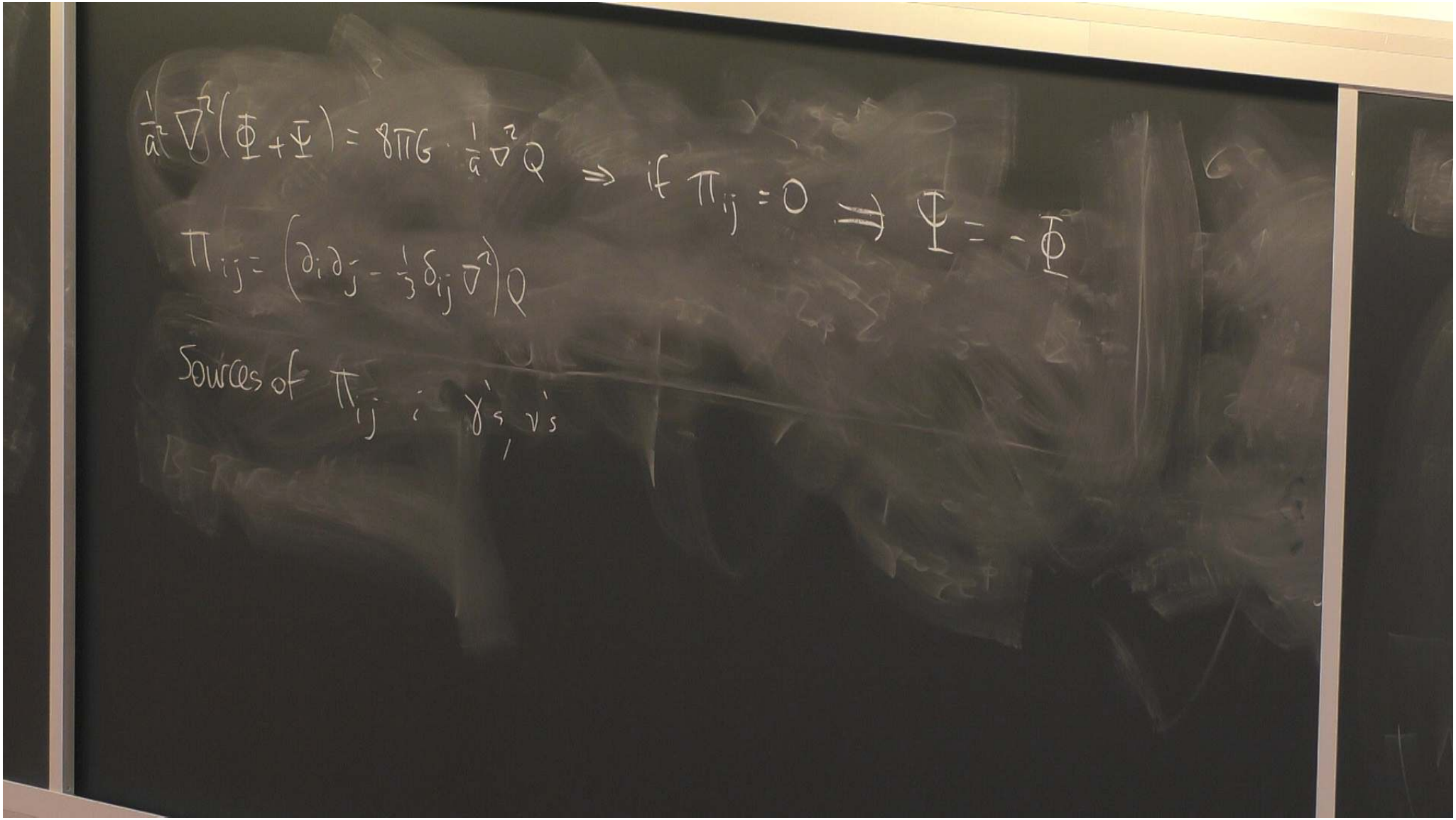
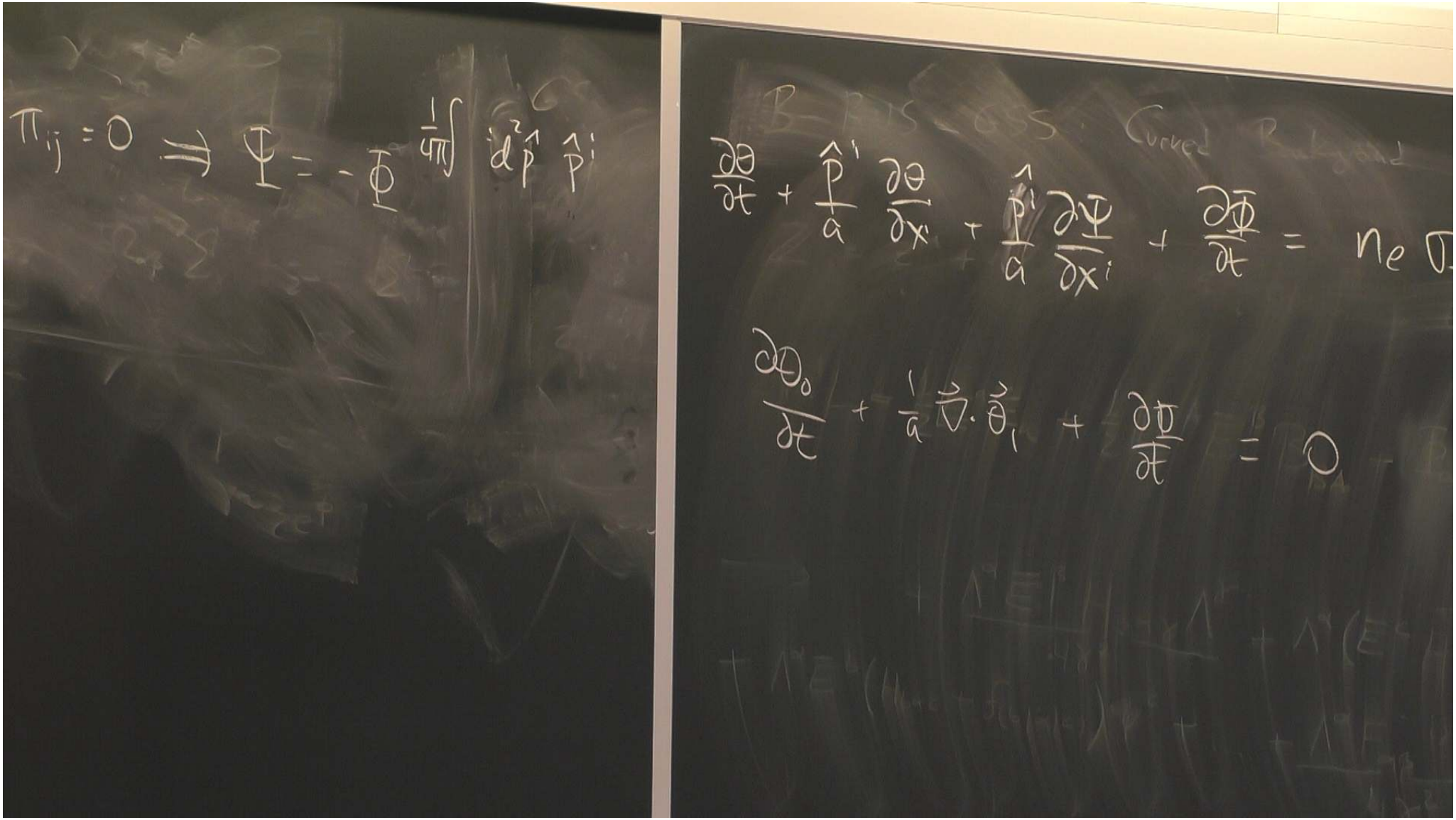
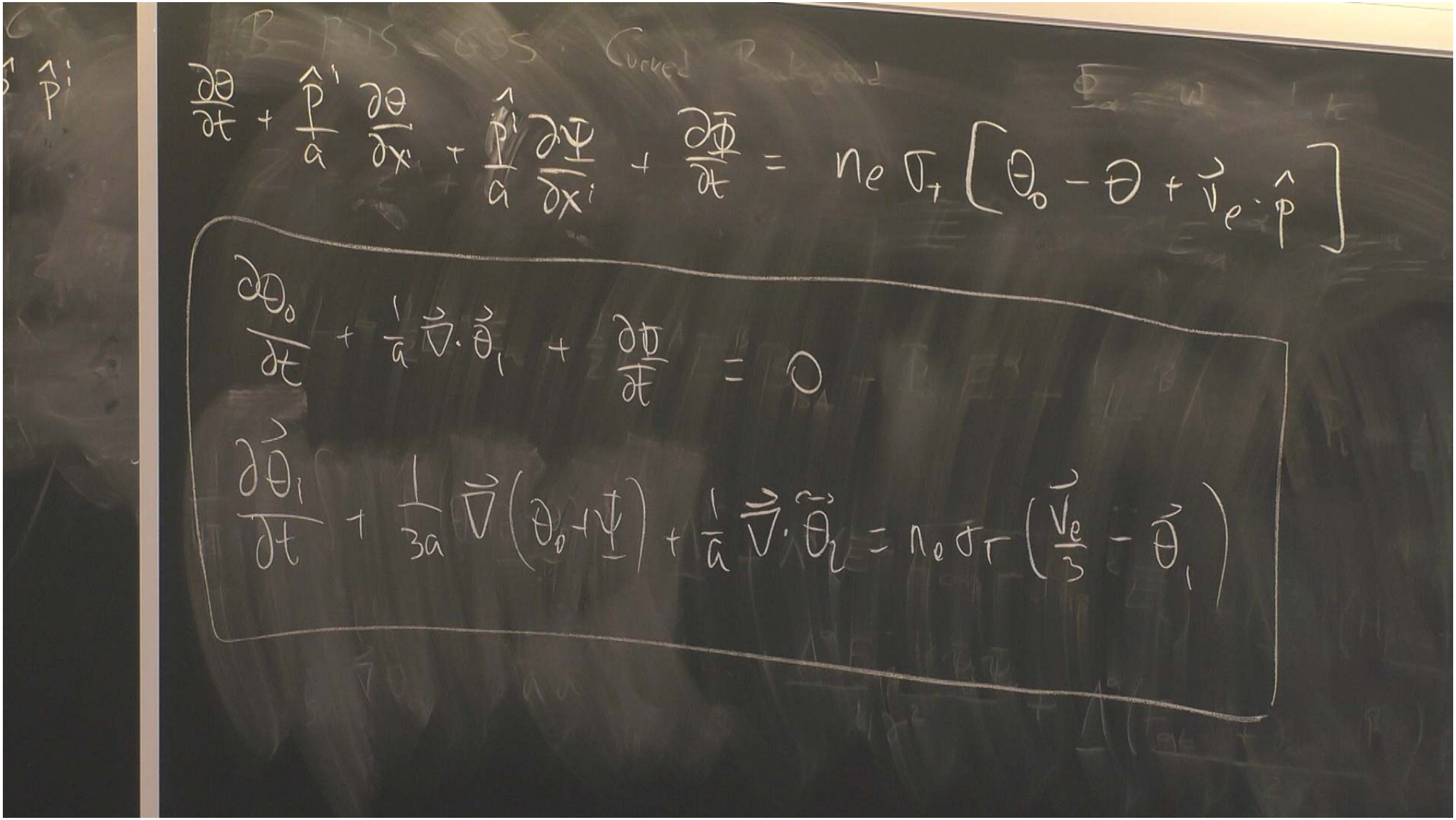
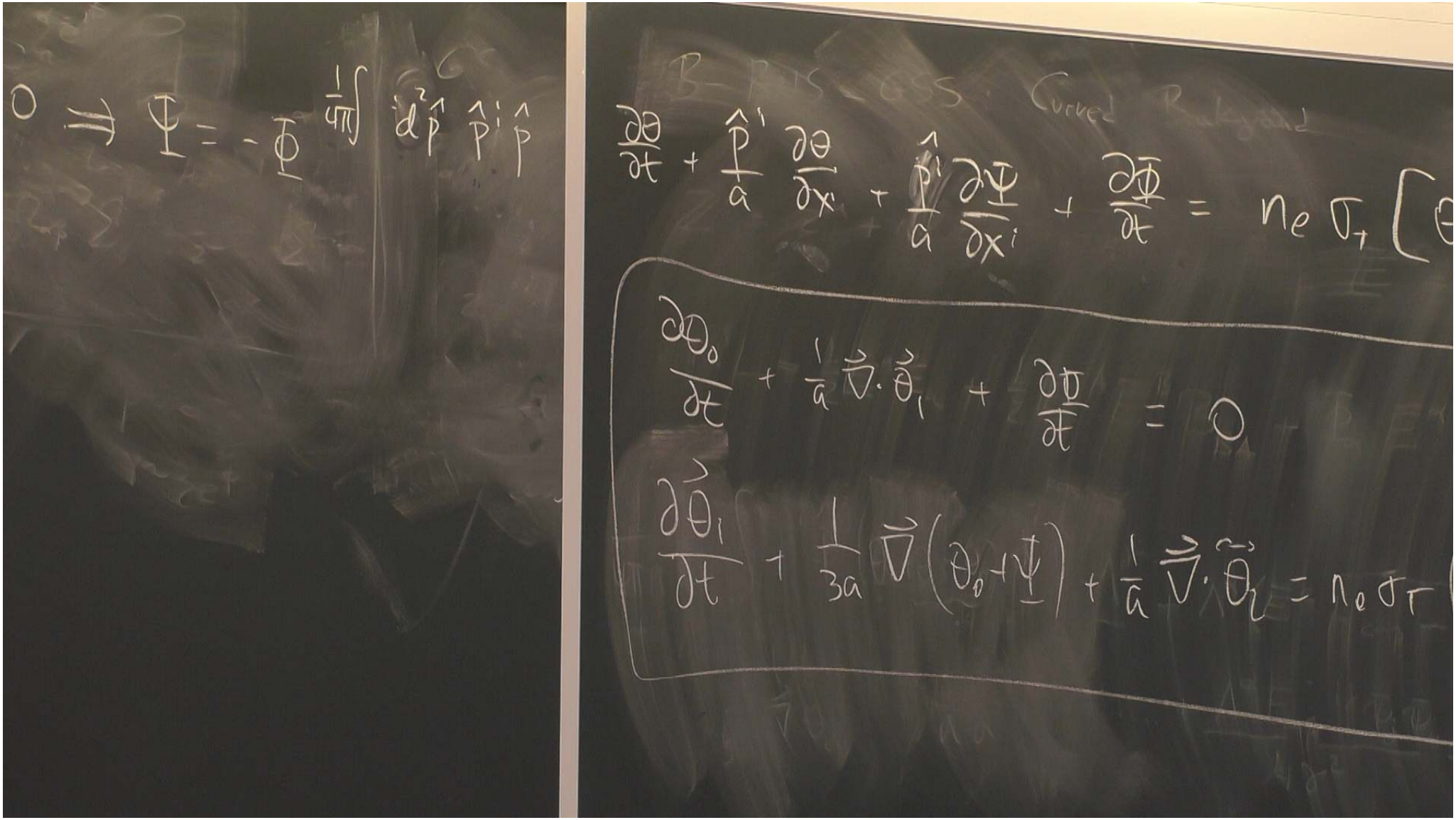

$a^{-2}\nabla^2\Phi + 3\left(H^2\Psi - H\frac{\partial\Phi}{\partial t}\right) = -4\pi G\,\delta\rho$
$\delta G^0{}_i = \frac{2}{a}\frac{\partial}{\partial x^i}\left[\frac{\partial\Phi}{\partial t} = H\Psi\right]$
$= 8\pi G\, T^0{}_i = 8\pi G(\rho_0 + P_0)\, v_i = 8\pi G(\rho_0 + P_0)\cdot\frac{1}{a}\frac{\partial V}{\partial x^i}$
$\frac{\partial\Phi}{\partial t} - H\Psi = 4\pi G(\rho_0 + P_0)V$

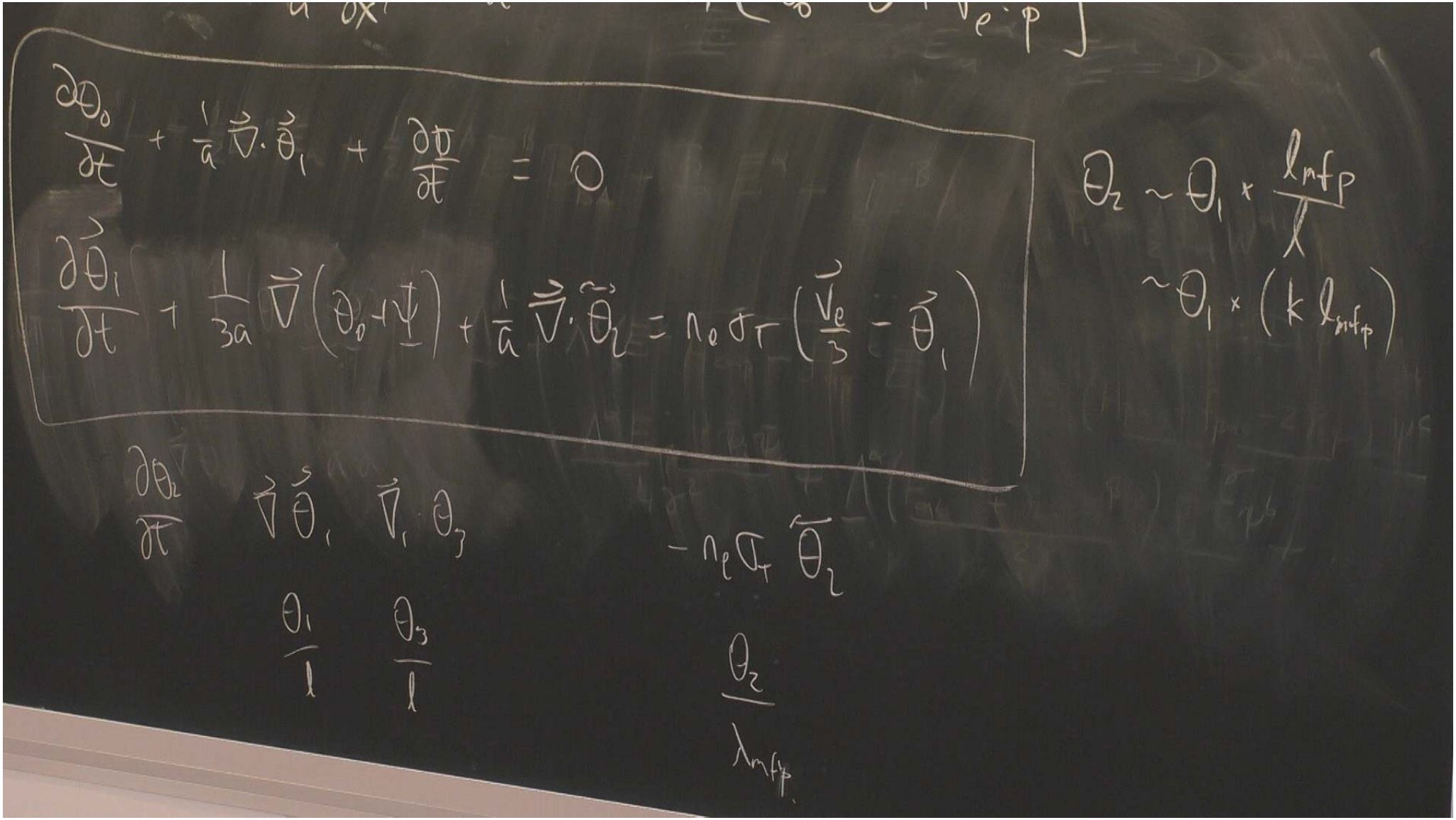

$\delta G^0{}_i = \frac{2}{a}\frac{\partial}{\partial x^i}\left[\frac{\partial \Phi}{\partial t} - H\Psi\right]$
$= 8\pi G\, T^0{}_i = 8\pi G(\rho_0 + P_0)\, v_i = 8\pi G(\rho_0 + P_0)\cdot\frac{1}{a}\frac{\partial V}{\partial x^i}$
$\frac{\partial \Phi}{\partial t} - H\Psi = 4\pi G(\rho_0 + P_0)V \Rightarrow \frac{1}{a^2}\nabla^2\Phi = -4\pi G\left[\delta\rho - 3H(\rho_0 + P_0)V\right]$

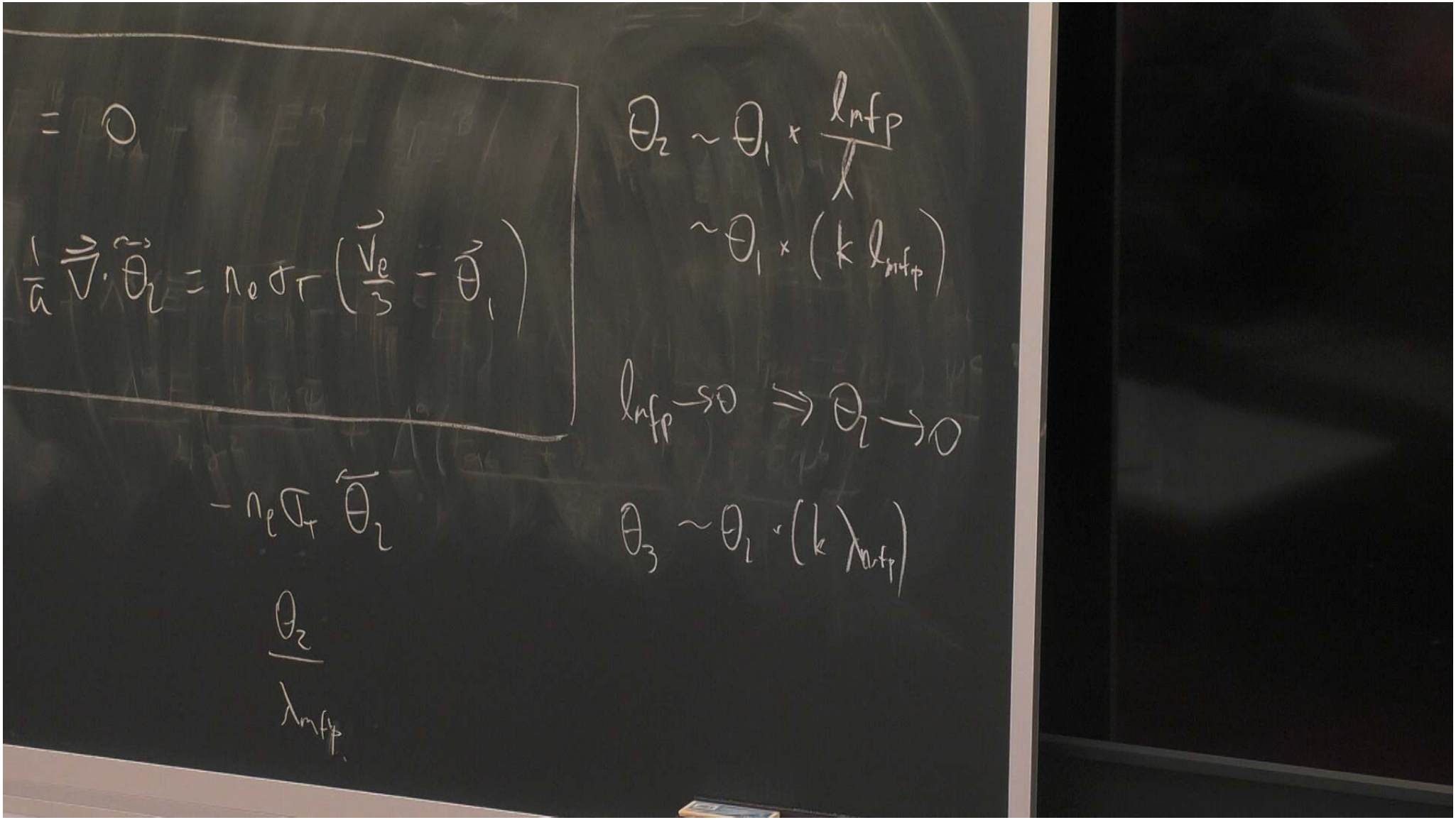

$\frac{1}{a^2}\nabla^2(\Phi+\Psi) = 8\pi G \cdot \frac{1}{a}\nabla^2 Q \Rightarrow$ if $\pi_{ij}=0 \Rightarrow \Psi = -\Phi$
$\pi_{ij} = \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)Q$
Sources of π_{ij} : γ's, v's

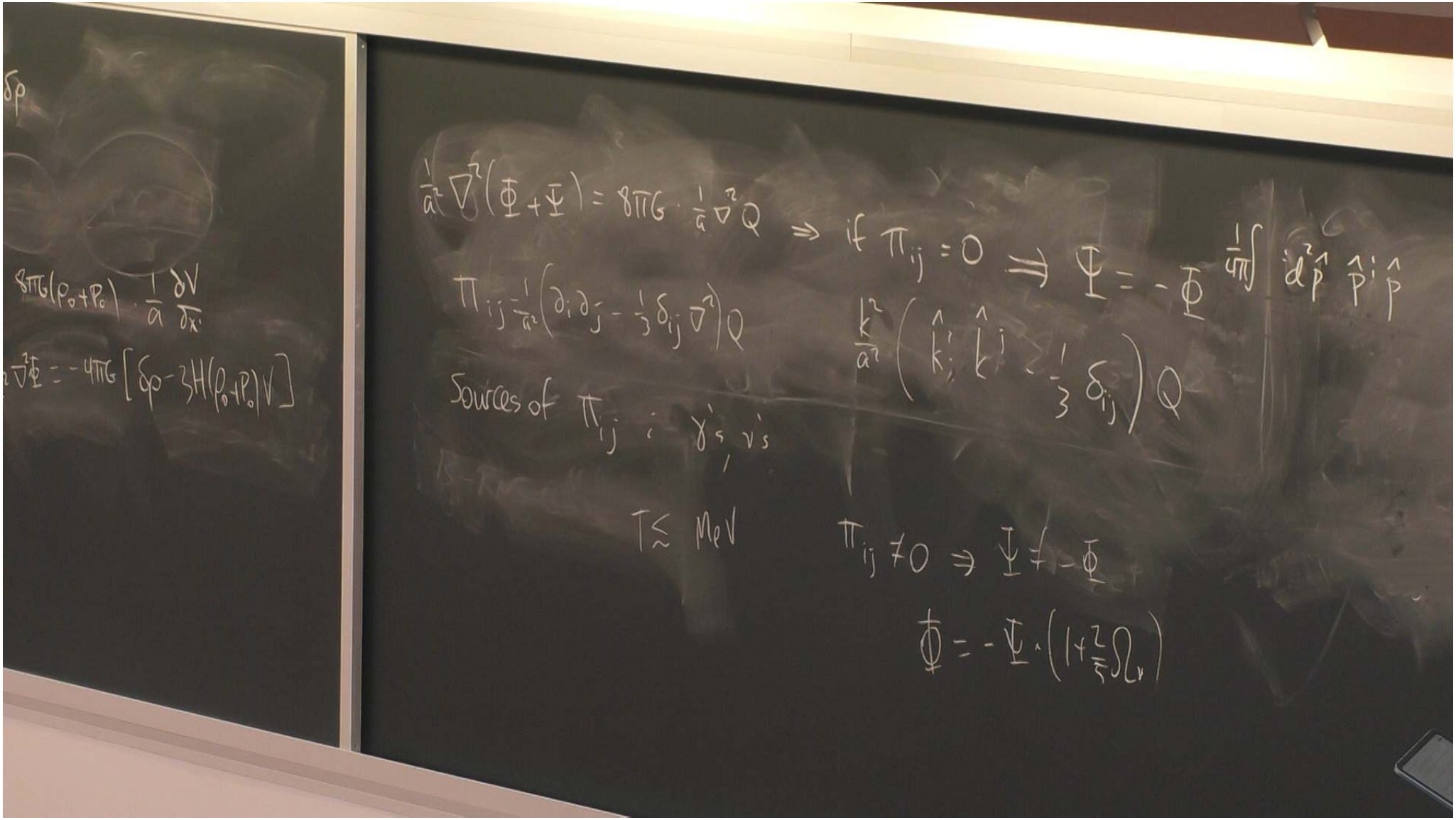

$\pi_{ij} = 0 \Rightarrow \Psi = -\Phi$
$\frac{1}{4\pi}\int d^2\hat{p}\ \hat{p}^i$
$\frac{\partial \Theta}{\partial \tau} + \frac{\hat{p}^i}{a}\frac{\partial \Theta}{\partial x^i} + \frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i} + \frac{\partial \Phi}{\partial \tau} = n_e \sigma$
$\frac{\partial \Theta_0}{\partial \tau} + \frac{1}{a}\vec{\nabla}\cdot\vec{\Theta}_1 + \frac{\partial \Psi}{\partial \tau} = 0$


$\frac{\partial \Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \Theta}{\partial x^i} + \frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i} + \frac{\partial \Phi}{\partial t} = n_e \sigma_T \left[\Theta_0 - \Theta + \vec{v}_e \cdot \hat{p}\right]$
$\frac{\partial \Theta_0}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{\Theta}_1 + \frac{\partial \Phi}{\partial t} = 0$
$\frac{\partial \vec{\Theta}_1}{\partial t} + \frac{1}{3a}\vec{\nabla}\left(\Theta_0 + \Psi\right) + \frac{1}{a}\vec{\nabla}\cdot\vec{\Theta}_2 = n_e \sigma_T \left(\frac{\vec{v}_e}{3} - \vec{\Theta}_1\right)$


$0 \Rightarrow \Psi = -\Phi$
$\frac{1}{4\pi}\int d^2\hat{p}\,\hat{p}^i\hat{p}$
$\frac{\partial\Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial\Theta}{\partial x^i} + \frac{\hat{p}^i}{a}\frac{\partial\Psi}{\partial x^i} + \frac{\partial\Phi}{\partial t} = n_e\sigma_T[$
$\frac{\partial\Theta_0}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{\Theta}_1 + \frac{\partial\Psi}{\partial t} = 0$
$\frac{\partial\vec{\Theta}_1}{\partial t} + \frac{1}{3a}\vec{\nabla}(\Theta_0+\Psi) + \frac{1}{a}\vec{\nabla}\cdot\vec{\Theta}_2 = n_e\sigma_T$


$\frac{\partial \Theta_0}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{\Theta}_1 + \frac{\partial \Psi}{\partial t} = 0$
$\frac{\partial \vec{\Theta}_1}{\partial t} + \frac{1}{3a}\vec{\nabla}(\Theta_0 + \Psi) + \frac{1}{a}\vec{\nabla}\cdot\tilde{\Theta}_2 = n_e \sigma_T \left(\frac{\vec{v}_e}{3} - \vec{\Theta}_1\right)$
$\Theta_2 \sim \Theta_1 \times \frac{l_{mfp}}{\lambda}$
$\sim \Theta_1 \times (k\, l_{mfp})$
$\frac{\partial \Theta_2}{\partial t}$
$\vec{\nabla}\vec{\Theta}_1$
$\vec{\nabla}_1 \cdot \Theta_3$
$-n_e \sigma_T \tilde{\Theta}_2$
$\frac{\Theta_1}{1}$
$\frac{\Theta_3}{1}$
$\frac{\Theta_2}{\lambda_{mfp}}$


$= 0$
$\frac{1}{a}\vec{\nabla}\cdot\overleftrightarrow{\Theta_2} = n_e \sigma_T \left(\frac{\vec{v}_e}{3} - \vec{\Theta}_1\right)$
$-n_e \sigma_T \vec{\Theta}_2$
$\frac{\Theta_2}{\lambda_{mfp}}$
$\Theta_2 \sim \Theta_1 \times \frac{l_{mfp}}{\lambda}$
$\sim \Theta_1 \times (k l_{mfp})$
$l_{mfp} \to 0 \Rightarrow \Theta_2 \to 0$
$\Theta_3 \sim \Theta_1 \cdot (k \lambda_{mfp})$


δρ
$8\pi G(\rho_0+P_0)\cdot\frac{1}{a}\frac{\partial V}{\partial x_i}$
$\nabla^2\Phi = -4\pi G\left[\delta\rho - 3H(\rho_0+P_0)V\right]$
$\frac{1}{a^2}\nabla^2(\Phi+\Psi) = 8\pi G\cdot\frac{1}{a}\nabla^2 Q \Rightarrow$ if $\pi_{ij}=0 \Rightarrow \Psi = -\Phi$
$\pi_{ij} = \frac{1}{a^2}\left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)Q$
$\frac{k^2}{a^2}\left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)Q$
$\frac{1}{4\pi}\int d^2\hat{p}\;\hat{p}_i\hat{p}$
Sources of π_{ij} : γ_s, ν_s
$T \lesssim$ MeV
$\pi_{ij}\neq 0 \Rightarrow \Psi \neq -\Phi$
$\Phi = -\Psi\cdot\left(1+\frac{2}{5}\Omega_\nu\right)$

$\Rightarrow \boxed{\Psi = -\Phi}$

$\hat{k}^i \hat{k}^j - \frac{1}{3}\delta_{ij}\Big) Q$

$\Rightarrow \Psi \neq -\Phi$

$= -\Psi \cdot \left(1 + \frac{2}{5}\Omega_\nu\right)$

Coupled, linear PDE's

$\delta_d, \delta_b, \Theta_0 \ldots$

$\vec{v}_d, \vec{v}_b, \vec{\Theta}_1$

$\vec{v}_d = \frac{1}{a}\vec{\nabla} v_d$

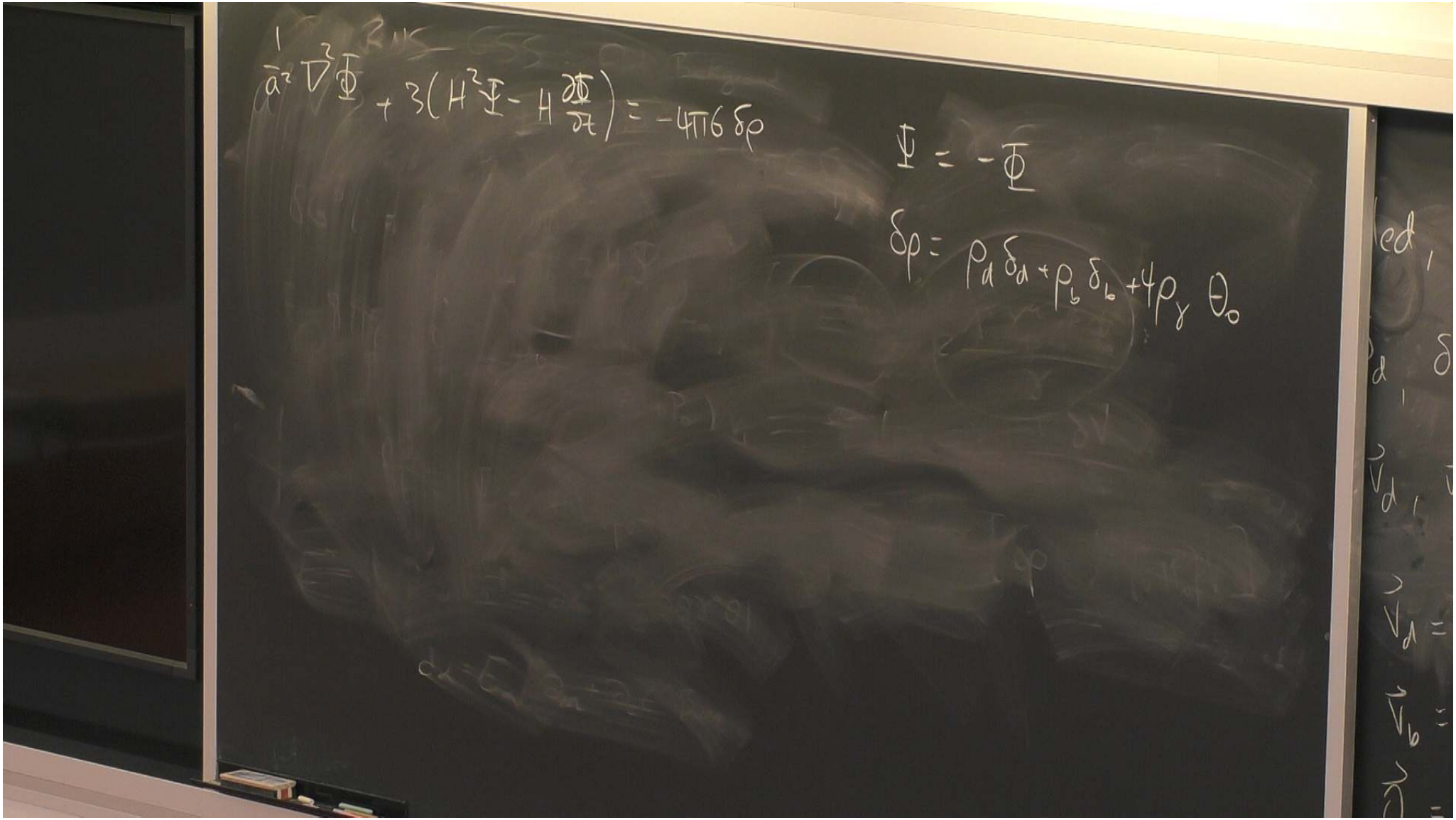
$\vec{v}_b = \frac{1}{a}\vec{\nabla} v_b$

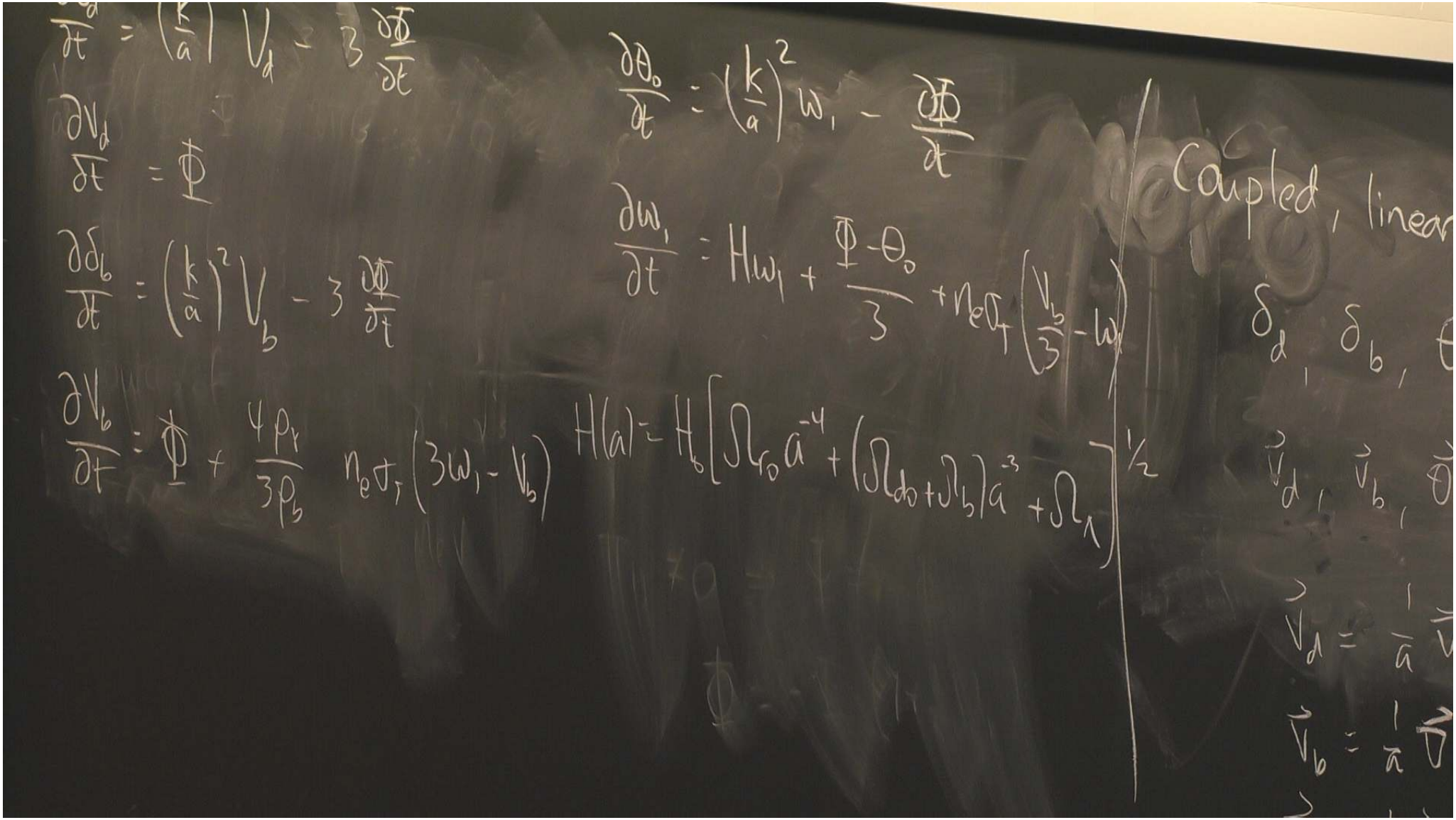
$\vec{\Theta}_1 = \frac{1}{a}\vec{\nabla} \omega_1$

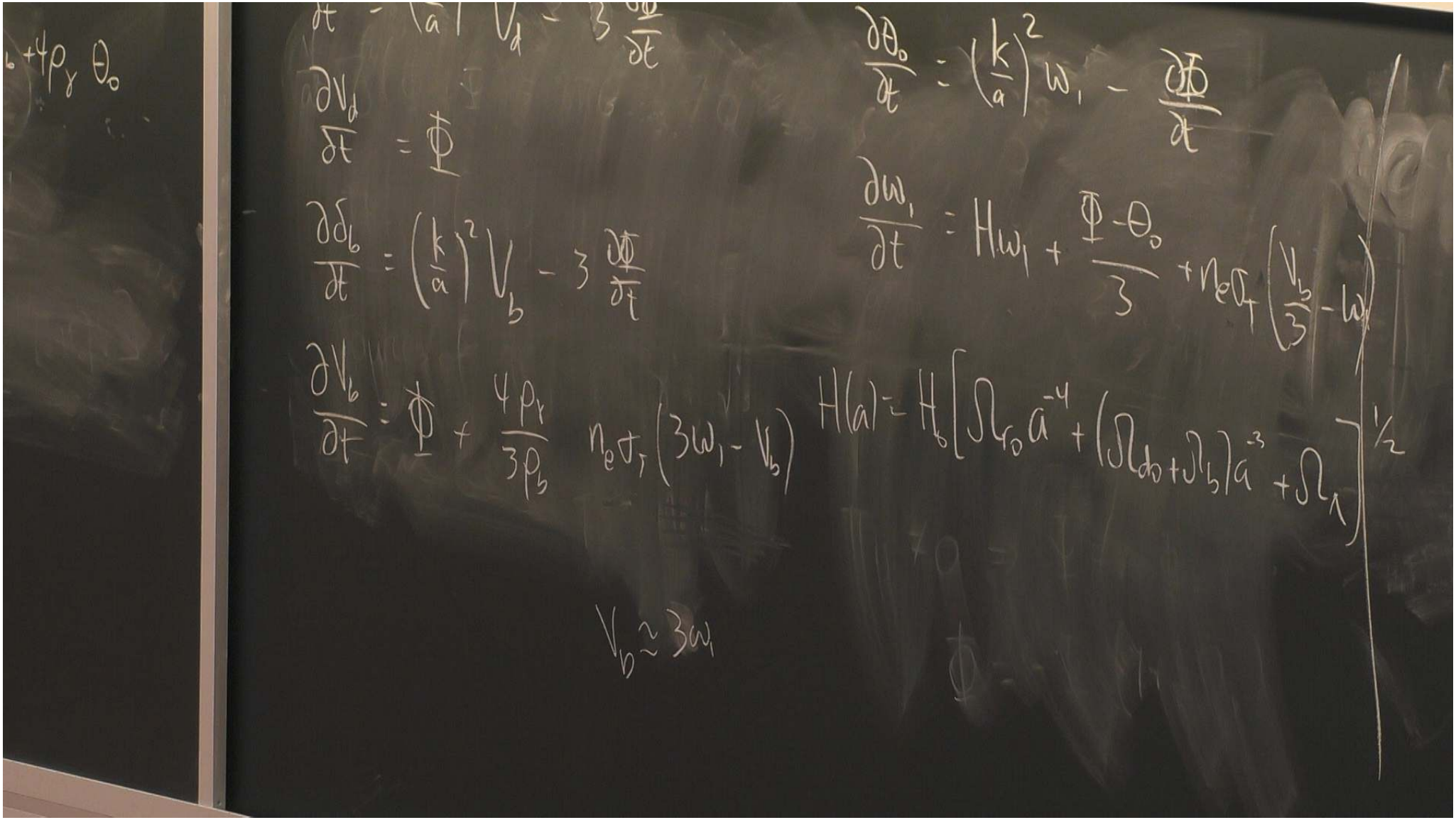
$\frac{\partial \delta_d}{\partial t} + \frac{1}{a^2}\nabla^2 v_d + 3\frac{\partial \Phi}{\partial t} = 0$

$\frac{\partial \delta_d}{\partial t} - \frac{k^2}{a^2} v_d + 3\frac{\partial \Phi}{\partial t} = 0$

FT $\Rightarrow$ different k's indep.


$\frac{1}{a^2}\nabla^2\Phi + 3\left(H^2\Psi - H\frac{\partial\Phi}{\partial\tau}\right) = -4\pi G\,\delta\rho$
$\Psi = -\Phi$
$\delta\rho = \rho_c\,\delta_c + \rho_b\,\delta_b + 4\rho_\gamma\,\Theta_0$


$\frac{\partial \delta_d}{\partial t} = \left(\frac{k}{a}\right) V_d - 3\frac{\partial \Phi}{\partial t}$
$\frac{\partial V_d}{\partial t} = \Phi$
$\frac{\partial \delta_b}{\partial t} = \left(\frac{k}{a}\right)^2 V_b - 3\frac{\partial \Phi}{\partial t}$
$\frac{\partial V_b}{\partial t} = \Phi + \frac{4\rho_\gamma}{3\rho_b} n_e \sigma_T (3w_1 - V_b)$
$\frac{\partial \theta_0}{\partial t} = \left(\frac{k}{a}\right)^2 w_1 - \frac{\partial \Phi}{\partial t}$
$\frac{\partial w_1}{\partial t} = H w_1 + \frac{\Phi - \theta_0}{3} + n_e \sigma_T \left(\frac{V_b}{3} - w_1\right)$
$H(a) = H_0 \left[\Omega_{r0} a^{-4} + (\Omega_{d0} + \Omega_{b0}) a^{-3} + \Omega_\Lambda\right]^{1/2}$
Coupled, linear
δ_d, δ_b,
$\vec{V}_d$, $\vec{V}_b$,
$\vec{V}_d = \frac{1}{a}$
$\vec{V}_b = \frac{1}{a}$

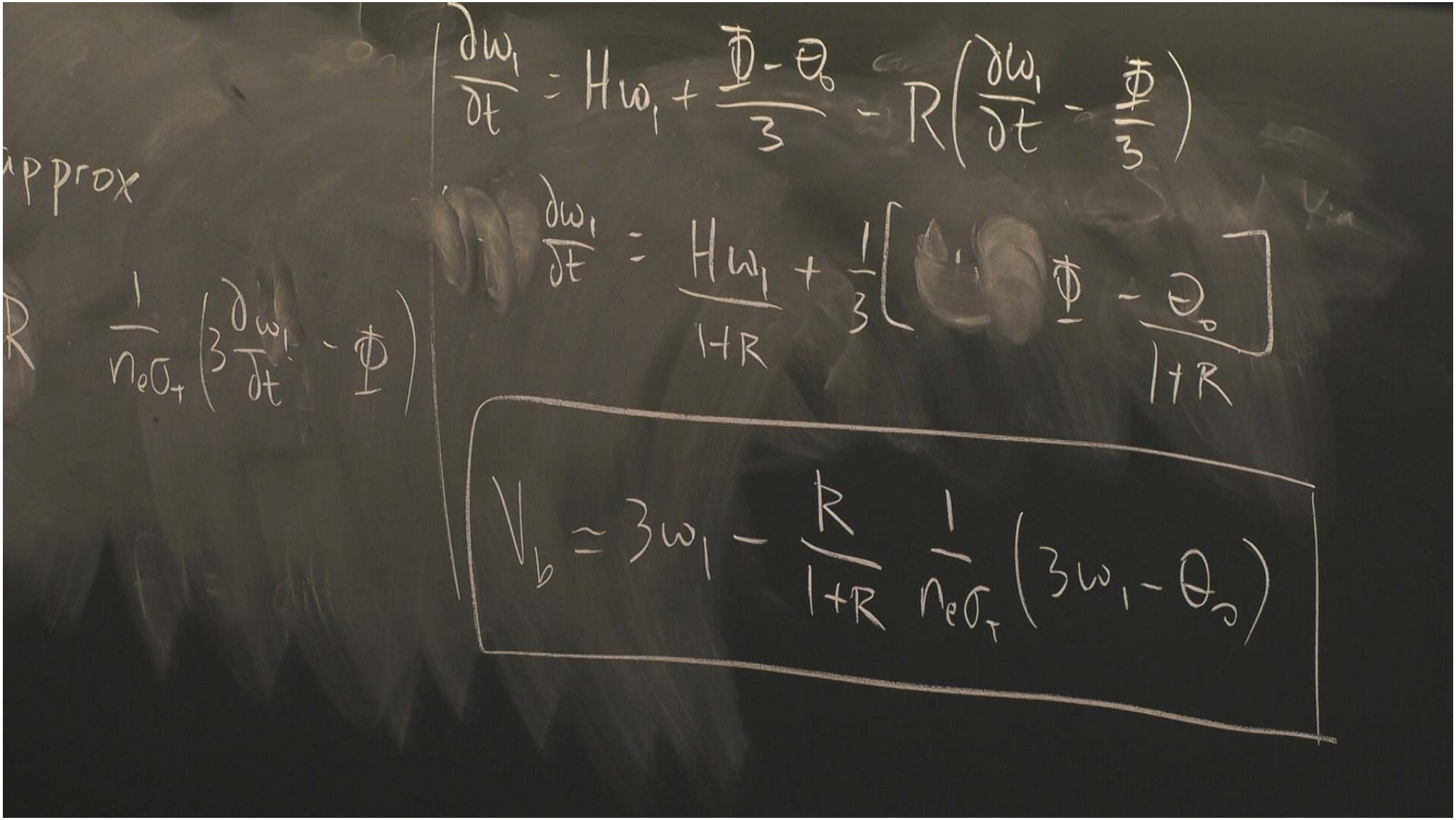

$+4\rho_\gamma\,\Theta_0$
$\frac{\partial V_d}{\partial t} = \Phi$
$\frac{\partial \delta_b}{\partial t} = \left(\frac{k}{a}\right)^2 V_b - 3\frac{\partial \Phi}{\partial t}$
$\frac{\partial V_b}{\partial t} = \Phi + \frac{4\rho_\gamma}{3\rho_b} n_e \sigma_T (3\omega_1 - V_b)$
$V_b \simeq 3\omega_1$
$\frac{\partial \Theta_0}{\partial t} = \left(\frac{k}{a}\right)^2 \omega_1 - \frac{\partial \Phi}{\partial t}$
$\frac{\partial \omega_1}{\partial t} = H\omega_1 + \frac{\Phi - \Theta_0}{3} + n_e \sigma_T \left(\frac{V_b}{3} - \omega_1\right)$
$H(a) = H_0\left[\Omega_{r0} a^{-4} + (\Omega_{d0} + \Omega_b) a^{-3} + \Omega_\Lambda\right]^{1/2}$

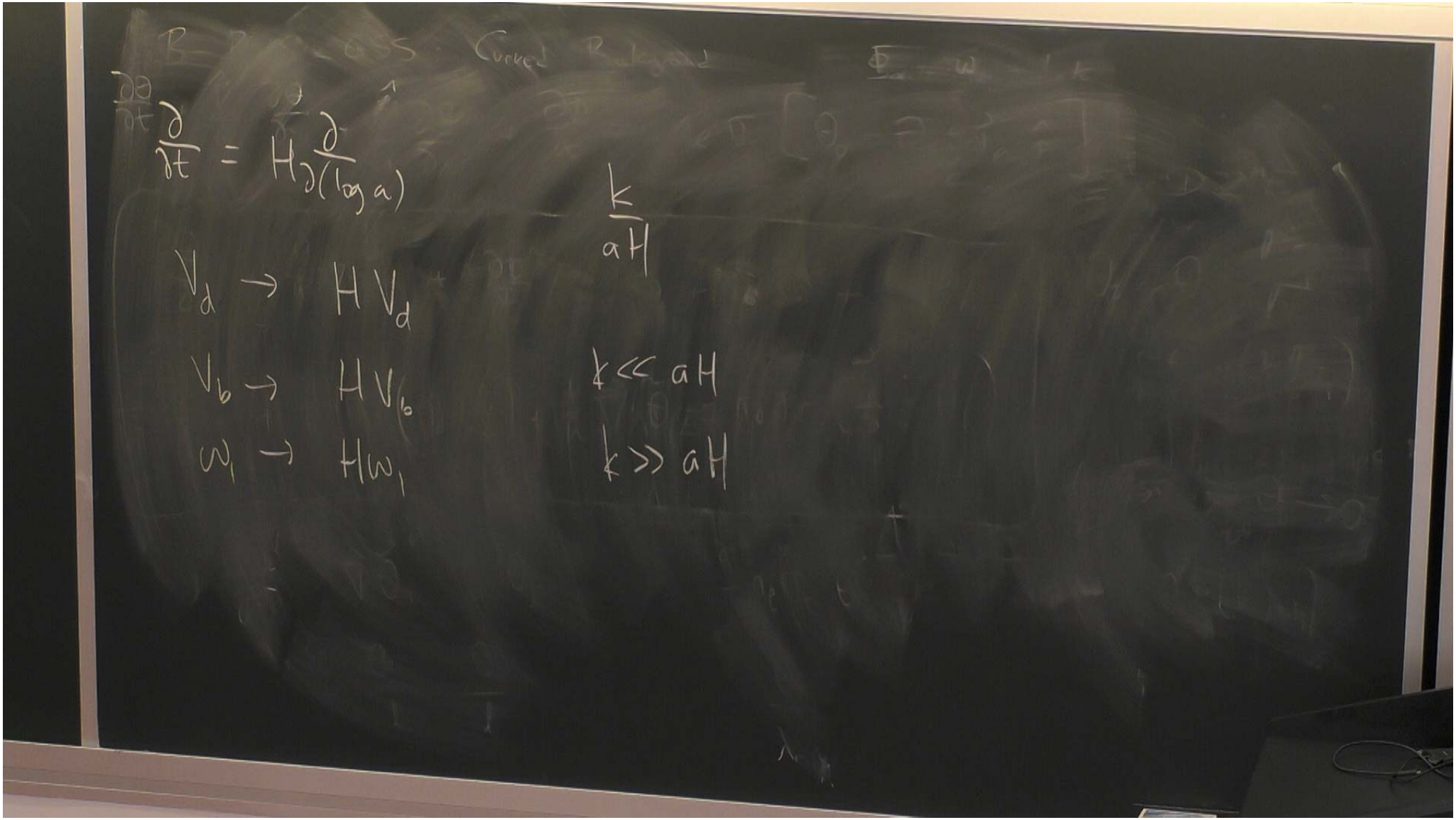
$$\Delta t < \frac{1}{n_e \sigma_T}$$

Tight coupling approx

$$V_b = 3\omega_1 - R \frac{1}{n_e \sigma_T}\left(3\frac{\partial \omega_1}{\partial t} - \Phi\right)$$

$$R \equiv \frac{3\rho_b}{4\rho_\gamma}$$


approx
$\frac{\partial \omega_1}{\partial t} = H\omega_1 + \frac{\Phi - \Theta_0}{3} - R\left(\frac{\partial \omega_1}{\partial t} - \frac{\Phi}{3}\right)$
$\frac{\partial \omega_1}{\partial t} = \frac{H\omega_1}{1+R} + \frac{1}{3}\left[\Phi - \frac{\Theta_0}{1+R}\right]$
$R \quad \frac{1}{n_e \sigma_T}\left(3\frac{\partial \omega_1}{\partial t} - \Phi\right)$
$V_b \simeq 3\omega_1 - \frac{k}{1+R}\frac{1}{n_e \sigma_T}(3\omega_1 - \Theta_0)$


$\frac{\partial}{\partial t} = H \frac{\partial}{\partial (\log a)}$
$\frac{k}{aH}$
$V_d \to H V_d$
$V_b \to H V_b$
$\omega_1 \to H \omega_1$
$k \ll aH$
$k \gg aH$