Title: Cosmology Lecture (230425)

Speakers: Neal Dalal

Collection: Cosmology (2022/2023)

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Photons–baryons collisions

$$\frac{df}{dt} = C[f]$$

$$\int d^3p' \, d^3q' \, d^3q \, [\quad] \, f(p') \, f_e(q') \, [1 - f_e(q)] \times [1 + f(p)]$$

$$\underset{q'}{e^-} + \underset{p'}{\gamma} \rightarrow \underset{q}{e^-} + \underset{p}{\gamma}$$

Photon + baryon collisions

$$\frac{df}{dt} = C[f]$$

$$\int d^3p' \, d^3q' \, d^3q \, [\quad] \, f(p') f_e(q') - f(p) f_e(q)$$

$$e^- + \gamma \rightarrow e^- + \gamma$$

q' p' q p

Photon-baryon collisions

$$\frac{df}{dt} = C[f]$$

$$\int d^3p' \; d^3q \; [\;\;] f(p') f_e(q') - f(p) f_e(q)$$

$$e^- + \gamma \to e^- + \gamma$$

$$q' \quad p' \quad q \quad p$$

$$q' = q + p - p'$$

$$|M|^2$$

$$\sigma_T \times$$

Photon-baryon collisions

$$\frac{df}{dt} = C[f]$$

$$\int d^3p' \, d^3q \, [\quad] \, f(p') f_e(q') - f(p) f_e(q)$$

$$\underset{q'}{e^-} + \underset{p'}{\gamma} \rightarrow \underset{q}{e^-} + \underset{p}{\gamma}$$

$$E_p \approx E_{p'}$$

$$E_q \approx E_{q'}$$

$$q' = q + p - p'$$

$$\Delta E \approx \vec{v}_e \cdot (\vec{p} - \vec{p}\,')$$

$$|M|^2 \, \delta^{(3)}(\vec{p} + \vec{q} - \vec{p}\,' - \vec{q}\,') \, \delta(E_p + E_q - E_p' - E_{q'})$$

$\sigma_T \times$ factors

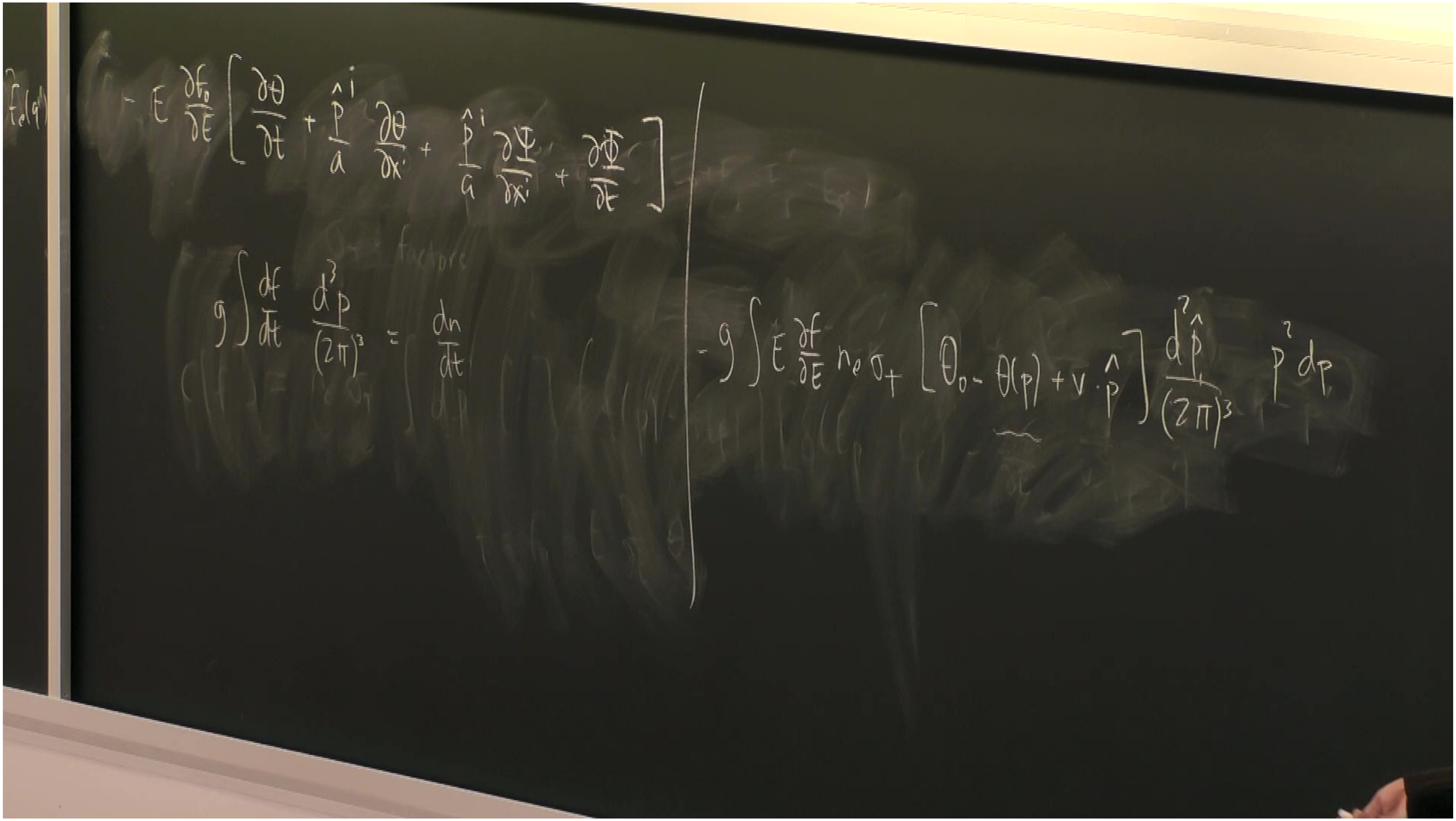
$$\simeq \delta(E_p + \quad - E_p' - \quad) + \frac{\partial \delta}{\partial E} \times \vec{v}_e \cdot (\vec{p} - \vec{p}\,')$$

$$C[f] \simeq n_e \sigma_T \int d^3p' \, [\ \ ] \left(f(p') - f(p) \right) \left[\delta(E_p - E_p') - \frac{\partial \delta}{\partial E} \cdot v_e (p - p') \right.$$

$$f = f_0 - E \frac{\partial f_0}{\partial E} \Theta$$

$$C[f] = -E \frac{\partial f_0}{\partial E} n_e \sigma_T \left[\Theta_0 - \Theta(\vec{p}) + \vec{v}_e \cdot \hat{p} \right]$$

$$\Theta_0 = \frac{1}{4\pi} \int \Theta(E, \hat{p}) \, d^2\hat{p}$$


$\tilde{f}_e(q')$
$-E\frac{\partial f_0}{\partial E}\left[\frac{\partial\Theta}{\partial t}+\frac{\hat{p}^i}{a}\frac{\partial\Theta}{\partial x^i}+\frac{\hat{p}^i}{a}\frac{\partial\Psi}{\partial x^i}+\frac{\partial\Phi}{\partial t}\right]$
$g\int\frac{df}{dt}\frac{d^3p}{(2\pi)^3}=\frac{dn}{dt}$
$-g\int E\frac{\partial f}{\partial E}n_e\sigma_T\left[\Theta_0-\Theta(p)+v\cdot\hat{p}\right]\frac{d^2\hat{p}}{(2\pi)^3}\;p^2dp$

$$- E \frac{\partial f_0}{\partial E}\left[\frac{\partial \Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \Theta}{\partial x^i} + \frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i} + \frac{\partial \Phi}{\partial t}\right]$$

$$g \int \frac{\partial f}{\partial t} \frac{d^3 p}{(2\pi)^3} = \frac{dn}{dt}$$

$$- g \int E \frac{\partial f}{\partial E} n_e \sigma_T \left[\Theta_0 - \Theta(p) + v \cdot \hat{p}\right] \frac{d^2 \hat{p}}{(2\pi)^3} \, p^2 dp$$

$$= 0$$

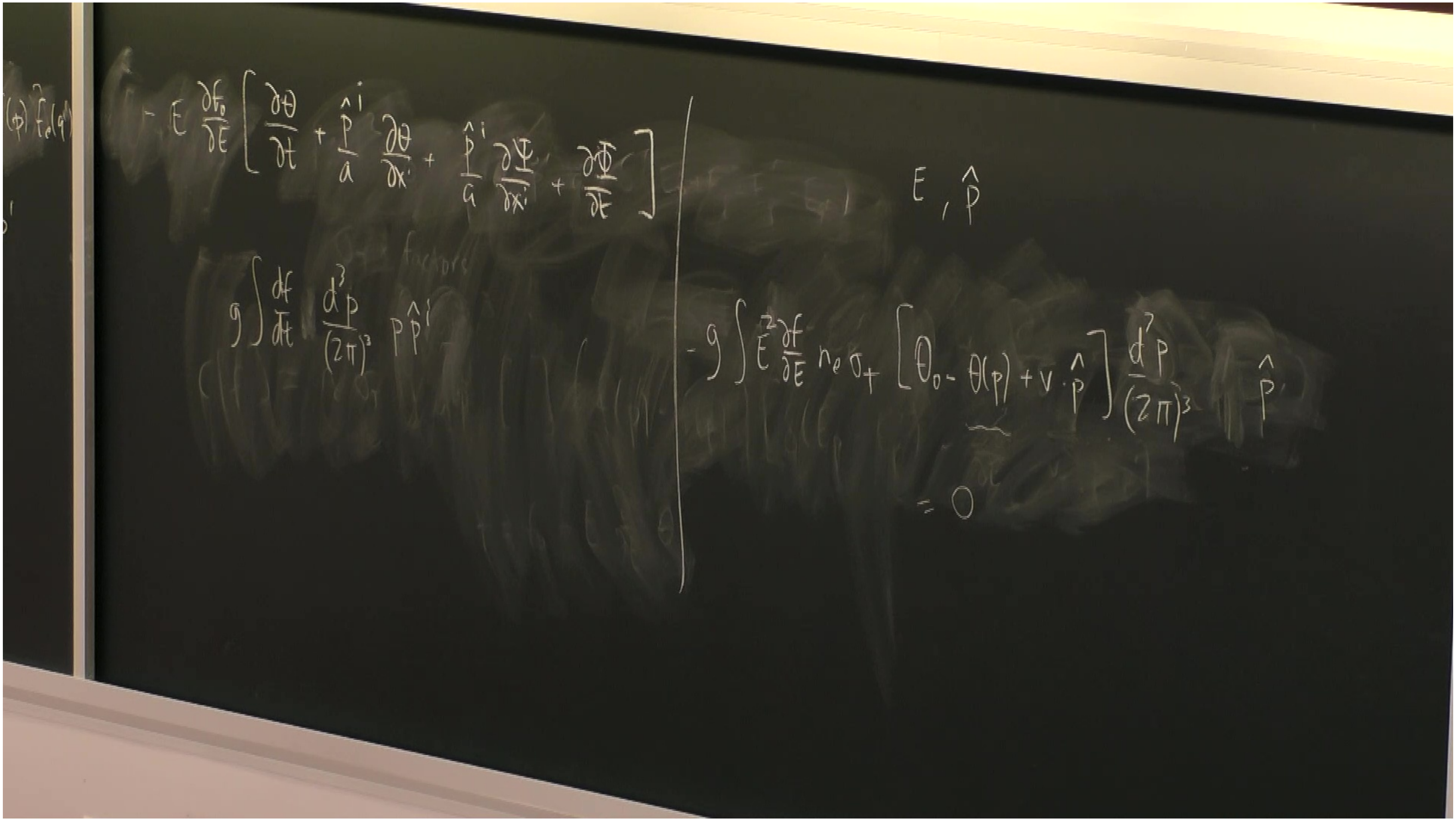
$$-E\frac{\partial f_0}{\partial E}\left[\frac{\partial\Theta}{\partial t}+\frac{\hat{p}^i}{a}\frac{\partial\Theta}{\partial x^i}+\frac{\hat{p}^i}{a}\frac{\partial\Psi}{\partial x^i}+\frac{\partial\Phi}{\partial t}\right]$$

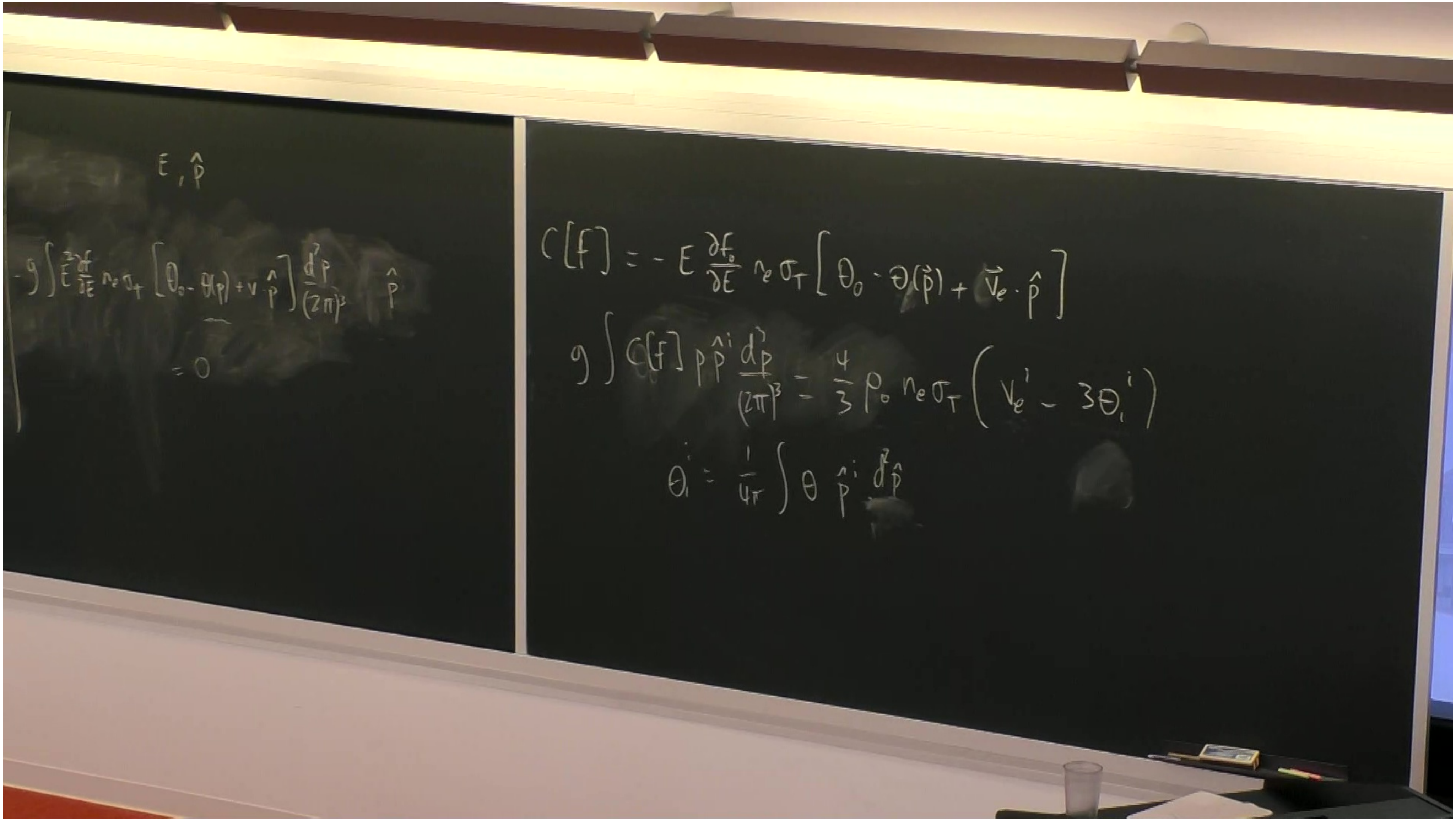
$$g\int\frac{df}{dt}\,E\,\frac{d^3p}{(2\pi)^3}$$

$E\,,\ \hat{p}$

$$-g\int E^2\frac{\partial f}{\partial E}\,n_e\sigma_T\left[\Theta_0-\Theta(p)+v\cdot\hat{p}\right]\frac{d^2\hat{p}}{(2\pi)^3}\quad p^2\,dp$$

$$=0$$


$-E \frac{\partial f_0}{\partial E}\left[\frac{\partial \Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \Theta}{\partial x^i} + \frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i} + \frac{\partial \Phi}{\partial t}\right]$
$g\int \frac{\partial f}{\partial t}\frac{d^3p}{(2\pi)^3}\, p\,\hat{p}^i$
$E\,,\ \hat{p}$
$-g\int E^2 \frac{\partial f}{\partial E}\, n_e \sigma_T \left[\Theta_0 - \Theta(p) + v\cdot\hat{p}\right]\frac{d^3p}{(2\pi)^3}\,\hat{p}$
$= 0$


$E, \hat{p}$
$-g \int E^2 \frac{\partial f}{\partial E} n_e \sigma_T \left[\Theta_0 - \Theta(p) + v \cdot \hat{p} \right] \frac{d^3p}{(2\pi)^3} \hat{p}$
$= 0$
$C[f] = -E \frac{\partial f_0}{\partial E} n_e \sigma_T \left[\Theta_0 - \Theta(\vec{p}) + \vec{v}_e \cdot \hat{p} \right]$
$g \int C[f]\, p\, \hat{p}^i \frac{d^3p}{(2\pi)^3} = \frac{4}{3} \rho_0 n_e \sigma_T \left(v_e^i - 3\Theta_1^i \right)$
$\Theta_1^i = \frac{1}{4\pi} \int \Theta\, \hat{p}^i\, d^2\hat{p}$

$$\frac{df}{\partial t} = C[f]$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{q} + 3\left(H + \frac{\partial \Phi}{\partial t}\right)(\rho + P)$$

$$\frac{\partial \vec{q}}{\partial t} + 4H\vec{q} + \frac{1}{a}\vec{\nabla}\cdot\overleftrightarrow{T} + \frac{1}{a}(\rho + P)\vec{\nabla}\Psi = \int C[f]\, p\,\hat{p}\, \frac{d^3p}{(2\pi)^3}$$

$$\frac{\partial \Theta}{\partial t} + \frac{\hat{p}^i}{a}\frac{\partial \Theta}{\partial x^i} + \frac{\hat{p}^i}{a}\frac{\partial \Psi}{\partial x^i} + \frac{\partial \Phi}{\partial t} = n_e \sigma_T \left[\Theta_0 - \Theta(\hat{p}) + \vec{v}_b \cdot \hat{p} \right]$$

photons

$$\frac{\partial \delta_{DM}}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{v}_{DM} + 3\frac{\partial \Phi}{\partial t} = 0$$

$$\frac{\partial v_{DM}}{\partial t} + H\vec{v}_{DM} + \frac{1}{a}\vec{\nabla}\Psi = 0$$

DM

$$\frac{\partial \delta_b}{\partial t} + \frac{1}{a}\vec{\nabla}\cdot\vec{v}_b + 3\frac{\partial \Phi}{\partial t} = 0$$

baryons

$$\frac{\partial v_b}{\partial t} + H\vec{v}_b + \frac{1}{a}\vec{\nabla}\Psi = \frac{4}{3}\frac{\rho_\gamma}{\rho_b} n_e \sigma_T \left(3\vec{\Theta}_1 - \vec{v}_b\right)$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$T^{\mu\nu} = g\int f(\vec{p})\,\frac{p^\mu p^\nu}{E}\,\frac{d^3p}{(2\pi)^3}$$

$$p^t = (1-\Psi)\,E$$

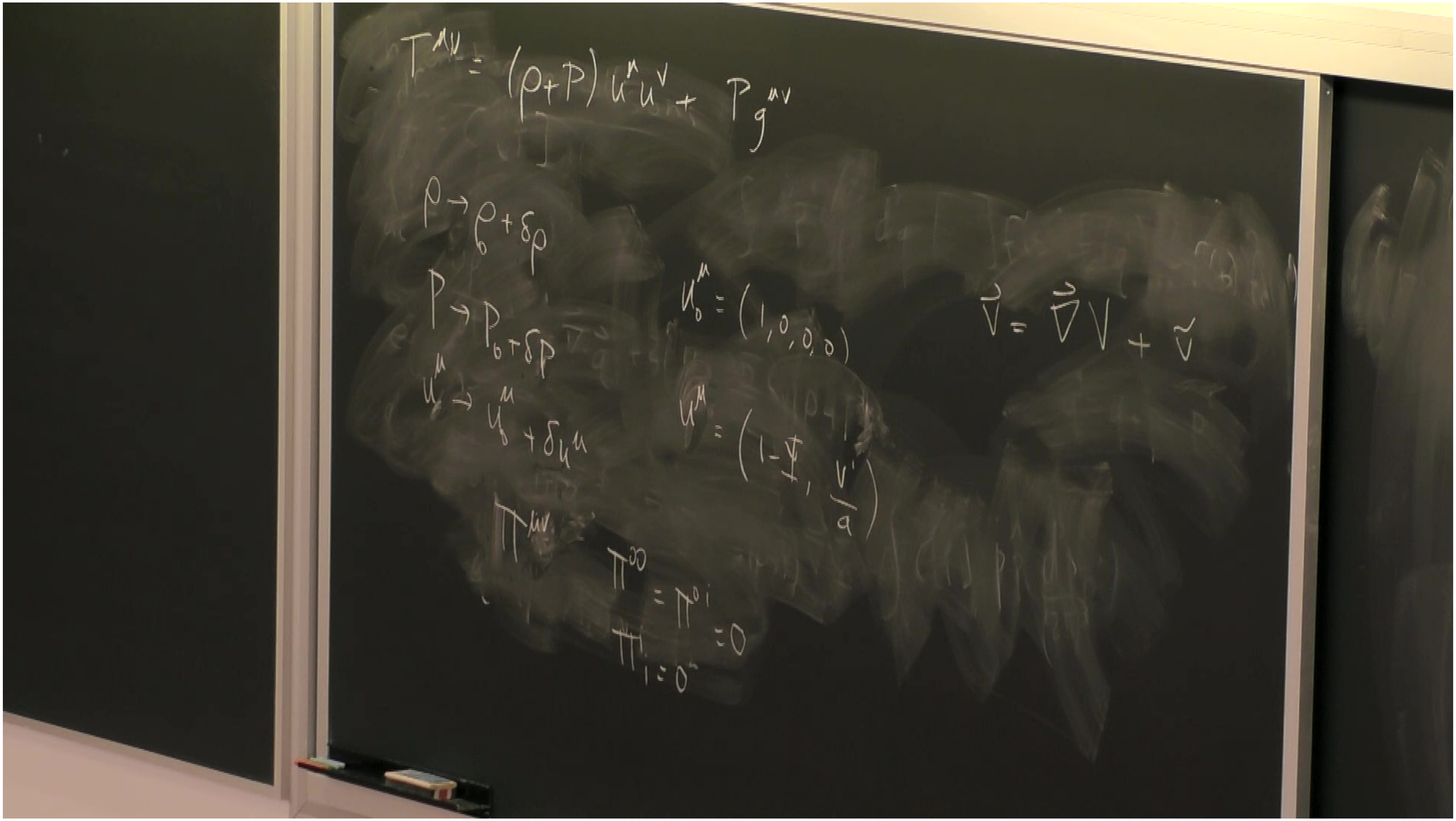
$$p^i = \frac{1-\Phi}{a}\,p\,\hat{p}^i$$

$$T^0{}_0 = -g\int f\,E\,\frac{d^3p}{(2\pi)^3} = -\rho$$

$$T^i{}_j = g\int f\,\frac{p^2}{E}\,\hat{p}^i\hat{p}^j\,\frac{d^3p}{(2\pi)^3}$$

$$T^i{}_i = 3P$$

$$\pi^i{}_j = T^i{}_j - P\,\delta^i{}_j \qquad T^0{}_i \neq 0$$


$T^{\mu\nu} = (\rho+P)u^\mu u^\nu + P g^{\mu\nu}$
$\rho \to \rho_0 + \delta\rho$
$P \to P_0 + \delta P$
$u^\mu \to u_0^\mu + \delta u^\mu$
$u_0^\mu = (1,0,0,0)$
$u^\mu = \left(1-\Psi, \frac{v^i}{a}\right)$
$\vec{v} = \vec{\nabla} V + \tilde{v}$
$\Pi^{\mu\nu}$
$\Pi^{00} = \Pi^{0i} = 0$
$\Pi^i_i = 0$

$$T^{\mu\nu} = (\rho + P)\, u^\mu u^\nu + P g^{\mu\nu}$$

$$\rho \to \rho_0 + \delta\rho$$

$$P \to P_0 + \delta P$$

$$u^\mu \to u_0^\mu + \delta u^\mu$$

$$u_0^\mu = (1, 0, 0, 0)$$

$$u^\mu = \left(1 - \Psi, \frac{v^i}{a}\right)$$

$$\vec{v} = -\frac{1}{a}\vec{\nabla} V + \tilde{v}$$

$$\Pi_{ij} = \frac{1}{a^2}\left(\partial_i \partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right) Q$$

$$\Pi^{\mu\nu}$$

$$\Pi^{00} = \Pi^{0i} = 0$$

$$\Pi^i{}_i = 0$$

$$\delta T^0{}_0 = -\delta\rho$$

$$\delta T^0{}_i = (\rho_0 + P_0)\, v_i$$

$$\delta T^i{}_j = \delta P\, \delta^i{}_j + \pi^i{}_j$$

$$\delta G^0{}_0 = \frac{1}{a^2}\nabla^2 \Phi + 6\left(H^2 \Psi - H \frac{\partial \Phi}{\partial t}\right) = 8\pi G T^0_0 = -4\pi G \rho$$

$$\delta G^0{}_i = \frac{2}{a}\frac{\partial}{\partial x^i}\left[\frac{\partial \Phi}{\partial t} - H\Psi\right] = 8\pi G T^0{}_i$$