Title: Cosmology Lecture (230421)

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Collection: Cosmology (2022/2023)

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$$g_{\mu\nu}\ ,\ f\ ,\ \rho\ ,\ P$$

$$\delta\rho \to \delta g_{\mu\nu}$$

$$|\delta| \ll 1$$

$$\rho(x,t) = \rho_0(t) + \delta\rho(x,t) = \rho_0(t)\left[1 + \delta(x,t)\right]$$

$$g_{\mu\nu}(x,t) = g^{FRW}_{\mu\nu} + \delta g_{\mu\nu}$$

$$ds^2 = -dt^2 + a^2\left[dr^2 + D^2(r)\,d\Omega^2\right] = -dt^2 + a^2\,\delta_{ij}\,dx^i\,dx^j$$

$$r^2$$

$$d\eta = dt/a$$

$$ds^2 = a^2\left[-d\eta^2 + \delta_{ij}\, dx^i dx^j\right]$$

$$= a^2(\eta)\left[-(1+A)\,d\eta^2 - 2B_i\, d\eta\, dx^i + (\delta_{ij} + h_{ij})\, dx^i dx^j\right]$$

Decompose A B h into scalar, vector, tensor

$$h_{ij} = C\delta_{ij} + \left(\nabla_i \nabla_j - \frac{1}{3}\delta_{ij}\nabla^2\right)E + \nabla_{(i}\tilde{E}_{j)} + \tilde{E}_{ij}$$

4 scalar: A, B, C, E − 2 gauge $\quad x^\mu \to x'^\mu$

4 vector: $\tilde{B}_i, \tilde{E}_i$

2 tensor $\tilde{E}_{ij}$

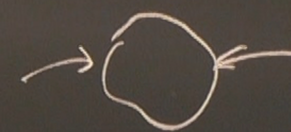
$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$$

not all 10 are dynamical d.o.f.

$$(x^0, x^i) \to (x'^0, x'^i) = (x^0 + T, x^i + L^i)$$

$+a^2 \delta_{ij}\, dx^i\, dx^j$

$$h_{ij} = \left(C\delta_{ij} + \left(\nabla_i \nabla_j - \tfrac{1}{3}\delta_{ij}\nabla^2\right)E + \nabla_{(i}\tilde{E}_{j)}\right) + \tilde{E}_{ij}$$



4 scalar : A, B, C, E − 2 gauge = 2 couple to density pert's

4 vector: $\tilde{B}_i, \tilde{E}_i$ − 2 gauge = 2 not generated, decay if present

2 tensor $\tilde{E}_{ij}$ = 2 polarizations of GWs

not all 10 are dynamical d.o.f.

$+ E_{ij}$

ouple to density pert's

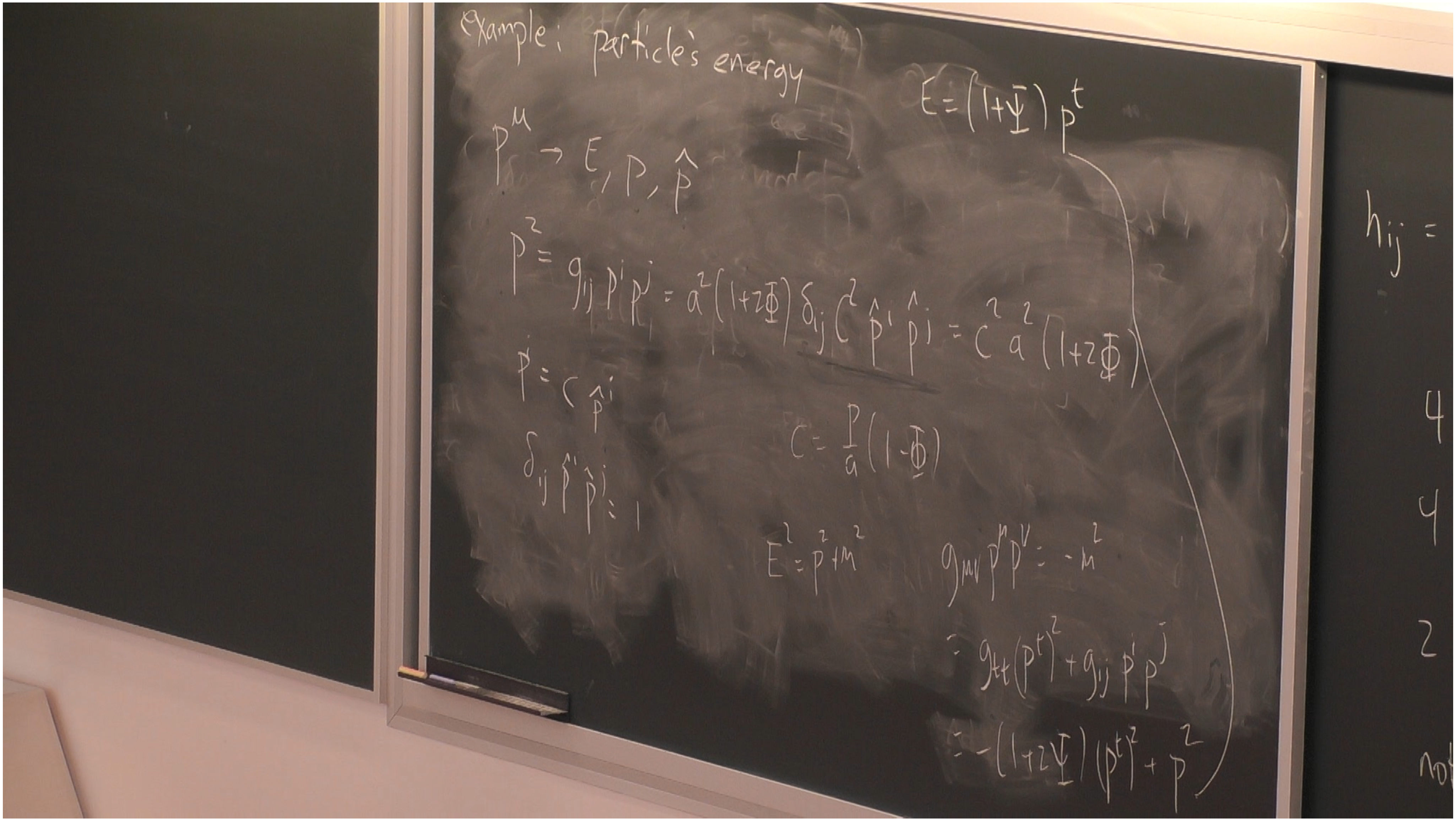
generated, decay if present

ations of GWs

synchronous: set $A=0$

Conformal Newtonian: set $B=E=0$

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1+2\Phi)\delta_{ij}\,dx^i dx^j\right]$$


Example: particle's energy
$E = (1+\Psi)\, p^t$
$p^\mu \to E, p, \hat{p}$
$p^2 = g_{ij} p^i p^j = a^2 (1+2\Phi) \delta_{ij} C^2 \hat{p}^i \hat{p}^j = C^2 a^2 (1+2\Phi)$
$p^i = C \hat{p}^i$
$C = \frac{p}{a}(1-\Phi)$
$\delta_{ij} \hat{p}^i \hat{p}^j = 1$
$E^2 = p^2 + m^2$
$g_{\mu\nu} p^\mu p^\nu = -m^2$
$= g_{tt} (p^t)^2 + g_{ij} p^i p^j$
$= -(1+2\Psi)(p^t)^2 + p^2$
$h_{ij} =$
4
4
2

$$\mu\nu \ \frac{p^\mu p^\nu}{p^t}$$

$$= 0$$

$$\Gamma^\alpha_{\mu\nu} \, p^\nu u^\mu$$

$$\frac{d\Psi}{dt} = \frac{\partial \Psi}{\partial t} + \frac{\partial \Psi}{\partial x^i}\frac{dx^i}{dt} = \frac{\partial \Psi}{\partial t} + \frac{\partial \Psi}{\partial x^i}\frac{p\,\hat{p}^i}{aE}$$

$$\frac{dx^i}{dt} = \frac{u^i}{u^t} = \frac{p^i}{p^t}$$

$$= \frac{p\,\hat{p}^i}{aE}(1-\Phi)(1+\Psi)$$

$$\boxed{\frac{dE}{dt} = \frac{p\hat{p}^i}{a}\frac{\partial\Psi}{\partial x^i} - \frac{p^2}{E}\left(H + \frac{\partial\Phi}{\partial t}\right)}$$

Perturbed Boltzmann eqn

$$\frac{dE}{dt} = \frac{dE}{dp}\cdot\frac{dp}{dt}$$

$$F\cdot v = \frac{dE}{dt}$$

$$\frac{d\Psi}{dt}$$