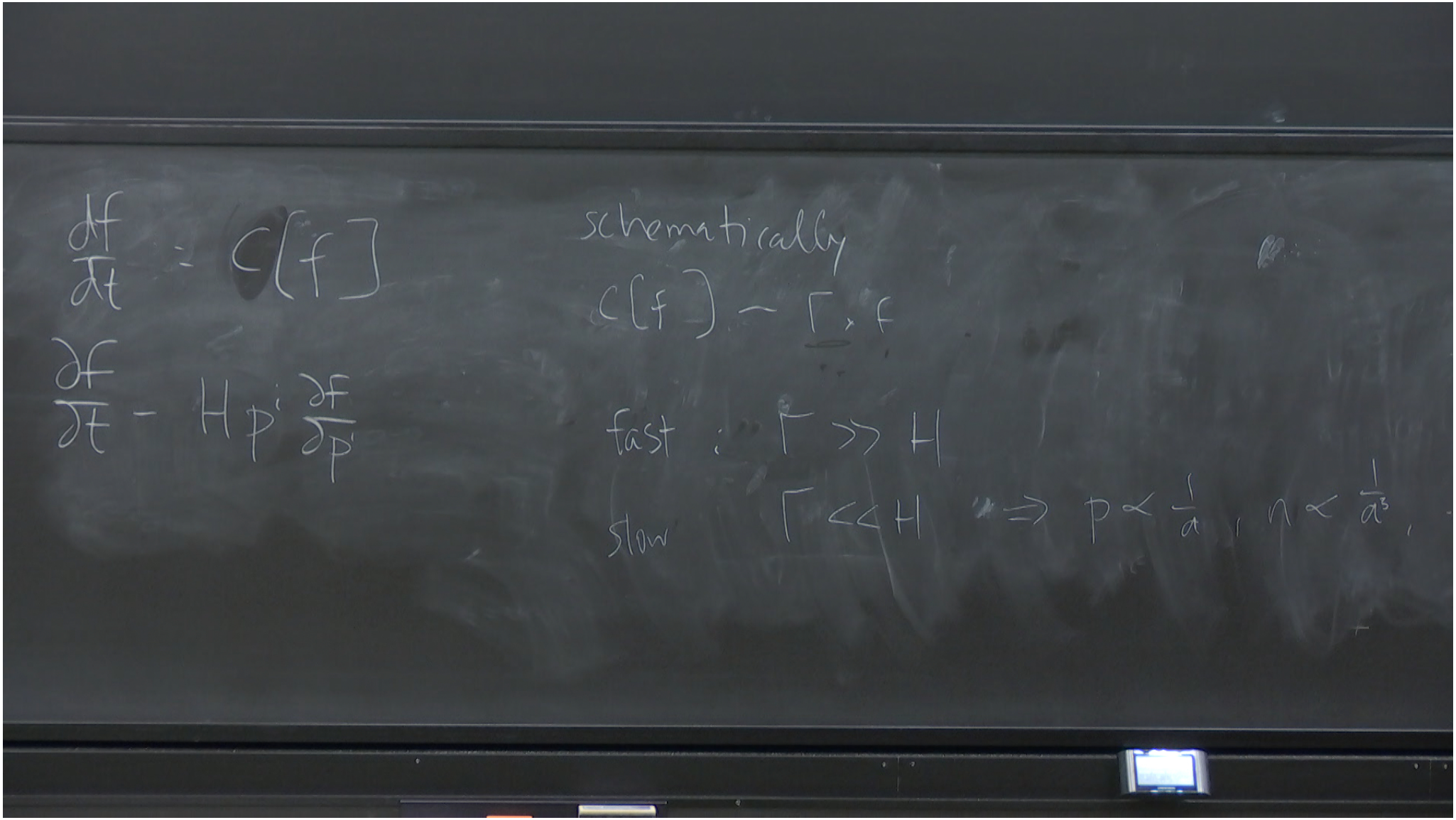
Title: Cosmology Lecture (230414)

Speakers: Neal Dalal

Collection: Cosmology (2022/2023)

Date: April 14, 2023 - 3:45 PM

URL: https://pirsa.org/23040057


$\frac{df}{dt} = C[f]$
$\frac{\partial f}{\partial t} - Hp^i \frac{\partial f}{\partial p^i}$
schematically
$C[f] \sim \Gamma \cdot f$
fast : $\Gamma \gg H$
slow $\Gamma \ll H \Rightarrow p \propto \frac{1}{a}, n \propto \frac{1}{a^3},$

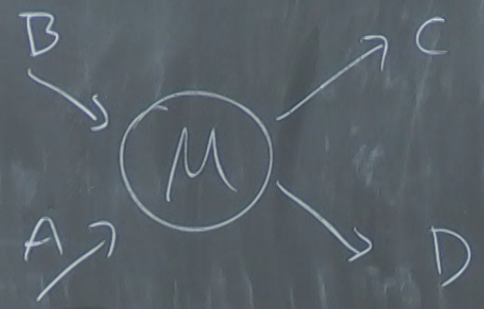
schematically

$C[f] \sim \Gamma \cdot f$

fast : $\Gamma \gg H \Rightarrow$ driven to equilibrium

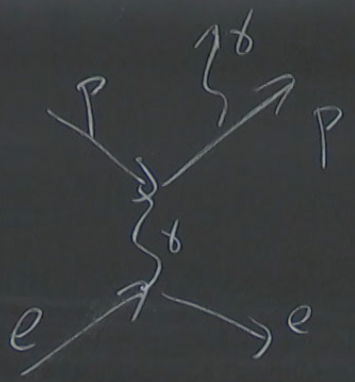
slow $\Gamma \ll H \Rightarrow p \propto \frac{1}{a}, n \propto \frac{1}{a^3}, \ldots$

$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$



$$T_A = T_B = T_C = T_D$$

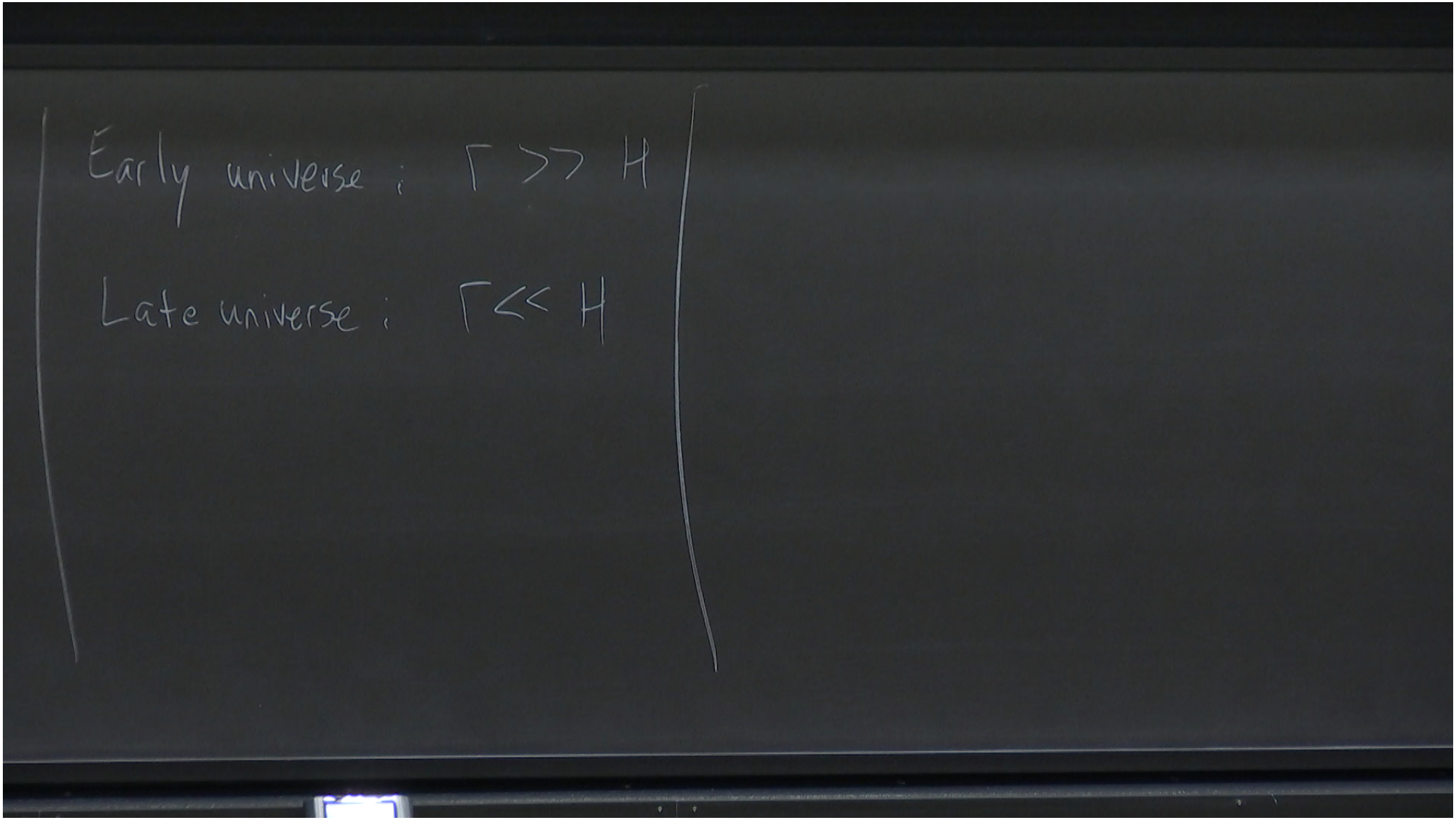
$$\mu_A + \mu_B = \mu_C + \mu_D$$



$$\mu_e + \mu_p = \mu_e + \mu_p + \mu_\gamma$$

$$\mu_\gamma = 0$$

$$e^+ + e^- \to 2\gamma \Rightarrow \mu_{e^-} = -\mu_{e^+}$$


Early universe : $\Gamma \gg H$
Late universe : $\Gamma \ll H$

$\Gamma \gg H$

$\Gamma \ll H$

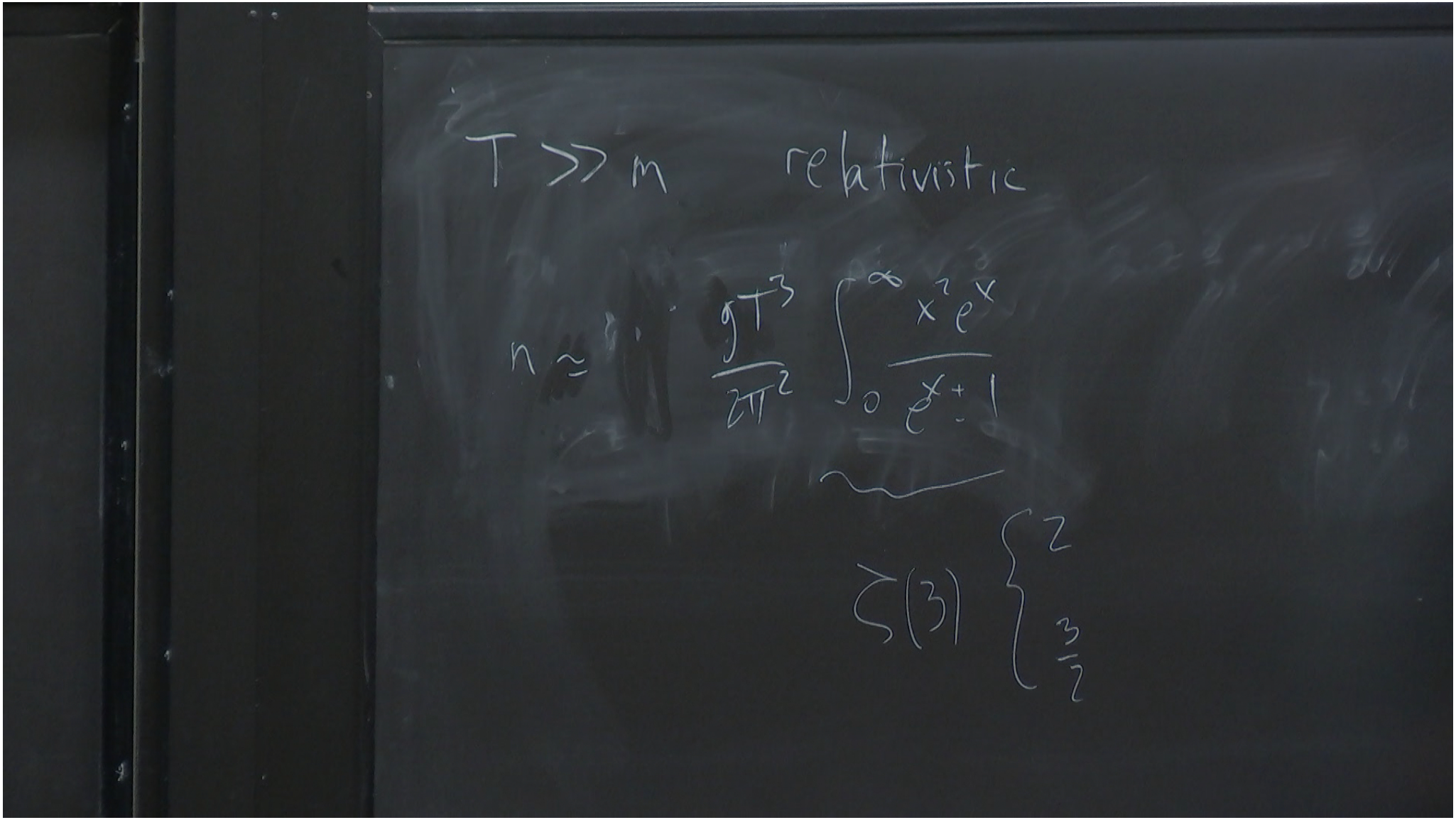
$$n = g \int \frac{d^3p}{(2\pi)^3} f(E) \qquad E^2 = m^2 + p^2$$

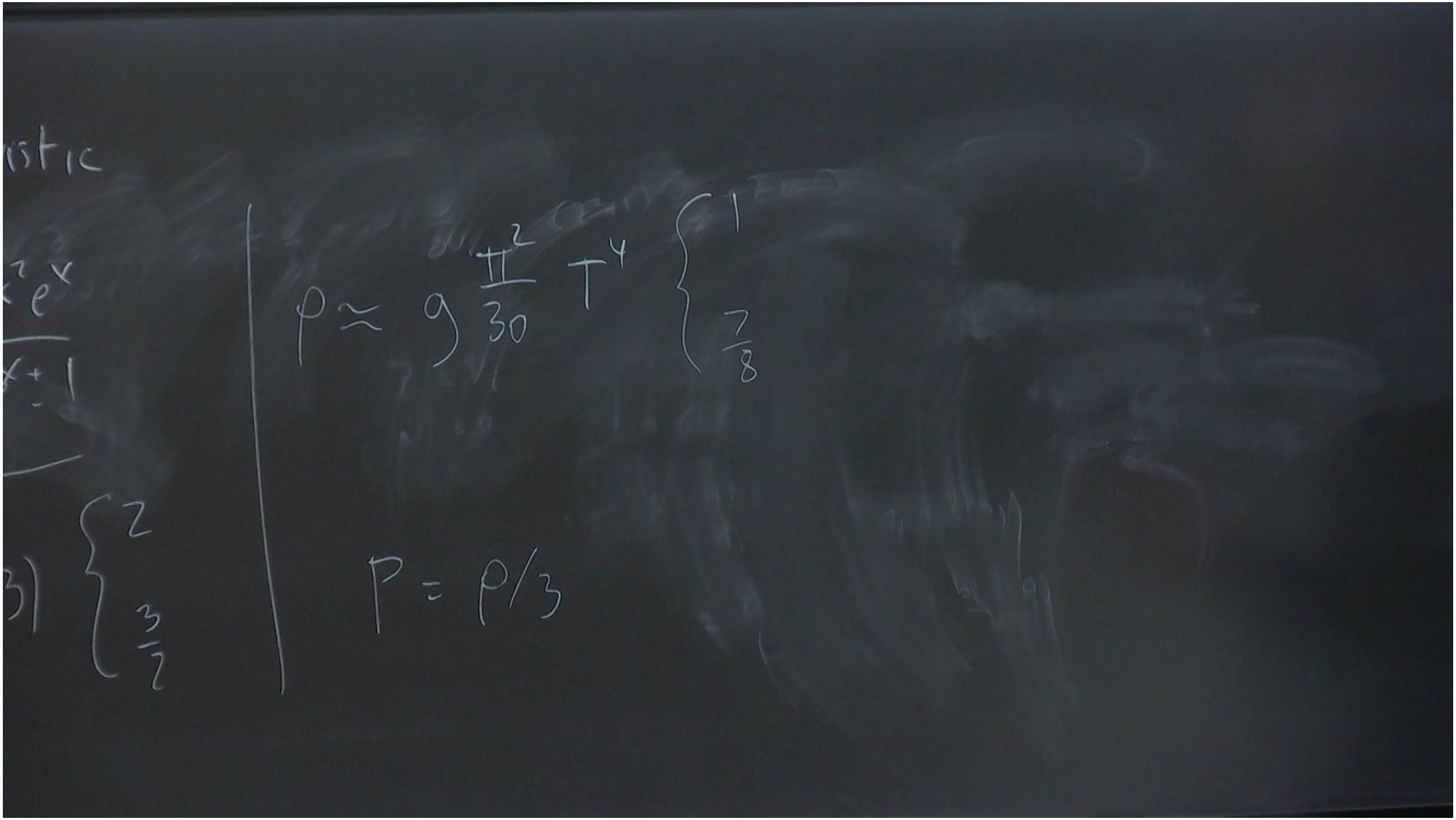
$$= g \int \frac{4\pi p^2 dp}{(2\pi)^3} f(E) \qquad E dE = p dp$$

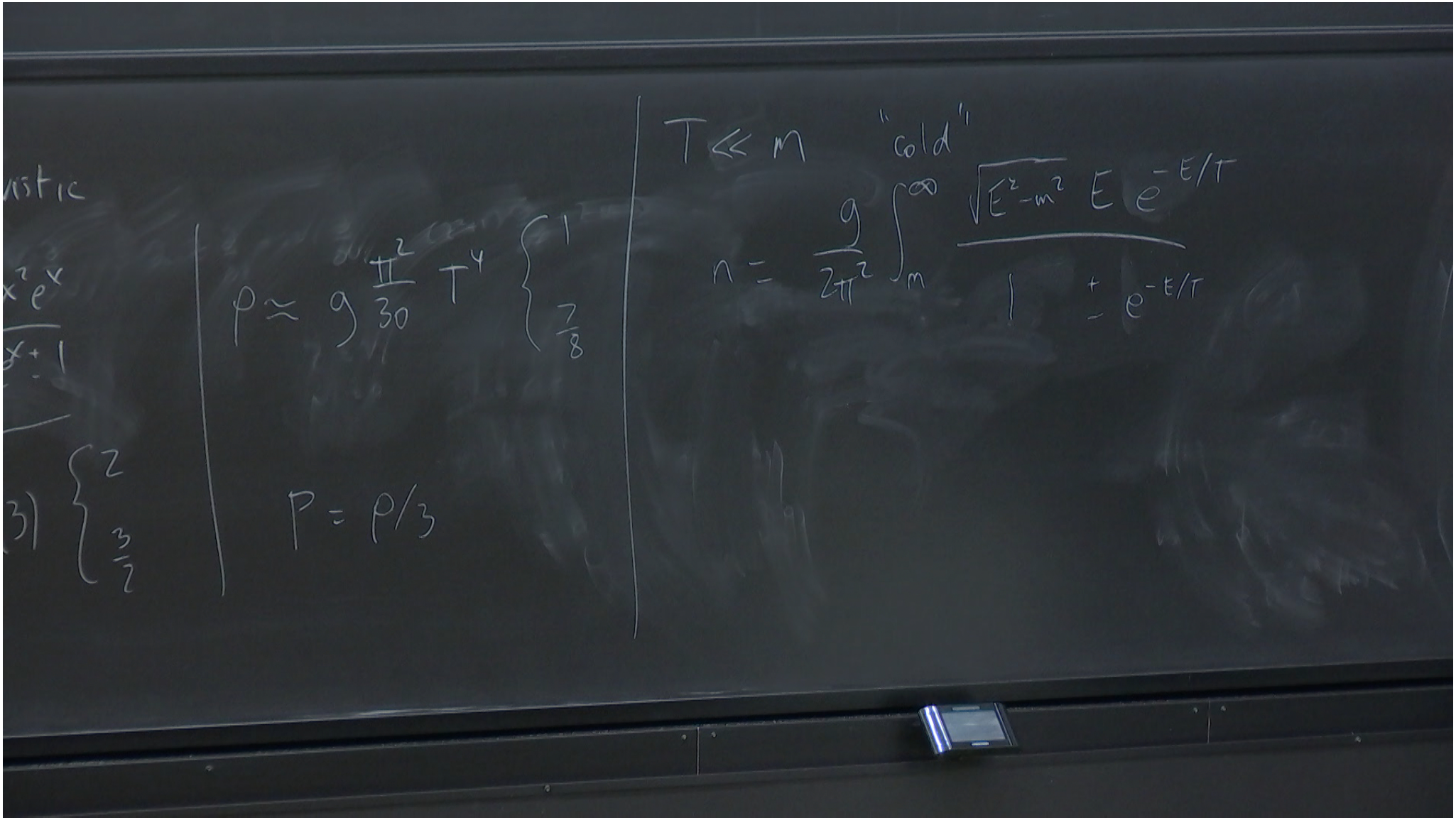
$$p^2 dp = p E dE$$

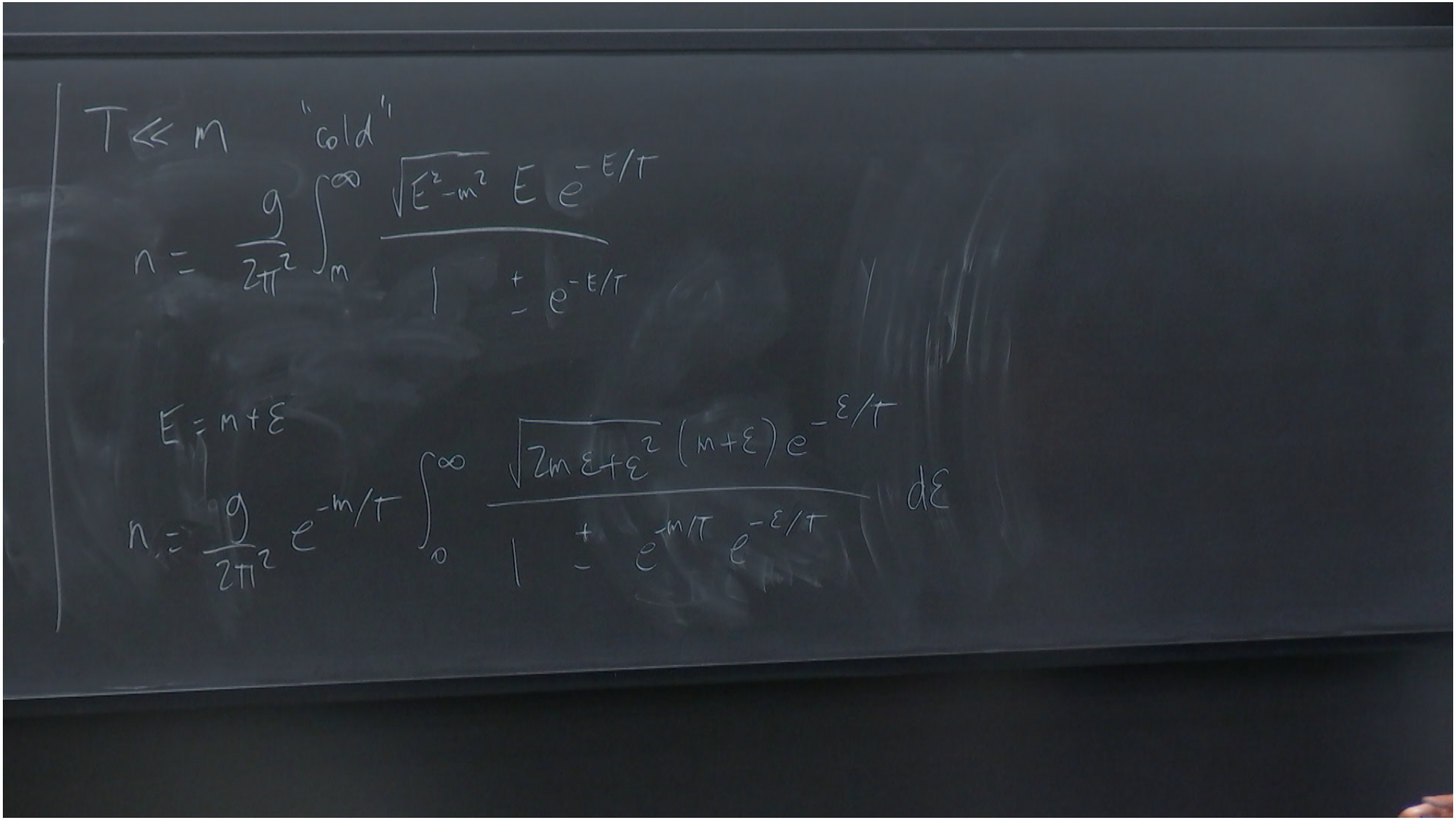
$$= \sqrt{E^2 - m^2}\, E dE$$

$$n = \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\, E dE}{e^{E/T} \pm 1}$$


T >> m relativistic
n ≃ g T^3 / 2π^2 ∫_0^∞ x^2 e^x / (e^x + 1)
ζ(3)
2
3/2


istic
$\rho \simeq g \frac{\pi^2}{30} T^4 \begin{cases} 1 \\ \frac{7}{8} \end{cases}$
$P = \rho/3$


istic
$x^2 e^x$
$e^x + 1$
3) $\begin{cases} 2 \\ \frac{3}{2} \end{cases}$
$\rho \simeq g \frac{\pi^2}{30} T^4 \begin{cases} 1 \\ \frac{7}{8} \end{cases}$
$P = \rho/3$
$T \ll m$ "cold"
$n = \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\, E\, e^{-E/T}}{1 \pm e^{-E/T}}$


T ≪ m "cold"
$n = \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\, E\, e^{-E/T}}{1 \pm e^{-E/T}}$
$E = m + \varepsilon$
$n = \frac{g}{2\pi^2} e^{-m/T} \int_0^\infty \frac{\sqrt{2m\varepsilon + \varepsilon^2}\,(m+\varepsilon)\, e^{-\varepsilon/T}}{1 \pm e^{-m/T} e^{-\varepsilon/T}}\, d\varepsilon$

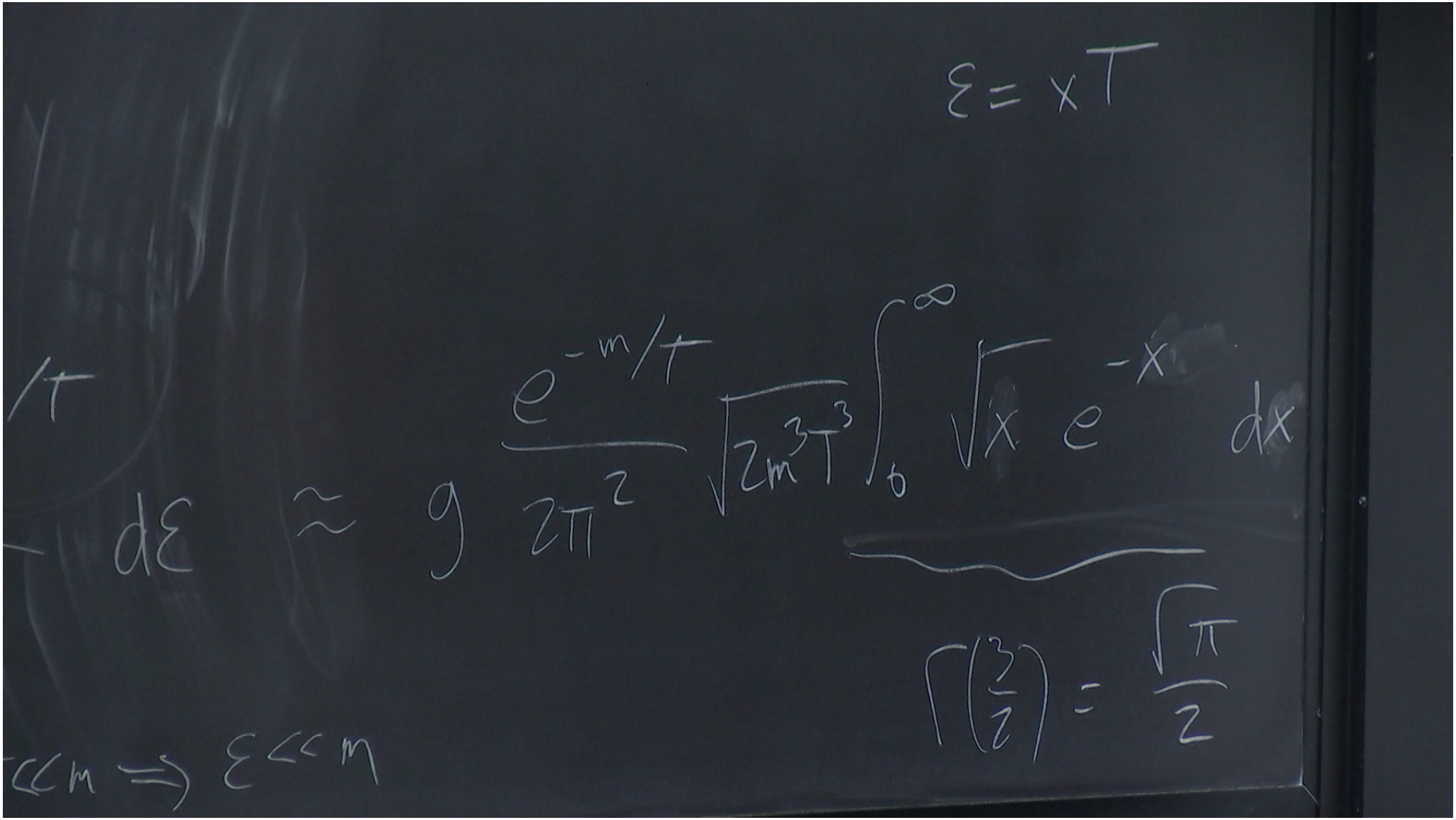
$T \ll m$ "cold"

$$n = \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\, E\, e^{-E/T}}{1 \pm e^{-E/T}}$$

$E = m + \varepsilon$

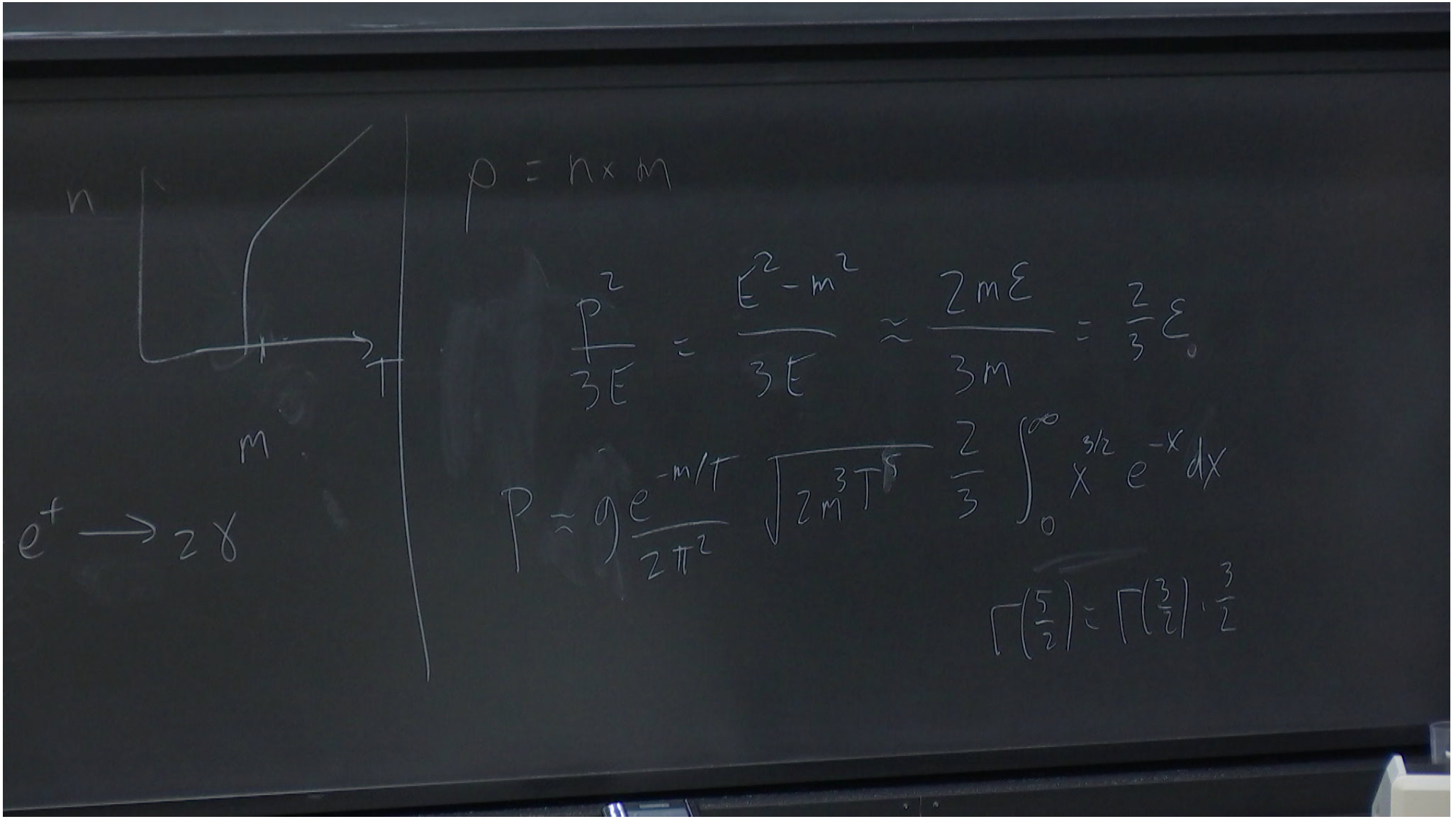
$$n = \frac{g}{2\pi^2} e^{-m/T} \int_0^\infty \frac{\sqrt{2m\varepsilon + \varepsilon^2}\,(m+\varepsilon)\, e^{-\varepsilon/T}}{1 \pm e^{-m/T} e^{-\varepsilon/T}} d\varepsilon \simeq g \frac{e^{-m/T}}{2\pi^2} \sqrt{2m^3}$$

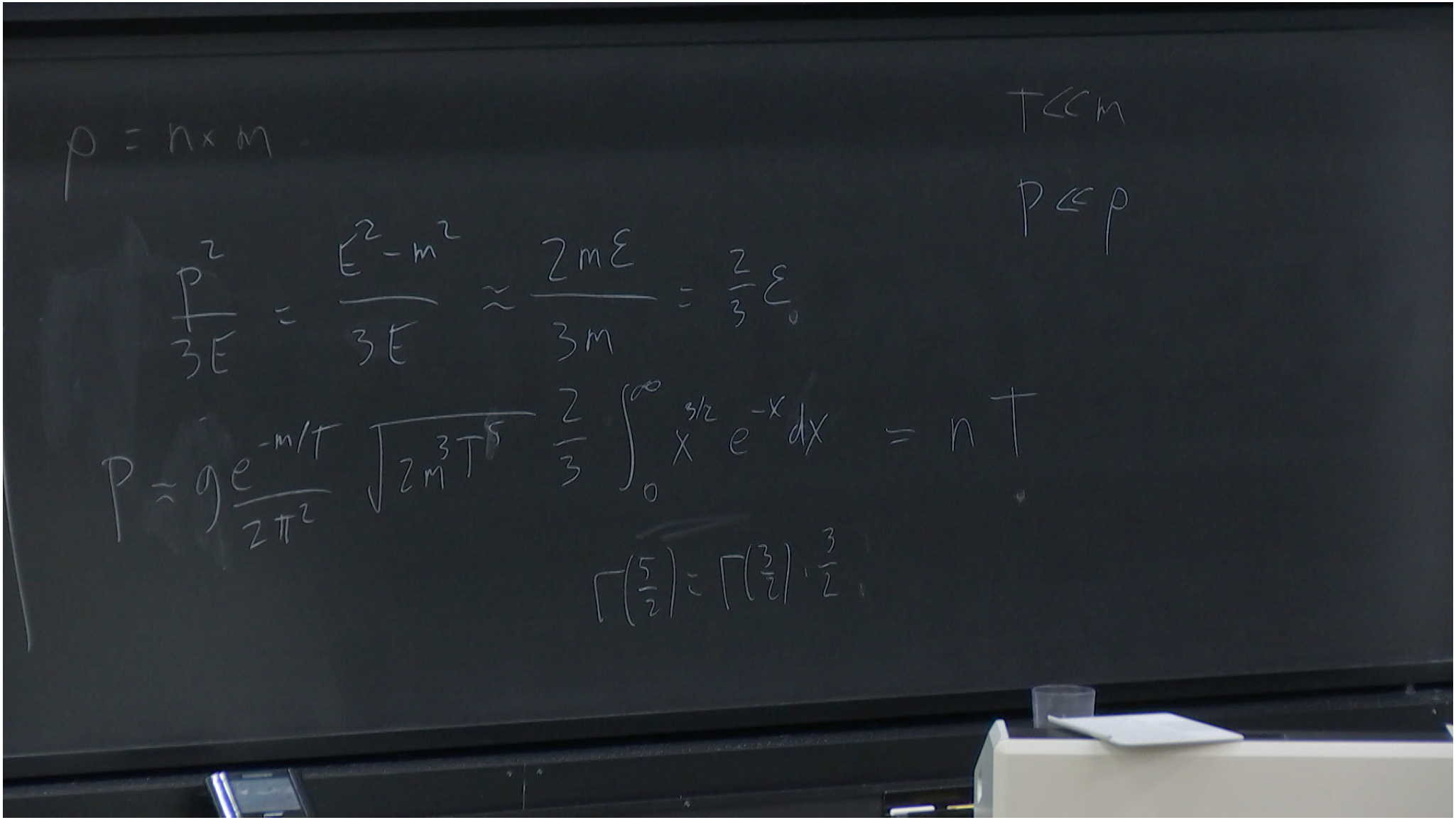
$\uparrow \ll 1$ $\quad T \ll m \Rightarrow \varepsilon \ll m$

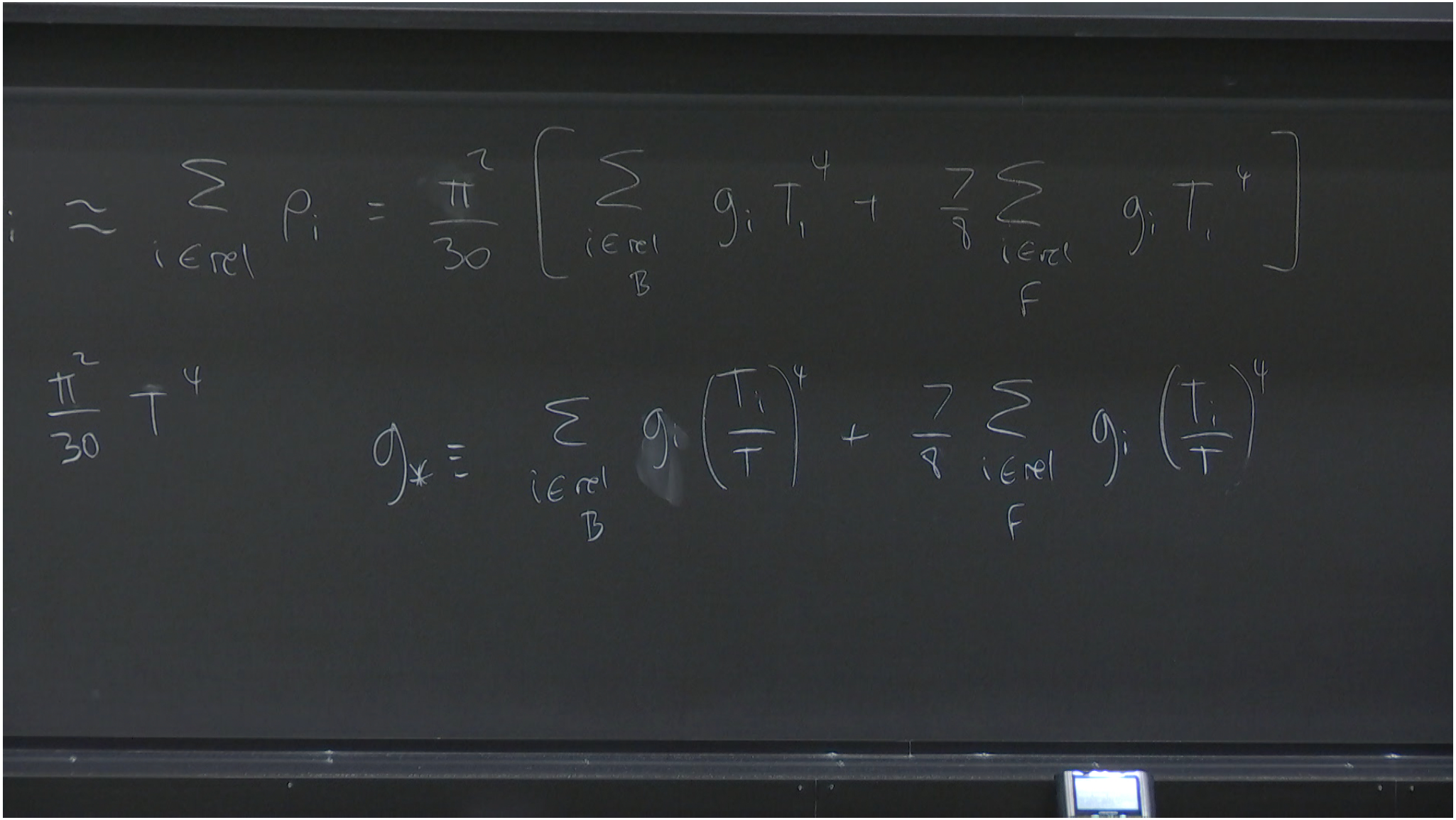

$\varepsilon = xT$
$d\varepsilon \simeq g \frac{e^{-m/T}}{2\pi^2} \sqrt{2m^3T^3} \int_0^\infty \sqrt{x}\, e^{-x}\, dx$
$\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$
$\varepsilon \ll m \Rightarrow \varepsilon \ll m$

$$n \approx g\, e^{-m/T} \left(\frac{mT}{2\pi}\right)^{3/2}$$

Boltzmann suppression


n
m
T
$e^+ \longrightarrow 2\gamma$
$\rho = n \times m$
$\frac{p^2}{3E} = \frac{E^2 - m^2}{3E} \approx \frac{2m\mathcal{E}}{3m} = \frac{2}{3}\mathcal{E}$
$P \approx g \frac{e^{-m/T}}{2\pi^2} \sqrt{2m^3 T^5} \frac{2}{3} \int_0^\infty x^{3/2} e^{-x} dx$
$\Gamma\left(\frac{5}{2}\right) = \Gamma\left(\frac{3}{2}\right) \cdot \frac{3}{2}$


$p = n \times m$
$T \ll m$
$p \ll p$
$\frac{p^2}{3E} = \frac{E^2 - m^2}{3E} \simeq \frac{2m\mathcal{E}}{3m} = \frac{2}{3}\mathcal{E}$
$P \simeq g \frac{e^{-m/T}}{2\pi^2} \sqrt{2m^3 T^5} \frac{2}{3} \int_0^\infty x^{3/2} e^{-x} dx = nT$
$\Gamma\left(\frac{5}{2}\right) = \Gamma\left(\frac{3}{2}\right) \cdot \frac{3}{2}$


$\rho \simeq \sum_{i \in \text{rel}} \rho_i = \frac{\pi^2}{30}\left[\sum_{\substack{i \in \text{rel} \\ B}} g_i T_i^4 + \frac{7}{8}\sum_{\substack{i \in \text{rel} \\ F}} g_i T_i^4\right]$
$\frac{\pi^2}{30} T^4$
$g_* \equiv \sum_{\substack{i \in \text{rel} \\ B}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{\substack{i \in \text{rel} \\ F}} g_i \left(\frac{T_i}{T}\right)^4$

$$\left. \sum_i g_i T_i^4 \right]$$

$$\sum_i g_i \left(\frac{T_i}{T}\right)^4$$

$$\frac{\partial \rho_{tot}}{\partial t} + 3H(\rho_{tot} + P_{tot}) = 0$$

$$\frac{dT}{dt} = \frac{d\rho/dt}{d\rho/dT}$$

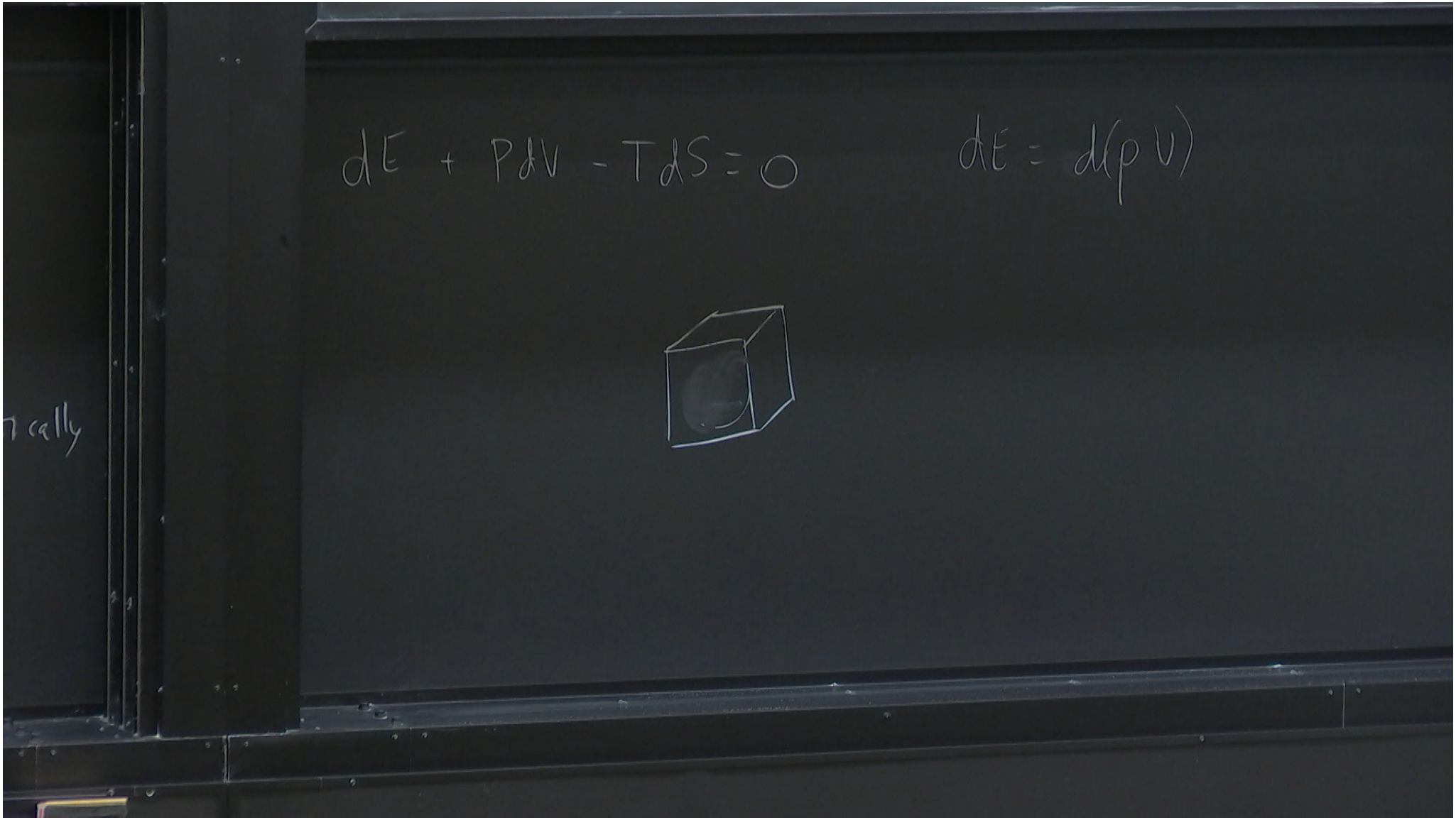
$$\frac{\partial \rho_{tot}}{\partial t} + 3H(\rho_{tot} + P_{tot}) = 0$$

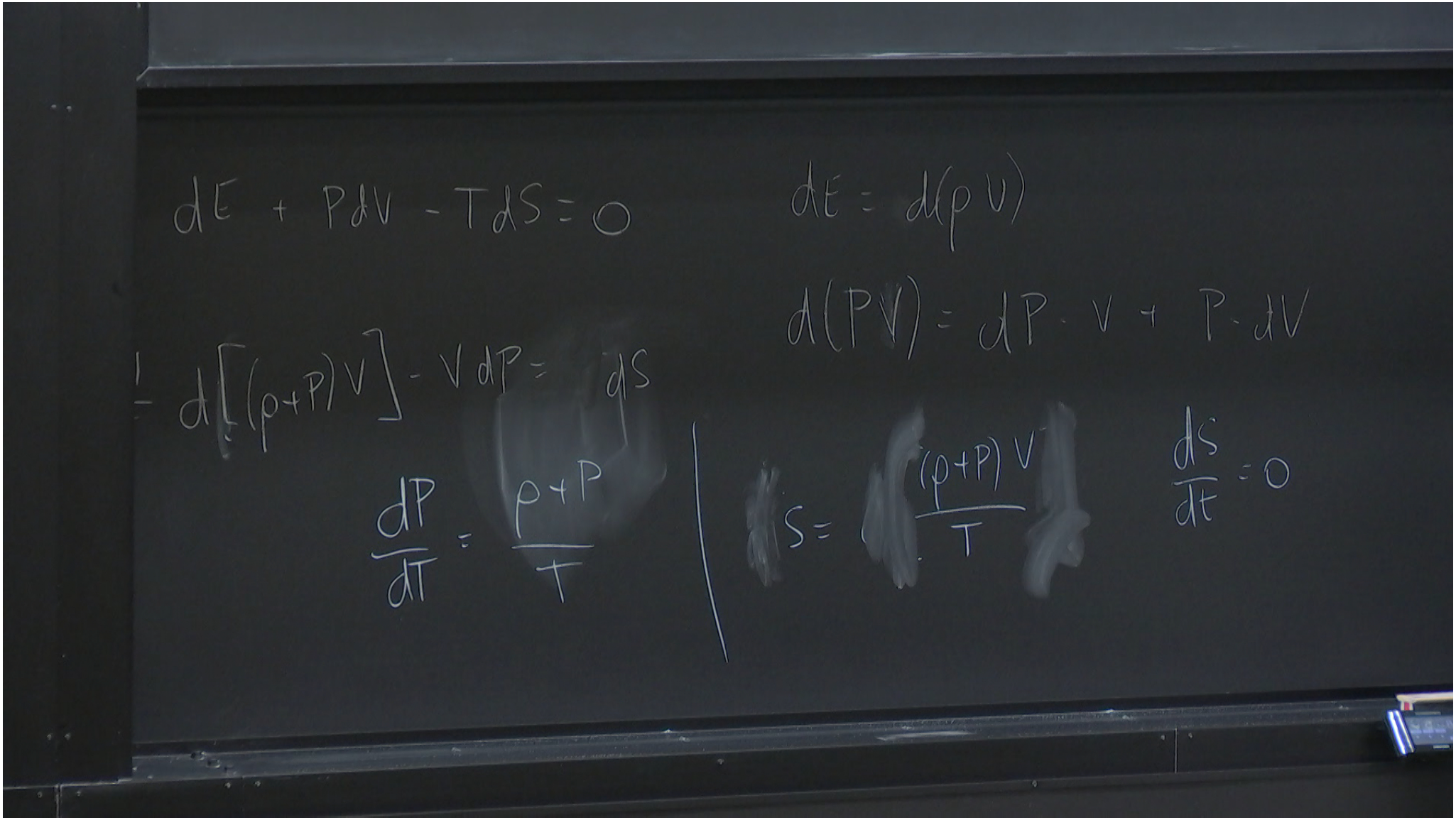
$$\frac{dT}{dt} = \frac{d\rho/dt}{d\rho/dT}$$

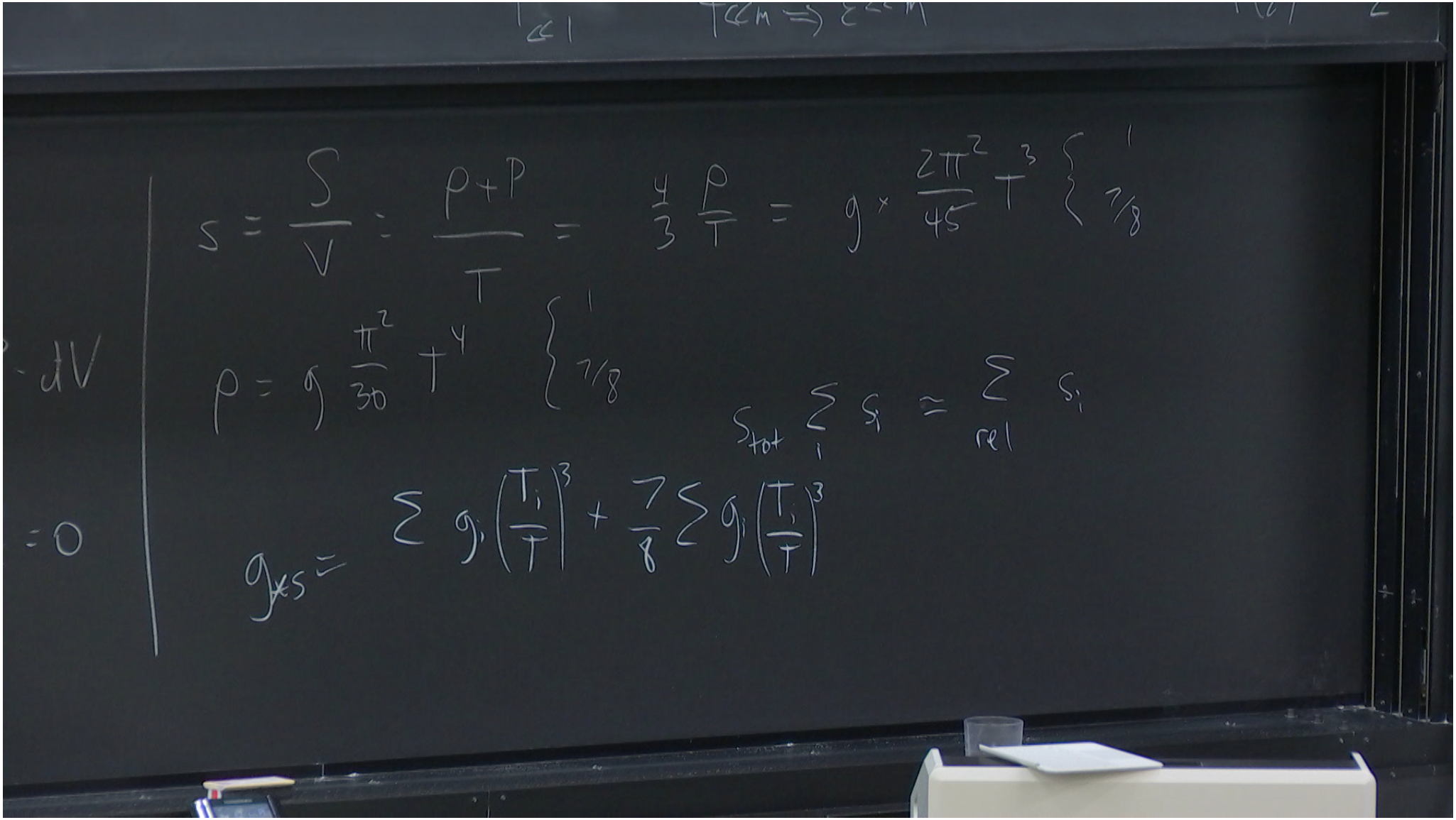
simpler

use T evolve adiabatically

$S = \text{const}$ $dS = 0$


dE + PdV - TdS = 0
dE = d(ρV)
cally


dE + PdV - TdS = 0
dE = d(ρV)
d(PV) = dP · V + P · dV
= d[(ρ+P)V] - V dP = dS
dP/dT = (ρ+P)/T
S = (ρ+P)V/T
dS/dt = 0


$s = \frac{S}{V} = \frac{\rho + P}{T} = \frac{4}{3}\frac{\rho}{T} = g_* \frac{2\pi^2}{45} T^3 \begin{cases} 1 \\ 7/8 \end{cases}$
$\rho = g \frac{\pi^2}{30} T^4 \begin{cases} 1 \\ 7/8 \end{cases}$
$S_{tot} \sum_i S_i \simeq \sum_{rel} S_i$
$g_{*S} = \sum g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum g_i \left(\frac{T_i}{T}\right)^3$
$\cdot dV$
$= 0$

$g_{*s} T^3 a^3 = \text{const}$

if $g_{*s} = \text{const} \Rightarrow T \propto \frac{1}{a}$

$g_{*s} = g_* = \sum g_i$