

Title: Cosmology Lecture (230411)

Speakers: Neal Dalal

Collection: Cosmology (2022/2023)

Date: April 11, 2023 - 3:45 PM

URL: <https://pirsa.org/23040056>

Last time: Friedmann eqns
solns; evolution determined by Ω 's
determined by: distances, CMB, LSS...

Λ CDM

matter: $\Omega_{m0} \sim 0.3$

vacuum: $\Omega_{\Lambda 0} \sim 0.7$

radiation: $\Omega_{r0} \sim 10^{-4}$

curvature: $\Omega_{k0} \approx 0 \pm 1\%$

Hubble : $H_0 \approx 70 \frac{\text{km/s}}{\text{Mpc}}$

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$$\text{Hubble: } H_0 = 70 \frac{\text{km/s}}{\text{Mpc}} \equiv h \cdot 100 \frac{\text{km/s}}{\text{Mpc}}$$

⇒ dark matter

evidence in: CMB, galaxies, clusters, grav. lensing...

DM: what is known?

1. gravitates

2. responds to gravity

3. no detected interactions besides gravity

presence in: CMB, galaxies, clusters, grav. lensing...

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$$\rho_{\text{crit}} = 10^{-29} \text{ g/cm}^3$$

$$\Omega_{\text{DM}} \approx 0.26$$

$$\text{if } M \approx 100 \text{ GeV} \Rightarrow n \approx 10^{-8} \text{ cm}^{-3}$$

DM: what is known?

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2. responds to gravity
3. no detected interactions besides gravity

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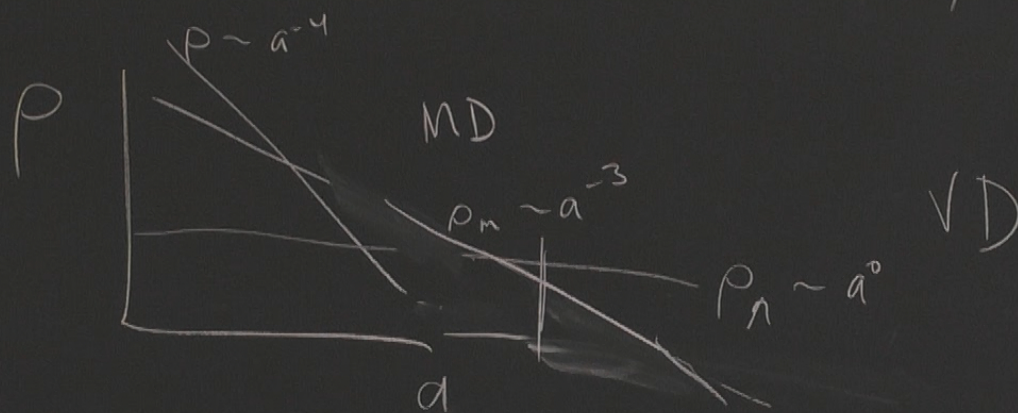
$$\Omega_{\text{DM}} \approx 0.26$$

$$\text{if } M \approx 100 \text{ GeV} \Rightarrow n = 10^{-8} \text{ cm}^{-3}$$

$$\text{baryons: } n \sim 2 \cdot 10^{-7} \text{ cm}^{-3}$$

$$n_{\gamma} \approx 400 \text{ cm}^{-3}$$

theory: expect $\frac{\rho_{\text{vac}}}{\rho_{\text{crit}}} = 10^{120}$, not $\sim O(1)$



$$\Omega_\Lambda \sim 1$$

$$\Omega_m, \Omega_r \ll 1$$

$$\Omega_k \sim \frac{1}{a^2}$$

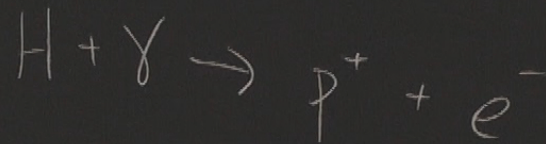
$$\rho_{\text{rad}}/\rho_m \sim \frac{1}{3000}$$

$$z \approx 3000 \quad \rho_r = \rho_m$$

$$\rightarrow \rho_r > \rho_m$$

$$T_{\text{CMB}} \approx 2.73 \text{ K today}$$

$$z \approx 10^3$$



no atoms, mostly ions
electrons

$$\approx 10^{-8} \text{ cm}^{-3}$$

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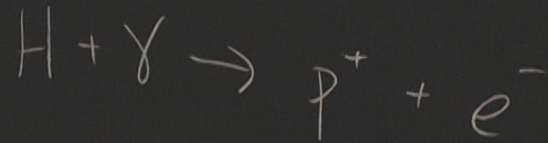
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$$T_\gamma > \text{MeV}$$

$$> 100 \text{ MeV}$$

theor

Thermal history of universe

distribution function $f(x, p, t) \frac{d^3x}{(2\pi\hbar)^3} \frac{d^3p}{(2\pi\hbar)^3} \sim$ # particles within d^3x of x
and d^3p of p

set $c = \hbar = 1$

$$[\text{length}] = [\text{time}] = [\text{mass}]^{-1}$$

$$n(x) = g \int f(x, p) \frac{d^3p}{(2\pi)^3} \quad \left| \quad \rho = g \int f(x, p) E \frac{d^3p}{(2\pi)^3}$$

ACDM
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$$E^2 = m^2 + p^2$$

$$T^{\mu\nu} = g \int \frac{p^\mu p^\nu}{E} f(x, p) \frac{d^3 p}{(2\pi)^3}$$

$$\rho = -T^0_0$$

$$P = \frac{1}{3} \text{Tr}(T^i_j) = \frac{T^i_i}{3}$$

$$= g \int \frac{p^2}{3E} f(x, p) \frac{d^3 p}{(2\pi)^3}$$

$f_x, f_e,$

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$f_x, f_e,$

evolution: Boltzmann equation

$$\frac{df_i}{dt} = C_i[\vec{f}]$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \sum_i \frac{\partial f}{\partial x^i} \frac{dx^i}{dt} - H$$

$$f(p, t)$$

$$\frac{dp^i}{dt} + H_{p^i} = 0$$

$$\frac{d^3 p}{(2\pi)^3}$$

$$[\vec{F}]$$

Suppose $C[f] = 0$

$$\int \frac{d^3 p}{(2\pi)^3} \left[E \frac{\partial f}{\partial t} - H \frac{\partial f}{\partial p^i} p^i E \right] = 0$$

$$\frac{\partial p}{\partial t} + H \int f \frac{\partial}{\partial p^i} (E p^i) \frac{d^3 p}{(2\pi)^3}$$

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$$\frac{\partial p}{\partial t} + H \int f \frac{\partial}{\partial p^i} (E p^i) \frac{d^3 p}{(2\pi)^3}$$

$$\int f \cdot \left(3E + \frac{p^2}{E} \right) \frac{d^3 p}{(2\pi)^3}$$

3p

$$\int \frac{\partial f}{\partial t} \frac{d^3 p}{(2\pi)^3} + H \int f \frac{\partial (E_p)}{\partial p_i} \frac{d^3 p}{(2\pi)^3} = 0$$

$$\frac{\partial \rho}{\partial t} + H \int f \frac{\partial (E_p)}{\partial p_i} \frac{d^3 p}{(2\pi)^3}$$

$$\int f \cdot \left(3E + \frac{p^2}{E} \right) \frac{d^3 p}{(2\pi)^3}$$

$$\frac{\partial \rho}{\partial t} + 3(\rho + P)H = 0$$