Title: Cosmology Lecture (230404)

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Collection: Cosmology (2022/2023)

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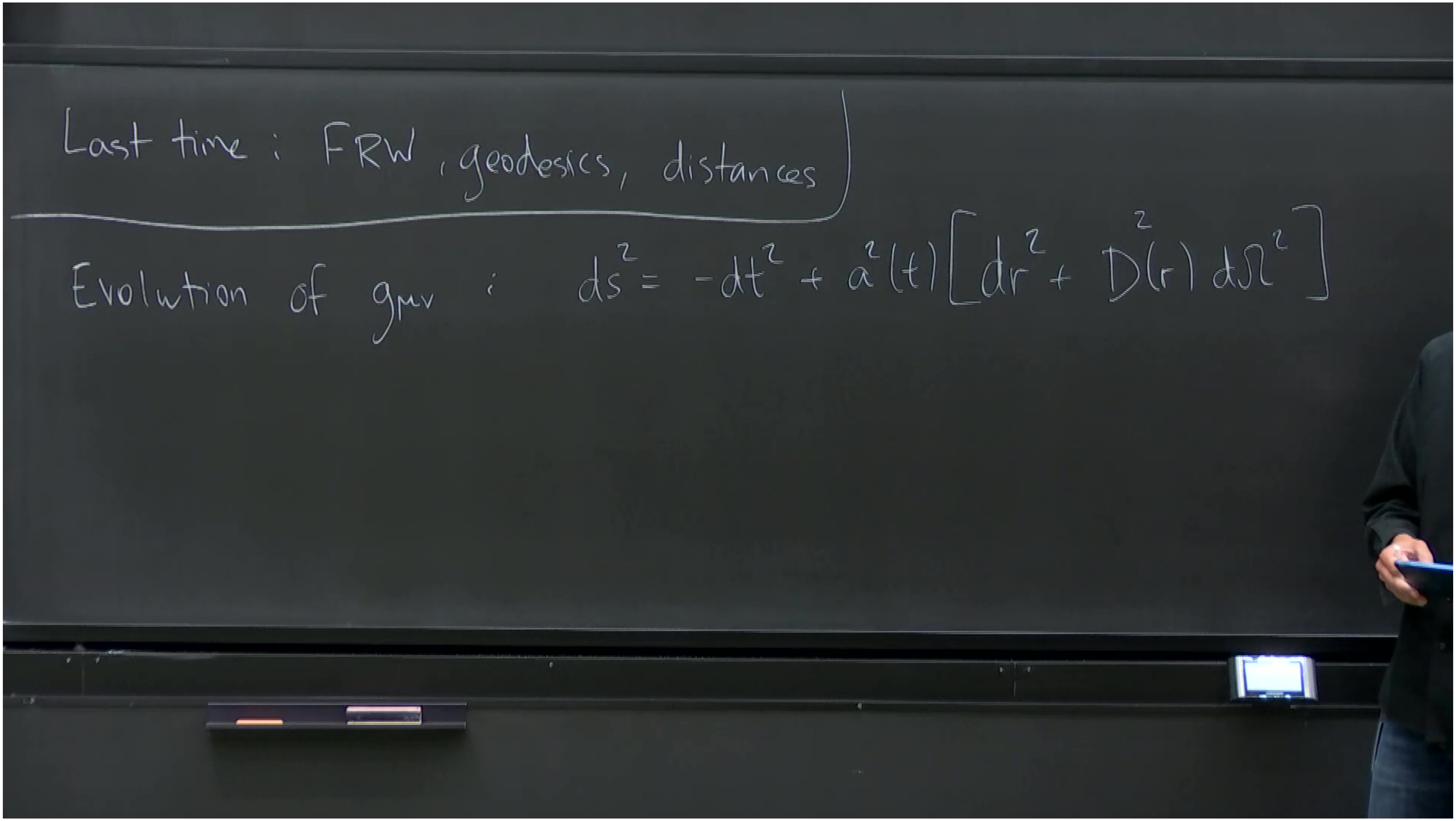
URL: https://pirsa.org/23040054

Last time : FRW, geodesics, distances

Evolution of $g_{\mu\nu}$: $ds^2 = -dt^2 + a^2(t)\left[dr^2 + D^2(r)\, d\Omega\right.$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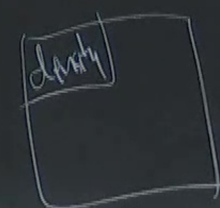
Evolution of $g_{\mu\nu}$: $ds^2 = -dt^2 + a^2(t)\left[dr^2 + D^2(r)\, d\Omega^2\right]$

Einstein eqns: $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$

density & flux of E, p



$\Omega^2]$

perfect fluid w/ ρ, P, u^μ

$$T^{\mu\nu} = (\rho + P) u^\mu u^\nu + P g^{\mu\nu}$$

frame where $u^\mu = (1,0,0,0)$

$$T^\mu{}_\nu = \begin{pmatrix} -\rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$$

perfect fluid w/ ρ, P, u^μ

$$T^{\mu\nu} = (\rho + P) u^\mu u^\nu + P g^{\mu\nu}$$

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$$T^\mu{}_\nu = \begin{pmatrix} -\rho & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}$$

$$\nabla_\nu T^{\mu\nu} = 0$$

$\mu = 0$: continuity eqn

$\mu = i$: Euler eqn

$$\frac{\partial \rho}{\partial t} + 3\frac{\dot{a}}{a}[\rho + P] = 0$$

$$P = w\rho \Rightarrow \dot{\rho} + 3\frac{\dot{a}}{a}(1+w)\rho = 0$$

$\boxed{\text{const. } w}$

$$\int \frac{d\rho}{\rho} = -3(1+w)\int \frac{da}{a} \Rightarrow \rho \propto a^{-3(1+w)}$$

NR: $P \ll \rho \Rightarrow w \simeq 0 \Rightarrow \rho \propto a^{-3}$

R: $P = \rho/3 \Rightarrow w = \frac{1}{3} \Rightarrow \rho \propto a^{-4}$


const. w
$\propto a^{-3(1+w)}$
$p \propto \frac{1}{a}$
$E = \sqrt{p^2+m^2}$

$$P = w\rho \Rightarrow \dot{\rho} + 3\frac{\dot{a}}{a}(1+w)\rho = 0 \qquad \boxed{\text{const. } w}$$

$$\int \frac{d\rho}{\rho} = -3(1+w)\int \frac{da}{a} \Rightarrow \rho \propto a^{-3(1+w)}$$

$$p \propto \frac{1}{a}$$

$$E = \sqrt{p^2 + m^2}$$

NR: $P \ll \rho \Rightarrow w \approx 0 \Rightarrow \rho \propto a^{-3}$

R: $P = \rho/3 \Rightarrow w = \frac{1}{3} \Rightarrow \rho \propto a^{-4}$

$$G^0{}_0 = -3\left[\left(\frac{\dot a}{a}\right)^2 + \frac{k}{(aR)^2}\right] = 8\pi G\, T^0{}_0 = -8\pi G \rho$$

$$G^i{}_j = -\left[2\frac{\ddot a}{a} + \left(\frac{\dot a}{a}\right)^2 + \frac{k}{(aR)^2}\right]\delta^i{}_j$$

$$\boxed{H^2 = \frac{8\pi G}{3}\rho - \frac{k}{(aR)^2}}$$

1st Friedmann

$+m^2$

$$\left[\frac{\ddot{a}}{a} + \underbrace{\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{k}{aR}\right)^2} \right] = -4\pi G P$$

2nd Friedmann

example

$$\frac{4\pi G \rho}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho - 4\pi G P = \boxed{-\frac{4\pi G}{3}(\rho + 3P) = \frac{\ddot{a}}{a}}$$

$$\left[\frac{\ddot{a}}{a} + \underbrace{\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{k}{aR}\right)^2}_{\frac{4\pi G\rho}{3}}\right] = -4\pi G P$$

2nd Friedmann

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho - 4\pi G P = \boxed{-\frac{4\pi G}{3}(\rho + 3P) = \frac{\ddot{a}}{a}}$$

example: $k=0$, $R\to\infty$, const w

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G\rho_0}{3} a^{-3(1+w)} = H_0^2\, a^{-3(1+w)}$$

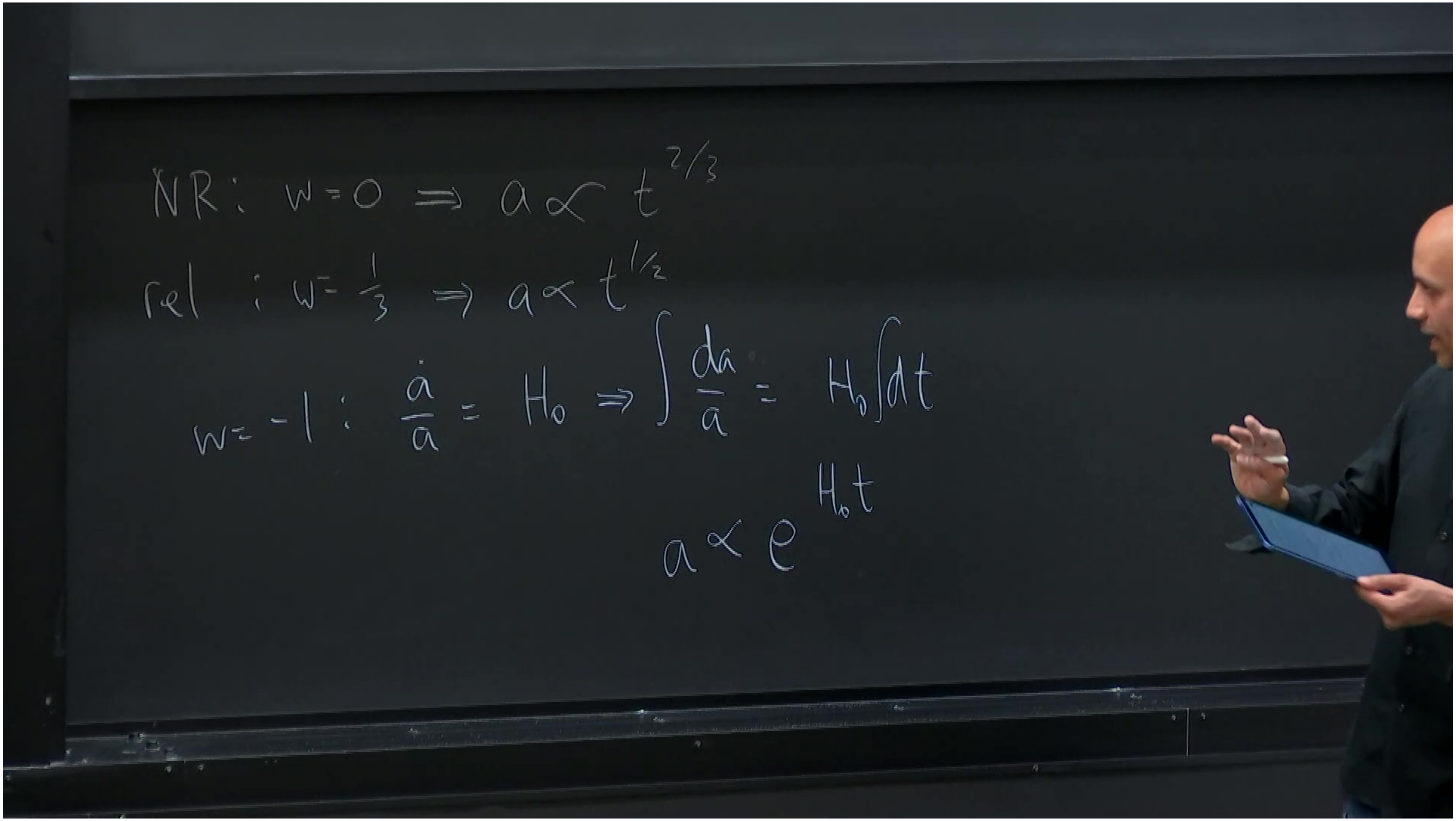
$$\rho = \rho_0\, a^{-3(1+w)}$$

$$\frac{\dot a}{a} = H_0\, a^{-\frac{3(1+w)}{2}} \Rightarrow \int a^{\frac{3}{2}(1+w)-1}\, da = H_0 \int dt$$

$$\frac{2}{3(1+w)}\, a^{\frac{3}{2}(1+w)} = H_0 t$$

NR: $w=0 \Rightarrow a \propto t^{2/3}$

rel: $w=\frac{1}{3} \Rightarrow a \propto t^{1/2}$


NR : $w = 0 \Rightarrow a \propto t^{2/3}$
rel : $w = \frac{1}{3} \Rightarrow a \propto t^{1/2}$
$w = -1 : \frac{\dot{a}}{a} = H_0 \Rightarrow \int \frac{da}{a} = H_0 \int dt$
$a \propto e^{H_0 t}$

$w \neq -1$

$H_0 \int dt$

$e^{H_0 t} \Rightarrow$ inflation DE.

Multiple component

critical density ρ_{crit}

$w \neq -1$

$H_0 \int dt$

$e^{H_0 t} \Rightarrow$ inflation DE.

Multiple component

critical density $\rho_{crit} \equiv \dfrac{3H^2}{8\pi G}$

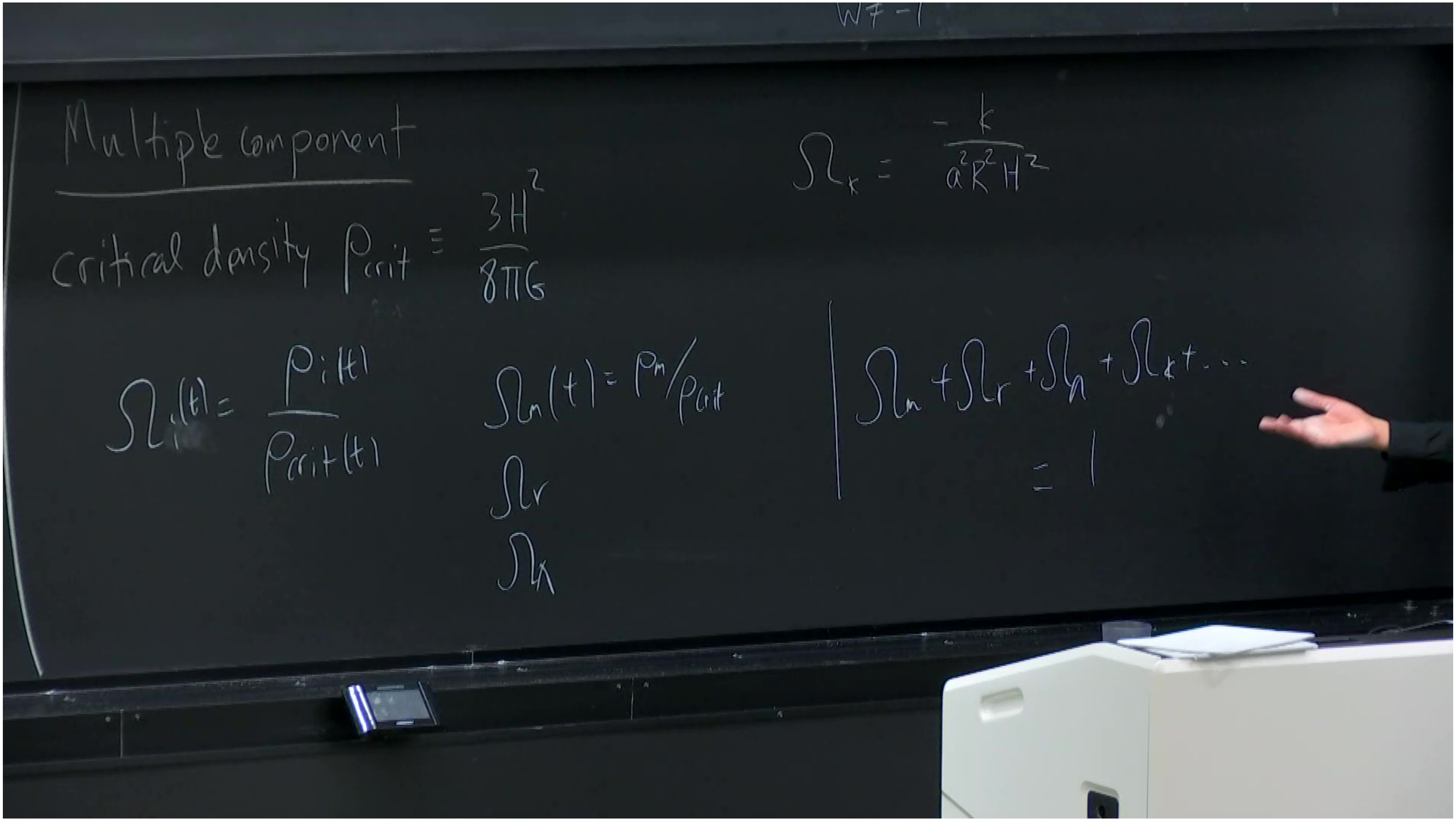
$$\Omega_k = \frac{-k}{a^2 R^2 H^2}$$

$$\Omega_i(t) = \frac{\rho_i(t)}{\rho_{crit}(t)}$$

$\Omega_m(t) = \rho_m / \rho_{crit}$

Ω_r

Ω_Λ

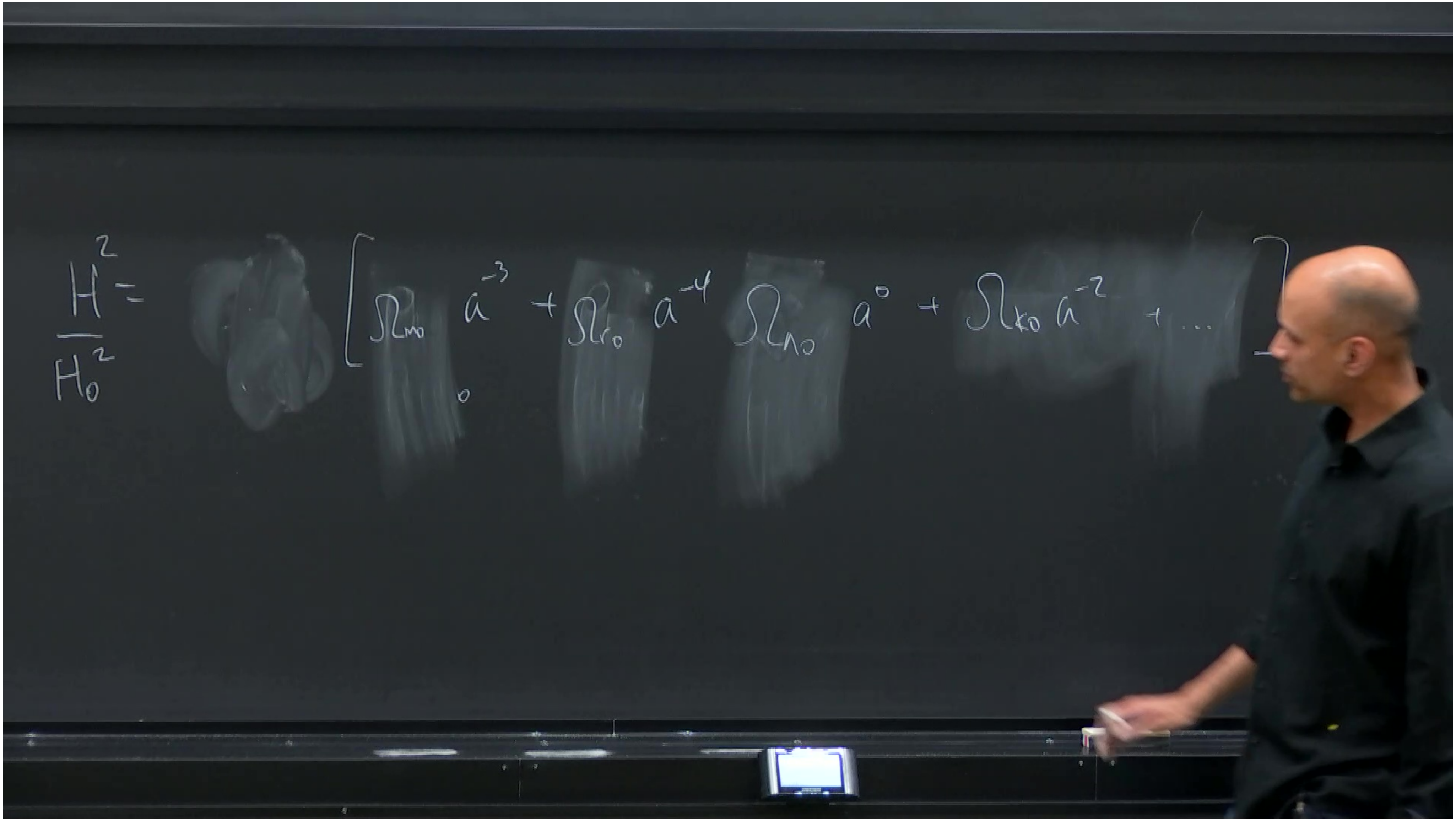

Multiple component
critical density $\rho_{crit} \equiv \frac{3H^2}{8\pi G}$
$\Omega_k = \frac{-k}{a^2 R^2 H^2}$
$\Omega_i(t) = \frac{\rho_i(t)}{\rho_{crit}(t)}$
$\Omega_m(t) = \rho_m / \rho_{crit}$
Ω_r
Ω_Λ
$\Omega_m + \Omega_r + \Omega_\Lambda + \Omega_k + \dots = 1$

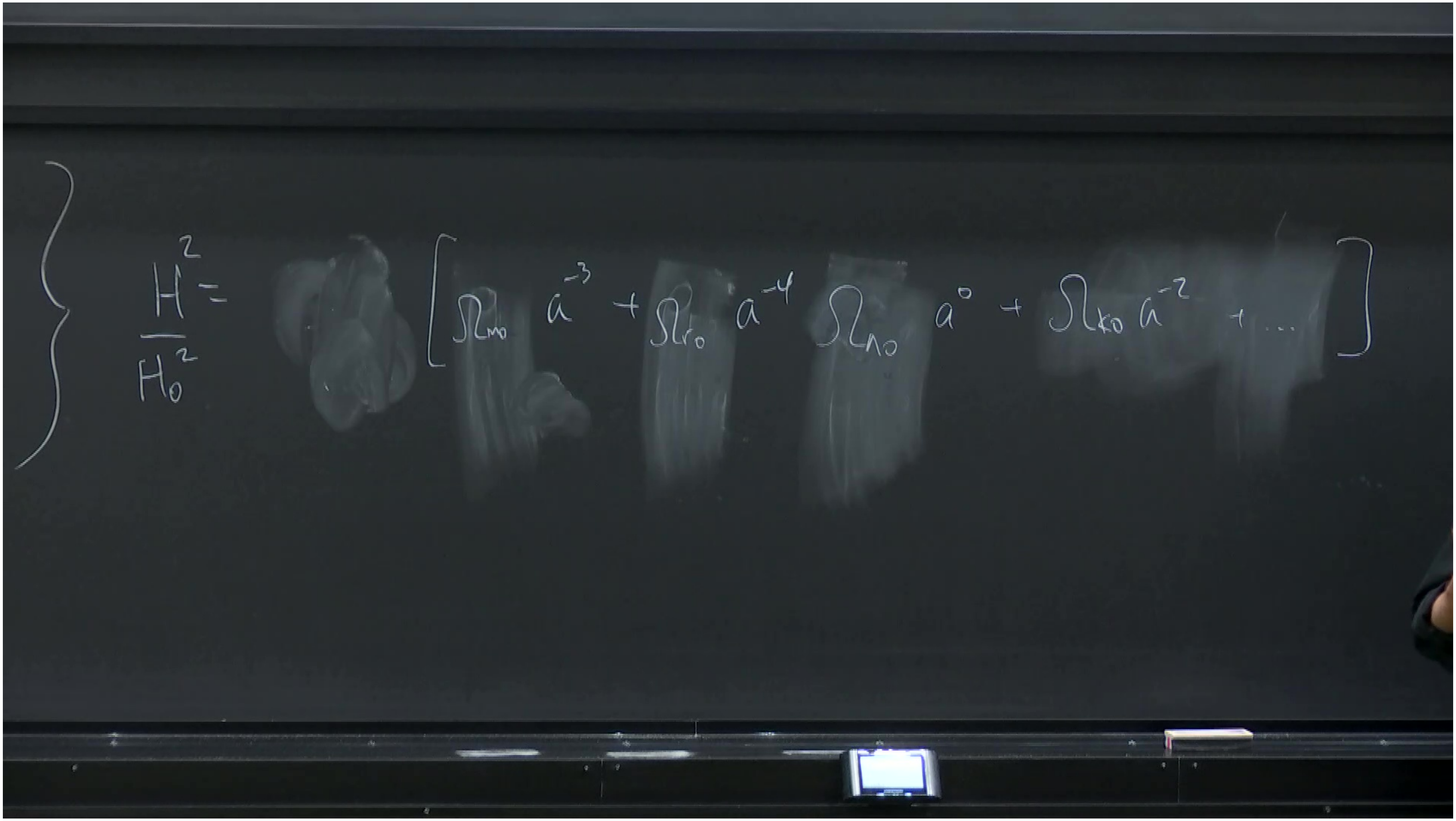
$$\rho_m = \rho_{m0}\, a^{-3}$$

$$\rho_r = \rho_{r0}\, a^{-4}$$

$$\frac{H^2}{H_0^2} = \frac{8\pi G}{3H_0^2}\left[\rho_{m0}\, a^{-3} + \rho_{r0}\, a^{-4} \quad \rho \ldots\right.$$

$$\frac{H^2}{H_0^2} = \left[\frac{\rho_{m0}}{\rho_{crit,0}} a^{-3} + \frac{\rho_{r0}}{\rho_{crit,0}} a^{-4} \quad \frac{\rho_{vac,0}}{\rho_{crit,0}} a^{0} + \dots \right) - \frac{k}{H_0^2 R^2 a^2}$$


$\frac{H^2}{H_0^2} = \left[\Omega_{m0} a^{-3} + \Omega_{r0} a^{-4} \ \Omega_{\Lambda 0} a^{0} + \Omega_{k0} a^{-2} + \ldots \right]$



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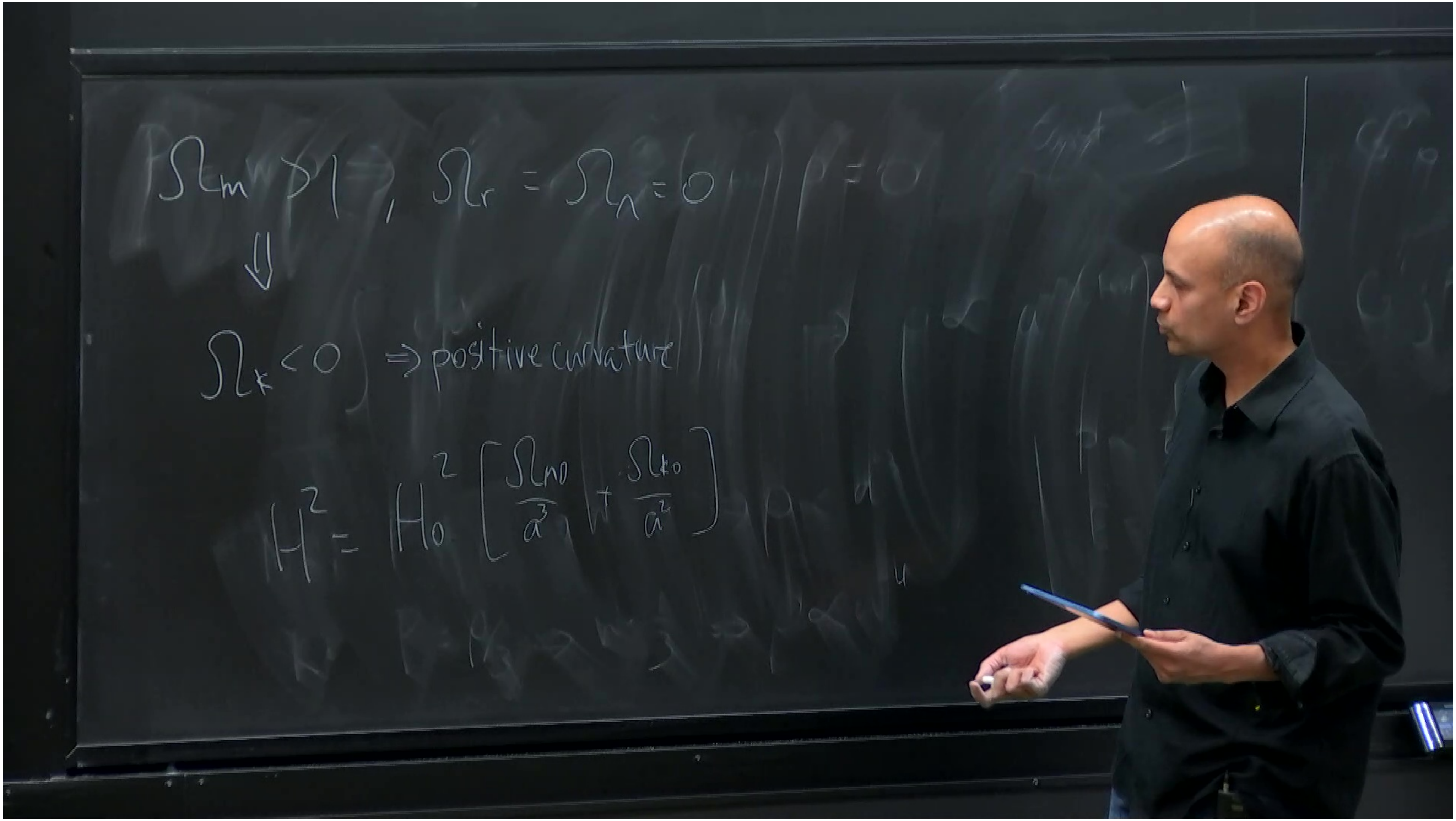

$\Omega_m > 1$, $\Omega_r = \Omega_\Lambda = 0$
$\Downarrow$
$\Omega_k < 0 \Rightarrow$ positive curvature
$H^2 = H_0^2 \left[\frac{\Omega_{m0}}{a^3} + \frac{\Omega_{k0}}{a^2} \right]$

$$\Omega_m > 1 \,,\quad \Omega_r = \Omega_\Lambda = 0 \quad \Rightarrow \Omega_{m0} = 1 + \delta_0 \quad \text{at } a = 1$$

$$\Downarrow \qquad\qquad \Omega_{k0} = -\delta_0$$

$\Omega_k < 0 \Rightarrow$ positive curvature

$$H^2 = H_0^2 \left[\frac{\Omega_{m0}}{a^3} + \frac{\Omega_{k0}}{a^2} \right] = H_0^2 \left[\frac{1+\delta_0}{a^3} - \frac{\delta_0}{a^2} \right]$$

$\Omega_m > 1$, $\Omega_r = \Omega_\Lambda = 0$ $\Rightarrow \Omega_{m0} = 1 + \delta_0$ at $a = 1$

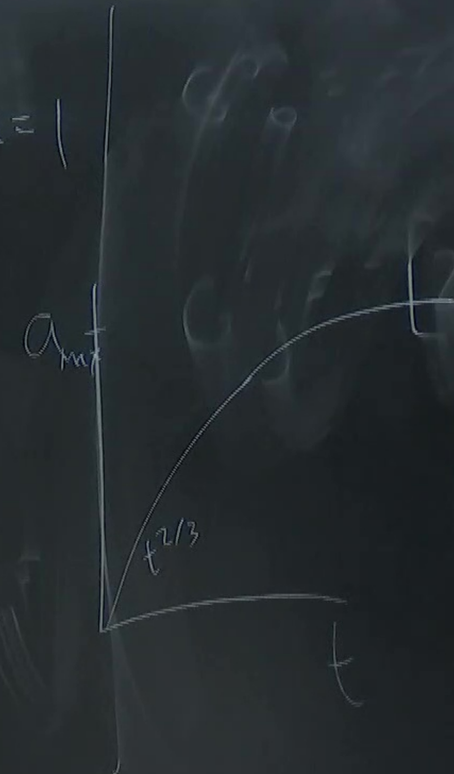
$\Downarrow$

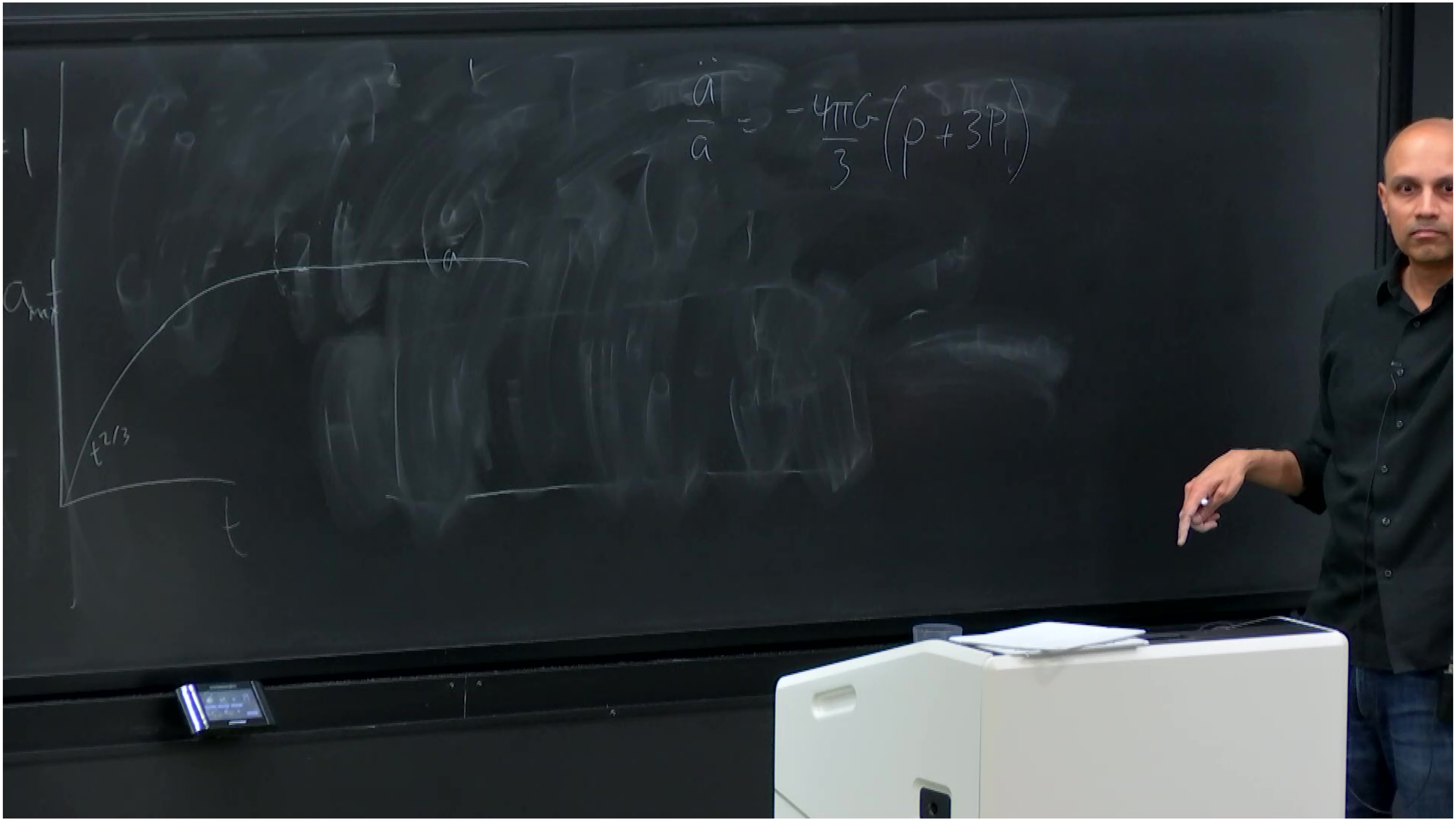
$\Omega_{k0} = -\delta_0$

$\Omega_k < 0 \Rightarrow$ positive curvature

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = H_0^2 \left[\frac{\Omega_{m0}}{a^3} + \frac{\Omega_{k0}}{a^2}\right] = H_0^2 \left[\frac{1+\delta_0}{a^3} - \frac{\delta_0}{a^2}\right]$$

$$a_{max} = \frac{1+\delta_0}{\delta_0}$$




$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$
$t^{2/3}$
t