Title: Mathematical Physics Lecture (230417)

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Last Time

$$A \in \Omega^{0,1}(\mathbb{PT}, \mathfrak{g})$$

$$B \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(-4) \otimes \mathfrak{g})$$

$$\int \mathrm{tr}\, B \wedge F^{0,2}(A)$$

On Space-time:

What are sol^{n}s to EOM?

Answer

EOM for this Lagrangian describe $A \in \Omega^1(\mathbb{R}^4) \otimes \mathfrak{g}$, gauge field with

$$F_-(A) = 0 \iff F(A) = * F(A)$$

And

$B \in$

d

And

$$B \in \Omega^2_-(\mathbb{R}^4) \otimes \mathfrak{g}$$

$$d_A B = 0$$

describe:

The eqn $F_-(A)=0$

$\implies$ the YM equation

$$d_A \star F(A) = 0$$

Because, if $F_-(A)=0$

then $\star F(A) = F(A)$

What if we work off-shell?

Answer: A 4d theory called Self-dual Yang-Mills.

Fields of SDYM:

$A \in \Omega^1(\mathbb{R}^4) \otimes \mathfrak{g}$

$B \in \Omega^2_-(\mathbb{R}^4) \otimes \mathfrak{g}$

Lagrangian is

$$\int_{\mathbb{R}^4} \mathrm{tr}(B \wedge F_-(A))$$

EOM:

$$F_-(A) = 0$$

$$d_A B = 0$$

Mason et al showed

SDYM = the theory we wrote on twistor space, at tree level

SDYM co

SDYM can be deformed to full YM.

If we add the term

$$-\frac{1}{4} g_{YM}^2 \, \mathrm{tr}\, B^2$$

to the Lagrangian, we get

$$\int \mathrm{tr}(B \wedge F_-(A)) - \frac{1}{4} g_{YM}^2 \, \mathrm{tr}\, B^2$$

Field redefinition

$$B \to B + 2 g_{YM}^2 F_-$$

the theory

twistor

ee level

term

we get

$$\frac{1}{g_{ym}^2}\int F_- \wedge F_- \; + \int B^2$$

n, we get

$$-\frac{1}{4} g_{ym}^2 \, tr \, B^2$$

B can be removed,
has no kinetic term,
only mass

$$\frac{1}{g_{ym}^2}\int F_- \wedge F_- \quad \text{is YM action}$$

up to a top' term.

F_-

$$\int F_- \wedge F_- = \frac{1}{4}\int (1-*)F \wedge (1-*)F$$

$$= \int F \wedge F + \int *F \wedge *F - \frac{1}{2}\int F \wedge *F$$

top'l = top'l YM

$$\int F_- \wedge F_- = \frac{1}{4}\int (1-*)F \wedge (1-*)F$$

$$= \underbrace{\int F\wedge F}_{\text{top}} + \underbrace{\int *F\wedge *F}_{= \text{ top}} - \underbrace{\frac{1}{2}\int F\wedge *F}_{\text{YM}}$$

$$F_{\alpha\dot\alpha\,\beta\dot\beta} = \varepsilon_{\alpha\beta}\, F^-_{\dot\alpha\dot\beta} + \varepsilon_{\dot\alpha\dot\beta}\, F^+_{\alpha\beta}$$

What is B^2 on twistor space?

$$\mathrm{tr}\, B^2 = \mathrm{tr}\left(B_{\dot\alpha\dot\beta}\, B_{\dot\gamma\dot\delta}\right) \varepsilon^{\dot\alpha\dot\gamma} \varepsilon^{\dot\beta\dot\delta}$$

What is $B_{\dot\alpha\dot\beta}$ on $\mathbb{PT}$?

Comes from $\mathcal{B} \in \Omega^{0,1}(\mathbb{PT}, \mathcal{O}(-4))$

$B_{\dot{\alpha}\dot{\beta}}(0)$ is a local operator.

$\Longleftrightarrow$ operator $\int_{\mathbb{CP}^1} \mathcal{B}$

$$B_{\dot{1}\dot{1}} = \int_{\mathbb{CP}^1} \mathcal{B} \, dz$$

$$B_{1\dot{2}} = \int_{\mathbb{CP}^1} \mathcal{B}\, z\, dz$$

$$B_{\dot{2}\dot{2}} = \int_{\mathbb{CP}^1} \mathcal{B}\, z^2\, dz$$

$$\mathrm{tr}(B^2(0))$$

$$= \int_{z, z' \in \mathbb{CP}^1} \mathrm{tr}(\mathcal{B}(z)\mathcal{B}(z'))\, dz\, dz' (z - z')^2$$

$$\int_{\mathbb{CP}^1} \mathcal{B}\; z\,dz$$

$$\int_{\mathbb{CP}^1} \mathcal{B}\; z^2\,dz$$

$$))$$

$$tr\left(\mathcal{B}(z)\mathcal{B}(z')\right) dz\, dz' \,\overbrace{(z - z')^2}$$

ε tensors in $tr\, \mathcal{B}^2$

$$z, z' \in \mathbb{CP}^1$$

$$tr($$

tor

$$B_{\dot 1 \dot 2} = \int_{\mathbb{CP}^1} \mathcal{B} \; z \, dz$$

$$B_{\dot 2 \dot 2} = \int_{\mathbb{CP}^1} \mathcal{B} \, z^2 \, dz$$

$$tr(B^2(0)) = \int_{z, z' \in \mathbb{CP}^1} tr(\mathcal{B}(z)\mathcal{B}(z'))\, dz\, dz' \overbrace{(z - z')^2}^{\epsilon \text{ tensors in } tr\, B^2}$$

Not Gauge invariant!

$$\delta B(z) = [\chi(z), B(z)]$$

$$\delta A = \bar{\partial}\chi + [\chi, A]$$

tensors in tr B^2

$z')^2$

Not Gauge invariant!

$$\delta B(z) = [\chi(z), B(z)]$$

$$\delta A = \bar{\partial}\chi + [\chi, A]$$

Σ tensors in $\mathrm{tr}\, B^2$

$$z\, dz' (z - z')^2$$

But we can make it gauge invariant
(Mason)
Add extra terms which are like
PExp in a Wilson line.
Full expression is

$$\sum_{n=2}^{\infty} \sum_{m=1}^{n} \int_{z_1 \ldots z_n}^{n} \Big[\mathrm{tr} \Big(B(z_1) A(z_2) A(z_3) \ldots \; B(z_m) A(z_{m+1}) \ldots A(z_n$$

$$\times \frac{(z_1 - z_m)^4}{(z_1 - z_2)(z_2 - z_3) \ldots (z_n - z_1)} \, dz_1 \ldots dz_n$$

This is Parke - Taylor!

$$(z_1)\, A(z_2)\, A(z_3) \ldots \; B(z_m)\, A(z_{m+1}) \ldots A(z_n) \Big)$$

$$\times \frac{(z_1 - z_m)^4}{(z_1 - z_2)(z_2 - z_3) \ldots (z_n - z_1)} \, dz_1 \ldots dz_n$$

– Taylor!

Let's check that this is gauge invariant

2 points:

$$\iint tr\left(B(z_1)B(z_2)\right)(z_1-z_2)^2\, dz_1\, dz_2$$

Gauge variation:

$$\iint \left(tr\left([X(z_1),B(z_1)]B(z_2) + B(z_1)[X(z_2),B(z_2)]\right)(z_1-z_2)^2\, dz_1\, dz_2\right.$$

3 point:

$$\int \left(B(z_1)A(z)B(z_2)\,\frac{(z_1-z_2)^4}{(z_1-z)(z-z_2)(z_2-z_1)} + B(z_1)B(z_2)A(z)\,\frac{(z_1-z_2)^4}{(z_1-z_2)(z_2-z)(z-z_1)} \right) dz_1\, dz_2$$

Apply gauge trans. to A

$$\delta A = \bar{\partial}\chi + [\chi, A]$$

focus on $\bar{\partial}\chi$

We'll get

$$\int B(z_1)\,\bar\partial X(z)\,B(z_2)\ \frac{(z_1-z_2)^4}{(z_1-z)(z-z_2)(z_2-z_1)} + \text{similar}$$

On shell

$\bar\partial B = 0$

$\int$ by parts, $\bar\partial$ hits $\frac{1}{z_1-z}$ or $\frac{1}{z-z_2}$, we get ar

$$B(z_1)\,X(z_1)\,B(z_2)\,\frac{(z_1-z_2)^4}{(z_1-z_2)^2}$$

$$-\,B(z_1)\,X(z_2)\,B(z_2)\,\frac{(z_1-z_2)^4}{(z_1-z_2)^2}$$

Include other terms, get

$$+B(z_1)B(z_2)X(z_2)(z_1-z_2)^2 - X(z_1)B(z_1)B(z_2)(z_1-z_2)^2$$

ψ quark

$$\delta \psi = [\lambda, \psi]$$

$$\delta\left(\psi(x)\,\psi(x')\right) = [\lambda$$

$$\psi(x) \quad \psi(x')$$

gauge invariant

$$\delta(\psi(x)\psi(x')) = (-\psi(x)\cdot\chi(x))\,\psi(x') + \psi(x)\,\chi(x')\,\psi(x')$$

Add $\displaystyle\int_{x<y<x'} \psi(x) A(y) \psi(x') + \ldots$

becomes gauge invariant