Title: AdS/CFT Lecture (230426)

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Collection: AdS/CFT (2022/2023)

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AdS/CFT: BASIC DICTIONARY

$$Z^{\lambda\to\infty,\ N_c\to\infty}_{\mathcal{N}=4\ SU(N_c)\ SYM}[\phi_0] = \int D\phi\, e^{-S_E + \int d^dx\, O(x)\phi_0(x)} = e^{-W[\phi_0]}$$

$$\| \text{AdS/CFT}$$

$$Z_{\substack{\text{IIB ST ON}\\ AdS_5\times S_5}}[\phi_0] \approx e^{-S_{\substack{\text{IIB SUGRA}\\ \text{ON } AdS_5\times S_5}}[\phi_0]}$$

ϕ_0 ← BOUNDARY VALUE OF CLASSICAL FIELD IN THE BULK

$$W[\phi_0] = S_{SUGRA}[\phi] \Big/ \lim_{z\to 0} \phi(z,x) z^{\Delta-d} = \phi_0(x)$$

RE

TOY EXAMPLE: SCALAR IN AdS_{d+1} 2) FOR:

$$\Rightarrow \langle O(x_1) \dots O(x_m) \rangle = - \frac{\delta^m W}{\delta \phi_0(x_1) \dots \delta \phi_0(x_m)}$$

RECIPE: 1) DETERMINE BULK FIELD ϕ DUAL TO O_Δ

2) SOLVE (SUGRA) EOM FOR ϕ, SUBJECT TO: $\phi(z,x) \sim z^{d-\Delta} \phi_0, \quad z \to 0$

3) PLUG BACK TO S_{SUGRA} ... GET

$S_{SUGRA}[\phi_0]$ ("HAMILTON'S FUNCTION")

EOM: $(\Box - m^2)\phi = 0, \quad \phi(z,x) = \int \frac{d^d k}{(2\pi)^d} e^{ik\cdot x} \phi(z,k)$

$\lim_{z \to 0} \phi(z,x)\, z^{\Delta_-} = \phi_0(x)$

TOY EXAMPLE: SCALAR IN AdS_{d+1}

2)

1) $$S = -\frac{c}{2} \int dz\, d^d x \sqrt{g} \left(g^{ab} \partial_a \phi \partial_b \phi + m^2 \phi^2 \right)$$

DUAL TO SCALAR PRIMARY $\mathcal{O}_\Delta$

YESTER

WHERE $\boxed{\Delta = \Delta_+}$

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2 \ell^2}$$

$= \phi_0(x)$

$S_{SUGRA}[\phi_0]$ ("HAMILTON'S FUNCTION")

2) EOM: $(\Box - m^2)\phi = 0$, $\phi(z,x) = \int \frac{d^d k}{(2\pi)^d} e^{ik\cdot x} \phi(z,k)$

$$z^2 \partial_z^2 \phi(k,z) - (d-1) z \partial_z \phi(k,z) - (m^2 l^2 + k^2 z^2)\phi(k,z) = 0 \quad (*)$$

YESTERDAY: NEAR $z=0$

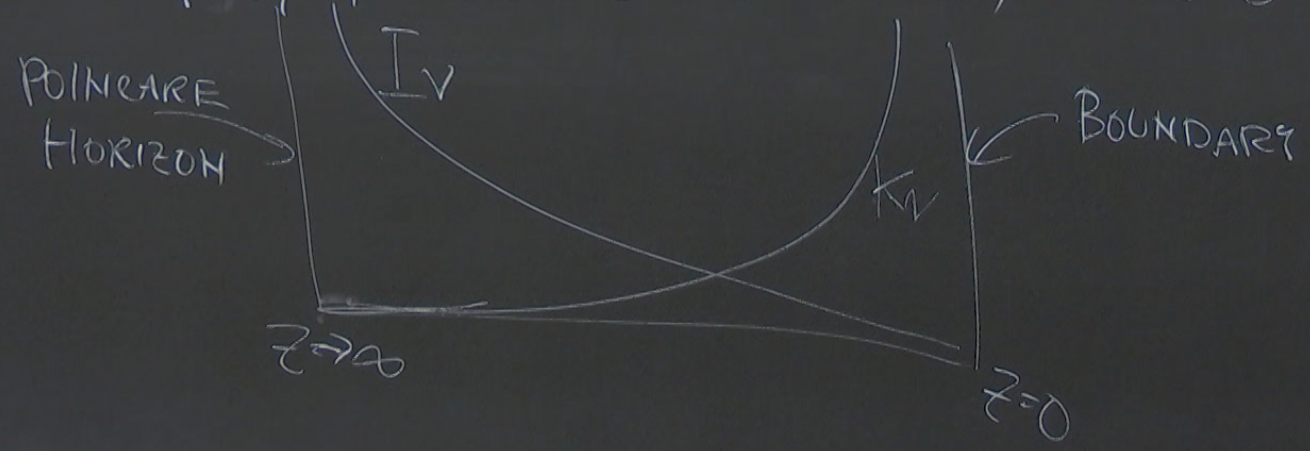
$$\phi \sim \phi_0 z^{\Delta_-} + \phi_+ z^{\Delta_+}$$

ϕ_0: SOURCE

ϕ_+: EXPECTATION VALUE $\langle O_{\Delta_+} \rangle$

FULL SOLUTION:

$$\Phi(z, k_z) = A k_z z^{d/2} K_\nu(z|k|) + B k_z z^{d/2} I_\nu(z|k|)$$



$\Delta_+ = \Delta, \quad \Delta_- = d - \Delta$

$+ B_k z^{d/2} I_\nu(z|k|)$ ~~I_ν~~

$\nu = \Delta - d/2 = \sqrt{\frac{d^2}{4} + m^2 l^2}$

AS $z \to 0$: $K_\nu \sim z^{-\nu} \sim z^{d-\Delta}$

BOUNDARY

$\Rightarrow A_k \ldots$ RELATED TO ϕ_0.

$z=0$

NORMALIZED SOLUTION:

$\Delta_+ = \Delta, \quad \Delta_- = d - \Delta$

$z^{d/2} I_\nu(z|x|)$ $\qquad \nu = \Delta - d/2 = \sqrt{\frac{d^2}{4} + m^2 \ell^2}$

AS $z \to 0$: $K_\nu \sim z^{-\nu} \sim z^{d-\Delta}$

ARY

$\Rightarrow A_k$ .. RELATED TO ϕ_0

NORMALIZED SOLUTION:

$$\phi(z,x) = \frac{z^{d/2} K_\nu(z|x|)}{\varepsilon^{d/2} K_\nu(\varepsilon|x|)} \phi_0(x) \varepsilon^{d-\Delta}$$

3) GET HAMILTON'S FUNCTION:

$$S = -\frac{c}{2}\int d^d x \sqrt{g}\, g^{zz} \left(\frac{\ell}{z}\right)^{d-1} \phi(z,x)\,\partial_z \phi(z,x)\Big|_{z=\varepsilon}^{z=\infty} = S[\phi_0]$$

SUPP OF ϕ IS LIMITED IN X-DIRS.

SOLUTION

$$= -\frac{c\,\ell^{d-1}}{2\varepsilon^{d-1}} \int \frac{d^d k}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} (2\pi)^d \delta(k+q)\, \phi(z,k)\,\partial_z \phi(z,q)\Big|_{z=\varepsilon}$$

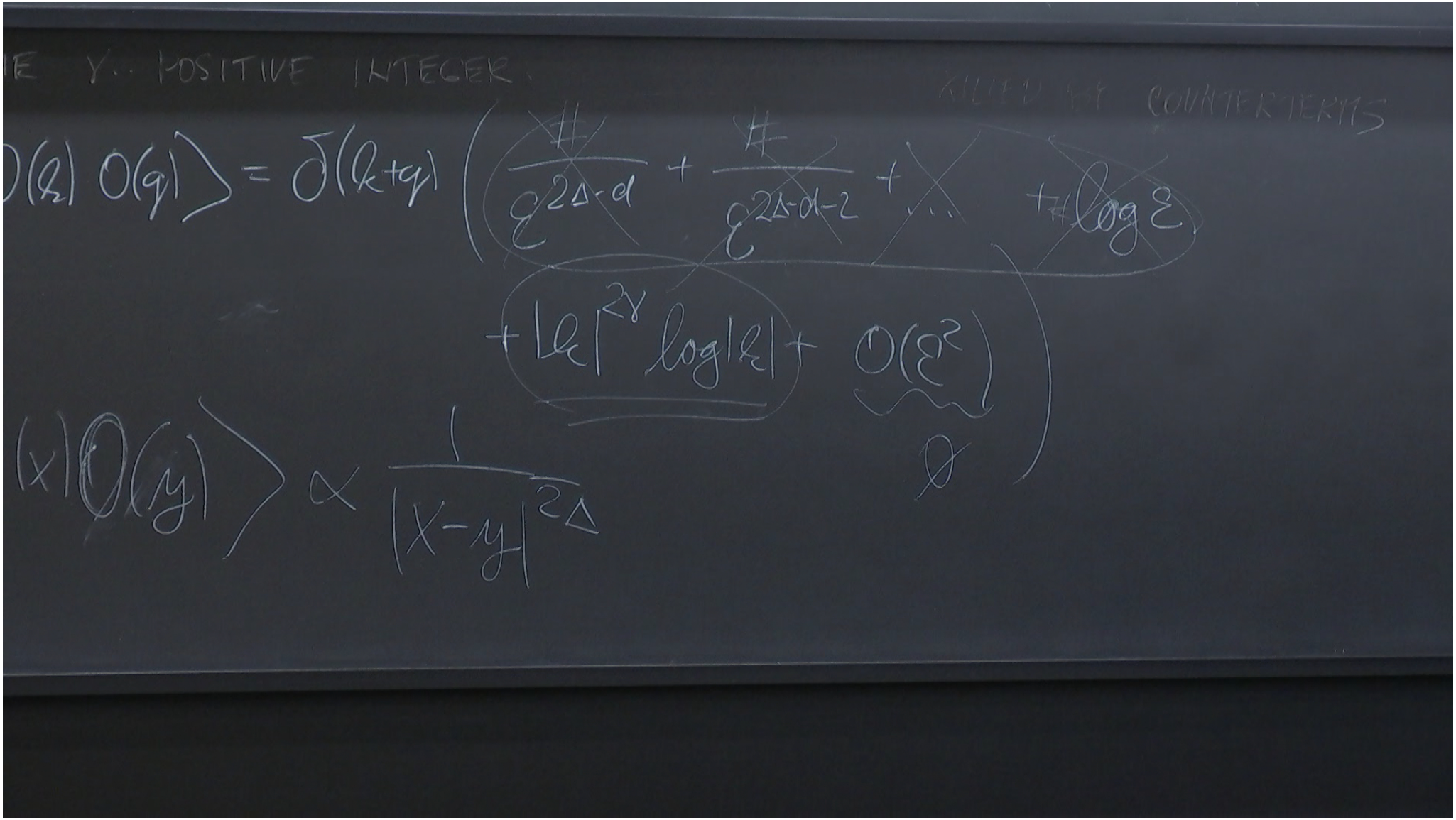
SPEC: 2-POINT FUNCTION

$$\int d^d x\, \mathcal{O}(x)|\phi_0(x)| = \int \frac{d^d k}{(2\pi)^d}\, \mathcal{O}(k)\,\phi_0(-k)$$

$$\langle \mathcal{O}(k)\,\mathcal{O}(q)\rangle = -(2\pi)^{2d}\,\frac{\delta^2 S[\phi_0]}{\delta\phi_0(-k)\,\delta\phi_0(-q)}$$

$$= \ldots = \left(\frac{d}{2} + \frac{\varepsilon|k|\,K_\nu'(z|k|)}{K_\nu(\varepsilon|k|)}\right)\frac{\delta(k+q)}{\varepsilon^{2\Delta-d}}$$

$$\ldots(z,q)\Big|_{z=\varepsilon}$$

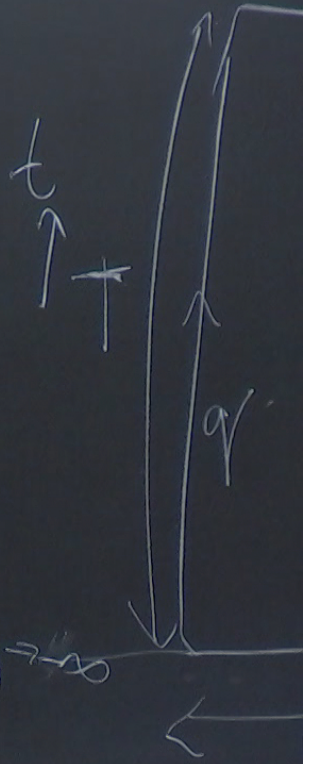

E Y. POSITIVE INTEGER.
COUNTERTERMS
$\langle O(x) O(y) \rangle \propto \frac{1}{|x-y|^{2\Delta}}$

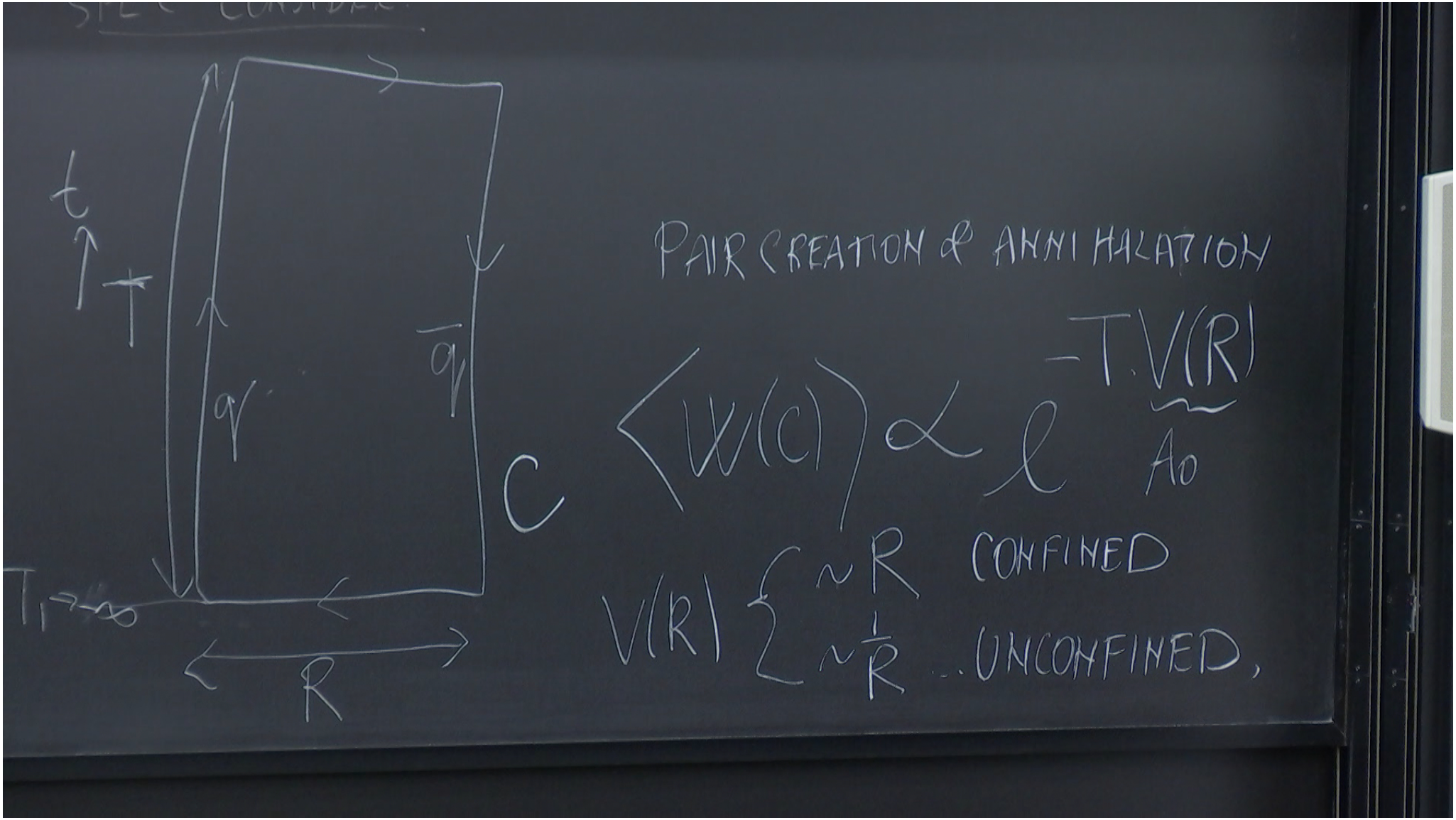
BE PARALLEL TRANSPORT OF QUARK

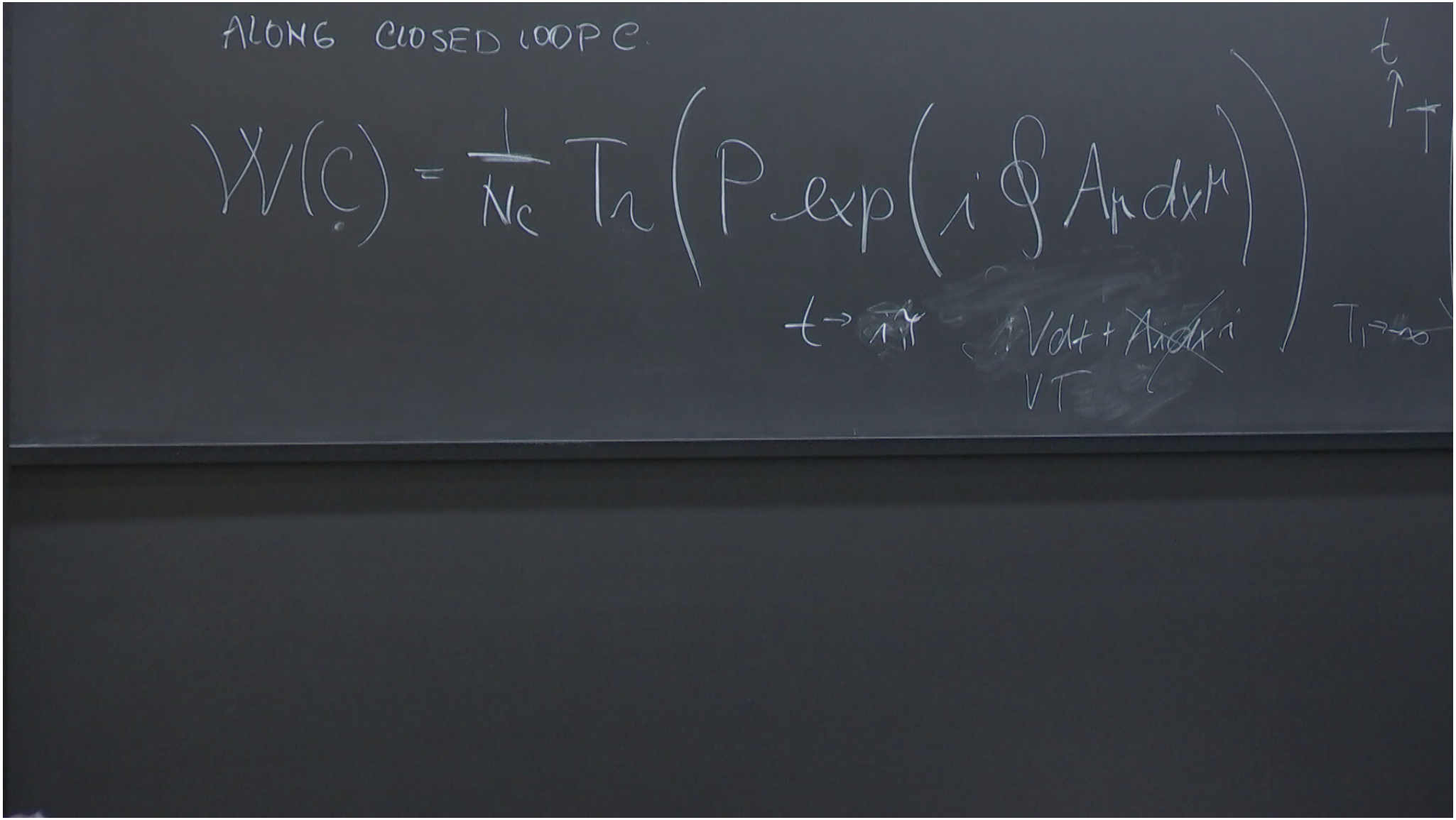
CLOSED LOOP C.

$$= \frac{1}{N_c} \mathrm{Tr}\left(P \exp\left(i \oint A_\mu dx^\mu \right) \right)$$

$t \to \infty$




t
q
$\bar{q}$
R
PAIR CREATION & ANNIHILATION
C
$W(C) \propto e^{-T.V(R)}$
A_0
$V(R) \begin{cases} \sim R & \text{CONFINED} \\ \sim \frac{1}{R} & \text{UNCONFINED.} \end{cases}$


ALONG CLOSED LOOP C
$W(C) = \frac{1}{N_c} \mathrm{Tr}\left(P \exp\left(i \oint A_\mu dx^\mu\right)\right)$
$t \to +T$
$\int V dt + \int A_i dx^i$
$T_i \to -\infty$
VT
t
$+T$

ALONG CLOSED LOOP C.

$$W(C) = \frac{1}{N_c} \mathrm{Tr}\left(P \exp\left(i \oint A_\mu dx^\mu \right) \right.$$

$t \to \infty$: Volt + $A_1 dx$

VT

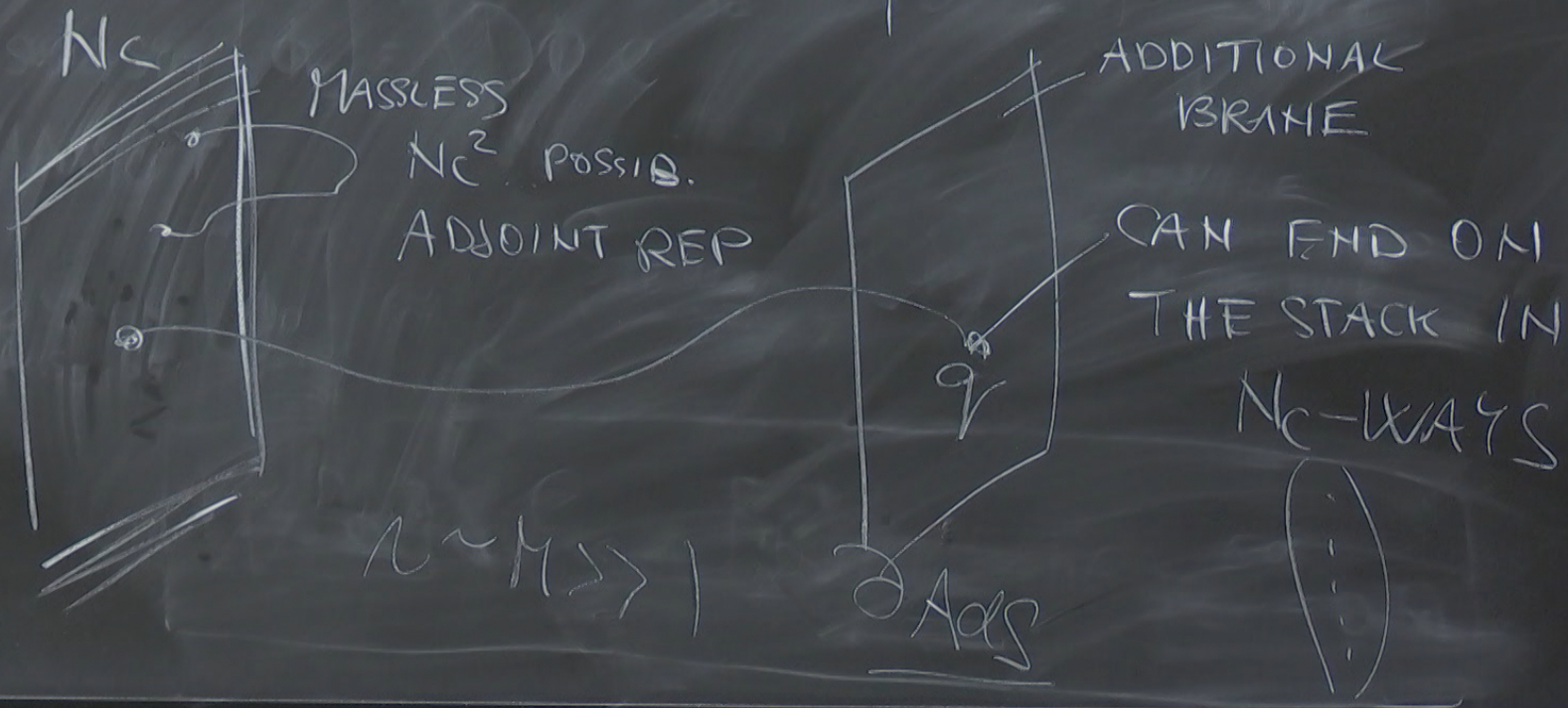
q... MASSIVE

FUNDAMENTAL REP OF SU(Nc)

WHERE $\Delta = \Delta_+$

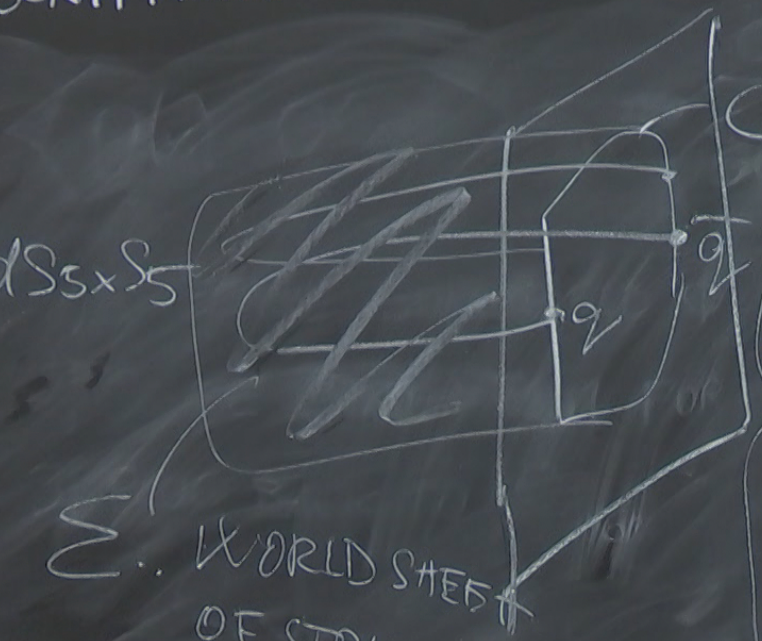
WHAT IS THE AdS PRESCRIPTION ?

- NEED MASSIVE QUARKS, FUNDAMENTAL REP.



$\phi \sim \phi_0 z + \phi_+ z$

PRESCRIPTION:

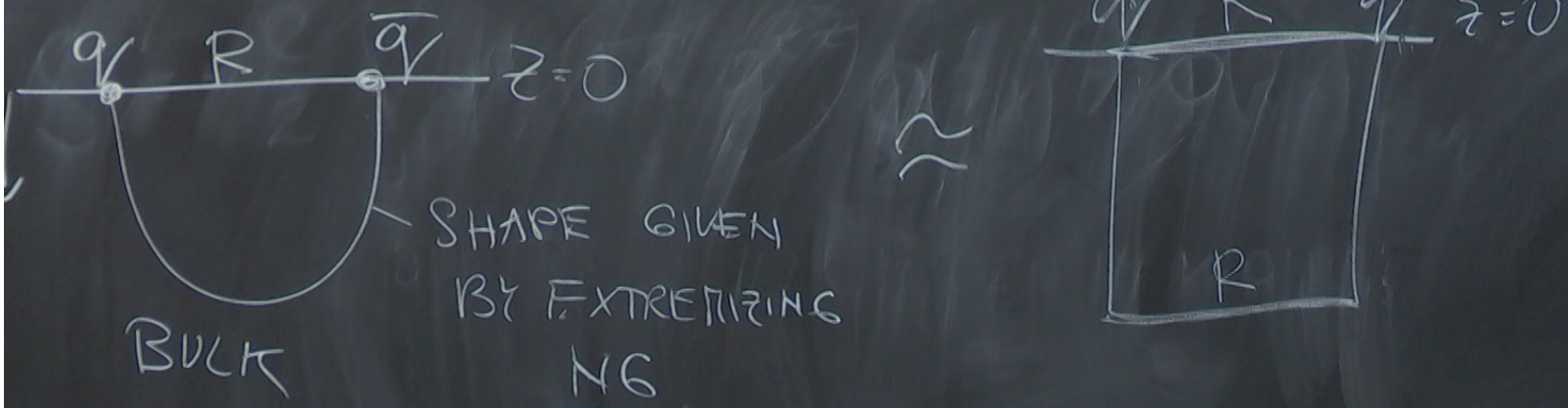


REQUIRE $\Sigma|_{bAdS} = C$

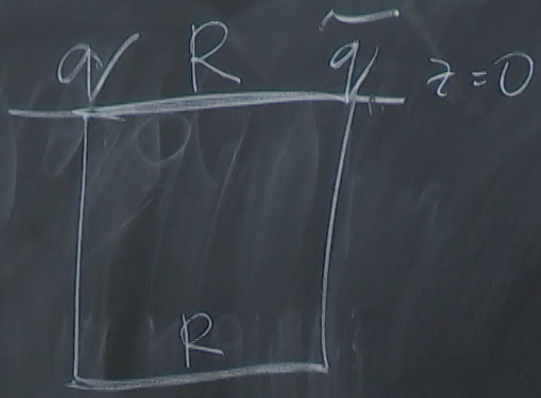
$\langle W(C) \rangle = e^{-S_{NG}}$

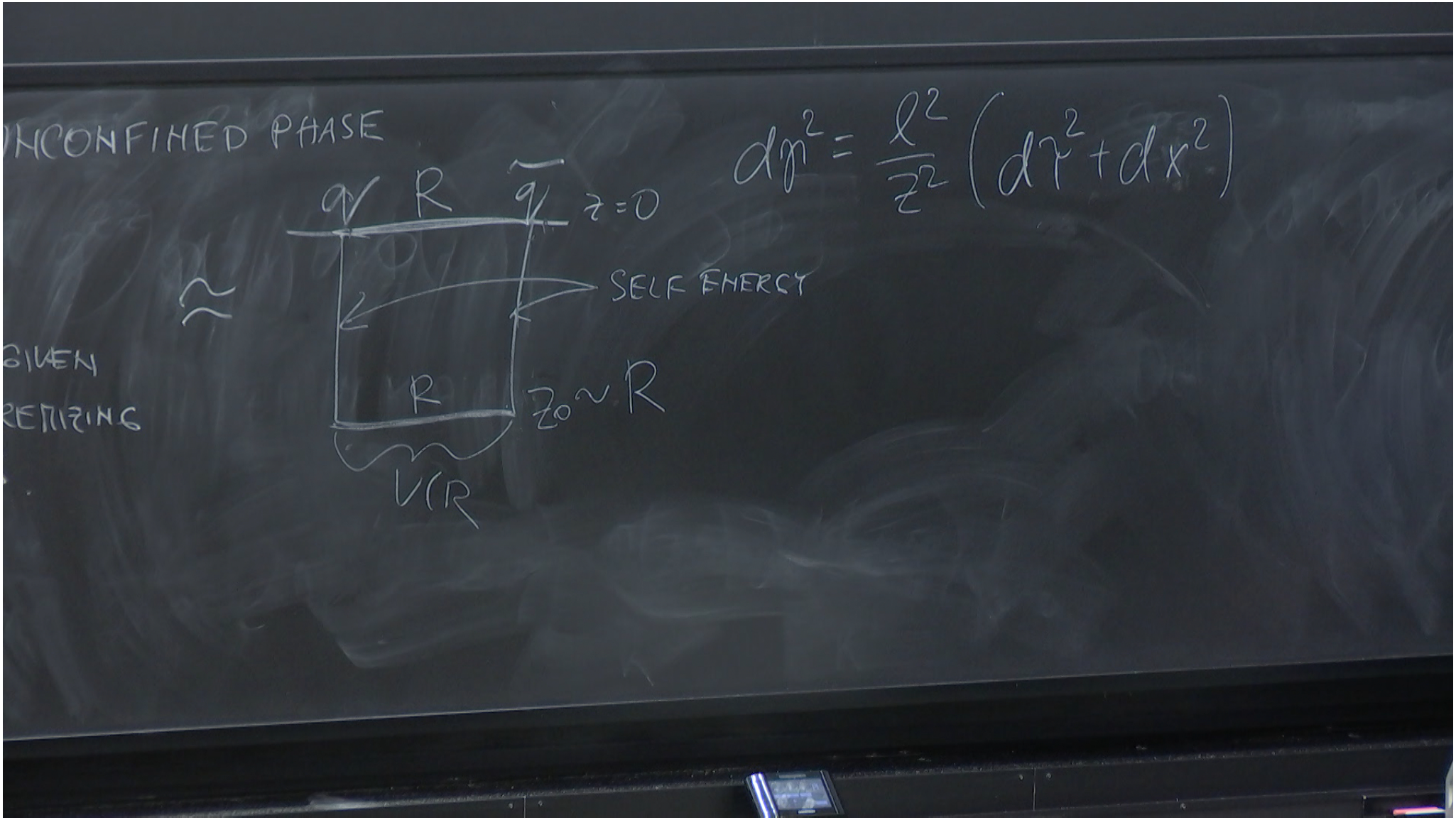
$S_{NG} = \frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{\det g_{AB}}$

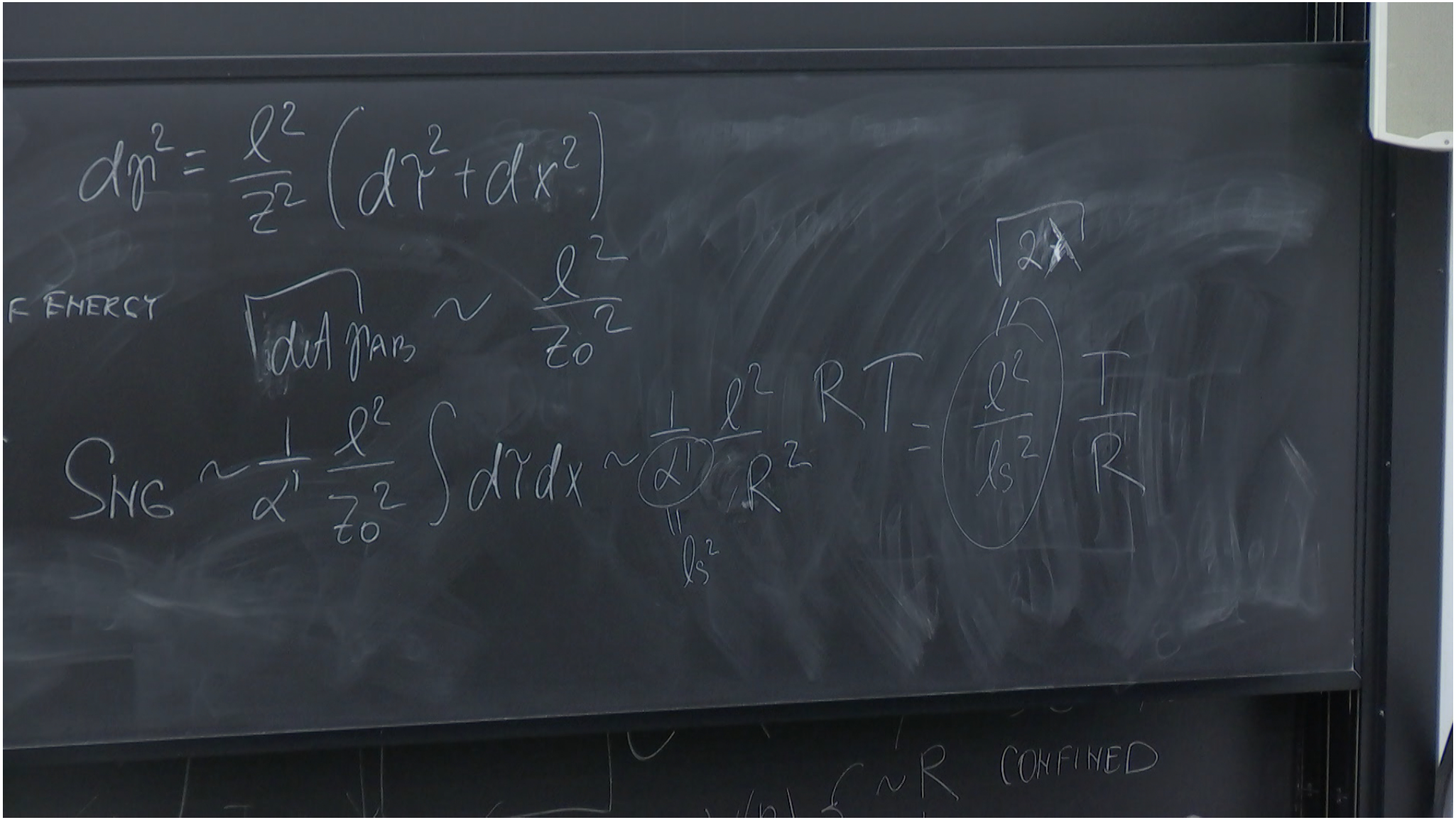
ALITATIVE ESTIMATE: UNCONFINED PHASE

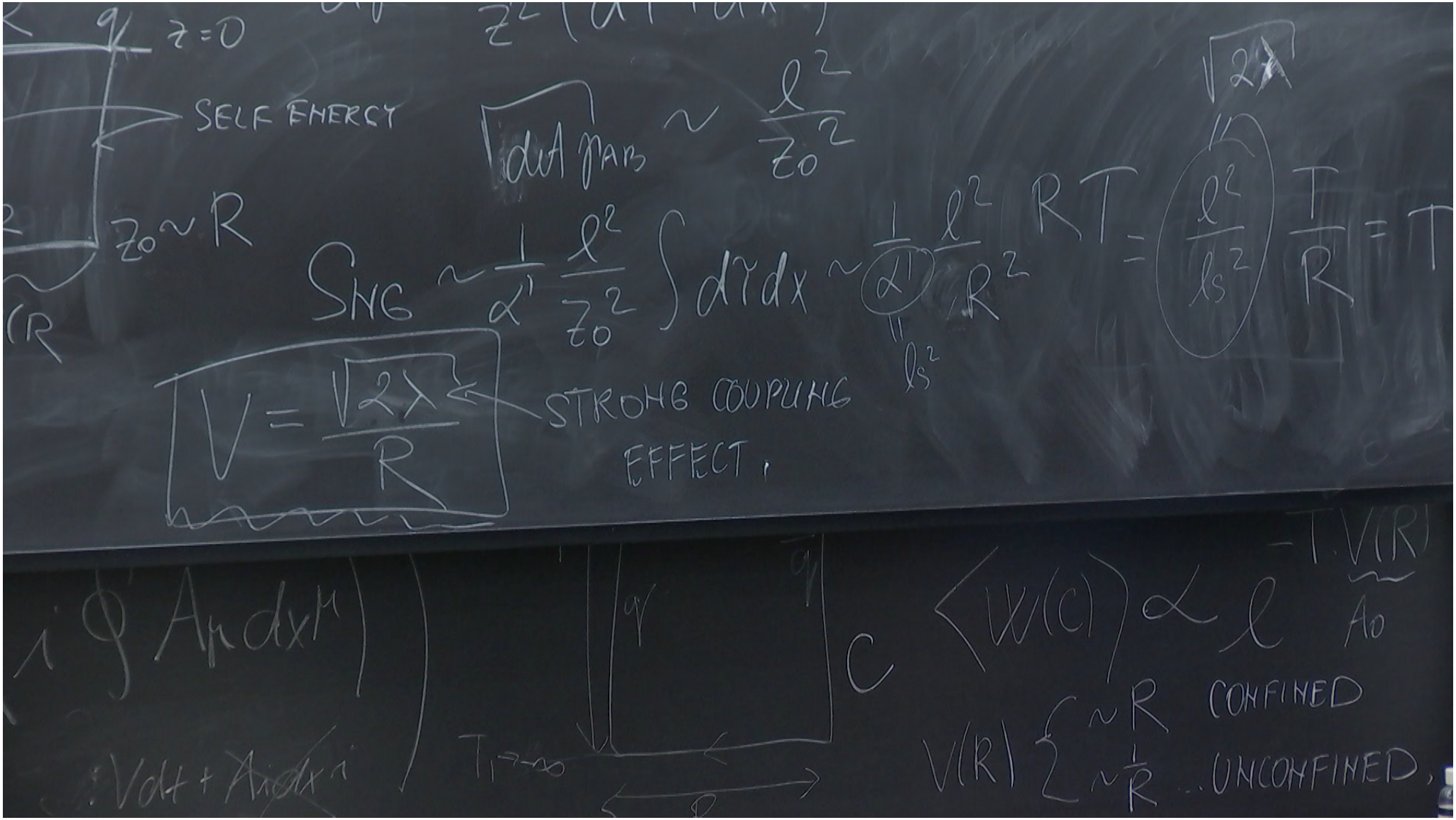


≈




UNCONFINED PHASE
q R $\bar{q}$ $z=0$
$ds^2 = \frac{l^2}{z^2}(dt^2 + dx^2)$
SELF ENERGY
GIVEN
REGULARIZING
R
$z_0 \sim R$
$V(R)$


$ds^2 = \frac{\ell^2}{z^2}(d\tau^2 + dx^2)$
F ENERGY
$\sqrt{det\, g_{AB}} \sim \frac{\ell^2}{z_0^2}$
$S_{NG} \sim \frac{1}{\alpha'}\frac{\ell^2}{z_0^2}\int d\tau dx \sim \frac{1}{\alpha'}\frac{\ell^2}{R^2} RT = \frac{\ell^2}{\ell_s^2}\frac{T}{R}$
ℓ_s^2
$\sqrt{2\lambda}$
CONFINED


z=0
SELF ENERGY
$z_0 \sim R$
$\int dA\, g_{AB} \sim \frac{\ell^2}{z_0^2}$
$S_{NG} \sim \frac{1}{\alpha'} \frac{\ell^2}{z_0^2} \int d\tau dx \sim \frac{1}{\alpha'} \frac{\ell^2}{R^2} RT = \frac{\ell^2}{\ell_s^2} \frac{T}{R}$
$\sqrt{2\lambda}$
$\alpha' = \ell_s^2$
$V = \frac{\sqrt{2\lambda}}{R}$
STRONG COUPLING EFFECT.
$i \oint A_\mu dx^\mu$
$T_{1,2} \to \infty$
$C \sim \langle W(C) \rangle \propto e^{-T \cdot V(R)}$
A_0
$V(R) \begin{cases} \sim R & \text{CONFINED} \\ \sim \frac{1}{R} & \text{UNCONFINED.} \end{cases}$