Title: AdS/CFT Lecture (230424)

Speakers: David Kubiznak

Collection: AdS/CFT (2022/2023)

Date: April 24, 2023 - 9:00 AM

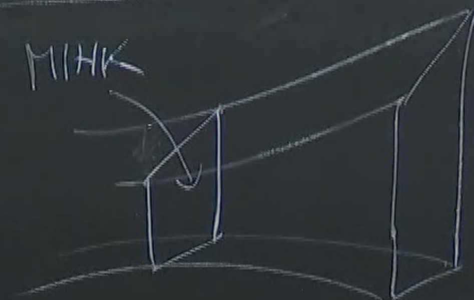
URL: https://pirsa.org/23040039

REMEMBER: AdS$_{d+1}$

Y

GLOBAL

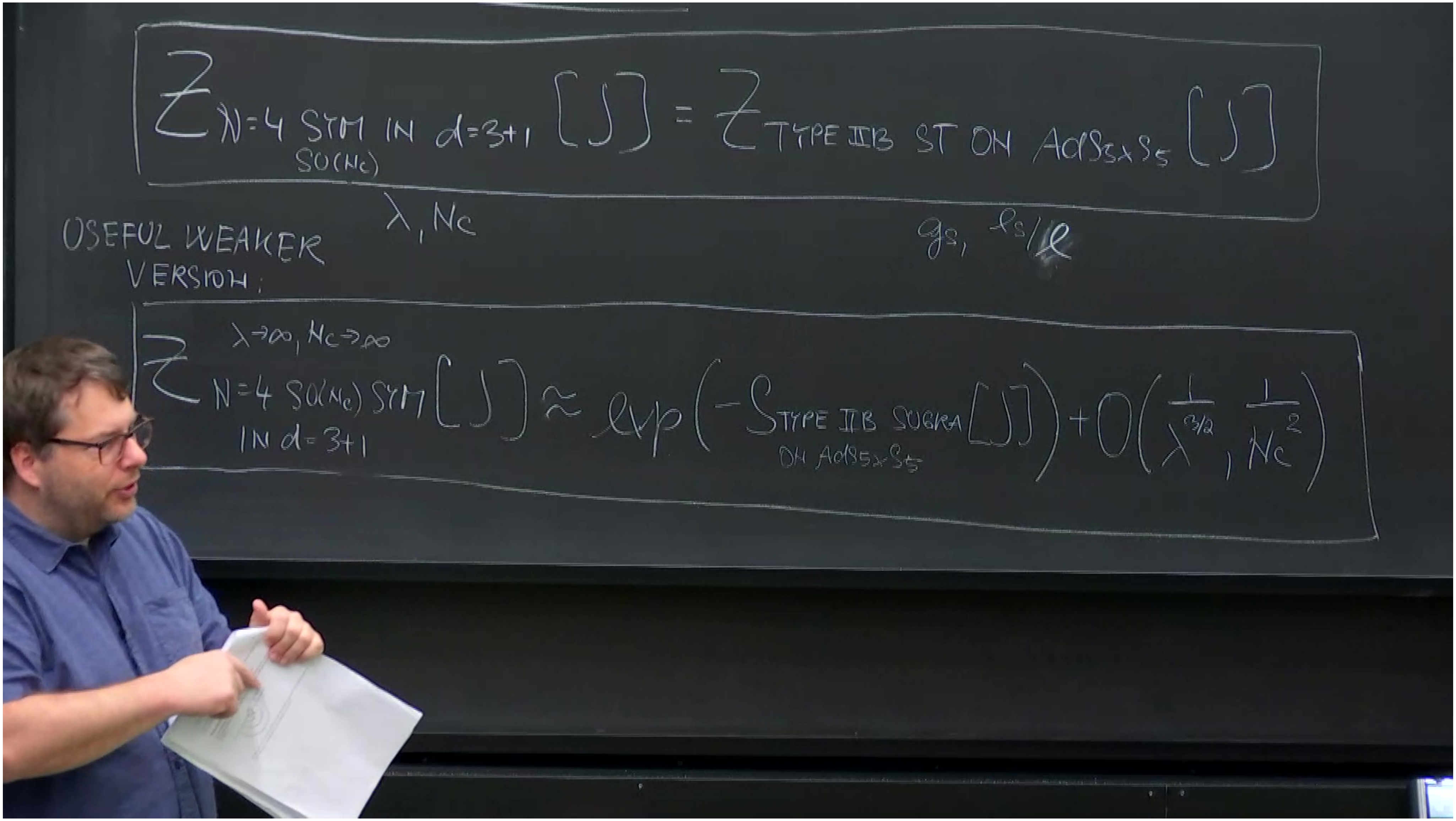
POINCARE

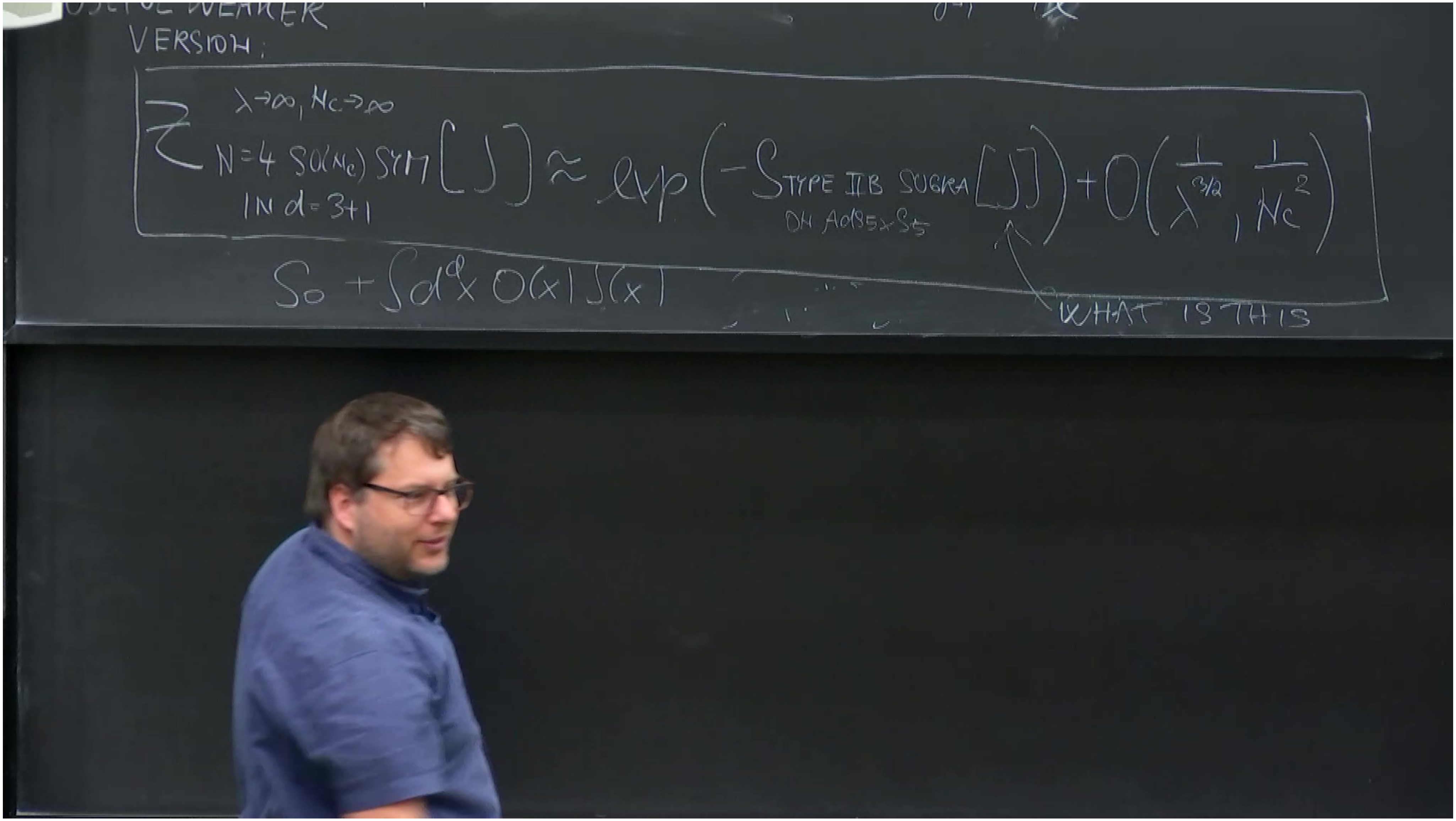


$z \to \infty$ (POINCARE HORIZON)

$z = 0$ (BOUNDARY)

$$\boxed{ds^2 = \frac{\ell^2}{z^2}\left(\eta_{\mu\nu}\, dx^\mu dx^\nu + dz^2\right)}$$


$Z_{\mathcal{N}=4 \text{ SYM IN } d=3+1}[J] = Z_{\text{TYPE IIB ST ON } AdS_5 \times S_5}[J]$
$SU(N_c)$
λ, N_c
$g_s, \ell_s/\ell$
USEFUL WEAKER VERSION:
$Z^{\lambda\to\infty, N_c\to\infty}_{\mathcal{N}=4 \, SU(N_c) \text{ SYM IN } d=3+1}[J] \approx \exp\left(-S_{\text{TYPE IIB SUGRA ON } AdS_5\times S_5}[J]\right) + O\left(\frac{1}{\lambda^{3/2}}, \frac{1}{N_c^2}\right)$


VERSION:
Z λ→∞, Nc→∞ N=4 SO(Nc) SYM [J] IN d=3+1 ≈ exp(−S TYPE IIB SUGRA ON AdS5×S5 [J]) + O(1/λ^3/2, 1/Nc^2)
So + ∫d^d x O(x) J(x)
WHAT IS THIS

BUILDING THE DICTIONARY: STATE-OPERATOR CORRESPONDENCE

- ON CFT SIDE: OPERATORS CHARACTERIZED BY SPIN & SCALING DIMENSION Δ

REMIND: $x \to \lambda x \qquad O_\Delta \to \lambda^{-\Delta} O_\Delta$

SPEC: PRIMARY SCALAR OP

$$\langle O_\Delta(x)\, O_\Delta(y) \rangle = \frac{1}{|x-y|^{2\Delta}}$$

ONDENCE

- TO FIND WHAT THIS CORRESPONDS TO ON AdS SIDE
  WE "MATCH SYMMETRIES"
- EXAMPLE: TO SCALAR OPERATOR $\mathcal{O}_\Delta$
  THE CORRESPONDING BULK FIELD .. SCALAR $\underline{\phi}$

BUILDING THE DICTIONARY: STATE-OPERATOR CORRESPONDENCE

- ON CFT SIDE: OPERATORS CHARACTERIZED BY SPIN OR SCALING DIMENSION Δ

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$$\langle \mathcal{O}_\Delta(x) \mathcal{O}_\Delta(y) \rangle = \frac{1}{|x-y|^{2\Delta}}$$

- TO FIND

WE "MA

- EXAMPLE

THE C

ACE

TO FIND WHAT THIS CORRESPONDS TO ON AdS SIDE

WE "MATCH SYMMETRIES"

EXAMPLE: TO SCALAR OPERATOR $\mathcal{O}_\Delta$

THE CORRESPONDING BULK FIELD . SCALAR $\underline{\phi}$ WITH

THE SPECIFIC MASS $\boxed{m^2 l^2 = \Delta(\Delta - d)}$

ACE

TO FIND WHAT THIS CORRESPONDS TO ON AdS SIDE
WE "MATCH SYMMETRIES"

EXAMPLE: TO SCALAR OPERATOR $\mathcal{O}_\Delta$

THE CORRESPONDING BULK FIELD SCALAR $\underline{\phi}$ WITH
THE SPECIFIC MASS $\boxed{m^2 \ell^2 = \Delta(\Delta - d)}$

ϕ SATISFIES $\boxed{\Box_{AdS}\, \phi = m^2 \phi}$

RE GENERALLY. USEFUL TABLE

OPERATOR (CFT)	FIELD (AdS)	m vs. Δ
SCALAR $\mathcal{O}_\Delta$	ϕ	$m^2\ell = \Delta(\Delta-d)$
$T_{\mu\nu}$	$h^{\mu\nu}$	$\Delta = d$, $m^2\ell^2 = 0$
J_μ	A^μ	$m^2\ell^2 = (\Delta-1)(\Delta$

ERALLY. USEFUL TABLE

OPERATOR (CFT)	FIELD (AdS)	m vs. Δ
ALAR O_Δ	ϕ	$m^2\ell = \Delta(\Delta-d)$
$\mu\nu$	$h^{\mu\nu}$	$\Delta = d, \; m^2\ell^2 = 0$
J_μ	A^μ	$m^2\ell^2 = (\Delta-1)(\Delta+1-d)$

$$\boxed{AdS_{d+1} / CFT_d}$$

<u>LET'S BE A BIT MORE CONCRETE</u>:

SCALAR IN AdS_{d+1} IN POINCARE COORDS.

$$S = -\frac{c}{2} \int dz\, d^d x \sqrt{g} \left(g^{ab} \partial_a \phi \partial_b \phi + m^2 \phi^2 \right)$$

$d)$

$\ell^2 = 0$

$(\Delta + 1 - d)$

EOM:

$$\left(\Box_{AdS_5} - m^2\right)\phi = 0, \quad \Box\phi = \frac{1}{\sqrt{g}}\left(\sqrt{g}\, g^{ab}\partial_b\phi\right)_{,a}$$

Ex:

$$\Box = \frac{1}{\ell^2}\left(z^2\partial_z^2 - (d-1)z\partial_z + z^2\delta^{\mu\nu}\partial_\mu\partial_\nu\right)$$

- $$\phi = \phi_k(z)\, e^{i k_\mu x^\mu}$$

$$\boxed{z^2\partial_z^2\phi_k - (d-1)z\partial_z\phi_k - \left(m^2\ell^2 + k^2 z^2\right)\phi_k = 0}$$

EOM: $\left(\Box_{AdS_5} - m^2\right)\phi = 0$, $\Box\phi = \frac{1}{\sqrt{g}}\left(\sqrt{g}\, g^{ab}\partial_b\phi\right)_{,a}$

Ex: $\Box = \frac{1}{\ell^2}\left(z^2\partial_z^2 - (d-1)z\partial_z + z^2\delta^{\mu\nu}\partial_\mu\partial_\nu\right)$

- $\phi = \phi_k(z)\, e^{i k_\mu x^\mu}$

$$\boxed{z^2\partial_z^2\phi_k - (d-1)z\partial_z\phi_k - \left(m^2\ell^2 + k^2 z^2\right)\phi_k = 0} \quad (*)$$

$$\left(\sqrt{g}\, g^{ab} \partial_b \phi\right)_{,a}$$

$$^{\mu\nu}\partial_\mu \partial_\nu \Big)$$

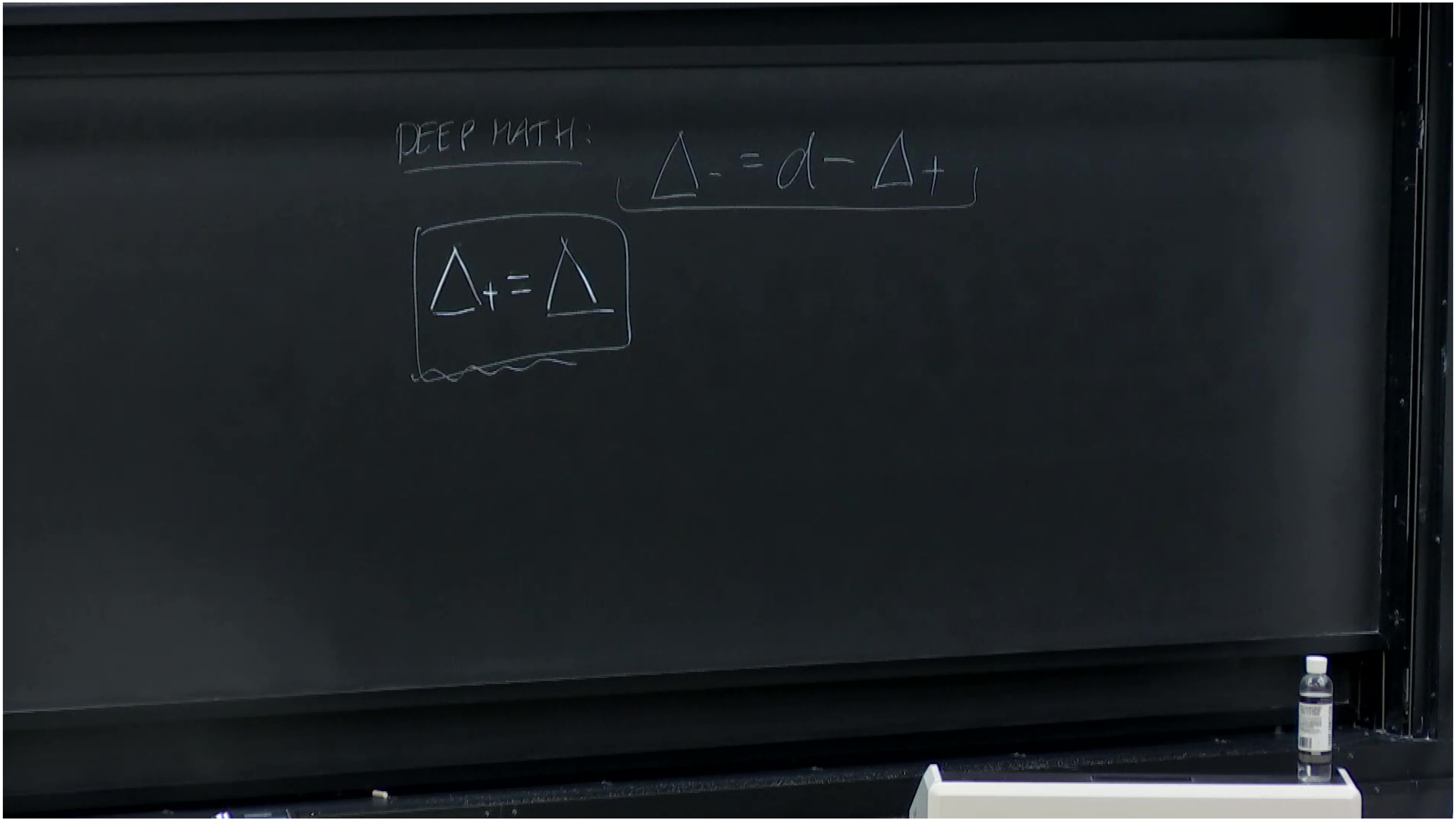
$$\boxed{\phi_R = 0} \quad (*)$$

• CLOSE TO BOUNDARY ($z=0$)

WE HAVE 2 INDEPENDENT SOLUTIONS

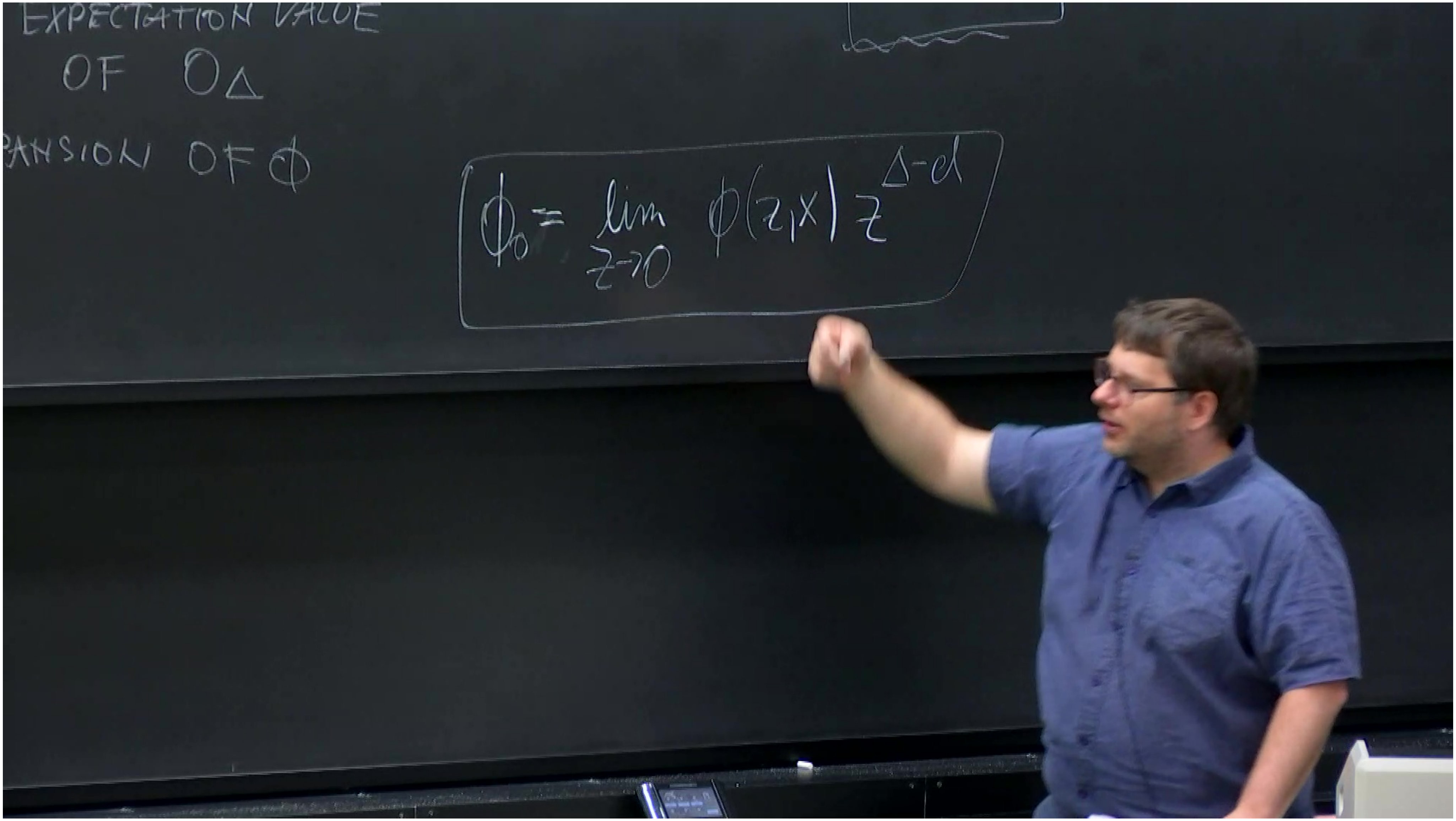
$$\phi_R = \begin{cases} z^{\Delta_+} & \dots \text{"NORMALIZABLE"} \\ z^{\Delta_-} & \dots \text{"NON-NORMALIZABLE"} \end{cases}$$

$$\boxed{\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2 \ell^2}}$$


DEEP MATH:
$\Delta_- = d - \Delta_+$
$\boxed{\Delta_+ = \Delta}$

$$\phi \sim \phi_0(x)\, z^{\Delta_-} + \phi_+(x)\, z^{\Delta_+} + \ldots$$

EXPECTATION VALUE OF O_Δ

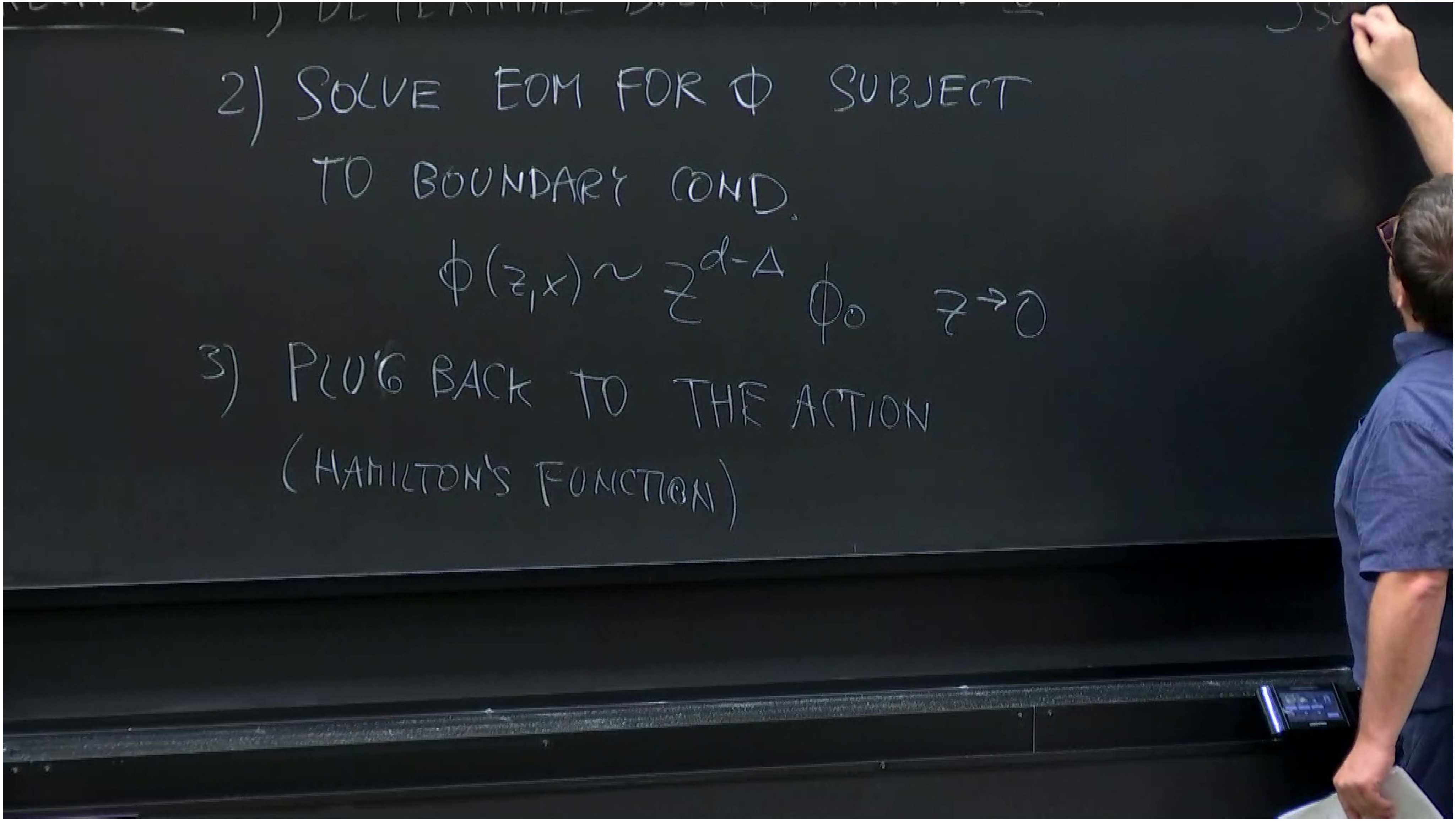

EXPECTATION VALUE
OF O_Δ
PANSION OF ϕ
$\phi_0 = \lim_{z \to 0} \phi(z,x)\, z^{\Delta - d}$

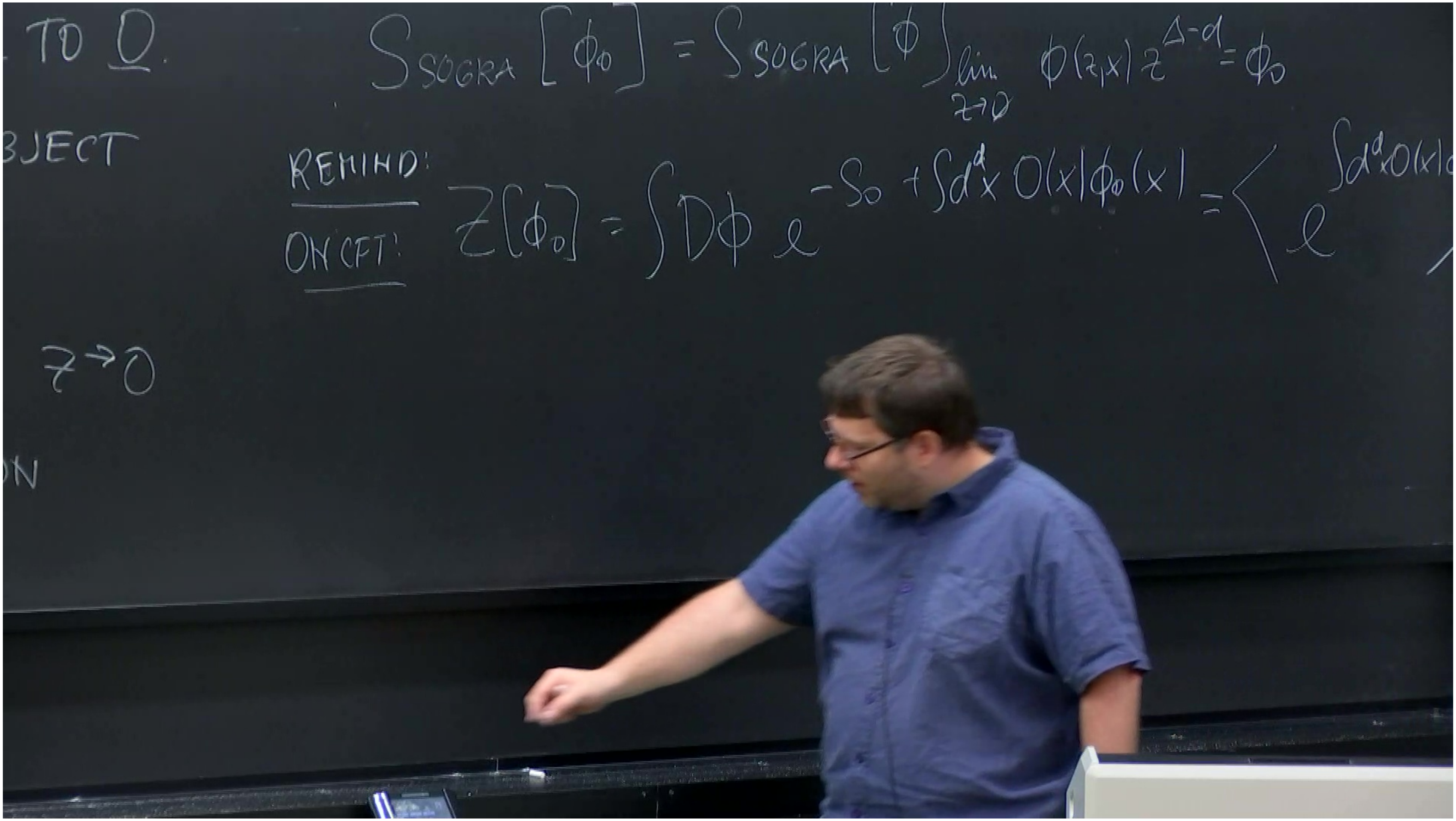
RECIPE: 1) DETERMINE BULK ϕ DUAL TO $\underline{O}$.

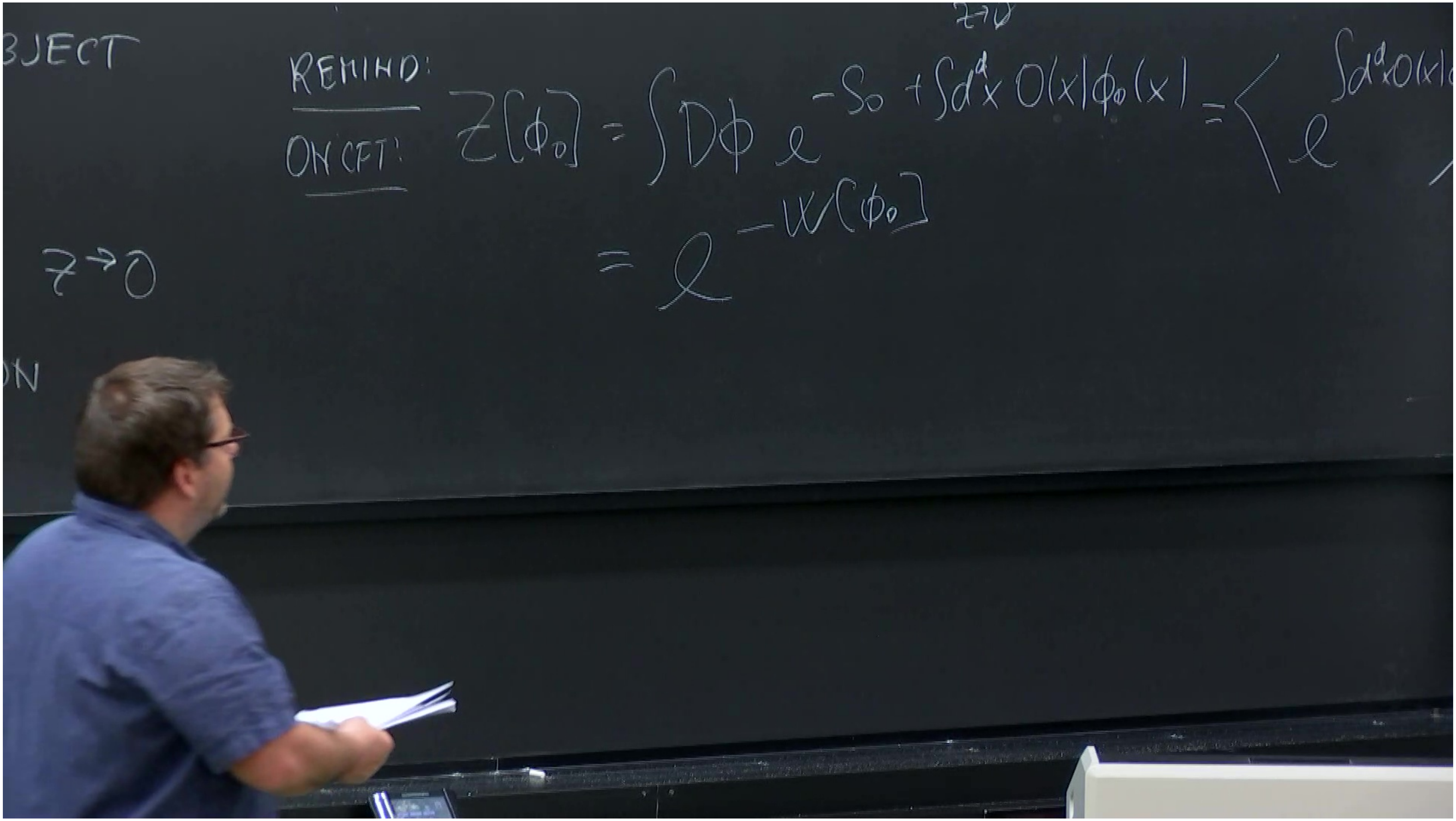
2) SOLVE EOM FOR ϕ SUBJECT TO BOUNDARY COND.

$$\phi(z,x) \sim z^{d-\Delta}\,\phi_0 \qquad z \to 0$$

3) PLUG BACK TO THE ACTION


2) SOLVE EOM FOR Φ SUBJECT
TO BOUNDARY COND.
$\phi(z,x) \sim z^{d-\Delta}\, \phi_0 \quad z \to 0$
3) PLUG BACK TO THE ACTION
(HAMILTON'S FUNCTION)


TO O.
S SUGRA [φ0] = S SUGRA [φ] lim z→0 φ(z,x) z^(Δ-d) = φ0
BJECT
REMIND:
ON CFT:
Z[φ0] = ∫Dφ e^(-S0 + ∫d^dx O(x) φ0(x)) = ⟨ e^(∫d^dx O(x) ...
z→0
ON



BJECT
REMIND:
ON CFT:
$Z[\phi_0] = \int D\phi \, e^{-S_0 + \int d^d x \, O(x) \phi_0(x)} = \langle e^{\int d^d x \, O(x)}$
$= e^{-W[\phi_0]}$
$z \to 0$
ON

TO $\underline{O}$.

BJECT

$z \to 0$

ON

$$S_{SUGRA}[\phi_0] = S_{SUGRA}[\phi]_{\lim_{z\to 0} \phi(z,x) z^{\Delta - d} = \phi_0}$$

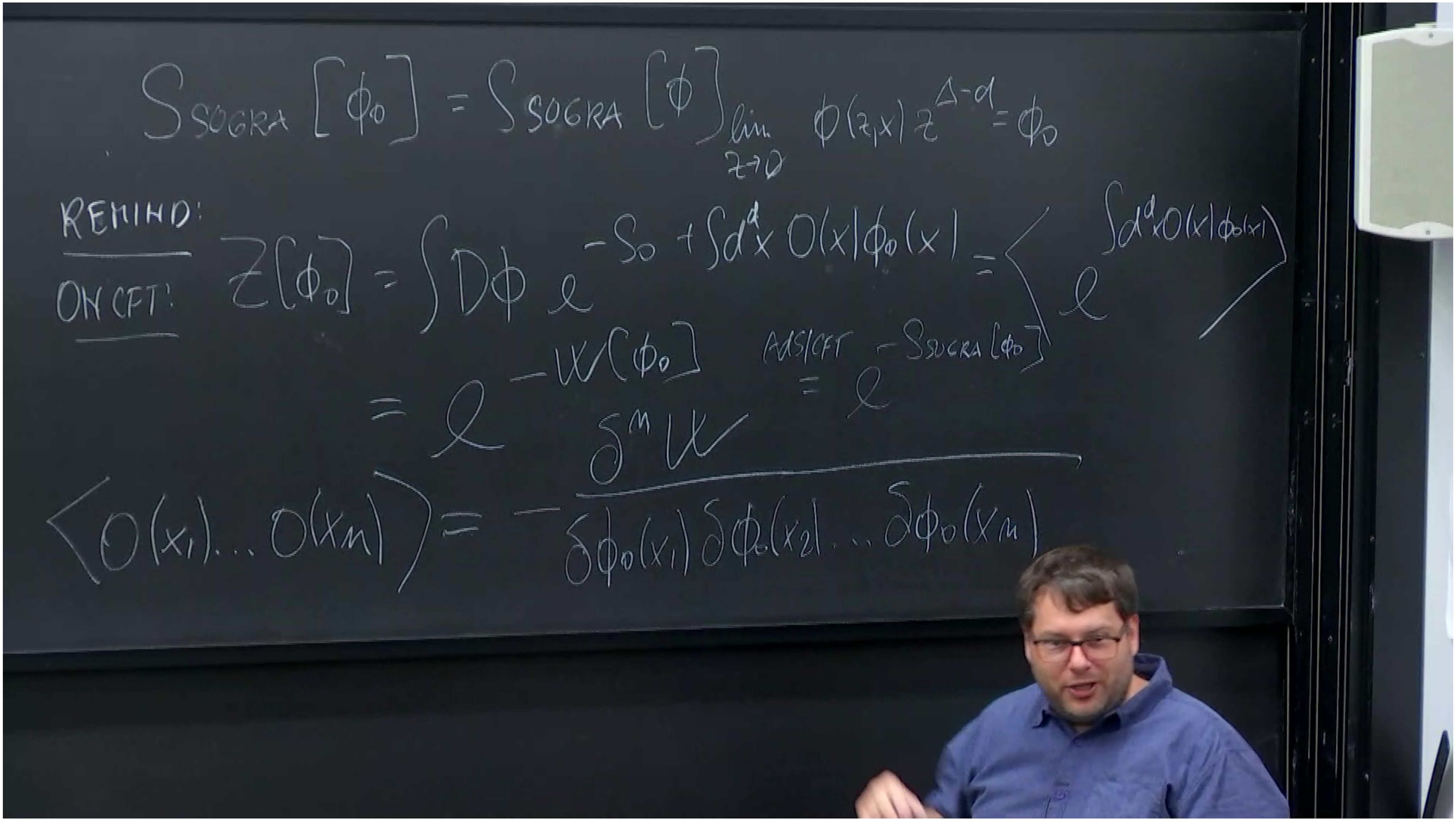
REMIND:

ON CFT:

$$Z[\phi_0] = \int D\phi \, e^{-S_0 + \int d^dx \, O(x)\phi_0(x)} = \left\langle e^{\int d^dx O(x)\phi_0(x)} \right\rangle$$

$$= e^{-W[\phi_0]}$$

$$\langle O(x_1) \dots O(x_n) \rangle = -\frac{\delta^n W}{\delta\phi_0(x_1)\,\delta\phi_0(x_2) \dots \delta\phi_0(x_n)}$$


$S_{SUGRA}[\phi_0] = S_{SUGRA}[\phi]_{\lim_{z\to 0} \phi(z,x) z^{\Delta-d} = \phi_0}$
REMIND:
ON CFT:
$Z[\phi_0] = \int D\phi \, e^{-S_0 + \int d^dx \, O(x)\phi_0(x)} = \langle e^{\int d^dx \, O(x)\phi_0(x)} \rangle$
$= e^{-W[\phi_0]} \overset{AdS/CFT}{=} e^{-S_{SUGRA}[\phi_0]}$
$\langle O(x_1)\ldots O(x_n)\rangle = -\frac{\delta^n W}{\delta\phi_0(x_1)\,\delta\phi_0(x_2)\ldots\delta\phi_0(x_n)}$

Δ

$(\Delta - d)$

$, \; m^2 \ell^2 = 0$

$(\Delta - 1)(\Delta + 1 - d)$

$$\boxed{AdS_{d+1} / CFT_d}$$

LET'S BE A BIT MORE CONCRETE

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$$S = -\frac{c}{2} \int dz\, d^d x \sqrt{g} \left(g^{ab} \partial_a \phi \partial_b \phi + m^2 \phi^2 \right)$$

$\boxed{AdS_{d+1} / CFT_d}$

$S[q_0, q_f]$

q_0 q_f

$\frac{\partial S}{\partial q^0} = p$

LET'S BE A BIT MORE CONCRETE

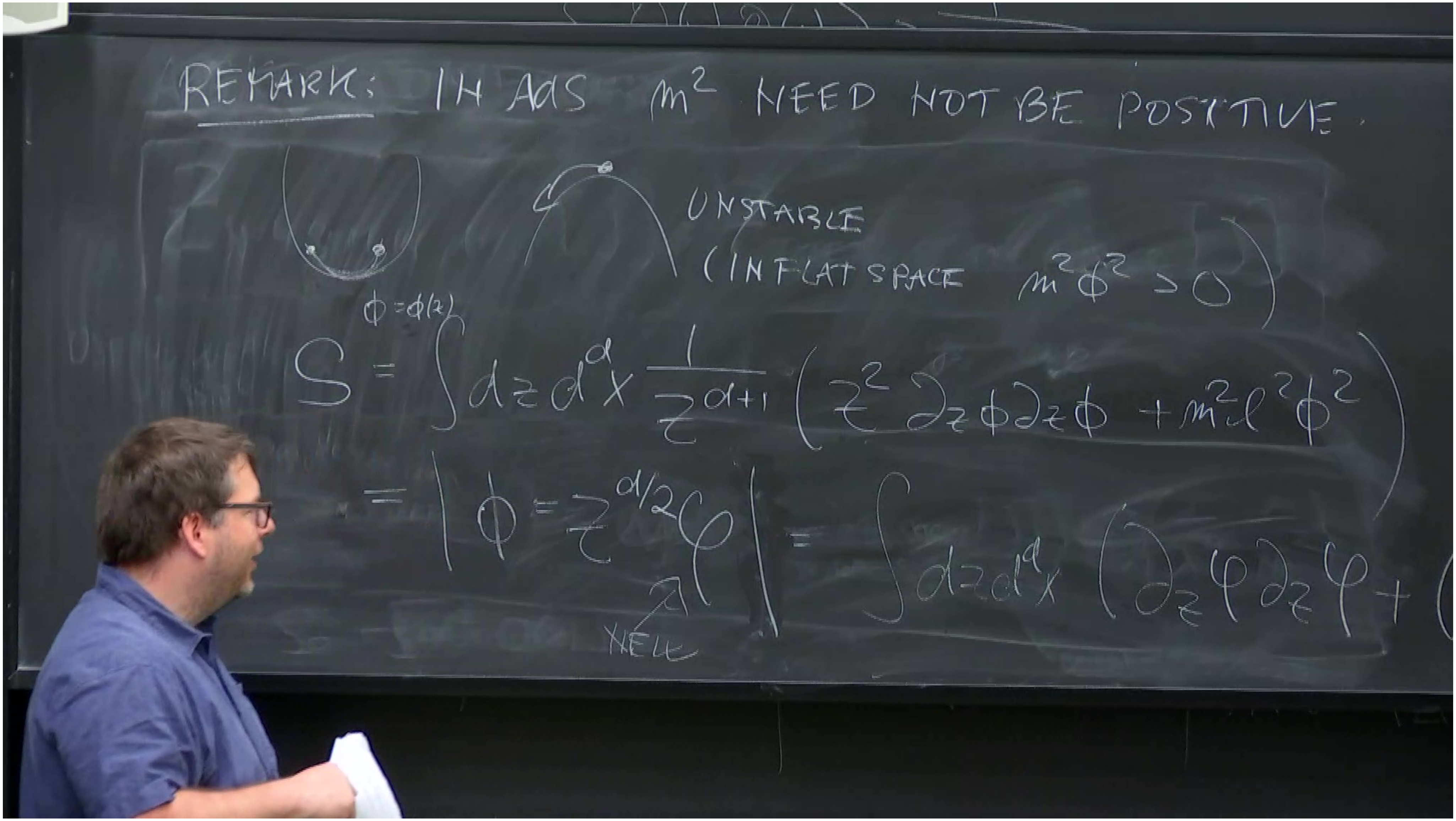
SCALAR IN AdS_{d+1} IN POINCARE COORDS.

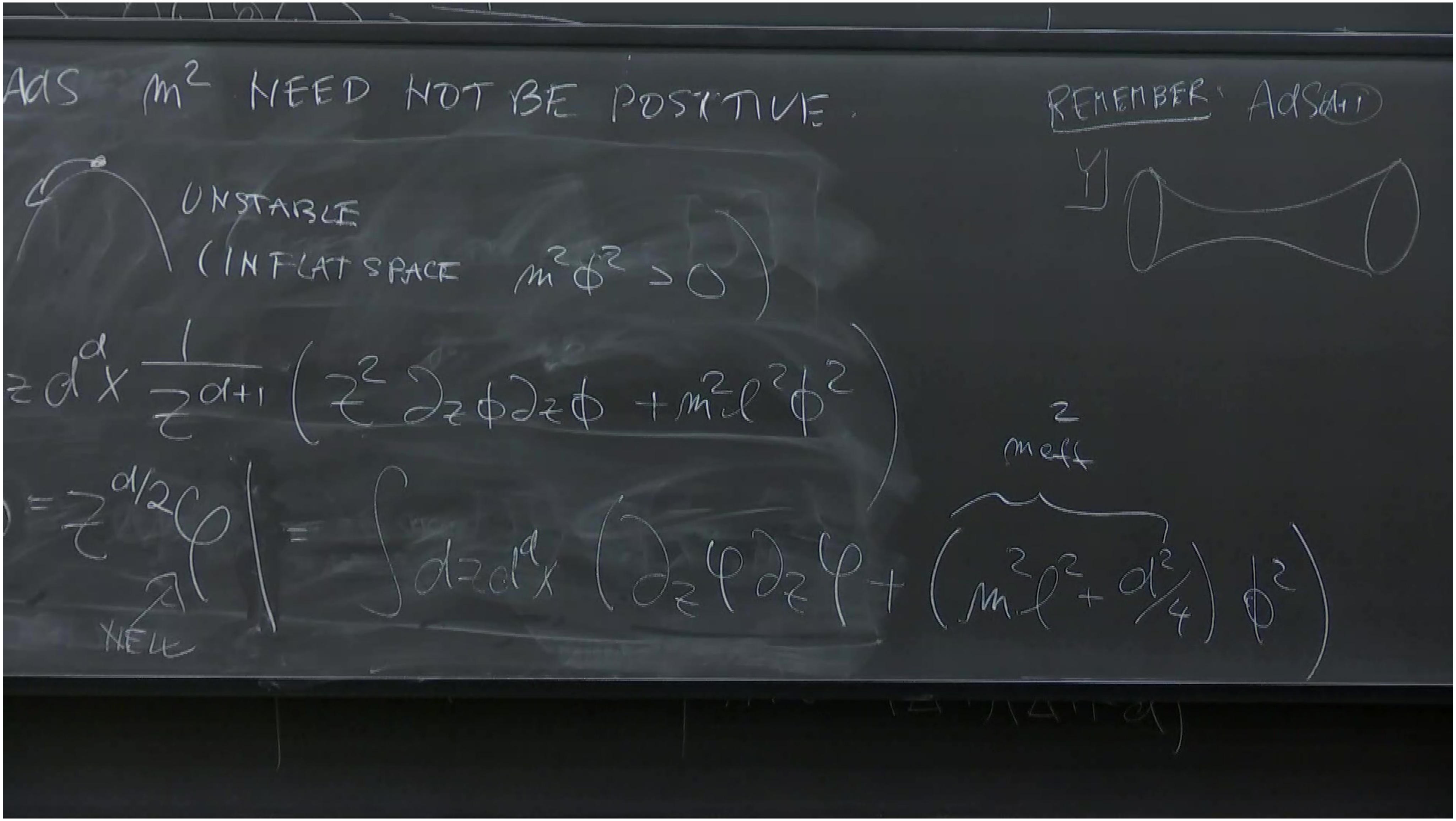
$$S = -\frac{C}{2} \int dz\, d^d x \sqrt{g} \left(g^{ab} \partial_a \phi \partial_b \phi + m^2 \phi^2 \right)$$

$\Delta - d)$

$m^2 \ell^2 = 0$

$-1)(\Delta + 1 - d)$


REMARK: IN AdS m^2 NEED NOT BE POSITIVE.
UNSTABLE
(IN FLAT SPACE $m^2\phi^2 > 0$)
$\phi = \phi(z)$
$S = \int dz\, d^dx \frac{1}{z^{d+1}} \left(z^2 \partial_z \phi \partial_z \phi + m^2 \ell^2 \phi^2 \right)$
$= \left| \phi = z^{d/2} \varphi \right| = \int dz\, d^dx \left(\partial_z \varphi \partial_z \varphi + (\right.$


AdS m^2 NEED NOT BE POSITIVE.
REMEMBER: AdS_{d+1}
y
UNSTABLE
(IN FLAT SPACE $m^2\phi^2 > 0$)
$z\, d^d x \frac{1}{z^{d+1}} \left(z^2 \partial_z\phi \partial_z\phi + m^2\ell^2\phi^2 \right)$
$= z^{d/2}\varphi$
NEW
$= \int dz\, d^d x \left(\partial_z\varphi \partial_z\varphi + \underbrace{\left(m^2\ell^2 + d^2/4 \right)}_{m_{eff}^2} \phi^2 \right)$

$$\underbrace{m^2\ell^2 + d^2/4}_{m_{eff}^2} \Big) \phi^2 \Big)$$

$$m_{eff}^2 \geq 0$$

$$\boxed{m^2\ell^2 \geq -\frac{d^2}{4}}$$

BREITENLOHNER-FREEDMAN

(BF) BOUND