Title: AdS/CFT Lecture (230421)

Speakers: David Kubiznak

Collection: AdS/CFT (2022/2023)

Date: April 21, 2023 - 9:00 AM

URL: https://pirsa.org/23040038

AdS$_d$ SPACETIME = MAXIMALLY SYMMETRIC SOLUTIONS OF EE WITH NEGATIVE $\Lambda = -\frac{(d-1)(d-2)}{\ell^2}$

1) EMBEDDING COORDS Y: USEFUL FOR RECOGNIZING ALL SYMMETRIES

$\mathbb{R}^{2,d-1}$

$$ds^2 = -dY_{-1}^2 - dY_0^2 + dY_1^2 + \dots + dY_{d-1}^2 = \eta_{AB}^{2,d-1} dY^A dY^B \qquad (1)$$

$$-\ell^2 = -Y_{-1}^2 - \underbrace{Y_0^2 + Y_1^2 + \dots + Y_{d-1}^2}_{Y_a} = -Y_{-1}^2 + \eta_{ab}^{1,d-1} Y^a Y^b \qquad (2)$$

RIES

(1)

(2)

$$\Rightarrow \boxed{g_{ab} = \eta^{1,d-1}_{ab} - \frac{Y_a Y_b}{\ell^2 + Y^a Y_a}}$$

• MAXIMALLY SYMMETRIC AS

$$\tilde{X}^A = \Lambda^A{}_B X^B : \eta^{2,d-1}_{AB} = \eta^{2,d-1}_{CD} \Lambda^C{}_A \Lambda^D{}_B$$

LEAVES (1) & (2) INVARIANT

GENERATORS

WRITE $\Lambda^a{}_b = \delta^A{}_B + \omega^A{}_B \Rightarrow \boxed{\omega_{AB} = -\omega_{BA}}$ $\binom{d+1}{2}$

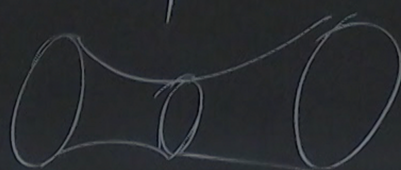
$SO(2, d-1)$

$$\Rightarrow \boxed{g_{ab} = \eta_{ab} - \frac{Y_a Y_b}{\ell^2 + Y^a Y_a}}$$

S ALL SYMMETRIES

$\eta^{2,d-1}_{AB}\, dY^A dY^B \quad (1)$

$+\eta^{1,d-1}_{ab}\, Y^a Y^b \quad (2)$



• MAXIMALLY SYMMETRIC AS

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LEAVES (1) & (2) INVARIANT

GE

WRITE $\Lambda^a{}_b = \delta^A{}_B + \omega^A{}_B \Rightarrow \boxed{\omega_{AB} = -\omega_{BA}}$

$SO(2,d-1)$

2) GLOBAL COORDINATES : SINGLE OUT Y_{-1} & Y_0.

$$Y_{-1} = \ell \cosh\tilde{\rho} \cos\tilde{t}, \quad Y_0 = \ell \cosh\tilde{\rho} \sin\tilde{t}$$

$$Y_i = \ell \sinh\tilde{\rho}\, \Omega_i, \quad \Omega_i \ldots \text{COORDS ON } S^{d-2}$$

$$\sum_i \Omega_i \Omega_i = 1$$

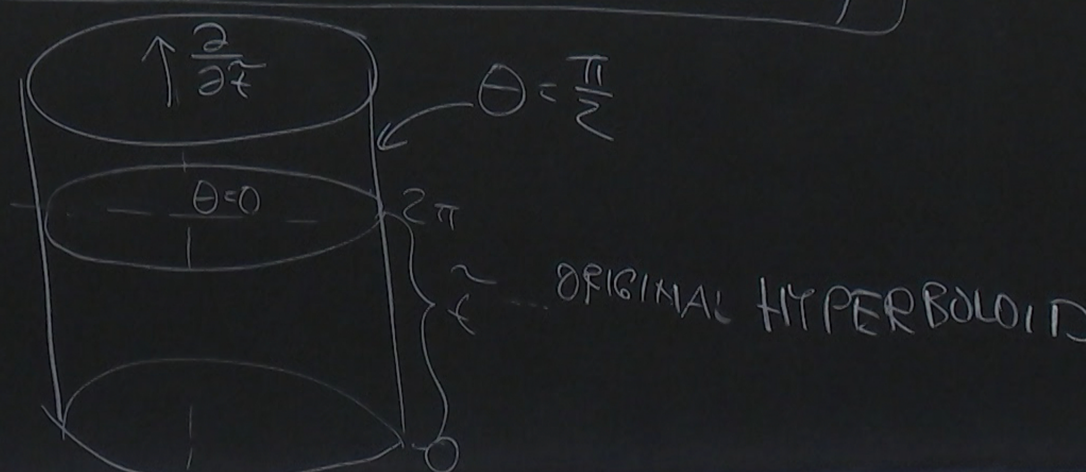
$SO(2, d-1)$

① COMPACTIFY $\tilde{\rho}$: $\sinh\tilde{\rho} = \tan\theta$

$$\boxed{ds^2 = \frac{\ell^2}{\cos^2\theta}\left(-d\tilde{t}^2 + d\theta^2 + \sin^2\theta \, d\Omega_{d-2}^2\right)}$$

$\theta \in (0, \frac{\pi}{2})$

BOUNDARY



$SO(2, d-1)$

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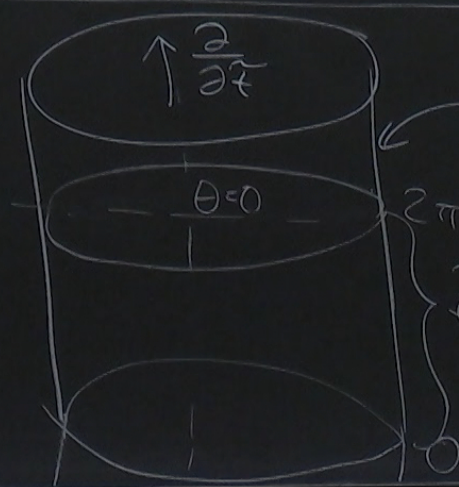
BOUNDARY



$\theta = \frac{\pi}{2}$

2π

$\tilde{t}$ ... ORIGINAL HYPERBOLOID

$\rightarrow$ COVERING SPACE $\tilde{t} \in (-\infty, \infty)$

0

① COMPACTIFY $\tilde{\rho}$: $\sinh\tilde{\rho} = \tan\theta$

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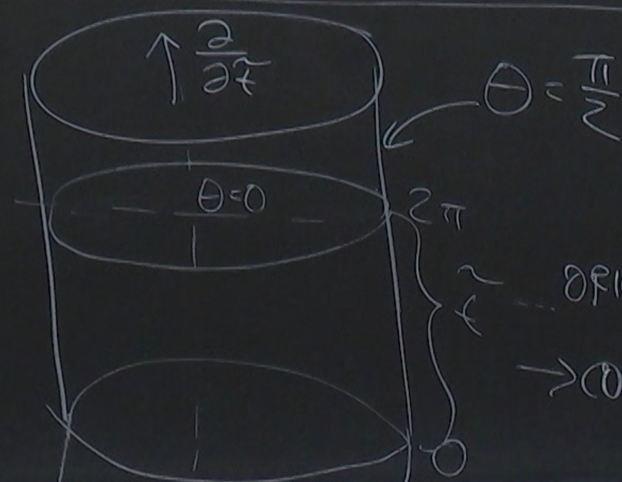
BOUNDARY



ORIGINAL HYPERBOLOID

$\rightarrow$ COVERING SPACE $\tilde{t} \in (-\infty, \infty)$

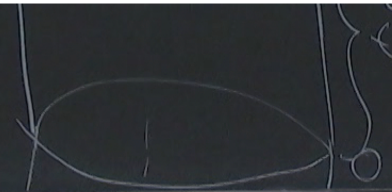
BOUNDARY: LOCATED AT $\Theta = \frac{\pi}{2}$

TAKE CONFORMAL COMPLETION

$$ds^2 \to \Omega^2 ds^2 = \tilde{ds}^2$$

CHOOSE THIS SO THAT THE LIMIT IS NICE

SPEC: $\Omega^2 = \frac{\cos^2\theta}{\ell^2}\,\omega^2(t, x_i)$



... ORIGINAL HYPERBOLOID

→ COVERING SPACE $\tau \in (-\infty, \infty)$

$$\boxed{ds^2/a = \underbrace{\left(-d\tau^2 + d\Omega_{d-2}^2\right)}_{\mathbb{R}^1 \times S^{d-2}} \omega^2(\tau, \Omega_i)}$$

ARBITRARY FUNCTION OF BOUNDARY COORDS.

... CONFORMAL BOUNDARY

EINSTEIN STATIC UNIVERSE

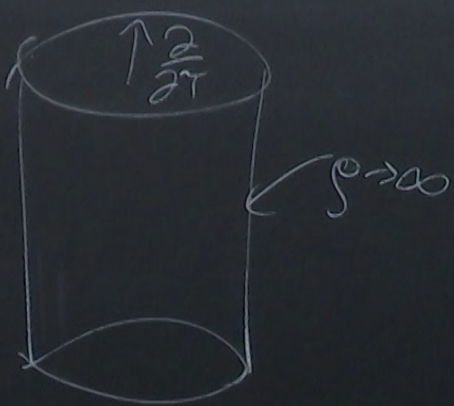
HAVING ARBITRARY ω^2 SHOULD NOT MATTER FOR THE CFT.

(2) INSTEAD OF COMPACTIFYING $\tilde{\rho}$

$$\rho = \ell \sinh \tilde{\rho} \,, \quad \tau = \ell \tilde{t}$$

$\Rightarrow$ USUAL GLOBAL COORDS

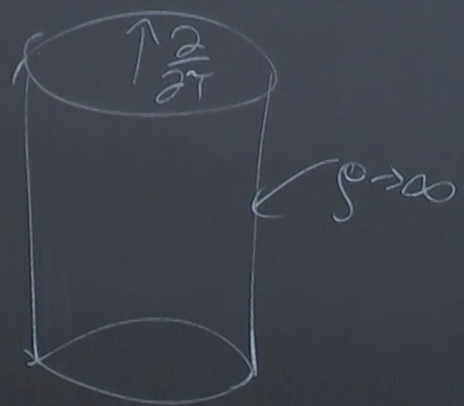
$$\boxed{ds^2 = -f\, d\tau^2 + \frac{d\rho^2}{f} + \rho^2 d\Omega_{d-2}^2 \qquad f = 1 + \frac{\rho^2}{\ell^2}}$$



3) POINCARE AdS : SINGLE OUT Y_{-1} & Y_{d-1}

$$Y_{-1} = \frac{1+z^2+\eta_{\mu\nu}x^\mu x^\nu}{2z}, \quad Y_{d-1} = \frac{1-z^2-\eta_{\mu\nu}x^\mu x^\nu}{2z}$$

$$Y_\mu = \frac{x_\mu}{z}$$



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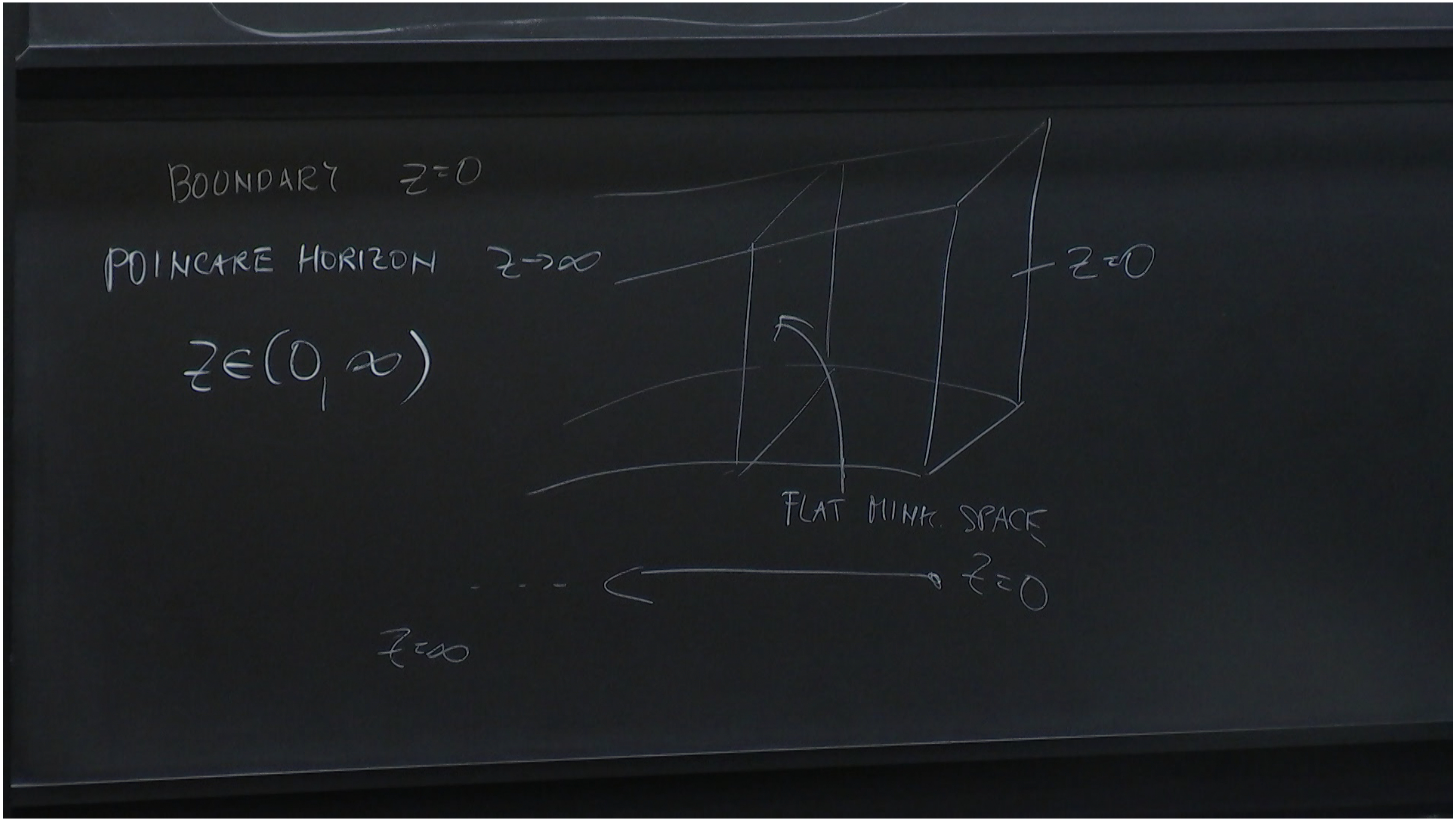
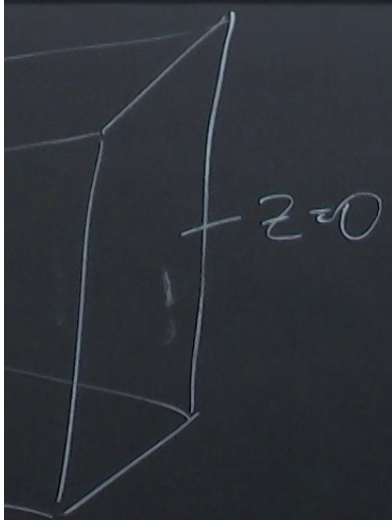
$$Y_\mu = \frac{x_\mu}{z}$$

$$\boxed{ds^2 = \frac{\ell^2}{z^2}\left(\underbrace{\eta_{\mu\nu}dx^\mu dx^\nu}_{-dt^2+dx_\mu dx^\mu} + dz^2\right)}$$


BOUNDARY z=0
POINCARE HORIZON z→∞
z∈(0,∞)
z=0
FLAT MINK. SPACE
z=0
z=∞



INK. SPACE (z = CONST)

→ z=0

- POINCARE AdS = VOLUME FILLING SLICES OF MINK.
- MANIFEST SCALING SYMMETRY

$$(t,\vec{x},z)\to\lambda(t,\vec{x},z)$$

- BOUNDARY $ds^2\big|_\partial = \omega^2(x^\mu)\,\eta_{\mu\nu}\,dx^\mu dx^\nu$

• POINCARE AdS = VOLUME FILLING SLICES OF MINK.

• MANIFEST SCALING SYMMETRY

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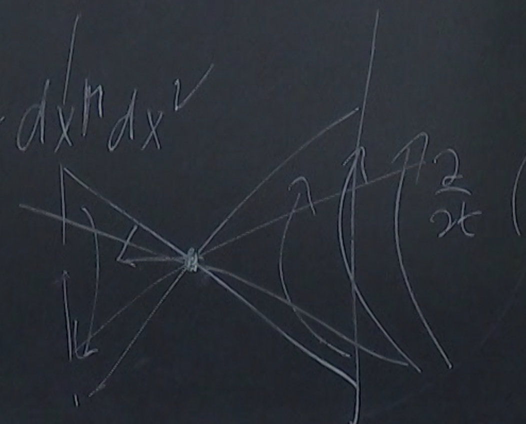
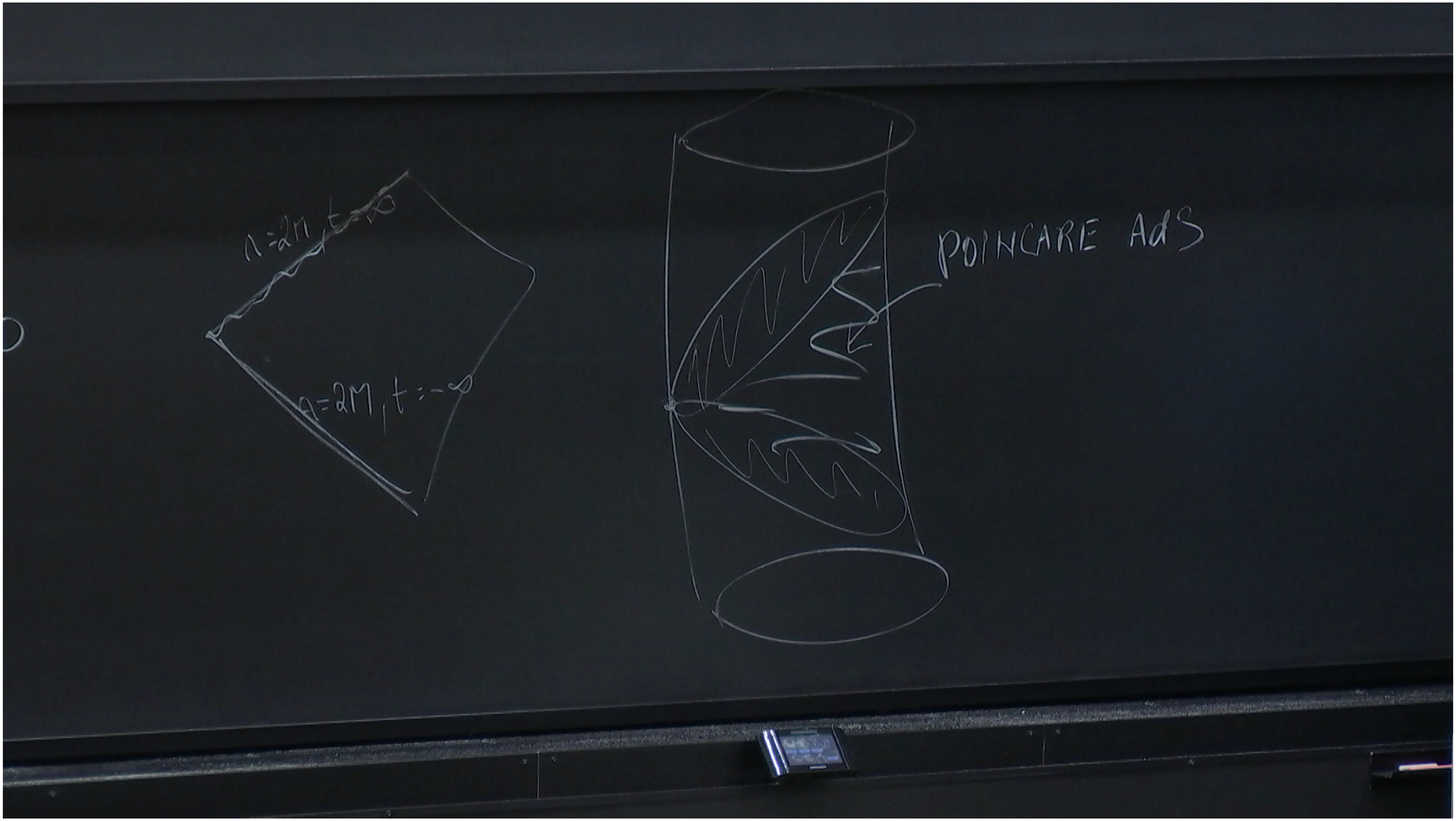
ONST)

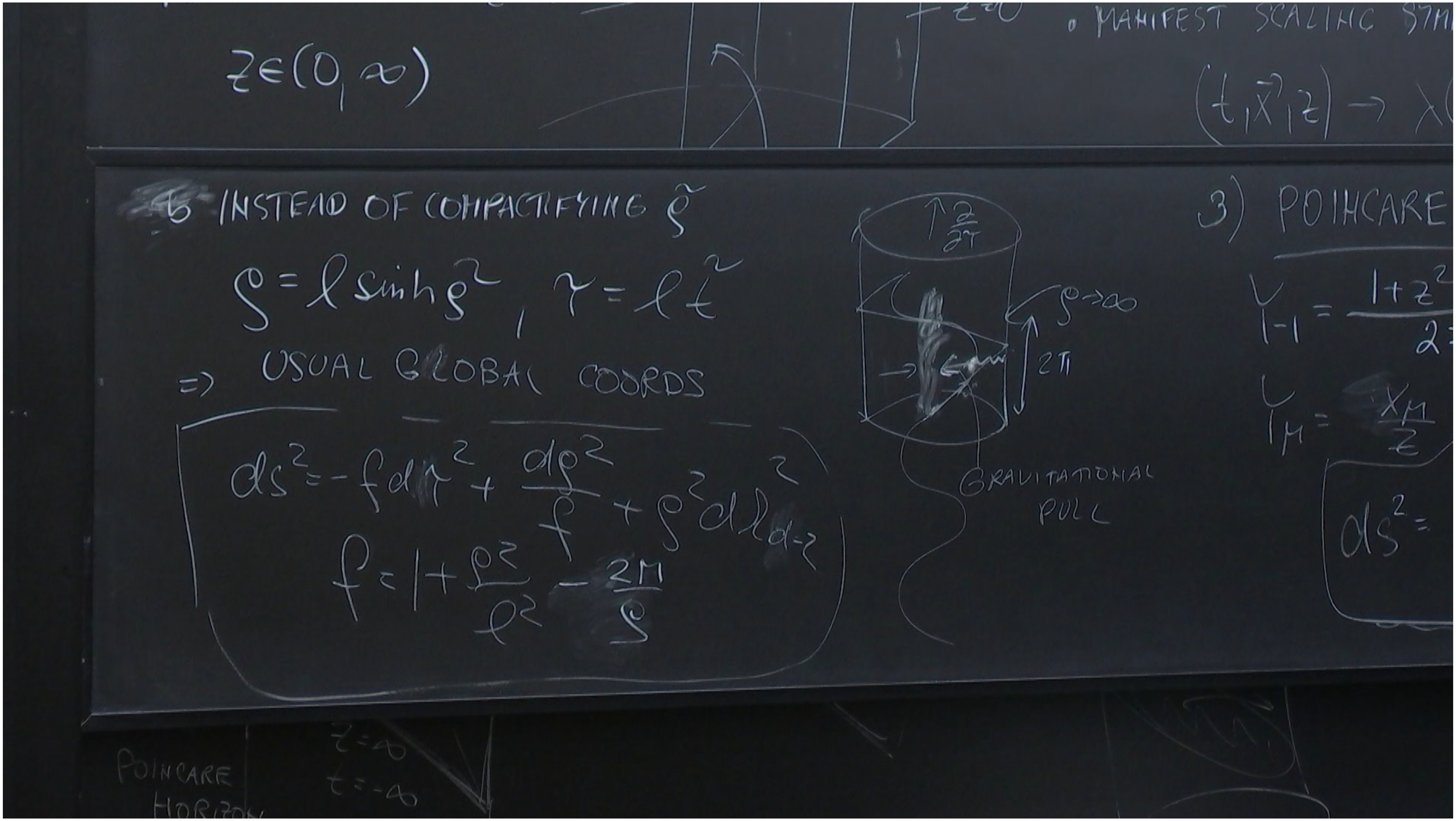
RINDLER

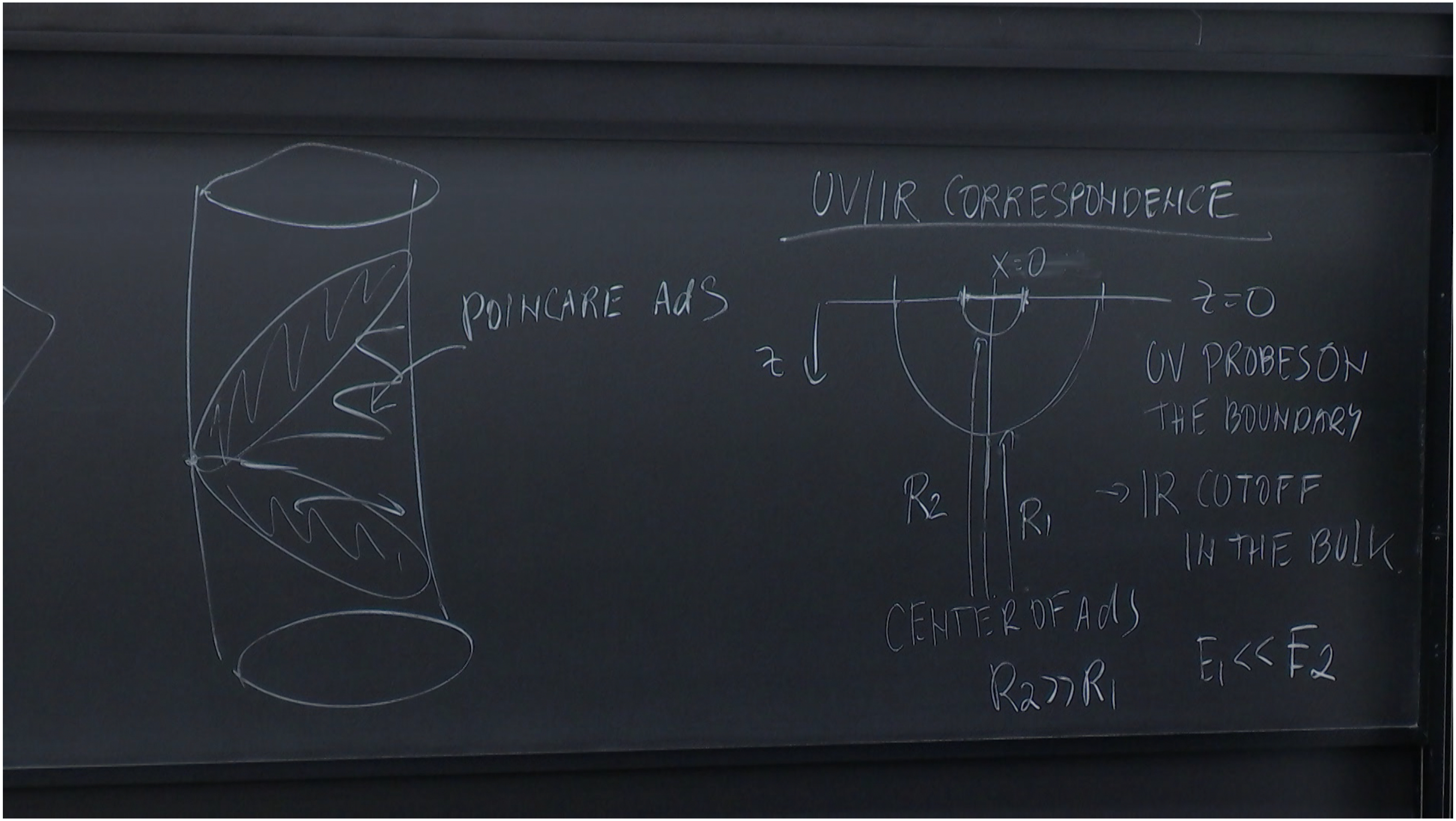


$\frac{\partial}{\partial t}$ (X = CONST)



r=2M, t=∞
r=2M, t=-∞
POINCARE AdS


$z \in (0, \infty)$
• MANIFEST SCALING SYM
$(t, \vec{x}, z) \to \lambda($
INSTEAD OF COMPACTIFYING $\tilde{\rho}$
$\rho = \ell \sinh \tilde{\rho}, \quad \tau = \ell \tilde{t}$
⇒ USUAL GLOBAL COORDS
$ds^2 = -f d\tau^2 + \frac{d\rho^2}{f} + \rho^2 d\Omega_{d-2}^2$
$f = 1 + \frac{\rho^2}{\ell^2} - \frac{2M}{\rho}$
$\rho \to \infty$
2π
GRAVITATIONAL PULL
3) POINCARE
$ds^2 =$
POINCARE HORIZON
$z = \infty$
$t = -\infty$


POINCARE AdS
UV/IR CORRESPONDENCE
$x=0$
$z=0$
z
UV PROBES ON THE BOUNDARY
R_2
R_1
→ IR CUTOFF IN THE BULK
CENTER OF AdS
$R_2 \gg R_1$
$E_1 \ll E_2$