Title: AdS/CFT Lecture (230419)

Speakers: David Kubiznak

Collection: AdS/CFT (2022/2023)

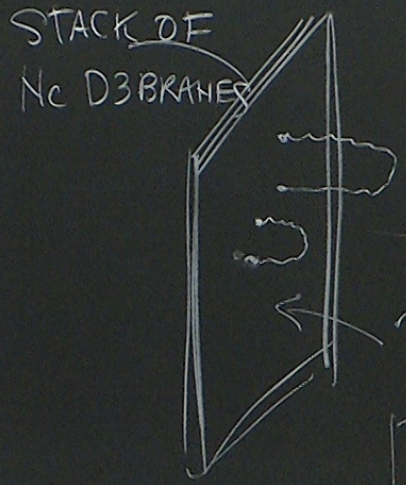
Date: April 19, 2023 - 9:00 AM

URL: https://pirsa.org/23040037

ADS/CFT CONJECTURE (MALDACENA 97): TYPE IIB SUPERSTRING TH

MOTIVATION:

OPEN STRING ($\lambda << 1$)



10D MINK

3+1 N=4 SU(Nc) SYM

LOW ENERGY

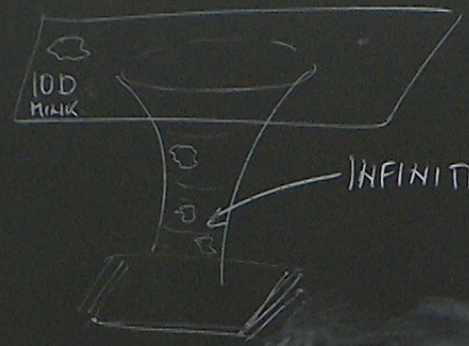
$N_c, \lambda = g_{YM}^2 N_c$

CLOSE

10D MINK

STRING THEORY ON $AdS_5 \times S_5$ IS DUAL TO $N=4$ $SU(N_c)$ SYM in (3+1)

CLOSED STRING ($\lambda \gg 1$)



INFINITE THROAT $AdS_5 \times S_5$ ... FULL SUPERSTRING THEORY
(MODES GET REDSHIFTED FOR ASYMPTOTIC OBSERVERS)

EXTREMAL MASSIVE
3-BRANE
(SOLUTION OF IIB SUGRA)

$g_s, \; L/l_s$

OPEN
$\lambda \ll 1$
||
CLOSED
$\gg 1$

DERIVED: $2\pi g_s = g_{YM}^2 = \frac{\lambda}{N_c}$

$\frac{L^4}{l_s^4} = 4\pi g_s N_c = 2\lambda$

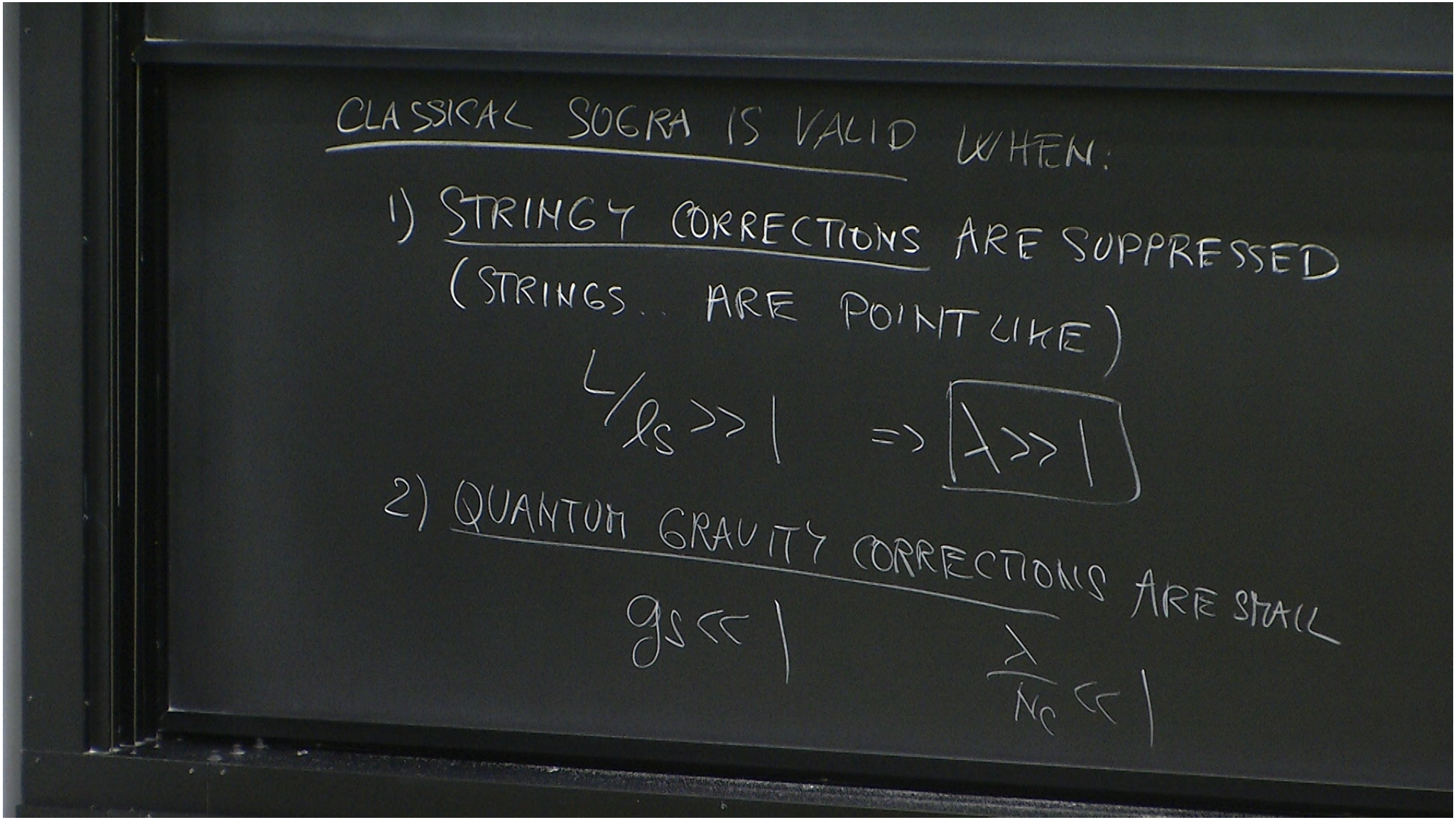
REMARKS: 1) DUALITY MEANS COMPLETE EQUIVALENCE.
NO EXPLICIT CHANGE OF VARIABLES IS KNOWN!

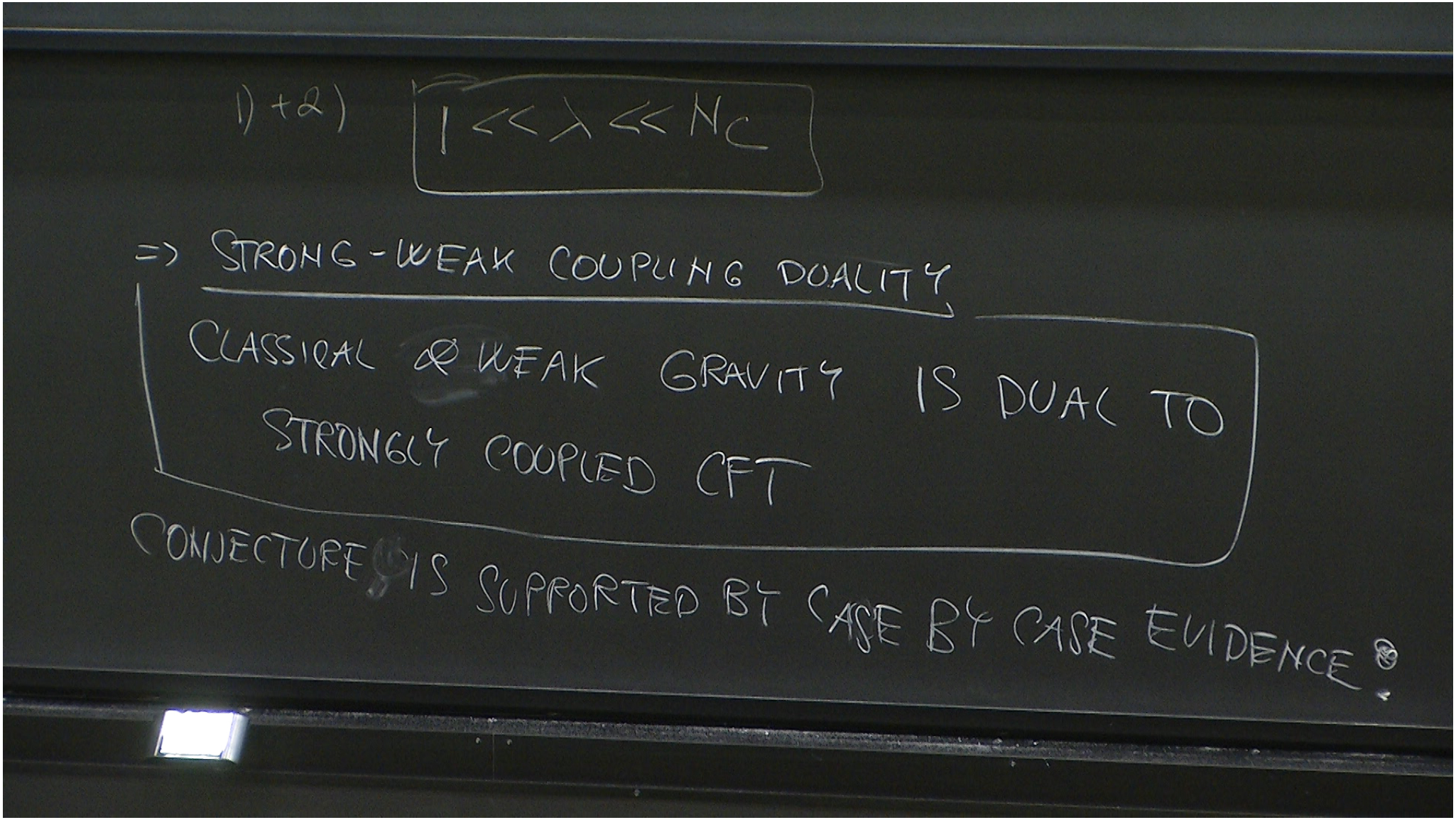
2) RELATE A THEORY OF QUANTUM GRAVITY
TO NON-GRAVITATIONAL QFT.

(SOLUTION OF IIB SUGRA)

3) HOLOGRAPHIC

SYM LIVES ON THE BOUNDARY OF AdS5


CLASSICAL SUGRA IS VALID WHEN:
1) STRINGY CORRECTIONS ARE SUPPRESSED
(STRINGS ... ARE POINT LIKE)
$L/\ell_s \gg 1 \Rightarrow \boxed{\lambda \gg 1}$
2) QUANTUM GRAVITY CORRECTIONS ARE STALL
$g_s \ll 1$
$\frac{\lambda}{N_c} \ll 1$


1) +2) $1 \ll \lambda \ll N_c$
=> STRONG-WEAK COUPLING DUALITY
CLASSICAL & WEAK GRAVITY IS DUAL TO STRONGLY COUPLED CFT
CONJECTURE IS SUPPORTED BY CASE BY CASE EVIDENCE

AS A QUICK CHECK: CHECK THE SYMMETRIES;

- $AdS_5 \times S^5$ ... $SO(4,2) \times SO(6)$
- SYM: CFT IN $d=4$ DIMS.

$$SO(4,2) \times SO(6)$$

↑ ROTATION OF 6 SCALARS.

A FEW REMARKS ON CFTs

CFT = FIELD THEORY WITH EXTRA SPACETIME SYMMETRIES.

T: CONFORMAL SYMMETRY IS THE LARGEST ONE FOR NONTRIVIAL QFTs.

IN D-DIMENSIONS: CFT IS INVARIANT UNDER $SO(2,d)$ CONF. TRANSF.

$$x^\mu \to x'^\mu(x) : \eta_{\mu\nu} \to \Omega^2(x)\eta_{\mu\nu}$$

CONF TRANSF: POINCARE + SPECIAL CONFORMAL + SCALING

$$t \to \lambda t, \quad \vec{x} \to \lambda \vec{x}$$

$$\phi \to \phi'(x') = \lambda^{-\Delta} \phi(x)$$

SCALING DIMENSION

- EXAMPLES:

1) MAXWELL $\partial_\mu F^{\mu\nu} = 0$

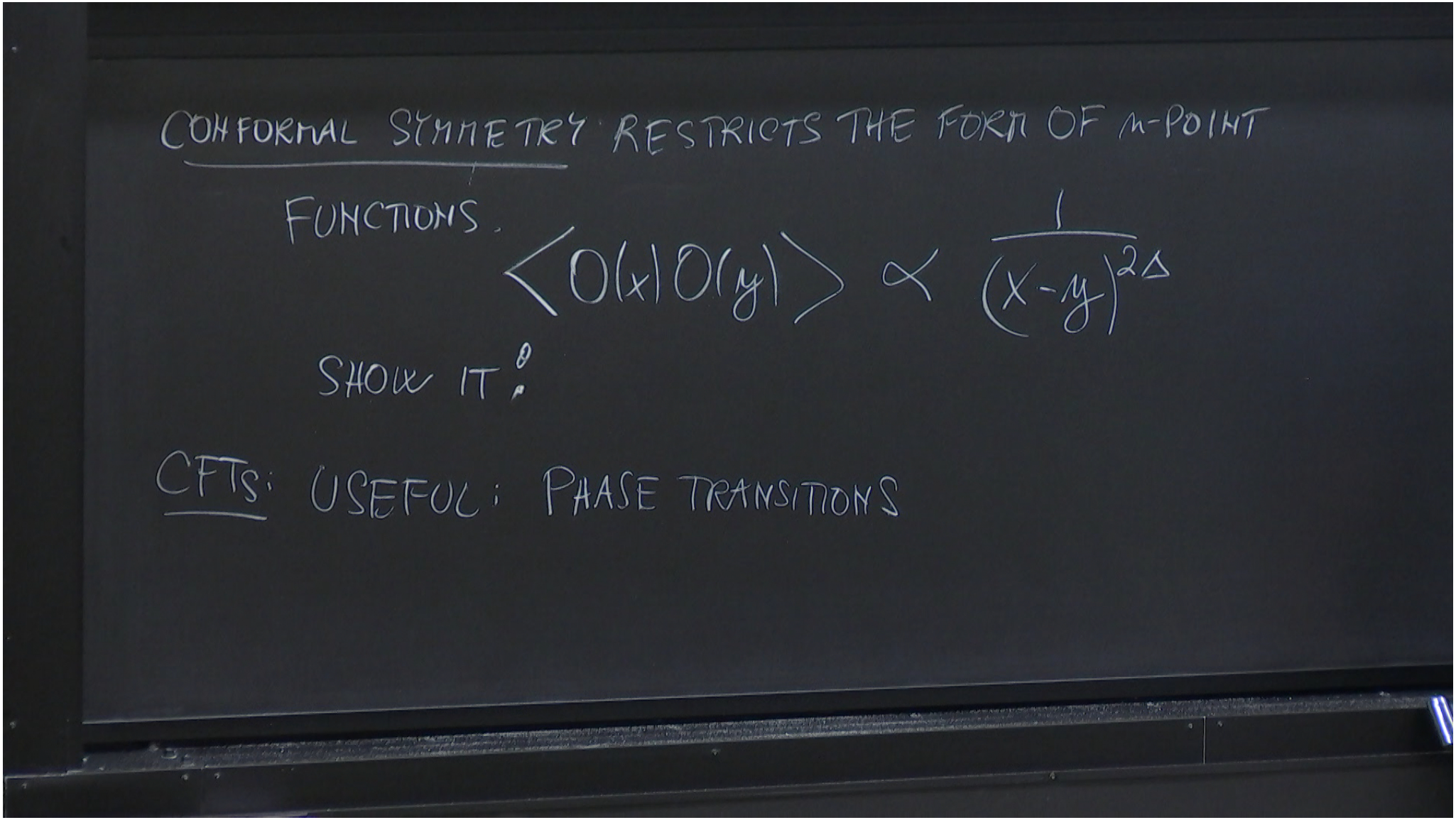
2) MASSLESS DIRAC: $\gamma^\mu \partial_\mu \psi = 0$

$B = 0$

3) λ^4 THEORY $\Box\phi = \frac{\lambda}{3!}\phi^3$

4) CLASSICAL YM: $D_\mu F^{\mu\nu} = 0$

} NOT QUANTUM MECHANICALLY


CONFORMAL SYMMETRY RESTRICTS THE FORM OF n-POINT
FUNCTIONS.
$\langle O(x) O(y) \rangle \propto \frac{1}{(x-y)^{2\Delta}}$
SHOW IT!
CFTs: USEFUL: PHASE TRANSITIONS

CONFORMAL SYMMETRY RESTRICTS THE FORM OF n-POINT FUNCTIONS.

$$\langle O(x) O(y) \rangle \propto \frac{1}{(x-y)^{2\Delta}}$$

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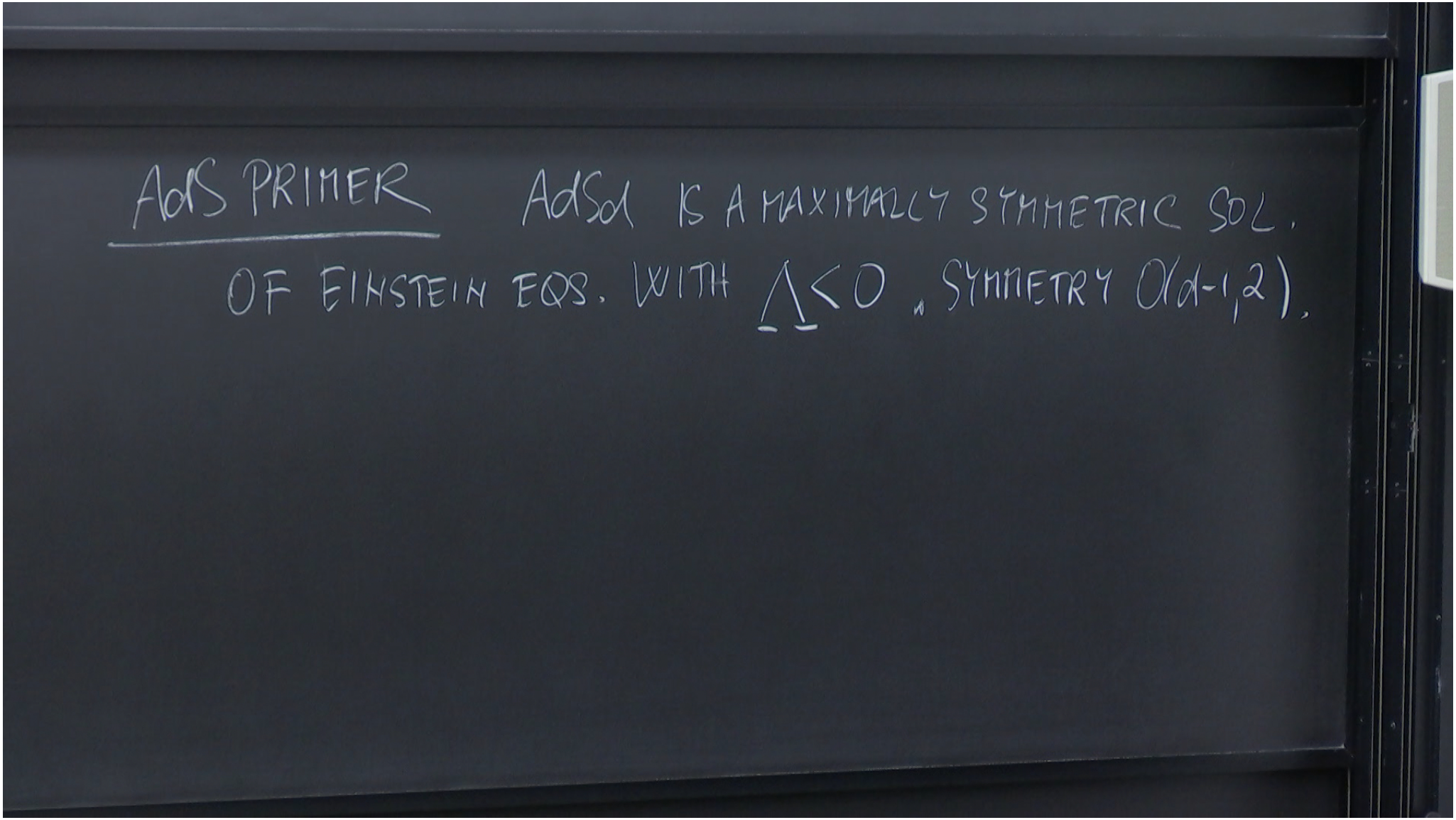
CFTs: USEFUL: PHASE TRANSITIONS $\{\alpha, \beta, \ldots\} \leftrightarrow \{\Delta\}$

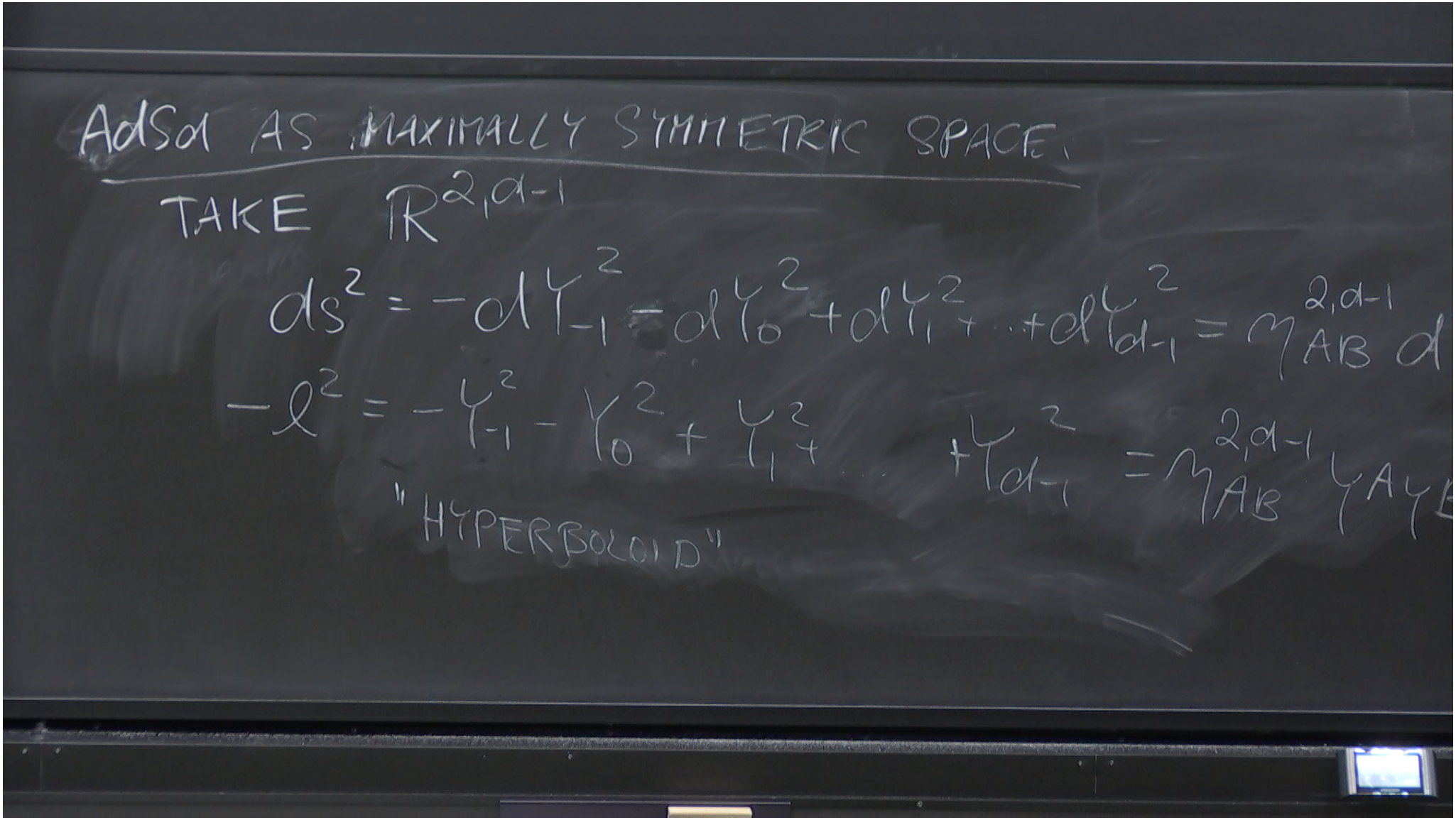
CONFORMAL SYMMETRY RESTRICTS THE FORM OF n-POINT FUNCTIONS.

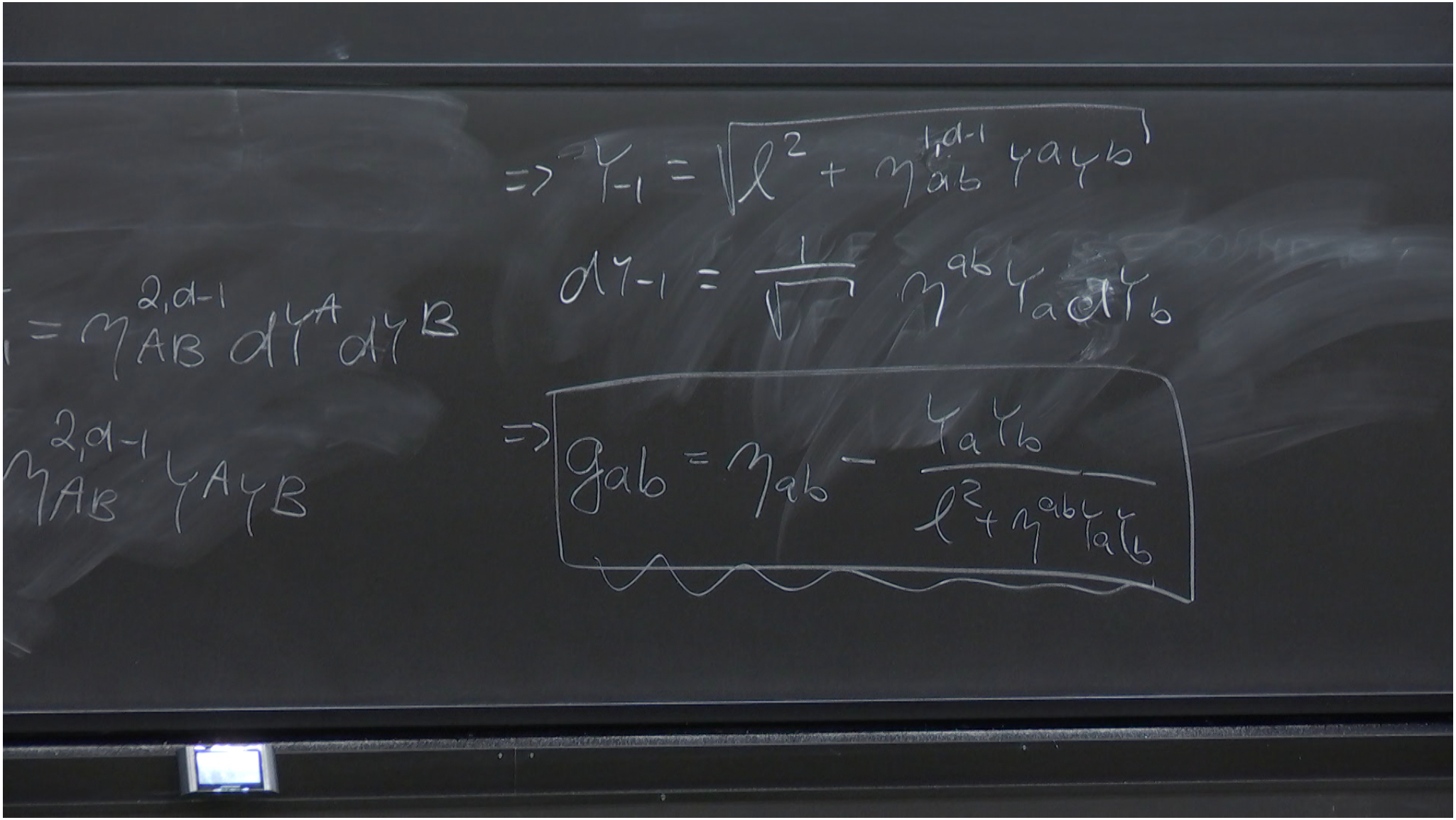
$$\langle O(x)O(y)\rangle \propto \frac{1}{(x-y)^{2\Delta}}$$

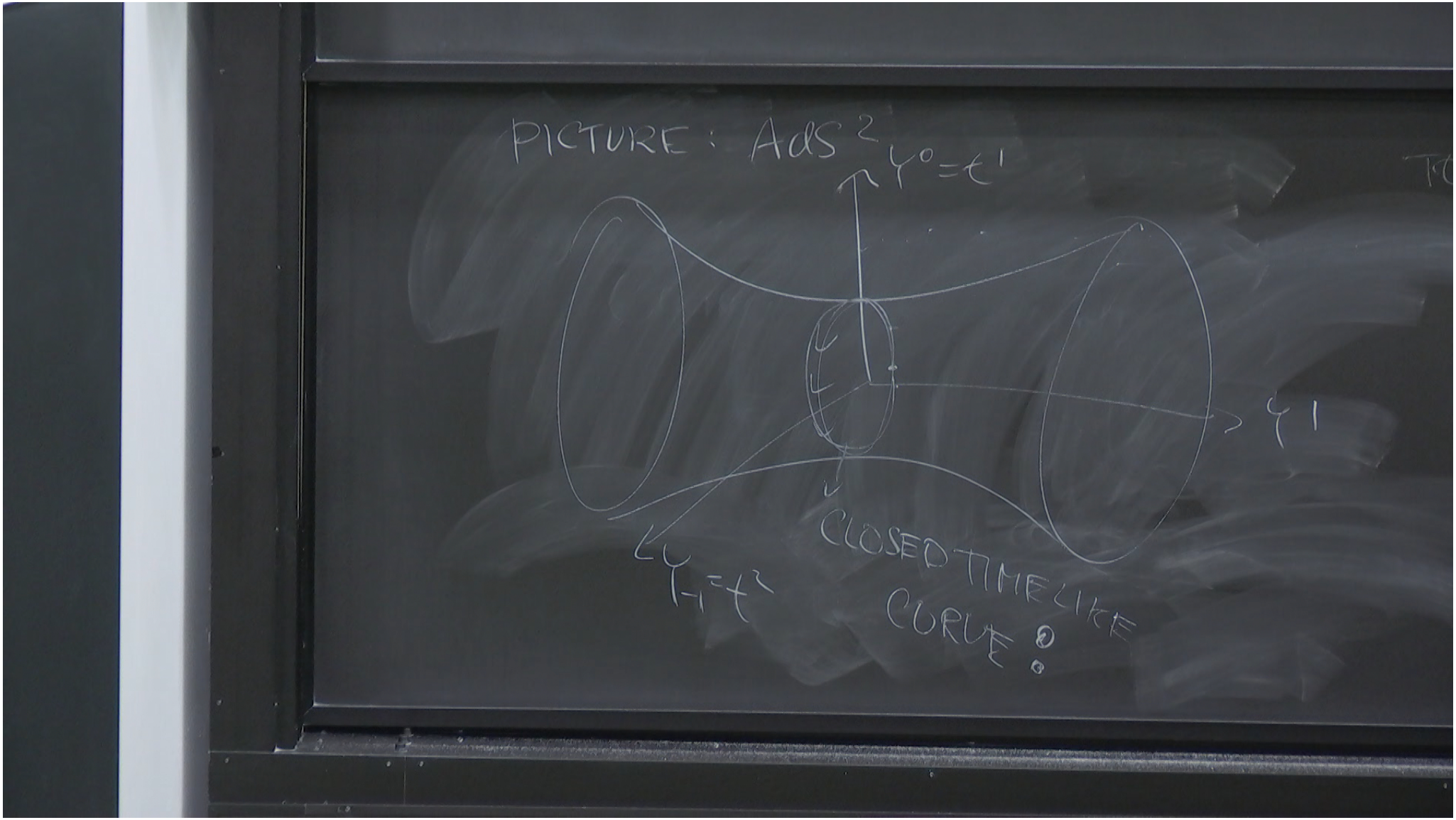
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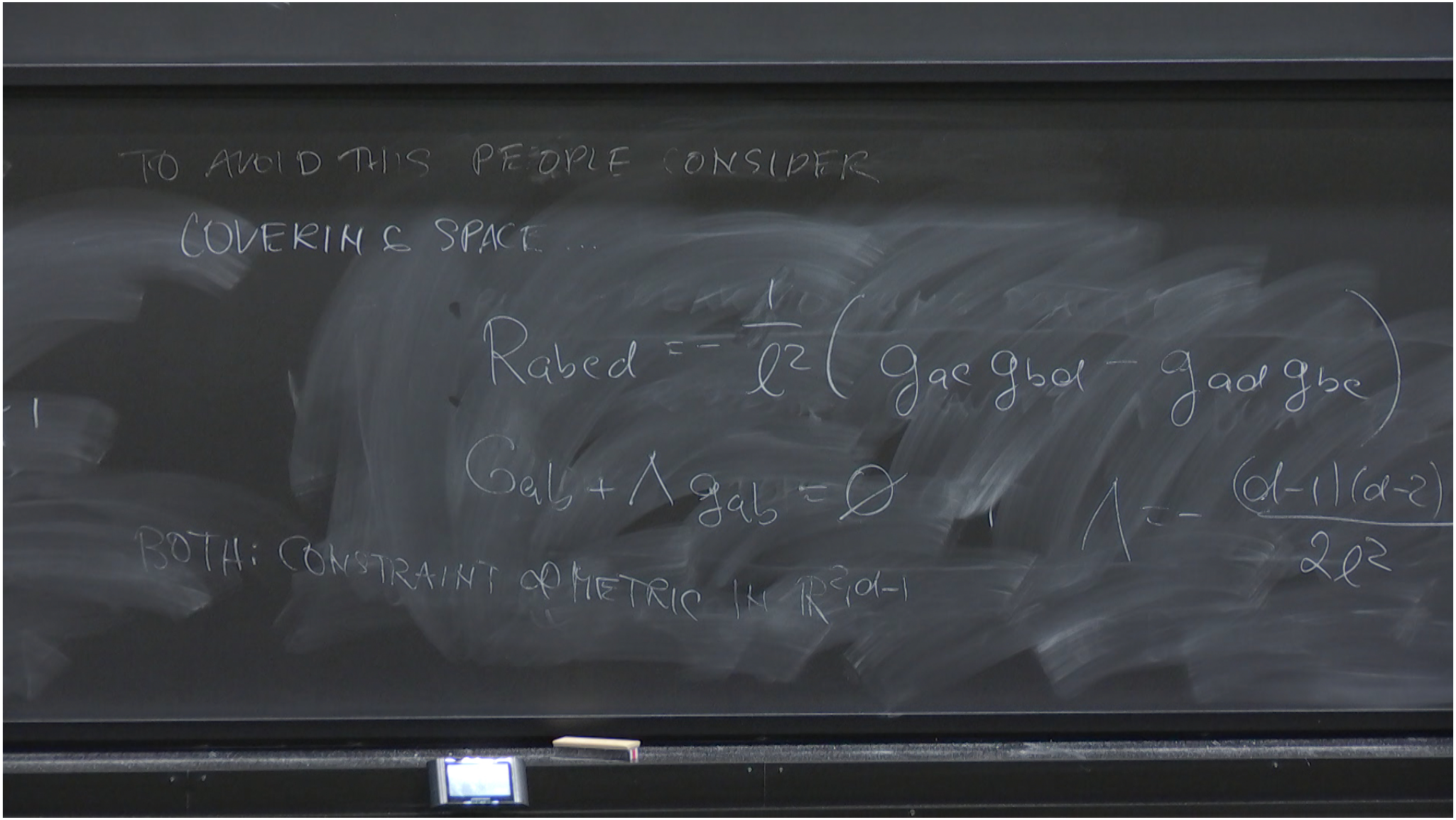
CFTs: USEFUL:
- PHASE TRANSITIONS $\{\alpha_i, \beta_i, \ldots\} \leftrightarrow \{\Delta\}$
- STRING THEORY
- MAPPING SPACE OF QFT's.

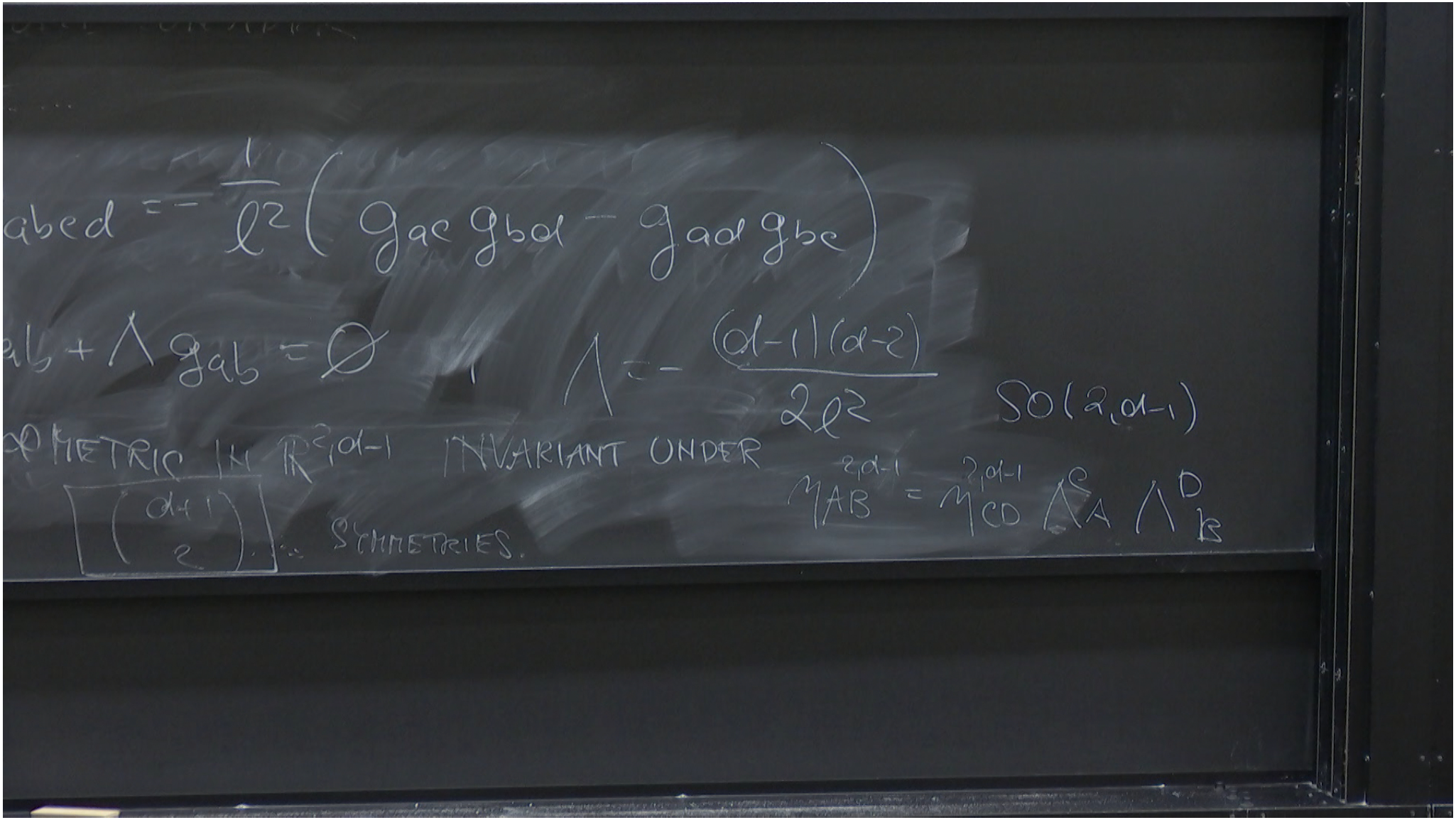

AdS PRIMER
AdS$_d$ IS A MAXIMALLY SYMMETRIC SOL.
OF EINSTEIN EQS. WITH $\Lambda < 0$, SYMMETRY $O(d-1,2)$.


AdS$_d$ AS MAXIMALLY SYMMETRIC SPACE.
TAKE $\mathbb{R}^{2,d-1}$
$ds^2 = -dY_{-1}^2 - dY_0^2 + dY_1^2 + \ldots + dY_{d-1}^2 = \eta^{2,d-1}_{AB} d$
$-\ell^2 = -Y_{-1}^2 - Y_0^2 + Y_1^2 + \ldots + Y_{d-1}^2 = \eta^{2,d-1}_{AB} Y^A Y^B$
"HYPERBOLOID"


$\Rightarrow Y_{-1} = \sqrt{l^2 + \eta^{1,d-1}_{ab} Y^a Y^b}$
$dY_{-1} = \frac{1}{Y_{-1}} \eta^{ab} Y_a dY_b$
$= \eta^{2,d-1}_{AB} dY^A dY^B$
$\eta^{2,d-1}_{AB} Y^A Y^B$
$\Rightarrow \boxed{g_{ab} = \eta_{ab} - \frac{Y_a Y_b}{l^2 + \eta^{ab} Y_a Y_b}}$


PICTURE : AdS 2
$Y^0 = t'$
Y^1
$Y^{-1} = t'$
CLOSED TIMELIKE CURVE!


TO AVOID THIS PEOPLE CONSIDER
COVERING SPACE ...
$R_{abcd} = -\frac{1}{\ell^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$
$G_{ab} + \Lambda g_{ab} = 0$
$\Lambda = -\frac{(d-1)(d-2)}{2\ell^2}$
BOTH: CONSTRAINT ON METRIC IN $\mathbb{R}^{2,d-1}$


$R_{abcd} = -\frac{1}{\ell^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$
$R_{ab} + \Lambda g_{ab} = 0$
$\Lambda = -\frac{(d-1)(d-2)}{2\ell^2}$
$SO(2,d-1)$
METRIC IN $\mathbb{R}^{2,d-1}$ INVARIANT UNDER
$\eta^{2,d-1}_{AB} = \eta_{CD}\Lambda^C_A\Lambda^D_B$
$\binom{d+1}{2}$ SYMMETRIES.