

Title: AdS/CFT Lecture (230417)

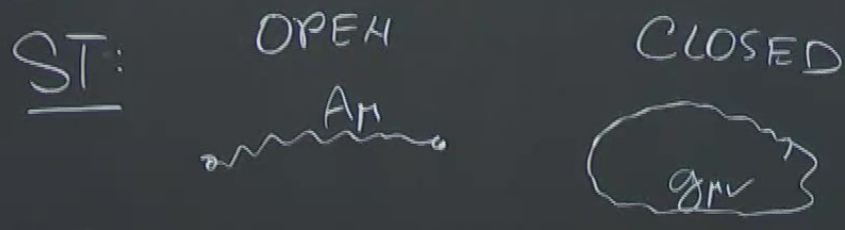
Speakers: David Kubiznak

Collection: AdS/CFT (2022/2023)

Date: April 17, 2023 - 9:00 AM

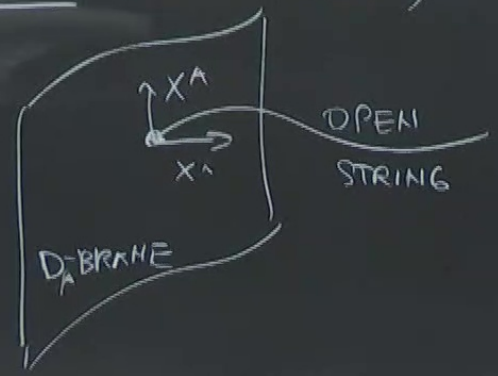
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# MOTIVATING AdS/CFT CORRESPONDENCE



$$l_s, g_s = \alpha' \phi$$

## D-BRANES (DIRICHLET)



$X^A$  ... NEUMANN DIRECTIONS ( $A=0, \dots, p$ )

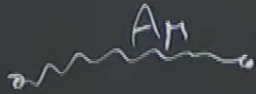
$X^a$  ... DIRICHLET DIRECTIONS ( $a=1, \dots, 9-p$ )

$A_M$   $\left\{ \begin{array}{l} A_A \dots \text{VECTOR FIELD ON D-BRANE} \\ \Phi_a \dots \text{SCALARS} \end{array} \right.$

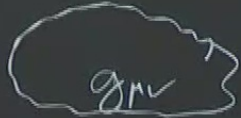
# MOTIVATING AdS/CFT CORRESPONDENCE

ST:

OPEN

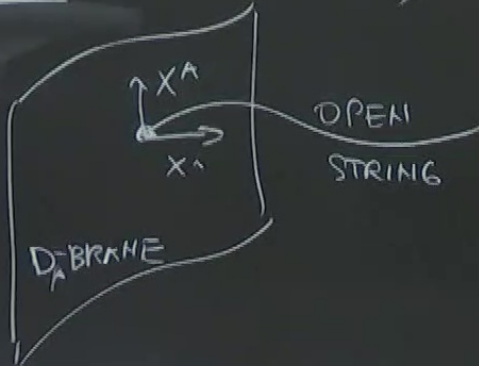


CLOSED



$$l_s, g_s = \alpha' \phi$$

D-BRANES (DIRICHLET)



$X^A$  ... NEUMANN DIRECTIONS ( $A=0, \dots, p$ )

$X^i$  ... DIRICHLET DIRECTIONS ( $i=1, \dots, g-p$ )

$A_M$   $\left\{ \begin{array}{l} A_A \dots \text{VECTOR FIELD ON D-BRANE} \\ \Phi_i \dots \text{SCALARS} \end{array} \right.$

• MOTION OF D-BRAVE GOVERNED BY DBI-ACTION.

$$S_{\text{DBI}} = -T_{\text{D}p} \int d\Sigma^{p+1} e^{-\phi} \sqrt{-\det(g_{AB}(g) + g_{AB}(B) + 2\pi\alpha' F_{AB})}$$

$$T_{\text{D}p} = \frac{1}{(2\pi)^p l_s^{p+1}}$$

• NOW CONSIDER MINKOWSKI,  $B=0$ ,  $e^{\phi} = g_s = \text{CONST}$

D-BRAVE

$$\mu_p = \frac{T_p}{g_s} \dots \text{"EFFECTIVE TENSION"}$$

$$\det(1+M) = 1 - \frac{1}{2} \text{Tr} M^2 + \dots$$

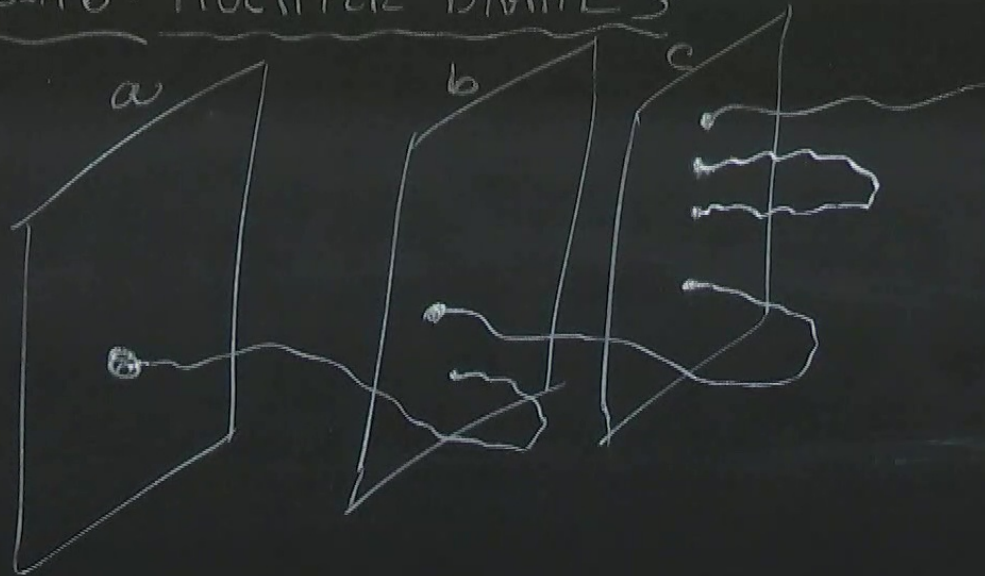
$$S_{\text{DBI}} \approx - \left( 2\pi \alpha' \right)^2 \frac{T_p}{g_s} \int d^{p+1} \xi F_{AB} F^{AB} + \dots$$

$$\alpha'^2 \sim \frac{1}{l_s^4} \quad \sim \frac{1}{g_{\text{YM}}^2}$$

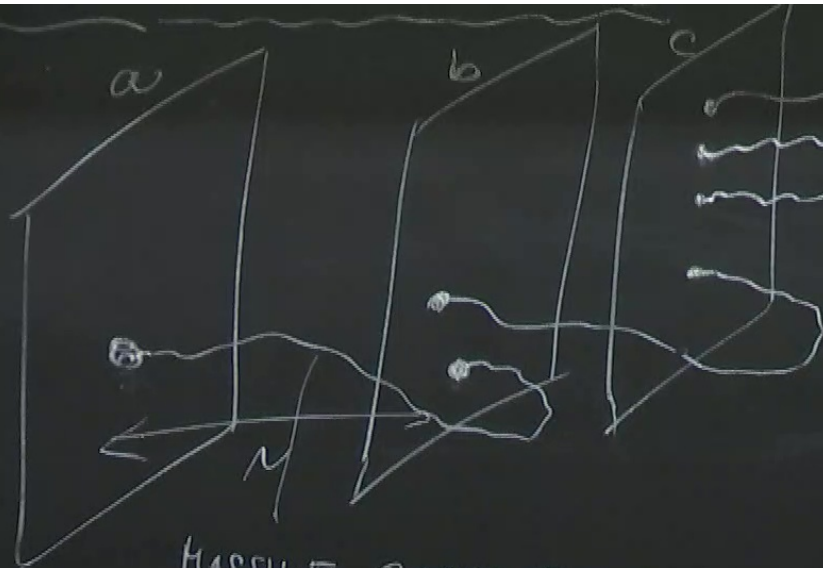
$$T_p \sim \frac{1}{l_s^{p+1}}$$

$$g_{\text{YM}}^2 = (2\pi)^{p-2} g_s l_s^{p-3}$$

# CARTOON 6: MULTIPLE BRANES



$S^1 \times S^{p-3}$



MASSIVE GAUGE FIELD  
 WITH  $m \propto \frac{1}{2\pi\alpha'}$

MASSLESS  $U(1)$  AA FIELD

MORE D-BRANES

WE HAVE EXTRA INDEX

$$(AA)^a_b$$

NON-ABELIAN  
 GAUGE TH.

SPECIFICALLY: CONSIDER  $N_c$  NUMBER OF  
COALESCENT D3 BRANES IN TYPE IIB  
ST. ... GIVES RISE TO  $\mathcal{N}=4$   $U(N_c)$   
SYM IN (3+1) DIMENSIONS



$$\mathcal{L} = -\frac{1}{g_{YM}^2} \text{Tr} \left( \frac{1}{4} F^{AB} F_{AB} + \frac{1}{2} D_A \phi^i D^A \phi^i + [\phi^i, \phi^j] \right)$$

+ FERMIONS,  $g_{YM}^2 = 2\pi g_s$

ALTERNATIVELY: D3 BRANES CAN BE VIEWED AS AS  
EXTREMAL BH SOLUTIONS IN IIB SUPERGRAVITY.

ASSUME ONLY  $g_{\mu\nu}$ ,  $F_5$ ,  $\phi = \text{CONST.}$

$$\Rightarrow R_{\mu\nu} = \frac{1}{96} F_{\mu\nu\alpha\beta\gamma\delta} F_{\alpha\beta\gamma\delta}$$

$$F_5 = *F_5$$

AS  
GRAVITY.

BLACK BRANE SOLUTION (NEAR EXTREMAL RN SOLUTION)

$$ds_{10}^2 = H^{-1/2} \left( -f dt^2 + dx^2 + dy^2 + dz^2 \right) + H^{1/2} \left( \frac{dn^2}{f} + n^2 ds_5^2 \right)$$



AS  
 REGRAVITY. BLACK BRANE SOLUTION (NEAR EXTREMAL RN SOLUTION)

$$ds_{10}^2 = H^{-1/2} \left( -f dt^2 + dx^2 + dy^2 + dz^2 \right) + H^{1/2} \left( \frac{dr^2}{f} + r^2 d\Omega_5^2 \right)$$

$$F_5 = -\frac{4L^2}{H^2 r^5} \sqrt{r_0^4 + L^4} (1 + *) dt \wedge dx \wedge dy \wedge dz \wedge dr$$



$$F_5 = -\frac{4L^2}{H^2 \lambda^5} \sqrt{a_0^4 + L^4} (1 + *) dt dx dy dz dr$$

$$H = 1 + \frac{L^4}{\lambda^4}, \quad f_0 = 1 - \frac{a_0^4}{\lambda^4}$$

2 HORIZONS  $\Lambda = 0$ ,  $\Lambda = 10$   
 $\uparrow$   
OUTER.

$\phi$  ... GRAVITATION

FOCUS ON EXTREMAL CASE

D3 BRANE ( $\Lambda_0 = 0$ )

$$\underline{f_0 = 1}$$

$$\Lambda = \Lambda_{\text{OUTER}}$$

\*  $\phi$ ... GRAVITATIONAL POTENTIAL

$$\phi \sim \frac{G_{10} M_{\text{TOT}}}{r^{d-p-3}} = \frac{G_{10} M_{\text{TOT}}}{r^4}$$

•  $\phi$ ... GRAVITATIONAL POTENTIAL

$$\phi \sim \frac{G_{10} M_{\text{TOT}}}{r^{d-p-3}} = \frac{G_{10} M_{\text{TOT}}}{r^4} \sim$$

$$\frac{\overset{G_{10}}{g_s^2 l_s^8}}{\overset{M_{\text{TOT}}}{N c \mu_3}} \sim \frac{14}{14}$$



# GRAVITATIONAL POTENTIAL

$$\sim \frac{G_{10} M_{\text{TOT}}}{r^{d-p-3}} = \frac{G_{10} M_{\text{TOT}}}{r^4} \sim \frac{\overset{G_{10}}{g_s^2 l_s^8} \overset{M_{\text{TOT}}}{N c \mu_3}}{r^4} \sim \frac{g_s^2 l_s^8}{r^4} N c \frac{1}{l_s^4}$$

$$\sim \frac{G_{10} M_{\text{tot}}}{\ell^{d-p-3}} = \frac{G_{10} M_{\text{tot}}}{\ell^4} \sim \frac{g_s^2 \ell^8}{\ell^4} (N_c \mu^3) \sim \frac{g_s^2 \ell^8}{\ell^4} N_c \frac{1}{\ell^4 g_s}$$

$$\sim (g_s N_c) \frac{\ell^4}{\ell^4}$$

$\lambda \dots$  'T' HOOFT COUPLING

2 HORIZONS  $\Lambda=0$ ,  $\Lambda=\Lambda_{\text{OUTER}}$   
FOCUS ON EXTREMAL CASE

D3 BRANE ( $\Lambda_0=0$ )

$$\frac{f_0}{f_0} = 1$$

①  $\lambda \ll 1$  ... MINK.

②  $\lambda \gg 1$  ... STRONG GRAVITY REGIME.

\*  $\phi$ ... GRAVITATION

$$\phi \sim \frac{G_{10} M_{\text{TOT}}}{r^{d-p-3}}$$

2 HORIZONS  $\Lambda = 0$ ,  $\Lambda = \Lambda_0$   
 $\Lambda_0$  OUTER.

US ON EXTREMAL CASE

D3 BRANE ( $\Lambda_0 = 0$ )

$$\frac{f_0}{f_0} = 1$$

①  $\lambda \ll 1$

MINK. ... "OPEN STRING"

②  $\lambda \gg 1$

... STRONG GRAVITY REGIME

... "CLOSED STRING" ... BH PICTURE

\*  $\phi$  ... GRAVITATIONAL POTENTIAL

$$\phi \sim \frac{G_{10} M_{\text{TOT}}}{\Lambda^{d-p-3}} = \frac{G_{10} M}{\Lambda^4}$$

$$H = 1 + \frac{L^4}{\Lambda^4} = 1 + \phi = 1 + \lambda \frac{\rho_S^4}{\Lambda^4}$$

$$\frac{L^4}{\rho_S^4} \sim \lambda$$

$$\lambda = g_{YM}^2 N_c \propto g_S N_c$$

$$H = 1 + \frac{L^4}{\Lambda^4} = 1 + \phi = 1 + \lambda \frac{\rho_s^4}{\Lambda^4}$$

$$\frac{L^4}{\rho_s^4} \sim \lambda$$

$$\lambda = g_{YM}^2 N_c \propto g_s N_c$$

$$|\rho_s, g_s|$$

• CONSIDER NEAR HORIZON LIMIT OF D3 BRANE.

$$\Lambda \rightarrow 0 \rightarrow H \sim \frac{L^4}{\Lambda}$$

$$ds_{10}^2 \simeq \frac{\Lambda^2}{L^2} \underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{-dt^2 + dx^2 + dy^2 + dz^2} + \frac{L^2}{\Lambda^2} dr^2 + \frac{L^2}{\Lambda^2} d\Omega_5^2$$

$$z = \frac{L^2}{\Lambda}$$

$$ds_{10}^2 = \frac{L^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right) + L^2 d\Omega_5^2$$

AdS<sub>5</sub> OF RADIUS L

S<sup>5</sup> OF RADIUS L

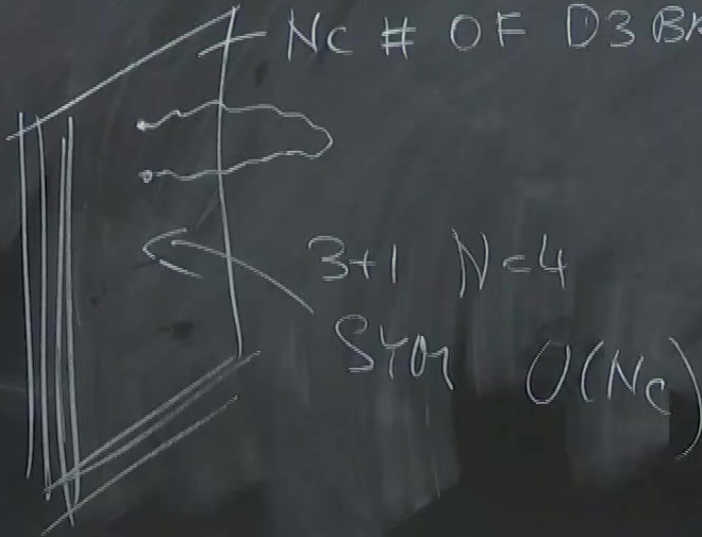
AdS<sub>5</sub> + S<sup>5</sup> GEOMETRY.



# CARTOON 7: AdS/CFT CONJECTURE

OPEN STRING ( $\lambda \ll 1$ )

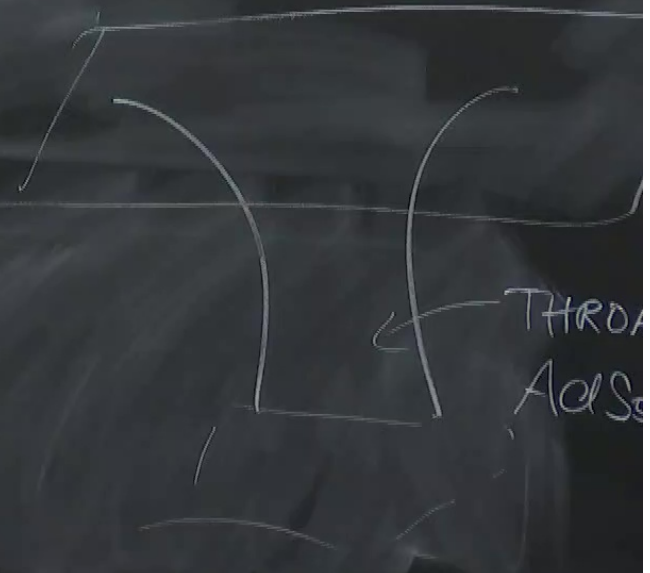
$N_c$  # OF D3 BRANES



3+1 N=4

SYM  $U(N_c)$

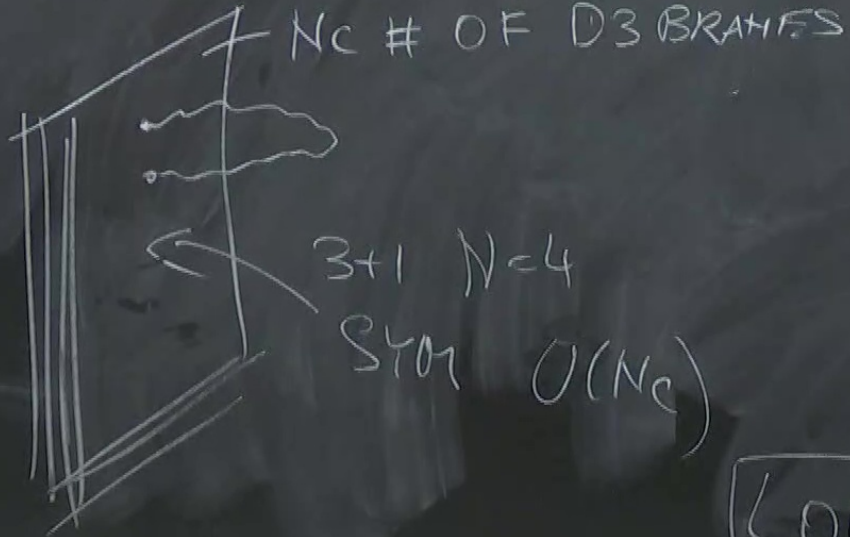
CLOSED STRING ( $\lambda$ )



THROUGH  
AdS

# ARTDUM 7: AdS/CFT CONJECTURE

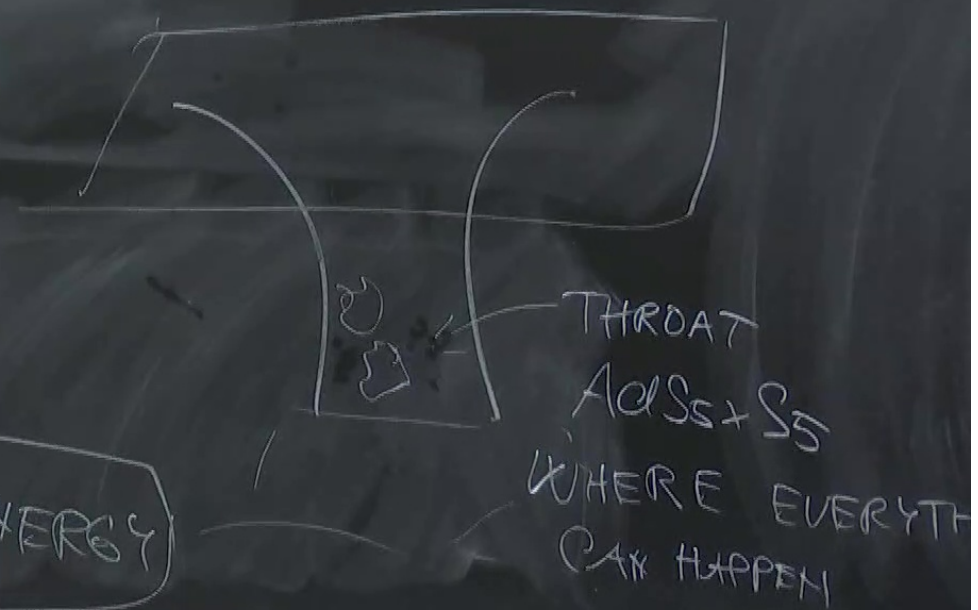
OPEN STRING ( $\lambda \ll 1$ )  
 $N_c$  # OF D3 BRANES



3+1 N=4  
SYM  $O(N_c)$

LOW ENERGY

CLOSED STRING ( $\lambda \gg 1$ )



THROAT  
AdS<sub>5</sub> x S<sup>5</sup>  
WHERE EVERYTHING  
CAN HAPPEN

CONJECTURE (MALDACEA 97): TYPE IIB SUPERSTRING  
ON  $AdS_5 \times S^5$  IS DUAL TO  $N=4$   $SU(N_c)$   
SYM IN  $d=(3+1)$  DIMS

1)

AT  
 $S^5$   
EVERYTHING  
HAPPEN