Title: AdS/CFT Lecture (230403)

Speakers: David Kubiznak

Collection: AdS/CFT (2022/2023)

Date: April 03, 2023 - 9:00 AM

URL: https://pirsa.org/23040030

ADS/CFT CORRESPONDENCE (GRAVITY & APPLICATIONS

A) LESSONS FROM BH TDS

CHARACTERISTICS OF BH - SCHWARZSCHILD

$$ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2 \,, \quad d\Omega^2 = \sin^2\theta\, d\varphi^2 + d\theta^2$$

$$f = 1 - \frac{2M}{r}$$

APPLICATIONS SIDE)

• <u>HORIZON</u>: LOCATED AT $r = r_+ = 2M$

$$\left(f(r_+) = 0 \right)$$

D

• ASYMPTOTIC MASS $\quad M = \frac{r_+}{2}$

CORRESPONDS TO TIME TRANSL. SYMMETRY

$$k = \frac{\partial}{\partial t}$$

$\ldots in^2\theta\, d\varphi^2 + d\theta^2$

$(f(r_+) = 0)$

- ASYMPTOTIC MASS $\quad M = \frac{r_+}{2}$

CORRESPONDS TO TIME TRANSL. SYMMETRY

$$k = \frac{\partial}{\partial t}$$

$$M = -\frac{1}{8\pi} \int_S$$

$\ldots\sin^2\theta\, d\varphi^2 + d\theta^2$

$$(f(r_+) = 0)$$

- ASYMPTOTIC MASS $\quad M = \frac{r_+}{2}$

CORRESPONDS TO TIME TRANSL. SYMMETRY

$$k = \frac{\partial}{\partial t}$$

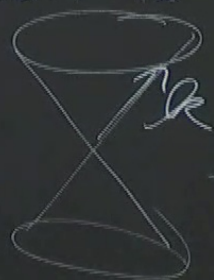
$$M = -\frac{1}{8\pi} \int_S * dk^b = M$$

D

$\sin^2\theta \, d\varphi^2 + d\theta^2$

• SURFACE GRAVITY ("GRAVITAT. ACCEL. ON HORIZON")

HORIZON IS GENERATED BY NULL KV $\underline{k}$

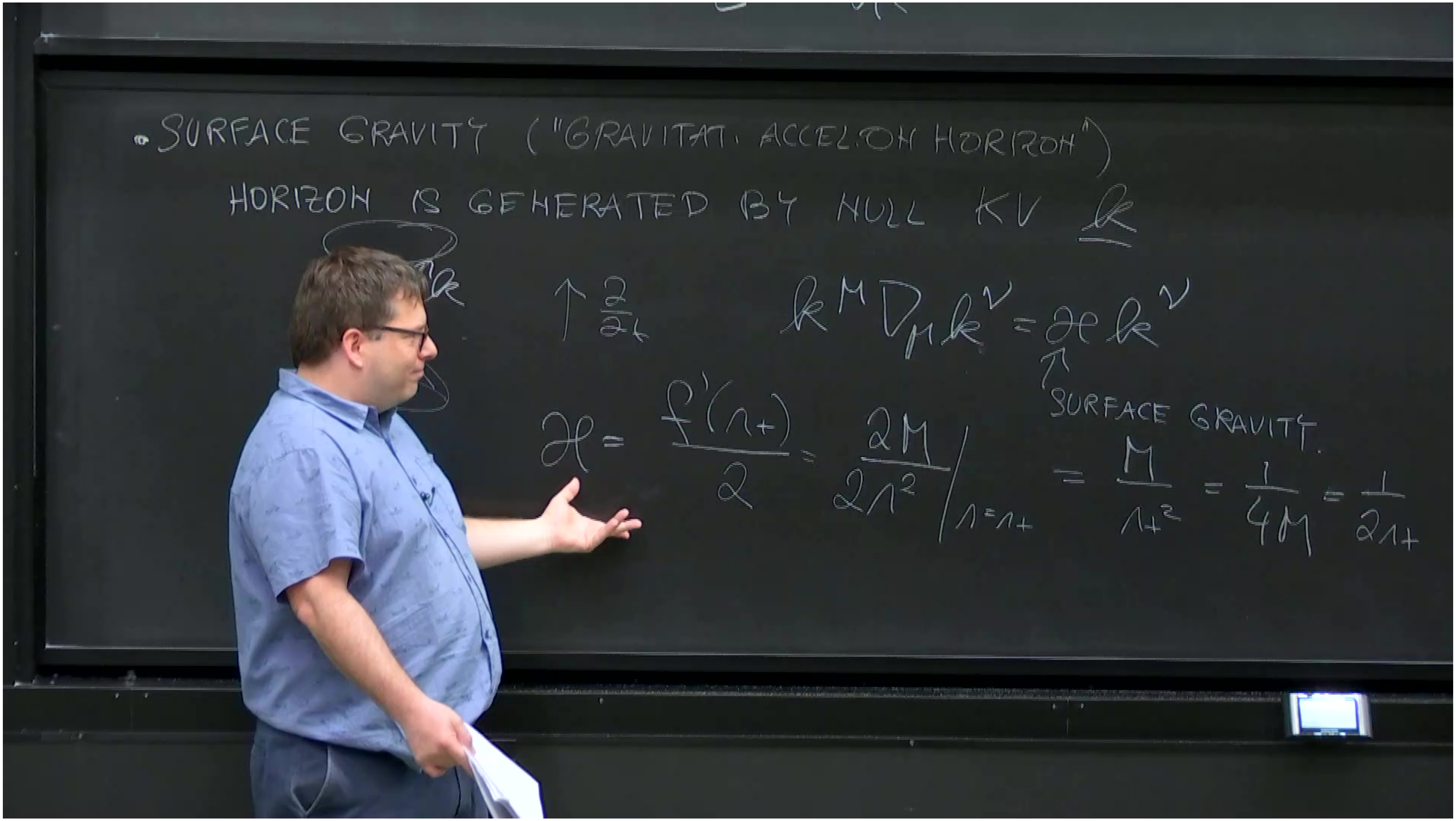


$\uparrow \frac{\partial}{\partial t}$

$$k^\mu \nabla_\mu k^\nu = \kappa k^\nu$$

SURFACE GRAVITY

$$\kappa = \frac{f'(r_+)}{2} = \frac{2M}{2r^2}\Big|_{r=r_+} = \frac{M}{r_+^2}$$


SURFACE GRAVITY ("GRAVITAT. ACCEL. ON HORIZON")
HORIZON IS GENERATED BY NULL KV k
∂/∂t
k^μ D_μ k^ν = ϰ k^ν
SURFACE GRAVITY.
ϰ = f'(r+)/2 = 2M/2r² |r=r+ = M/r+² = 1/4M = 1/2r+

("GRAVITAT. ACCEL. ON HORIZON")

HERATED BY NULL KV k

$\uparrow \frac{\partial}{\partial t}$ $\quad k^\mu D_\mu k^\nu = \varkappa k^\nu$

SURFACE GRAVITY.

$$\varkappa = \frac{f'(r_+)}{2} = \frac{2M}{2r^2}\Big|_{r=r_+} = \frac{M}{r_+^2} = \frac{1}{4M} = \frac{1}{2r_+}$$

COMPARE TO NEWTON

$$\varkappa_N = \frac{M}{r_+^2} \checkmark$$

ZON")

$\underline{k}$

$= \varkappa k^\nu$

SURFACE GRAVITY.

$= \frac{M}{r_+^2} = \frac{1}{4M} = \frac{1}{2r_+}$

COMPARE TO NEWTON

$$\varkappa_N = \frac{M}{r_+^2} \quad ✓$$

- $\underline{\text{HORIZON AREA}}$: $t = \text{CONST}, \; r = r_+ = \text{CONST}$.

$$ds^2 \to d\gamma^2 = r_+^2 d\Omega^2$$

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- HORIZON AREA: $t = \text{CONST}, \; r = r_+ = \text{CONST}.$

$$ds^2 \to d\gamma^2 = r_+^2 d\Omega^2$$

$$A = \int \sqrt{\det(\gamma)}\, d\theta\, d\varphi = 4\pi r_+^2$$

• INGENIOUS IDEA:

$$dA = 8\pi\, r_+\, dr_+$$

$$dM = \frac{1}{2}\, dr_+$$

$$\Rightarrow \boxed{dM = \frac{\kappa}{2\pi} \frac{dA}{4}}$$

$\ldots \lambda = \lambda_+ \quad \lambda_+^2 \quad 4M \quad 2\lambda_+ \qquad A = \int \sqrt{\det(\gamma)}\, d\theta\, d\varphi = 4\pi\lambda\ldots$

- **BCH - 1973** **LAWS OF BLACK HOLE MECHANICS**

STATIONARY BHS. BORING: DESCRIBED BY M (MASS), J, Q (CHARGE)

0: κ = CONST ON THE HORIZON

1: $$\boxed{dM = \frac{\kappa}{2\pi}\frac{dA}{4} + \Omega\, dJ + \Phi\, dQ}$$

2:

3:

$$\Big|_{r=r_+} = \frac{M}{r_+^2} = \frac{1}{4M} = \frac{1}{2r_+} \qquad A = \int \sqrt{\det(\gamma)}\, d\theta\, d\varphi = 4\pi r_+^2$$

- BCH – 1973   LAWS OF BLACK HOLE MECHANICS

STATIONARY BHS. BORING: DESCRIBED BY M (MASS), J (ANG. MOM.) Q (CHARGE)

0: κ = CONST ON THE HORIZON

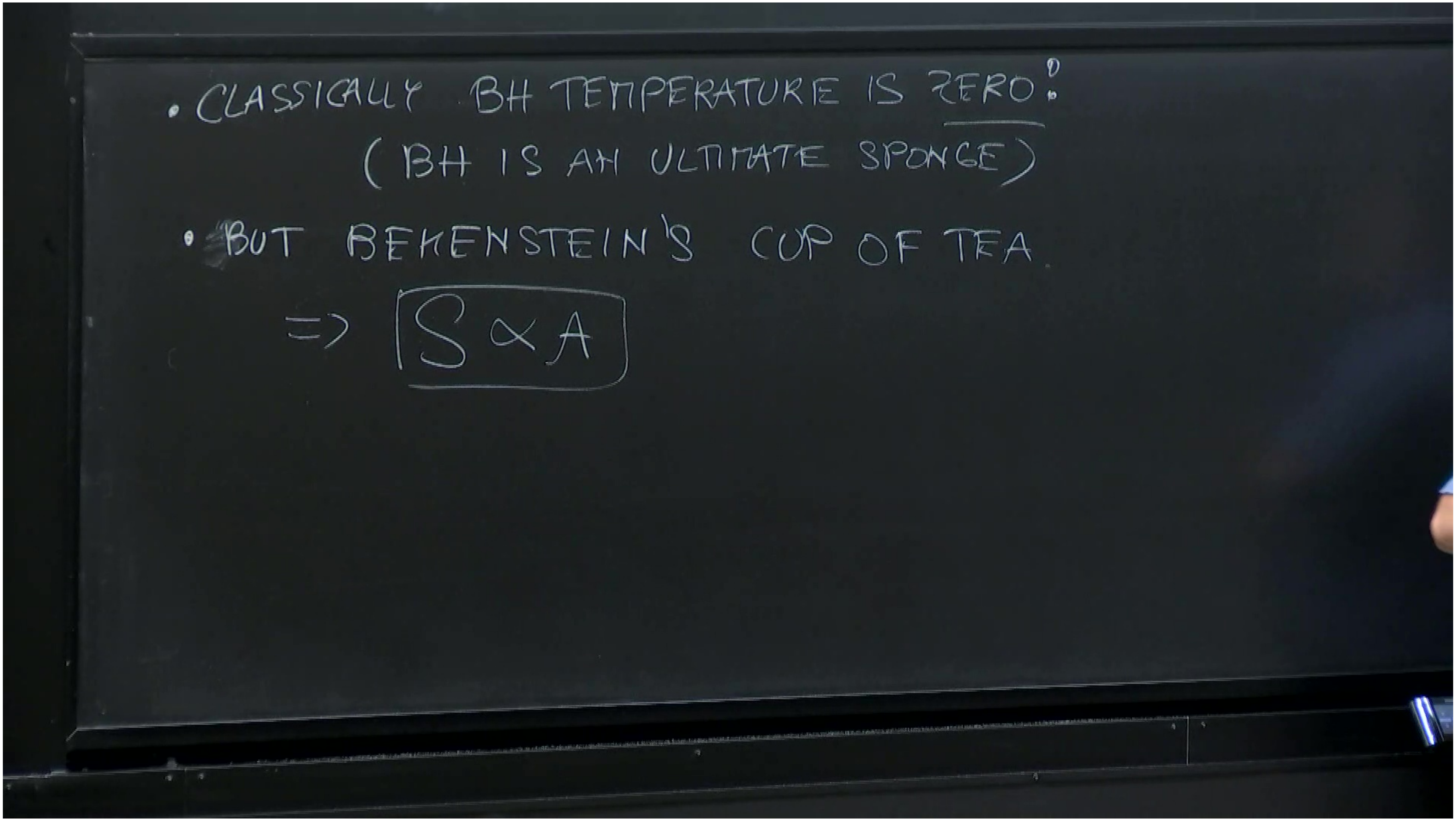
1: $\boxed{dM = \frac{\kappa}{2\pi}\frac{dA}{4} + \underbrace{\Omega\, dJ + \phi\, dQ}_{\text{WORK TERMS}}}$

2: $\boxed{dA \geq 0}$

3. $\kappa \not\to 0$ FINITE # OF STEPS

$\Omega = \frac{d\varphi}{dt}\Big|_H$ ... ANG. VEL.

ϕ ... EL STAT POT.


• CLASSICALLY BH TEMPERATURE IS ZERO!
(BH IS AN ULTIMATE SPONGE)
• BUT BEKENSTEIN'S CUP OF TEA
$\Rightarrow \boxed{S \propto A}$

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- PUT ON FIRM GROUND BY HAWKING 1974

$$T =$$

- BUT BEKENSTEIN'S CUP OF TEA

$$\Rightarrow \boxed{S \propto A}$$

- PUT ON FIRM GROUND BY HAWKING 1974

$$\boxed{T = \frac{\varkappa}{2\pi}} \ldots$$

HAWKING RAD

(QUANTUM)

- CLASSICALLY BH TEMPERATURE IS ZERO.

  (BH IS AN ULTIMATE SPONGE)

- BUT BEKENSTEIN'S CUP OF TEA

  $\Rightarrow \boxed{S \propto A}$

- PUT ON FIRM GROUND BY HAWKING 1974

  $\boxed{T = \frac{\varkappa}{2\pi}}$ ... HAWKING RAD $\Rightarrow \boxed{S = \frac{A}{4}}$

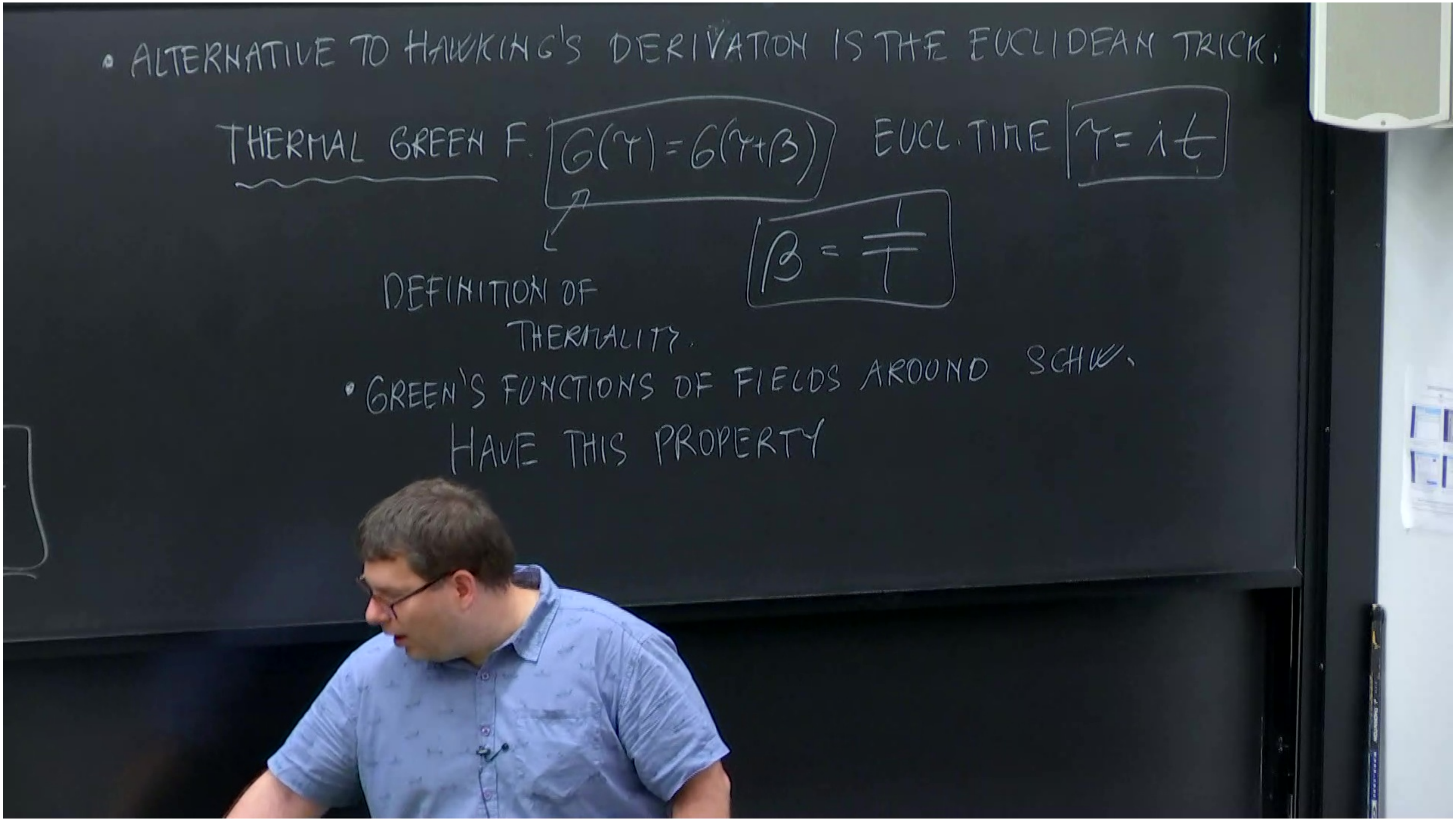
  (QUANTUM)

- ALTERNATIVE TO HAWKING'S DERIVATION IS THE EUCLIDEAN TRICK:

THERMAL GREEN F. $\boxed{G(\tau)=G(\tau+\beta)}$ EUCL. TIME $\boxed{\tau = i t}$

DEFINITION OF THERMALITY.

$$\boxed{\beta = \frac{1}{T}}$$


• ALTERNATIVE TO HAWKING'S DERIVATION IS THE EUCLIDEAN TRICK:
THERMAL GREEN F. $G(\tau) = G(\tau + \beta)$
EUCL. TIME $\tau = i t$
$\beta = \frac{1}{T}$
DEFINITION OF THERMALITY.
• GREEN'S FUNCTIONS OF FIELDS AROUND SCHW.
HAVE THIS PROPERTY

• WHAT ABOUT GEOMETRY ITSELF ?

$$ds_E^2 = + f\, d\tau^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

ZOOM ON THE HORIZON $r \to r_+$

$$f = \underbrace{f(r_+)}_{0} + (r - r_+)\, \underbrace{f'(r_+)} + \dots$$

• WHAT ABOUT GEOMETRY ITSELF?

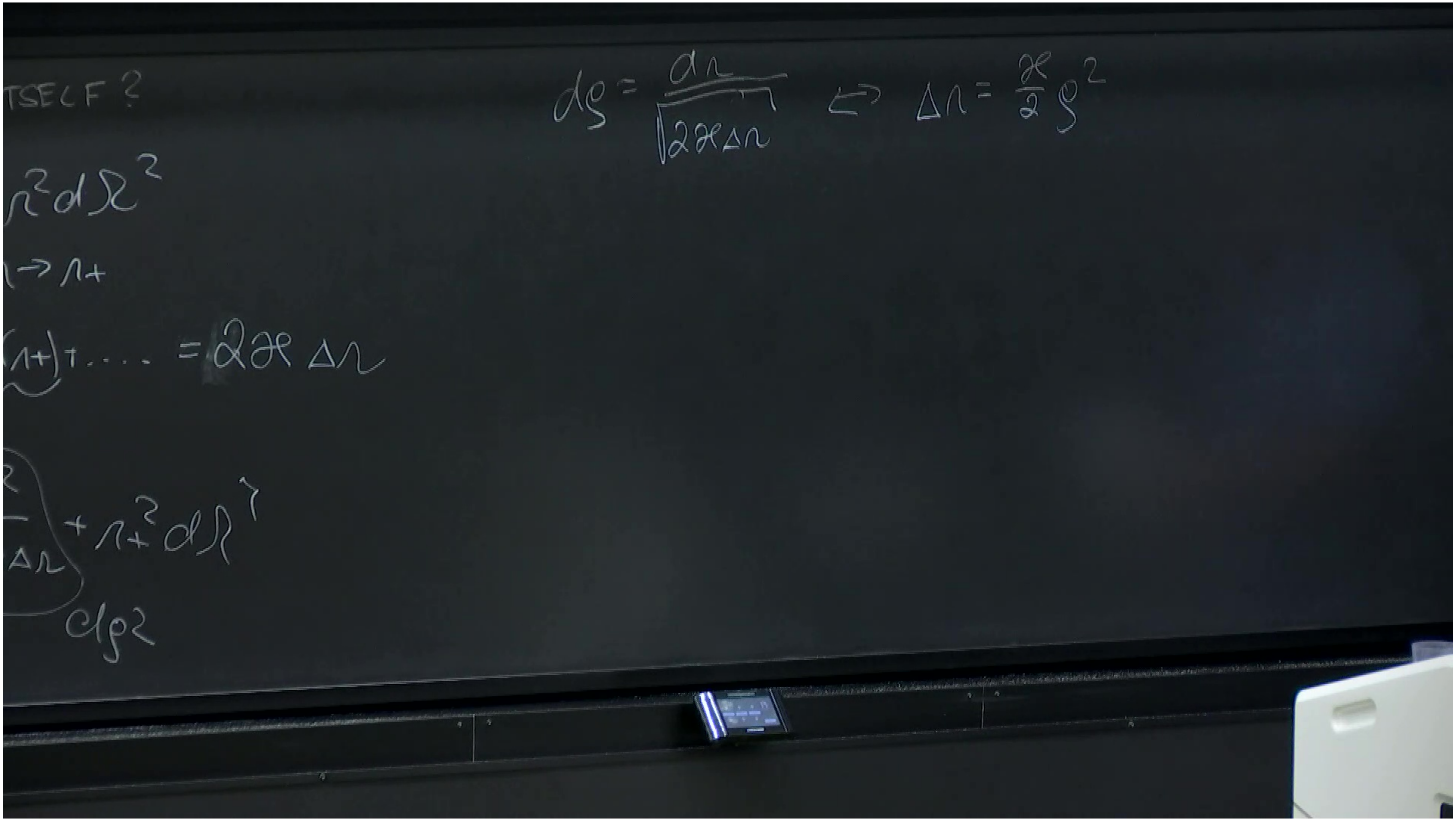
$$ds_E^2 = + f\, d\tau^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

ZOOM ON THE HORIZON $r \to r_+$

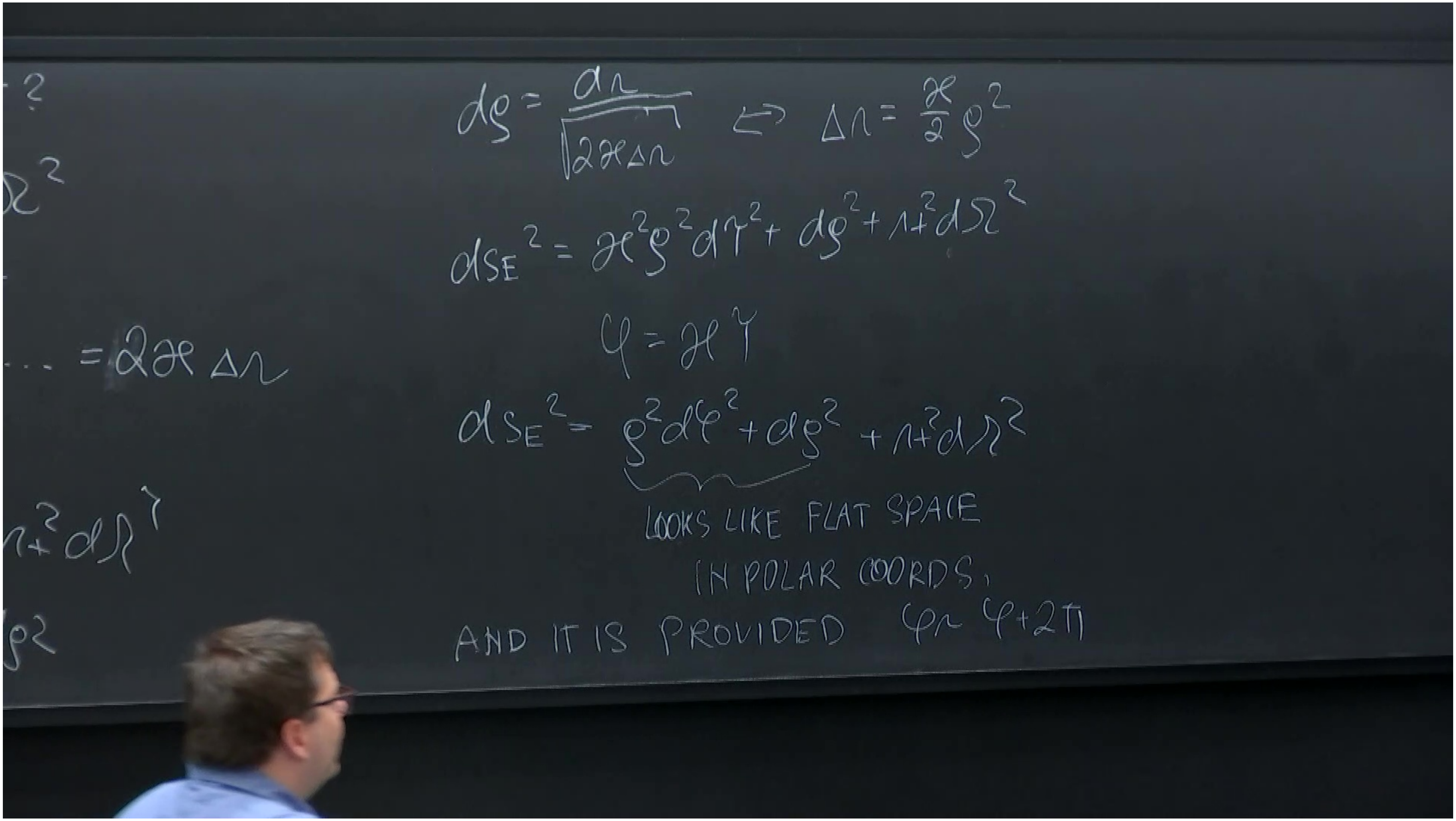
$$f = \underbrace{f(r_+)}_{0} + \underbrace{(r - r_+)}_{\Delta r} f'(r_+) + \ldots = 2\varkappa \Delta r$$

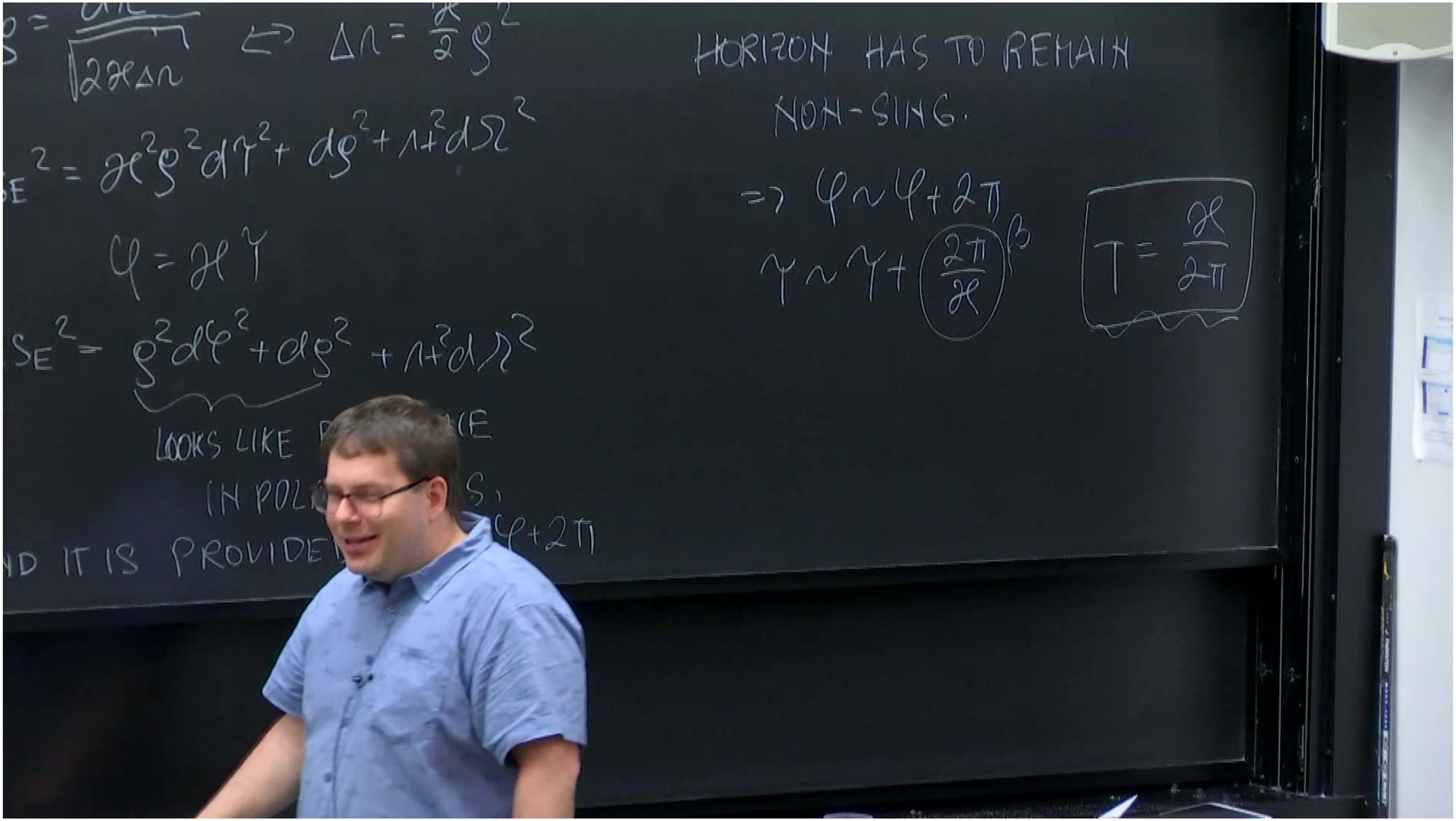
$$ds_E^2 = 2\varkappa \Delta r\, d\tau^2 + \underbrace{\frac{dr^2}{2\varkappa \Delta r}}_{d\rho^2} + r_+^2 d\Omega^2$$

$$d\rho = \frac{dr}{\sqrt{2\varkappa \Delta r}}$$



TSELF ?
$d\varrho = \frac{dr}{\sqrt{2\varkappa \Delta r}} \Leftrightarrow \Delta r = \frac{\varkappa}{2}\varrho^2$
$r^2 d\Omega^2$
$r \to r_+$
$(r_+) + \dots = 2\varkappa \Delta r$
$+ r_+^2 d\Omega^2$
$d\varrho^2$


$d\rho = \frac{dr}{\sqrt{2\varkappa \Delta r}} \leftrightarrow \Delta r = \frac{\varkappa}{2}\rho^2$
$ds_E^2 = \varkappa^2\rho^2 d\tau^2 + d\rho^2 + r_+^2 d\Omega^2$
$\varphi = \varkappa\tau$
$ds_E^2 = \underbrace{\rho^2 d\varphi^2 + d\rho^2}_{} + r_+^2 d\Omega^2$
LOOKS LIKE FLAT SPACE
IN POLAR COORDS,
AND IT IS PROVIDED $\varphi \sim \varphi + 2\pi$
$\dots = 2\varkappa \Delta r$



HORIZON HAS TO REMAIN
NON-SING.
LOOKS LIKE
$T = \frac{\varkappa}{2\pi}$

$$\boxed{T = \frac{\kappa}{2\pi} \frac{\hbar c^3}{k_B}}$$

ASTRO BHS: $6 \times 10^{-6}\,K$

ASTRO BHS. $6 \times 10^{-6}\,K \ll T_{CMB}$

BHs SIZE OF MOON $T \simeq T_{CMB}$

SMALL BHS ... CAN HAVE HUGE TEMP.

(LHC)

$$\frac{dM}{dt} \sim \sigma T^4 A \sim \frac{1}{M^2}$$

$$\boxed{t_{EVAP} \simeq \left(\frac{M}{M_S}\right)^3 \times 10^{71}\, s}$$

BH EVAP.

AP.