Title: Quantum Gravity Lecture (230427)

Speakers: Aldo Riello

Collection: Quantum Gravity (2022/2023)

Date: April 27, 2023 - 9:00 AM

URL: https://pirsa.org/23040025

Recap

- $\underline{\alpha} \in \Omega^{k,l}(M \times F)$ is general covariant (backgrnd indep) iff $\mathbb{L}_{\rho(\xi)} \underline{\alpha} = L_\xi \underline{\alpha} \quad \forall \xi \in \mathfrak{X}^1(M)$ (diffeo)

- In a general covariant theory ($\mathbb{L}_{\rho(\xi)} \underline{L} = L_\xi \underline{L}$)

we have: $i_{\rho(\xi)} \Omega_\Sigma = -\delta Q_\Sigma(\xi) + \int_\Sigma i_\xi \underline{E} + \int_{\partial\Sigma} i_\xi \underline{\theta}$ (1) (2)

(1) non zero only if ξ transverse to Σ & zero on-shell

(2) non zero only if ξ transverse to $\partial\Sigma$, does not vanish on shell, but zero if $\partial\Sigma = \emptyset$

Ref
Wal
the
qua

ovariant (backgrnd indep)

$\in \mathcal{E}'(M)$

diffeo)

$(D_{\varphi(\xi)} \underline{f} = L_\xi \underline{f})$

$\underline{\quad}(\xi) + \int_\Sigma i_\xi \underline{E} + \int_{\partial\Sigma} i_\xi \underline{\underline{\mu}}$

① ②

set to Σ

e to $\partial\Sigma$,

zero if $\partial\Sigma = \emptyset$

Ref

Wald 1999 "Gravitation thermodynamics and quantum theory"

(background indep)

)

$\underline{\mathcal{L}} = L_\xi \underline{\mathcal{L}}$)

$i_\xi \underline{E} + \int_{\partial\Sigma} i_\xi \textcircled{4}$

① ②

$\Sigma = \emptyset$

Ref

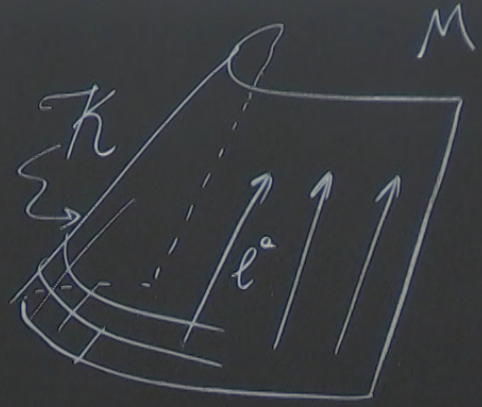
Wald 1999 "Gravitation thermodynamics and quantum theory"

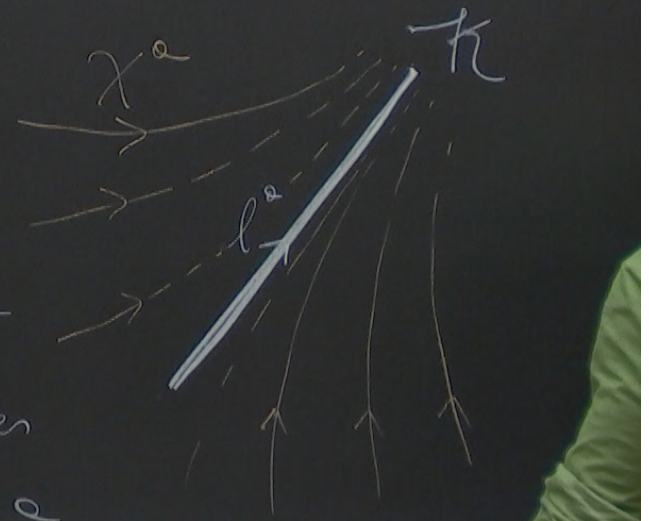
A BLACK HOLE DETOUR

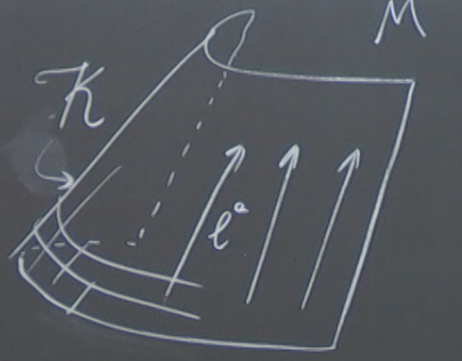
(M, g_{ab}) is a Lorentzian mfd

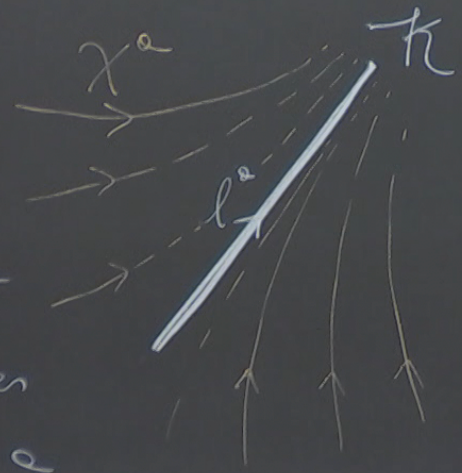
DEF [Killing horizon $\mathcal{K}$]

$\mathcal{K}$ is a null surface such that its null generator ℓ^a coincides with the restriction to $\mathcal{K}$ of a Killing v.f. χ^a on (M, g_{ab})









hat
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of a

Ex Rindler horizon in Minkowski

Rmk null vectors are weird!

ℓ^a is tangent to $\mathcal{K}$,

but it is also normal to $\mathcal{K}$.

↑ $\ell^a v_a = 0 \;\forall\, v^a$ tangent to $\mathcal{K}$

(bc ℓ^a is parallel & normal to itself)

$\Sigma = f^{-1}(0)$

v is tangent to Σ iff $i_v df = 0$

Consequence of Rmk:

χ^a is killing, $\chi^e \chi_{a;e}|_{\mathcal{K}} = 0$

$\hookrightarrow \nabla_a (\chi^b \chi_b) = \underline{-2\kappa} \chi_a$ at $\mathcal{K}$

DEF κ so defined is called the SURFACE GRAVITY of $\mathcal{K}$

Rmk: alternatively: $\chi^b \nabla_b \chi_a = \boxed{\kappa} \chi_a$ at $\mathcal{K}$

↑ "renormalized" acceleration of an observer along χ^a very close to $\mathcal{K}$

DEF [Bifurcate $\mathcal{K}$]

- $\mathcal{K}_1$, $\mathcal{K}_2$ killing hor. for the SAME χ^a
- $\mathcal{C} = \mathcal{K}_1 \cap \mathcal{K} \neq \emptyset$
  [codim 2 surface called the BIFURCATION SURFACE.

$\mathcal{C}_2$

...e at $\mathcal{K}$

...normalized"

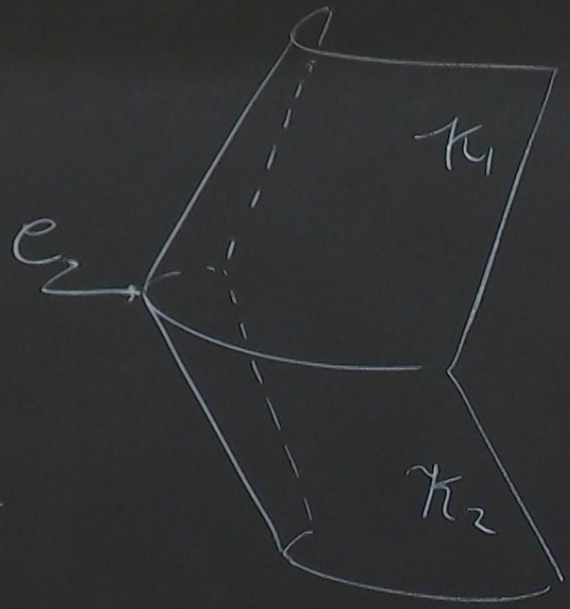
...bifurcation of

for the

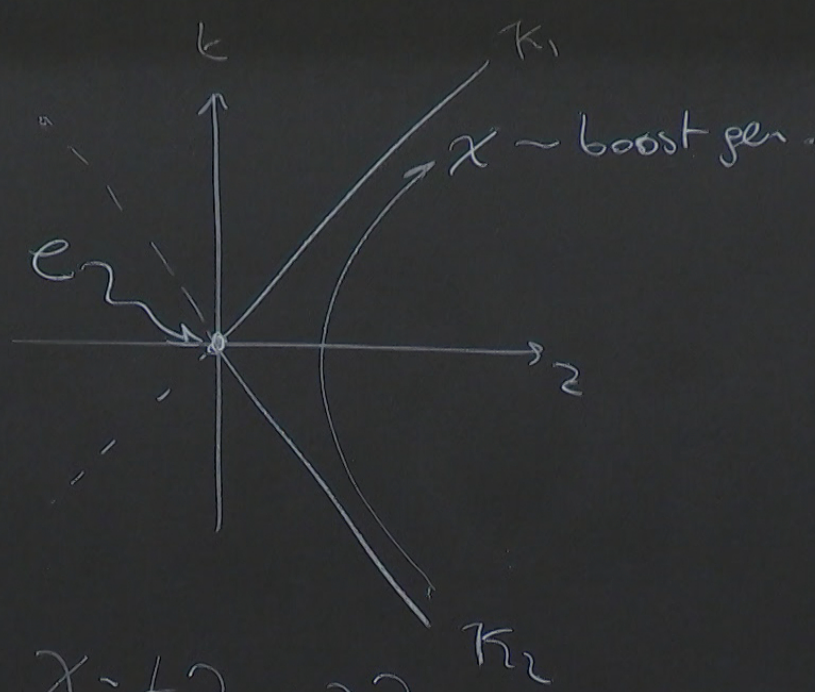
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RFACE –



Rindler



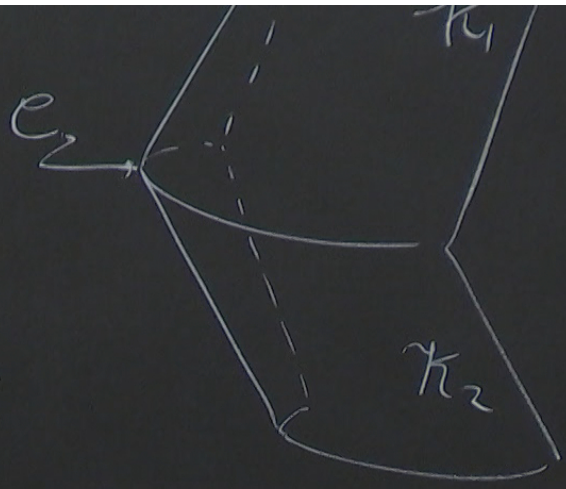
$\chi \sim t\partial_z + z\partial_t$

$= 0$ @ e

SAME χ^e

$C = \mathcal{K}_1 \cap \mathcal{K} \neq \emptyset$

↑ codim 2 surface called the BIFURCATION SURFACE.

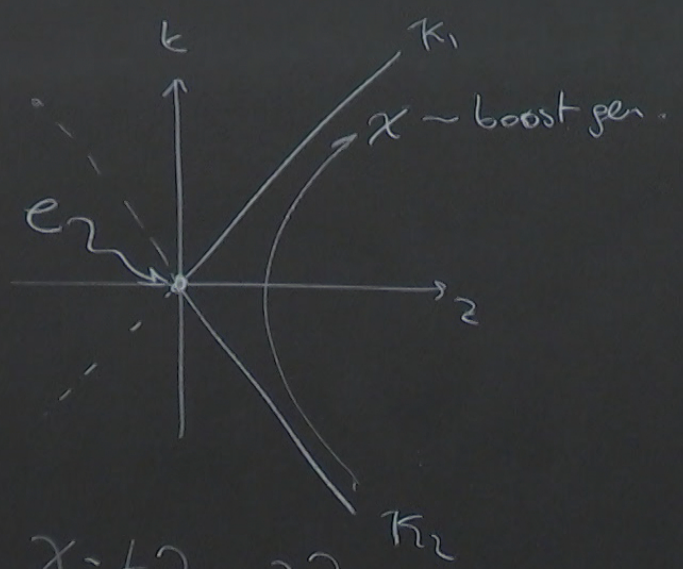


PROP: $\chi^e|_C = 0$.

PROP: if $\exists C$ codim 2 s.t. $\chi^e|_C = 0$

$\Rightarrow C$ bifurcation surface

Rindler.



$\chi = t\partial_z + z\partial_t$

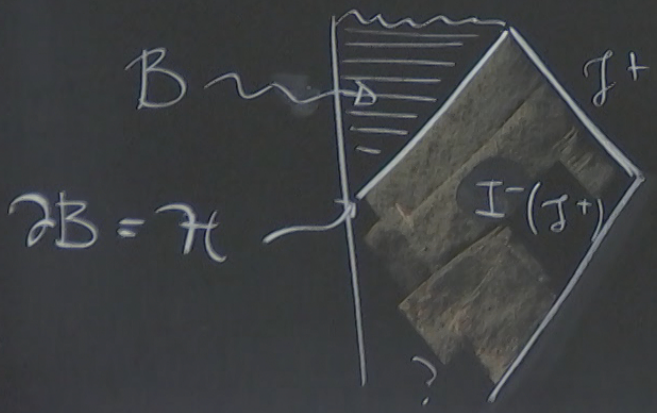
$= 0$ @ e

Killing horizons are relevant when studying stationary (late time) BH solutions

DEF: In an asymptotically flat spt. M, the black hole region is

$$\mathcal{B} := M \setminus I^-(\mathcal{I}^+)$$

(region that does not sit in the past of asympt. null infinity)



DEF: Bh horizon

$$\mathcal{H} = \partial \mathcal{B}$$

DEF: BH horizon

$\mathcal{H} = \partial \mathcal{B}$

Rigidity Theorems

Thm [Carter & Hawking]

$\mathcal{B}$ stationary $\Rightarrow$ $\mathcal{H}$ is a Killing horiz

and (in nice cases):

$$\chi^a = t^a + \omega_H \phi^a$$

angular vel of $\mathcal{H}$

t^a: Killing v.f. that at $\mathcal{J}^+$ coincides with Minkowski's ∂_t

ϕ^a: Killing v.f. $\rightarrow$ rotations at $\mathcal{J}^+$ 2π

Thm [Zeroth law, Carter & Bardeen]

- κ is constant on $\mathcal{H}$ (kin. statement)
- ——— h ——— on $\mathcal{H}$ (on shell of E.eq.)

$\rightsquigarrow$ κ is "thermalized" on $\mathcal{H}$ $\rightarrow$ temperature

↓

what happens if $\kappa = 0$?

DEF: BH extremal iff $\kappa = 0$

$\exists$ results that say that extremal BH are unstable (Dafermos)

t on $\mathcal{H}$ (kin. statement)
on $\mathcal{K}$ (on shell of E.eq.)

ted" on $\mathcal{K}$ → temperature
↓
what happens
if $\mathcal{K}=0$?

emal iff $K=0$

that extremal ⇝ we will not consider them
("not physical")

(Dafermos)

Thm [Racz & Wald]

$\mathcal{H} = \partial B$, $\kappa \neq 0$ then

the "maximal extended" spt.

representing a stationary BH

is s.t. $\mathcal{H}$ is a killing horizon

that bifurcates.



$\mathcal{H} = \partial D$, $n+$

the "maximal extended" spt.

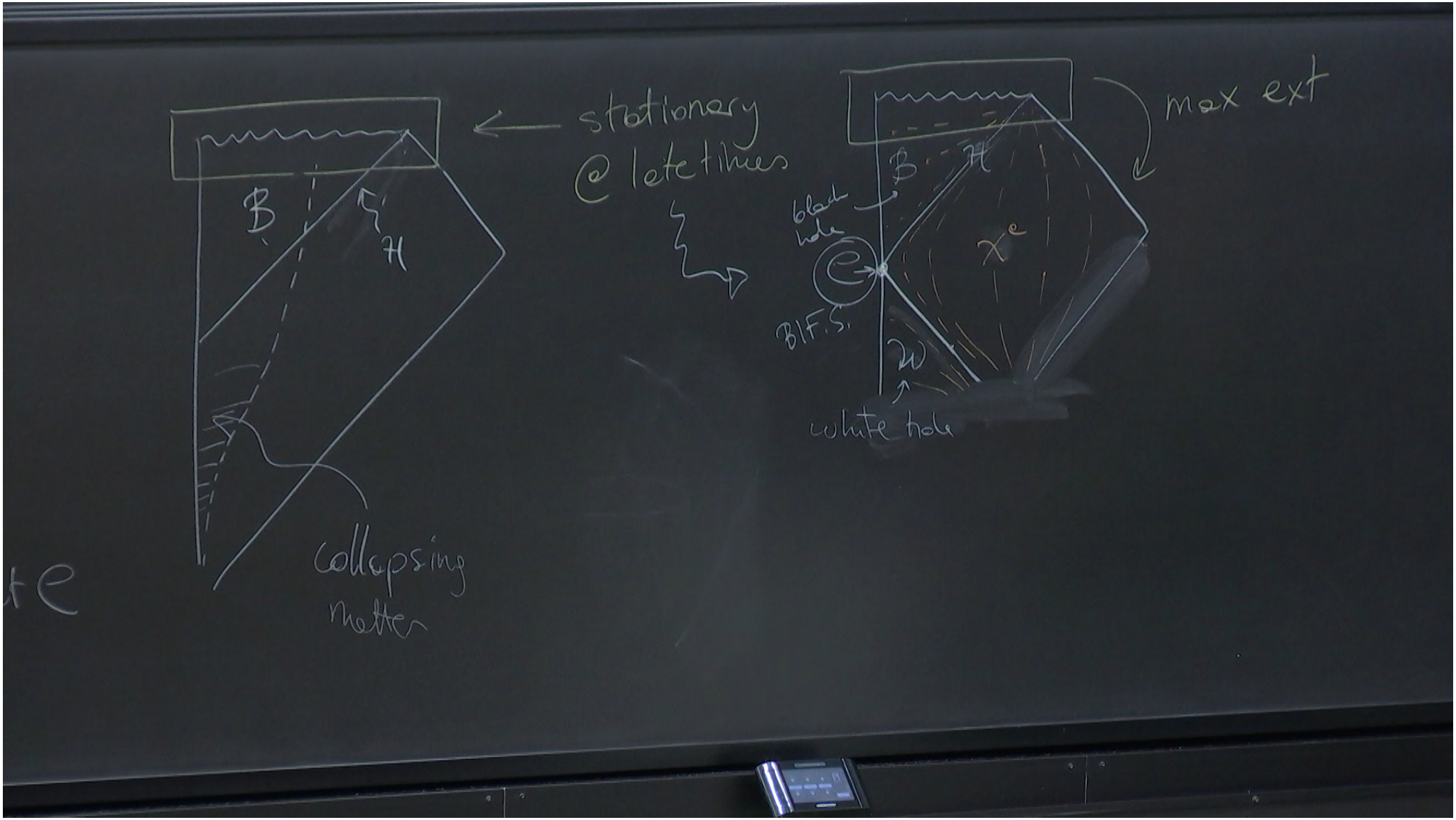
representing a stationary BH

is s.t. $\mathcal{H}$ is a Killing horizon

that bifurcates.

$$\chi^a|_{\mathcal{C}} = 0 \qquad \boxed{\nabla_a \chi_b = \kappa \varepsilon_{ab}} \text{ at } \mathcal{C}$$




stationary
@ late times
max ext
B
H
collapsing
matter
black
hole
B
H
$\mathcal{I}^+$
B.H.S.
$\mathcal{I}^-$
white hole