Title: Quantum Gravity Lecture (230425)

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Collection: Quantum Gravity (2022/2023)

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GENERAL RELATIVITY

$$\underline{L} = \left(\tfrac{1}{2}R - \Lambda\right)\underline{\epsilon}$$

$$\hookrightarrow \begin{cases} \underline{E} = -\frac{1}{2}\left(G_{ab} + \Lambda g_{ab}\right) \delta g^{ab}\, \underline{\epsilon} \\ \underline{\Theta} = -\frac{1}{2}\left(\nabla_b \delta g^{ab} - \nabla^a \delta g\right) \underline{\epsilon}_a \end{cases}$$

$$\underline{\epsilon} = \sqrt{g}\, dx^0 \wedge dx^1 \wedge \cdots \wedge dx^{n-1}$$

e.g. $(\cdots) \equiv J$

$$\underline{\epsilon}_a = i_{\partial_a} \underline{\epsilon}$$

$$\int_\Sigma (\cdots)^a \underline{\epsilon}_a = \int_\Sigma \sqrt{h}\, n_a (\cdots)^a = \int_M \sqrt{g}\, \nabla_a (\cdots)^a = \int_M d(\underline{\cdots}) = \int_{\partial\Sigma} (\underline{\cdots})$$

Noether current ?

First, though, GENERAL COVARIANCE
(aka background independence)

$$\mathbb{L}_{\rho(\xi)} \underline{L} = L_\xi \underline{L}$$

Ex : scalar field on a background spt :

$$\underline{L}_{SF}(\varphi) = \frac{1}{2}\sqrt{g}\, g^{ab}\, \partial_a \varphi\, \partial_b \varphi$$

$$\mathbb{L}_{\rho(\xi)} \underline{L}_{SF}(\varphi) \neq L_\xi \underline{L}$$

acts only on φ, not on g_{ab}!

ANCE
(independence)

d spt g_{ab} :

$\varphi \; d^4x$

g_{ab}!

Def $\alpha \in \Omega^{k,\ell}(M \times \mathcal{F})$ is general covariant

iff $\boxed{\mathbb{L}_{\rho(\xi)}}\,\alpha = \boxed{L_\xi}\,\alpha$

Noether:

$\underline{J}(\xi) = \mathring{I}_{\rho(\xi)}\underline{\theta} - \underline{R}(\xi)$ $\qquad \mathbb{L}_{\rho(\xi)}\underline{L} = d\underline{R}(\xi)$

backgrnd indep: $d\underline{R}(\xi) = L_\xi \underline{L} = i_\xi \underbrace{\cancel{d\underline{L}}}_{\equiv 0} + d i_\xi \underline{L}$

we can take $\underline{R}(\xi) = i_\xi \underline{L}$

$= \left(\frac{1}{2}R - \Lambda\right)\xi^a \underline{\epsilon}_a$

$$\mathbb{I}_{\varphi(\xi)}\, d g_{ab} = \delta_\xi g_{ab} = \mathcal{L}_\xi g_{ab} = \nabla_a \xi_b + \nabla_b \xi_a$$

$$\boxed{J^a(\xi) = C^a_b\, \xi^b + \nabla_b\, j^{ba}(\xi)}$$

$$C^a_b = -\left(G^a_b + \Lambda\, \delta^a_b\right) \approx 0$$

$$\left[\; C_{\Sigma a} = n_a C^a_b \quad \text{(constraints)} \;\right]$$

$$\boxed{\begin{array}{l} j^{ba}(\xi) \equiv j^{[ba]} = -\frac{1}{2}\left(\nabla^b \xi^a - \nabla^a \xi^b\right) \\ \text{KOMAR CURRENT} \end{array}}$$

$j(\xi) = -$

$\xi_g \equiv$

$\nabla_b \xi_a$

$$j(\xi) = -\frac{1}{2} * d\,\underline{\xi}_g = \frac{1}{2} j^{ab}(\xi)\,\underline{\epsilon}_{ab}$$

$$\underline{\xi}_g \equiv g_{ab}\,\xi^b\,dx^a$$

$\Rightarrow$

COVARIANT HAMILTONIAN FLOW EQ?

Recall: $\mathbb{I}_{\rho(\xi)}\underline{\Omega} = -\mathbb{d}\,\underline{J}(\xi) + \left(\mathbb{I}_{\rho(\xi)}\underline{\Theta} - \mathbb{d}\,\underline{R}(\xi)\right)$

For any General cov theory, diffeos give this:

$$\underline{R}(\xi) = i_\xi \underline{L} \qquad \mathbb{d}\,\underline{R}(\xi) = i_\xi\,\mathbb{d}\underline{L} = i_\xi \underline{E} + i_\xi\,\mathbb{d}\underline{\Theta}$$

$$\mathbb{L}_{\rho(\xi)}\underline{\Theta} - L_\xi \Theta = d\,i_\xi \underline{\Theta} + i_\xi\,d\,\underline{\Theta}$$

$(\nabla^b \xi^a - \nabla^a \xi^b)$

$$\Rightarrow \quad \mathring{I}_{\rho(\xi)}\underline{\Omega} = -d\underline{J}(\xi) + \underbrace{i_\xi \underline{E}}_{①} + \underbrace{d\, i_\xi \underline{④}}_{②}$$

Integrate over $\Sigma \hookrightarrow M$

$$\mathring{I}_{\rho(\xi)}\Omega_\Sigma = -dQ_\Sigma(\xi) + \overbrace{\int_\Sigma i_\xi \underline{E}}^{①} + \overbrace{\int_{\partial\Sigma} i_\xi \underline{④}}^{②}$$

$dR(\xi))$

this:

$+ i_\xi d\underline{④}$

① Off-shell obstruction

$$\int_\Sigma i_\xi \underline{E} = \int_\Sigma (\dots)\, i_\xi \underline{\epsilon} = \int_\Sigma (\dots)\, \xi^a \underline{\epsilon}_a$$

$$= \int_\Sigma (\dots)\, \xi^a n_a \sqrt{h}$$

$$\Rightarrow \quad i_{\rho(\xi)}\underline{\Omega} = -d\underline{J}(\xi) + \underbrace{i_\xi \underline{E}}_{1} + \underbrace{d\, i_\xi \underline{L}}_{2}$$

Integrate over $\Sigma \hookrightarrow M$

$$i_{\rho(\xi)}\Omega_\Sigma = -dQ_\Sigma(\xi) + \overbrace{\int_\Sigma i_\xi \underline{E}}^{1} + \overbrace{\int_{\partial\Sigma} i_\xi \underline{L}}^{2}$$

1 Off-shell obstruction

$$\int_\Sigma i_\xi \underline{E} = \int_\Sigma (\dots)\, i_\xi \underline{\epsilon} = \int_\Sigma (\dots)\, \xi^a \underline{\epsilon}_a$$

transverse diffeos

$$= \int_\Sigma (\dots) \boxed{\xi^a n_a} \sqrt{h}$$

$$\Rightarrow \quad i_{\rho(\xi)} \underline{\Omega} = -d\underline{J}(\xi) - \underbrace{i_\xi \underline{E}}_{①} + \underbrace{d\, i_\xi \underline{④}}_{②}$$

Integrate over $\Sigma \hookrightarrow M$

$$i_{\rho(\xi)} \Omega_\Sigma = -dQ_\Sigma(\xi) - \overbrace{\int_\Sigma i_\xi \underline{E}}^{①} + \overbrace{\int_{\partial\Sigma} i_\xi \underline{④}}^{②}$$

$\underline{R}(\xi))$

...is:

$+ i_\xi \underline{d ④}$

① Off-shell obstruction

$$\int_\Sigma i_\xi \underline{E} = \int_\Sigma (\dots)\, i_\xi \underline{\epsilon} = \int_\Sigma (\dots)\, \xi^a \underline{\epsilon}_a$$

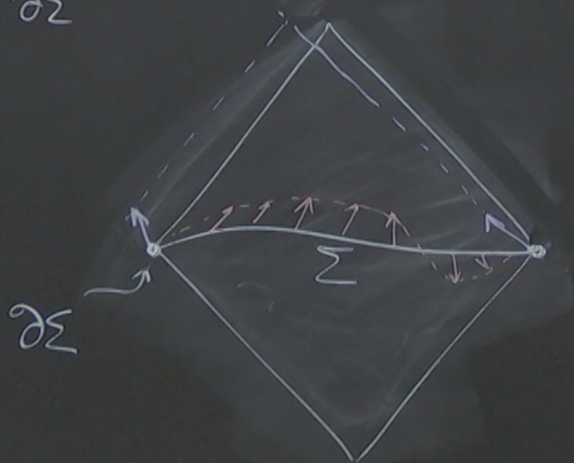
$$= \int_\Sigma (\dots) \boxed{\xi^a n_a} \sqrt{h}$$

transverse diffeos only

KOMAR CURRENT

(2) about corners ($\partial\Sigma \neq \phi$)

$\int_{\partial\Sigma} i_\xi \underline{\Theta} \neq 0$ only if ξ transverse to $\partial\Sigma$



In some cases, I might be able to impose "strong enough" boundary conditions @ $\partial\Sigma$ such that

$$\Omega^1(\mathcal{F}) \ni \int_{\partial\Sigma} i_\xi \underline{\omega} = \mathbb{d} \int_{\partial\Sigma} \underline{B} \quad , \underline{B} \in \Omega^{top,0}(\partial\Sigma \times \mathcal{F})$$

Imposing bdry conds means considering $\mathcal{F}_B \hookrightarrow \mathcal{F}$ and pulling back to this subset of config.

bsorbed
harge!

Teaser

- BH spacetimes ($\Lambda = 0$)
- use the Ham flow eq in $F_{BH} \hookrightarrow F$

to deduce 1st law of BH mech:

$$0 = -T\,\delta S_{BH} + \delta \mathcal{E} - \omega_{BH}\,\delta J$$

BH Entropy as a Noether charge (Wald-Iyer)

energy & angular mom. part of Noether charge associated to time transl. & rotations in adapted coords:

$$\Sigma = \{X^0 = 13\} \qquad \xi = \xi^0 \partial_0 + \xi^1 \partial_1 + \dots$$

$$\xi^e \underline{\epsilon}_e = \xi^0 dx^1 \wedge \dots \wedge dx^{n-1}$$
$$+ \xi^1 dx^0 \wedge dx^2 \wedge dx^3 \wedge \dots$$
$$+ \xi^2 dx^0 \wedge dx^1 \wedge dx^3 \wedge \dots$$
$$+ \dots$$

$$\iota^*_\Sigma (\xi^e \underline{\epsilon}_e) = \xi^0 dx^1 \wedge \dots \wedge dx^{n-1}$$
$$+ (\dots)\underbrace{(\iota^*_\Sigma dx^0)}_{\equiv 0}$$

$$i_\xi \underline{\Theta} = \Theta^a i_\xi \underline{\epsilon}_a$$
$$= \Theta^a \xi^b \underline{\epsilon}_{ab}$$

$$\underline{\epsilon}_{ab} \xrightarrow{\text{pull back to } \partial\Sigma} \underline{\epsilon}_{\partial\Sigma} \quad \epsilon_{ab}$$