Title: Quantum Matter Lecture (230426)

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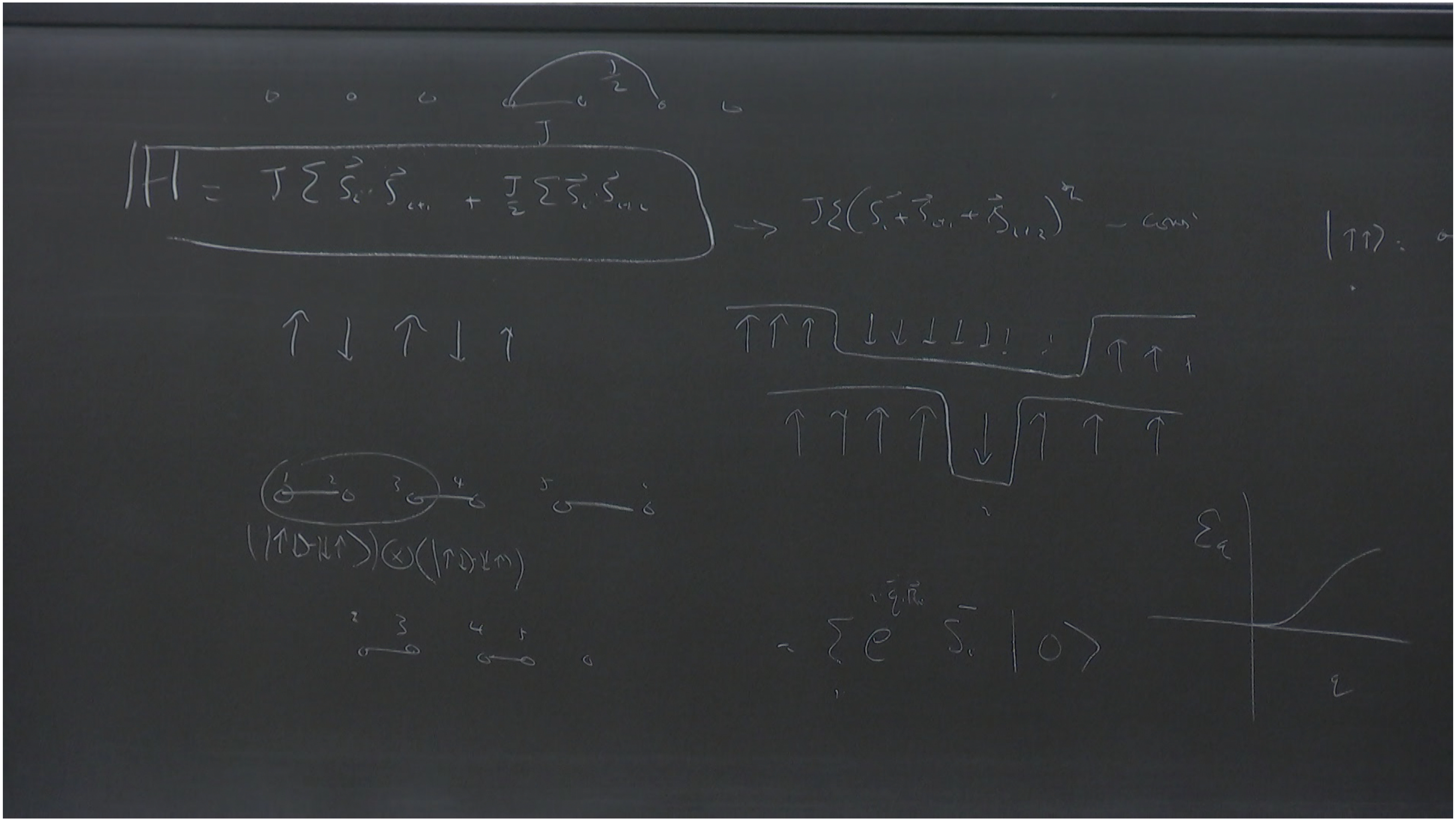
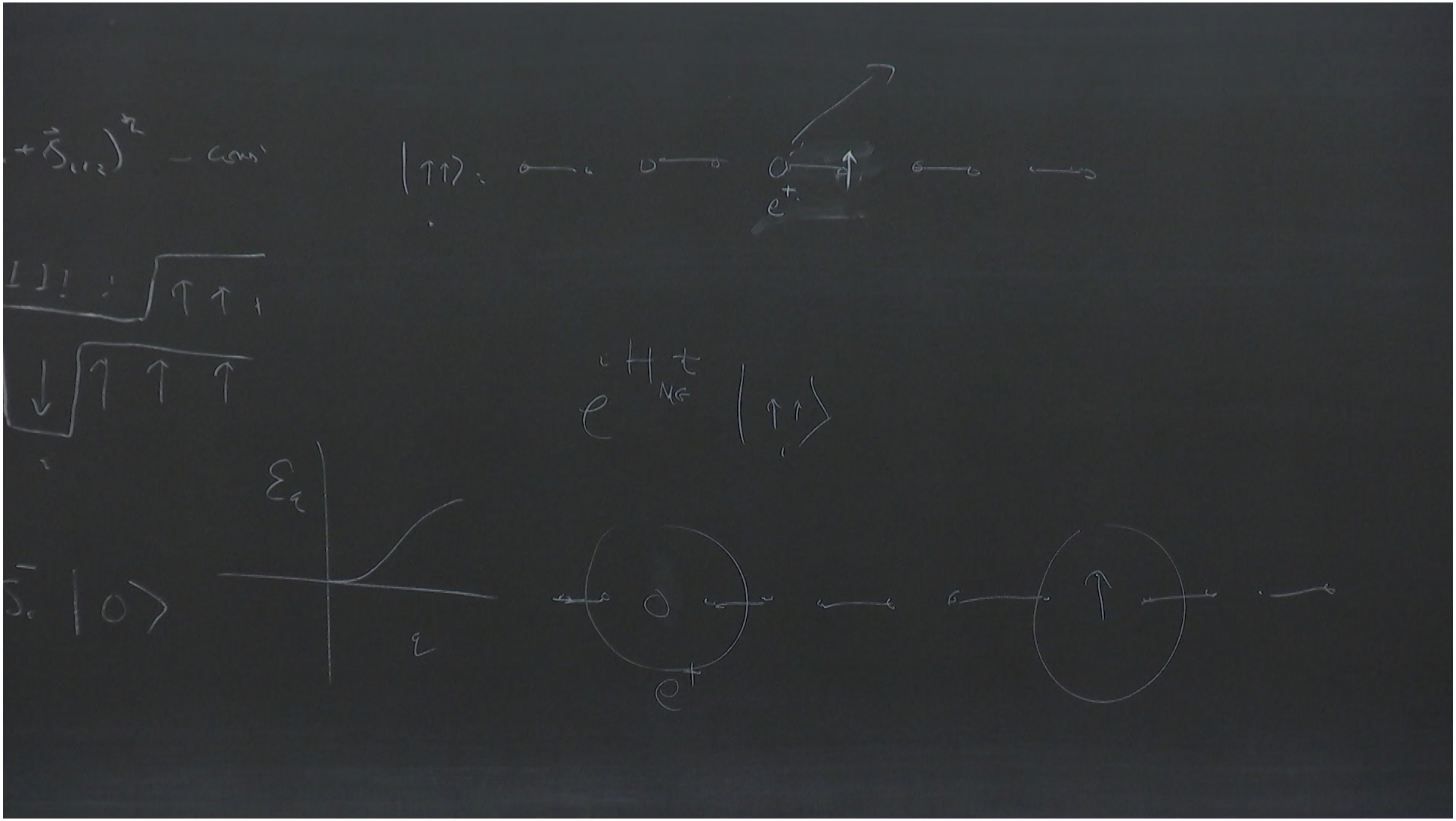
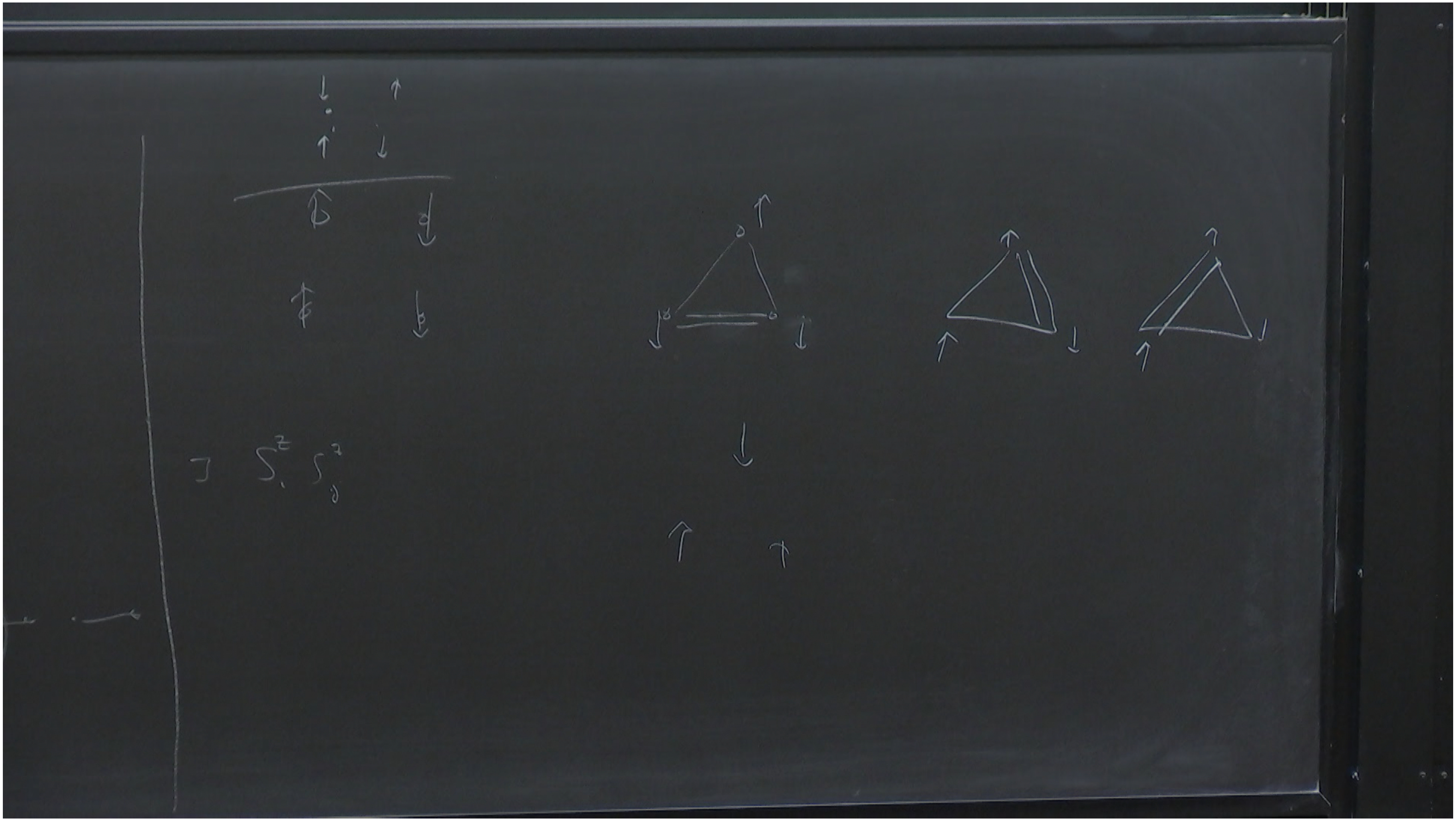
Collection: Quantum Matter (2022/2023)

Date: April 26, 2023 - 10:15 AM

URL: https://pirsa.org/23040016


J
$\frac{J}{2}$
$H = J\sum \vec{S}_i\cdot\vec{S}_{i+1} + \frac{J}{2}\sum \vec{S}_i\cdot\vec{S}_{i+2}$
$\rightarrow J\sum(\vec{S}_i+\vec{S}_{i+1}+\vec{S}_{i+2})^2 - \text{const}$
↑ ↓ ↑ ↓ ↑
↑↑↑ ↓↓↓↓↓↓ ↑↑↑
↑↑↑↑ ↓ ↑↑↑
1 2 3 4 5 6
$(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\otimes(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
2 3 4 5
$=\sum e^{i\vec{q}\cdot\vec{R}_i} S_i^- |0\rangle$
ε_q
q


|↑↑⟩ :
e+
e^{iH_NG t} |↑↑⟩
E_q
q
e+

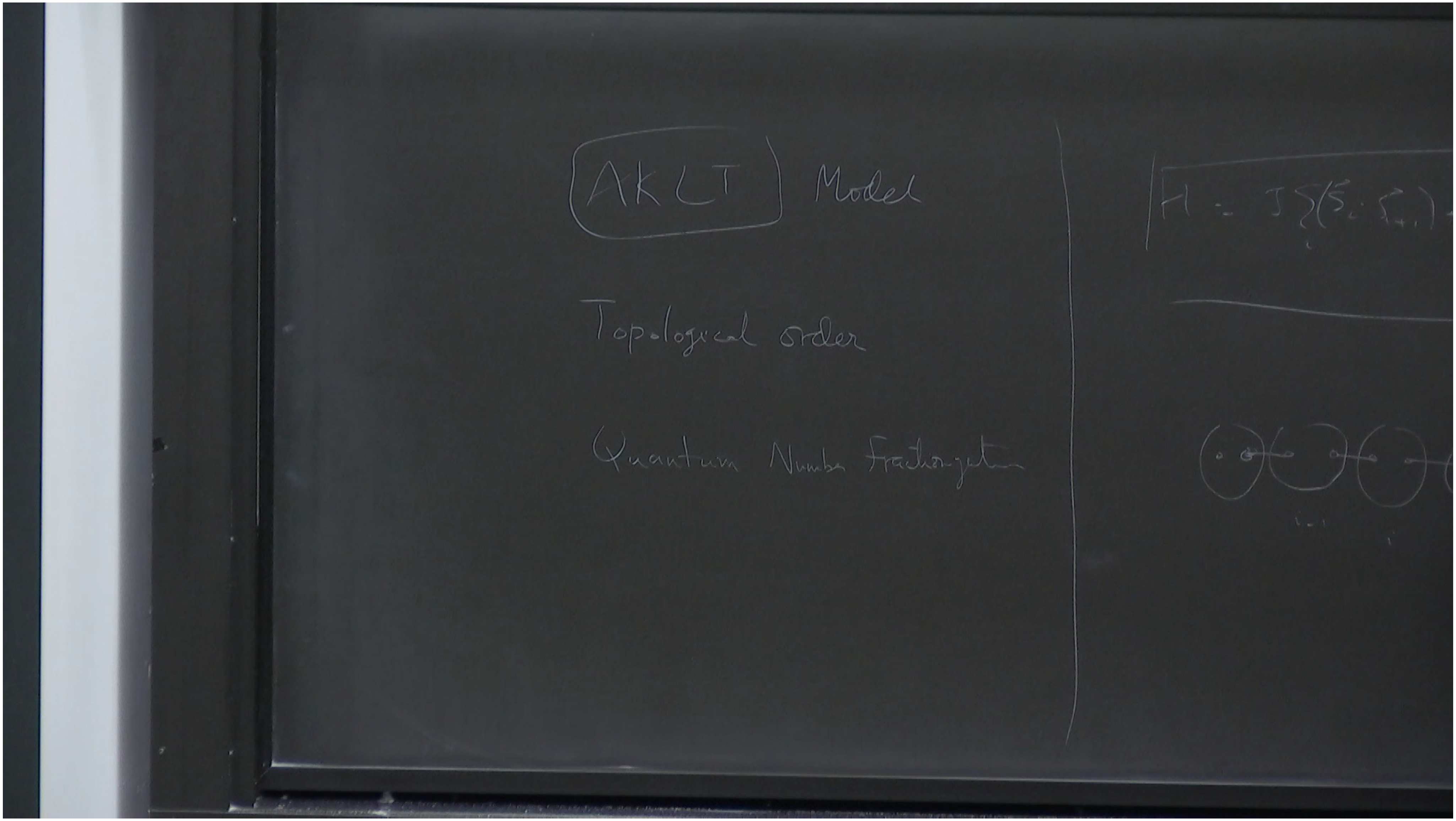


Wen-Wilczek-Zee

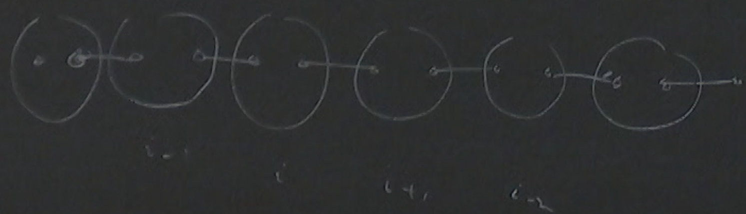
$$\boxed{\vec{S}_1 \cdot (\vec{S}_2 \times \vec{S}_3)} = \hat{\chi}_{123}$$

$$\hat{\chi}_{123} |+1\rangle = \left(|\downarrow\uparrow\uparrow\rangle + e^{\pm i\frac{2\pi}{3}} |\uparrow\downarrow\uparrow\rangle + e^{\pm i\frac{4\pi}{3}} |\uparrow\uparrow\downarrow\rangle \right)$$

$$\boxed{\hat{\chi}^2_{ijk} = \alpha(\vec{S}_i \cdot \vec{S}_j + \vec{S}_j \cdot \vec{S}_k + \vec{S}_k \cdot \vec{S}_i) + \text{const}}$$


AKLT Model
Topological order
Quantum Number fractionalization

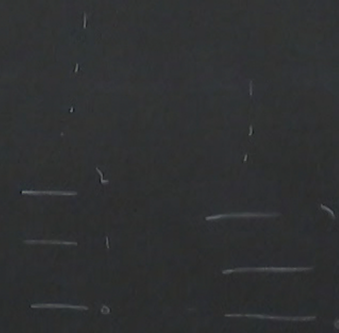
$$\boxed{H = J\sum_i (\vec{S}_i \cdot \vec{S}_{i+1}) + \frac{J}{3}\sum (\vec{S}_i \cdot \vec{S}_{i+1})^2}$$



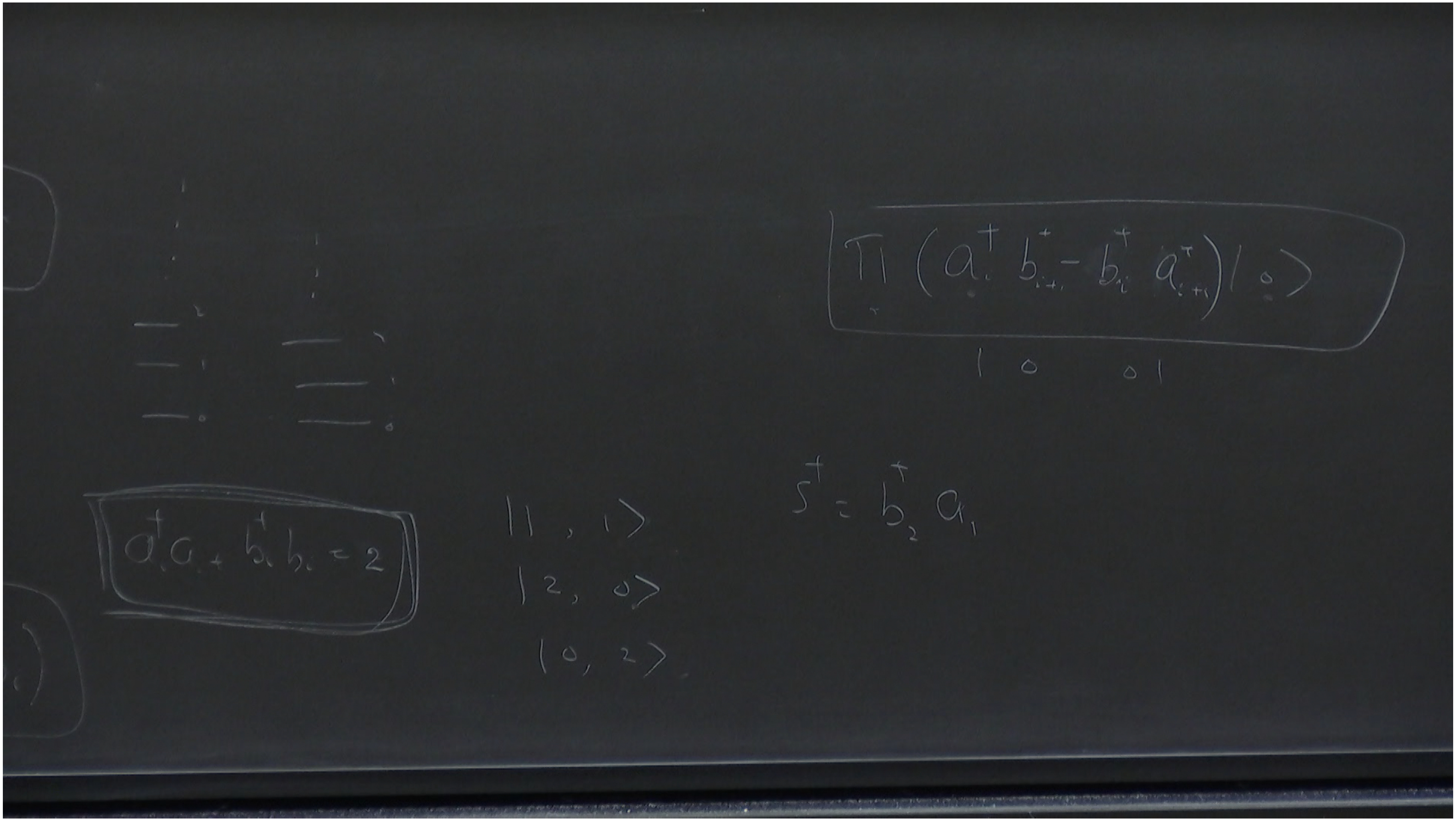
Schwinger Boson

$(a_i^\dagger, a_i)$

$(b_i^\dagger, b_i)$

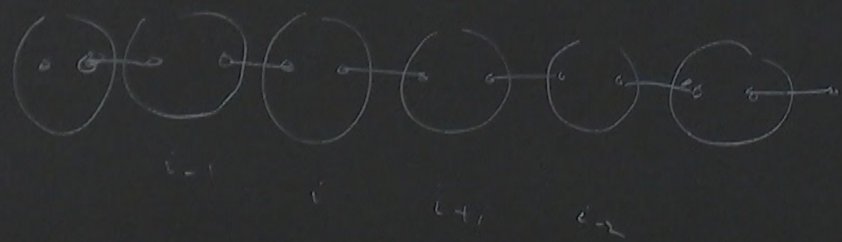


$$\boxed{a_i^\dagger a_i + b_i^\dagger b_i = 2}$$


$\prod_i (a_i^\dagger b_{i+1}^\dagger - b_i^\dagger a_{i+1}^\dagger)|0\rangle$
1 0 0 1
$a_i^\dagger a_i + b_i^\dagger b_i = 2$
$|1, 1\rangle$
$|2, 0\rangle$
$|0, 2\rangle$
$S^+ = b_2^\dagger a_1$

$$H = J\sum_i (\vec{S}_i \cdot \vec{S}_{i+1}) + \frac{J}{3}\sum (\vec{S}_i \cdot \vec{S}_{i+1})^2$$

Schwinger Boson



$(a_i^\dagger, a_i)$

$(b_i^\dagger, b_i)$

$$S_i^z = \frac{1}{2}(a_i^\dagger a_i - b_i^\dagger b_i)$$

$$|G\rangle = \sum \mathrm{Tr}\left(A_{s_1^z} A_{s_2^z} A \cdots N\right) |s_1^z s_2^z \cdots s_N^z\rangle$$

$(s_i^z = \pm 1, 0)$

$$\boxed{\prod_i \left(a_i^\dagger b_{i+1}^\dagger - b_i^\dagger a_{i+1}^\dagger\right)|0\rangle}$$

1 0   0 1 $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$A_{s^z=0} =$

$S^+ = b_2^\dagger a_1$

$A_{s^z=+1} = \begin{pmatrix} 0 & 1 \\ 0 & \end{pmatrix}$

$A_{s^z=-1} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$|1, 1\rangle$

$|2, 0\rangle$

$|0, 2\rangle$

2

$|\, 1\ 0\ 0\ 0\ -1\ -1\ 0\ 1\ -1\ -1\ -1\ \dots \rangle$

$|\, \underbrace{1\ 0\ 0\ 0\ 0\ 0}\ -1\ \underbrace{0\ 0\ 0\ 0}\ 1 \rangle$

$$\left\langle \boxed{S^z_x \, e^{i\pi \sum_{l>x}^{y} S^z_l} \, S^z_y} \right\rangle$$