Title: On Hikita-Nakajima conjecture for some quiver varieties and Slodowy slices

Speakers: Vasily Krylov

Series: Mathematical Physics

Date: March 31, 2023 - 11:00 AM

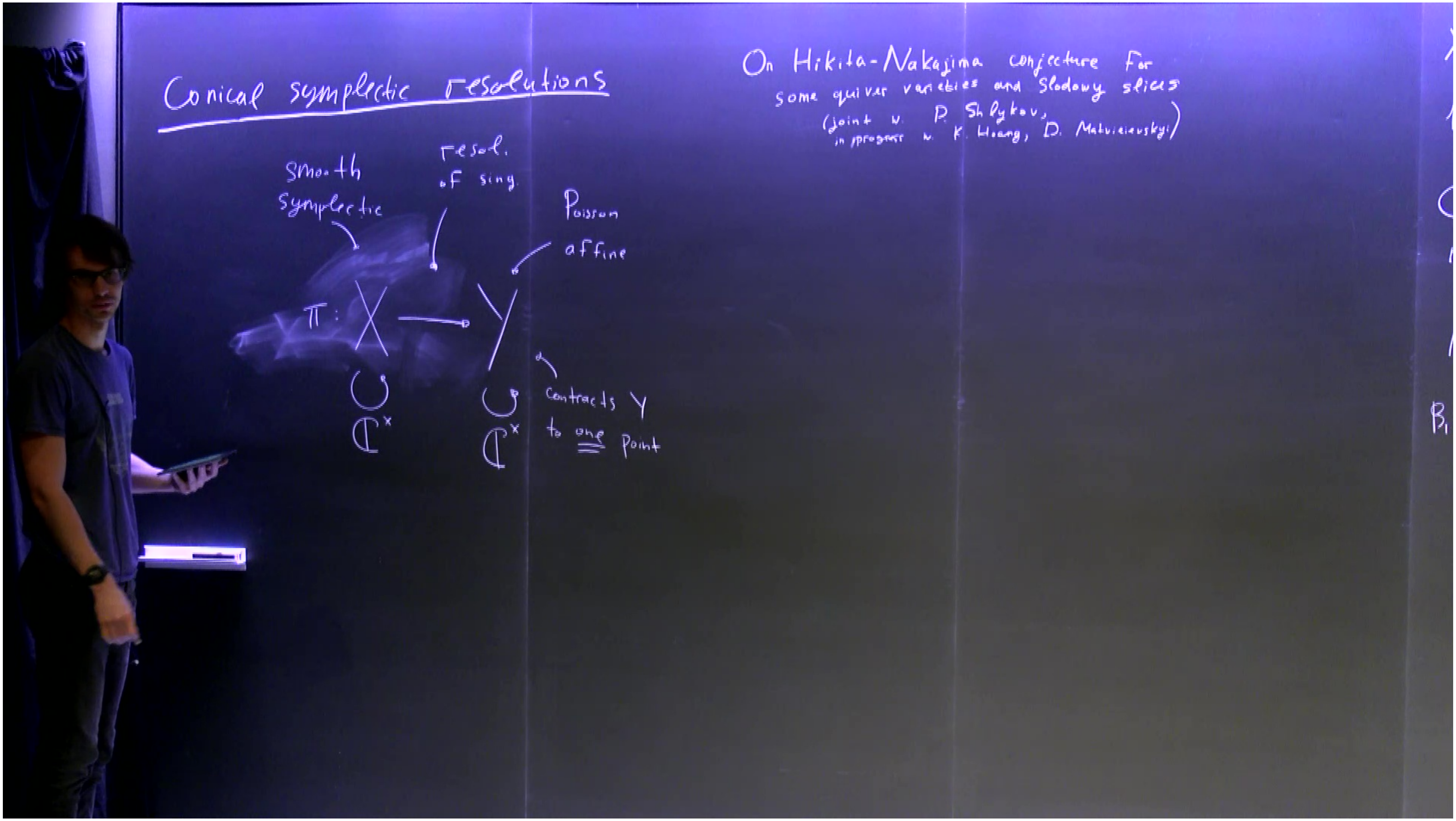
URL: https://pirsa.org/23030118

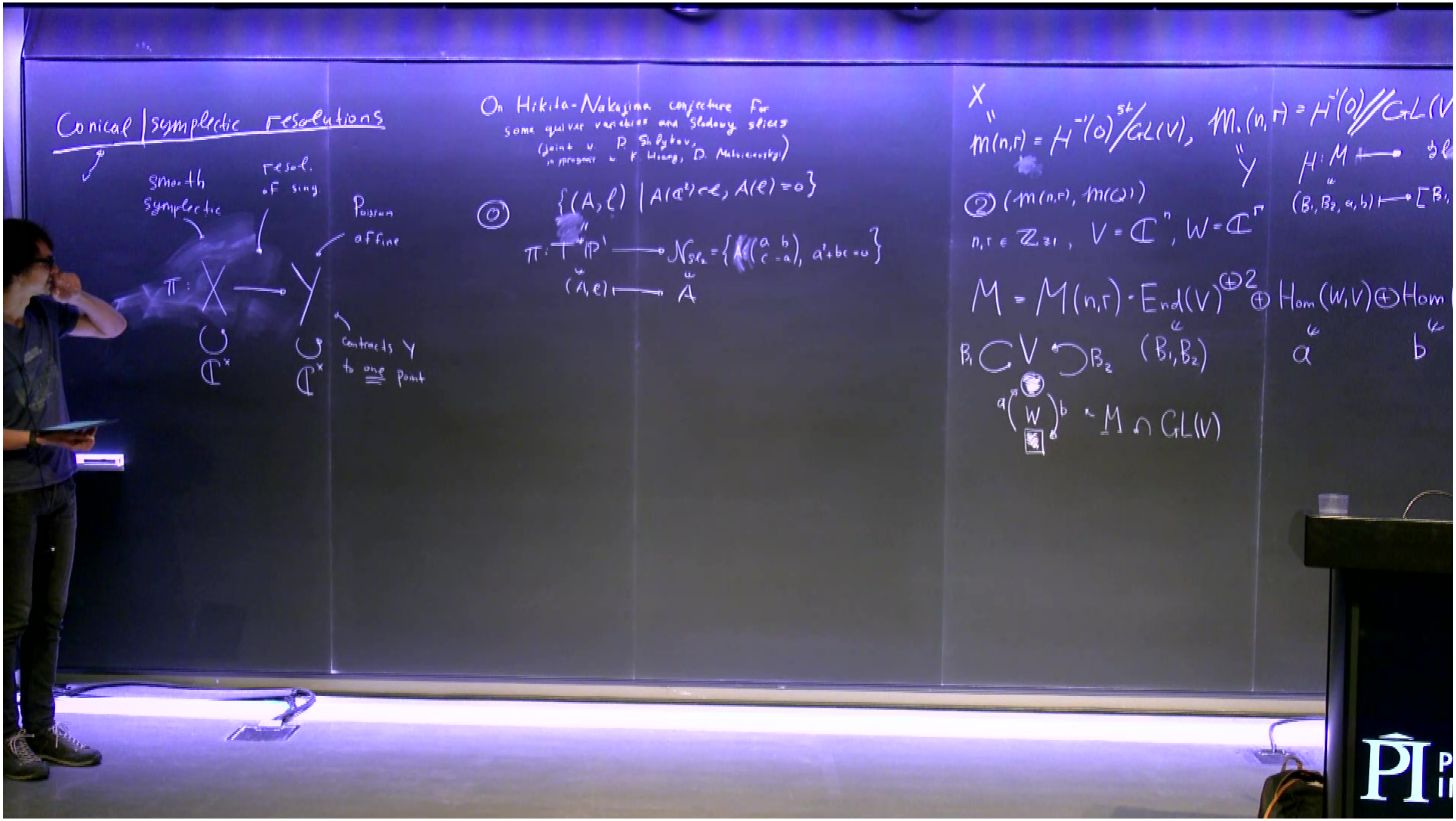
Abstract: Symplectic duality predicts that symplectic singularities should come in pairs. For example, Nakajima quiver varieties are conjecturally dual to BFN Coulomb branches (of the corresponding quiver theories). Another family of potentially symplectically dual pairs was described recently in the works of Losev, Mason-Brown, and Matvieievskyi: they describe symplectically duals to Slodowy slices to nilpotent orbits.

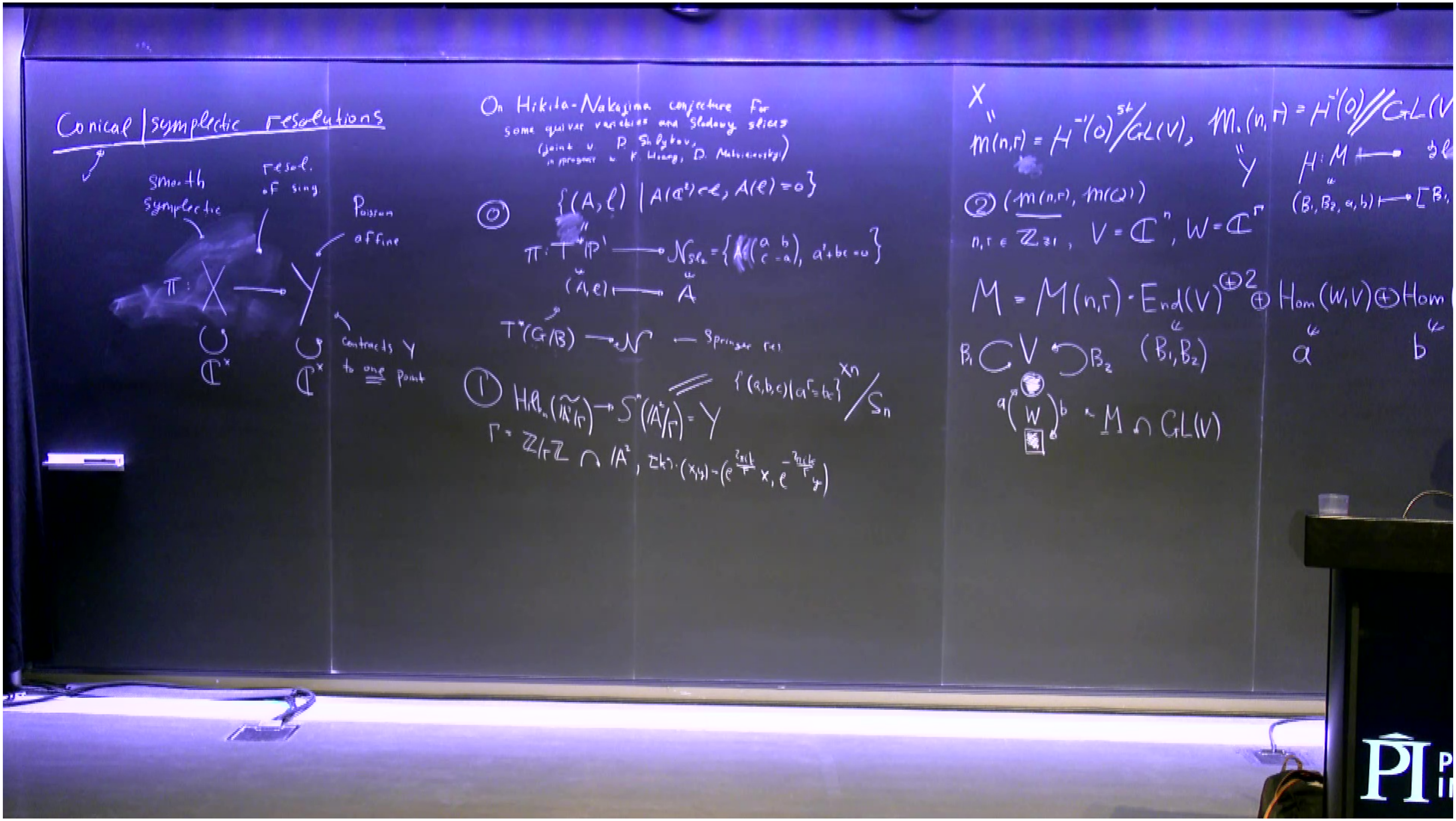
In this talk, we will discuss the Hikita-Nakajima conjecture that relates the geometry of symplectically dual varieties. We will restrict to the cases of certain quiver varieties and Slodowy slices and discuss the picture in these cases.

Based on the joint work with Pavel Shlykov (arXiv:2202.09934) and the work in progress with Do Kien Hoang and Dmytro Matvieievskyi.

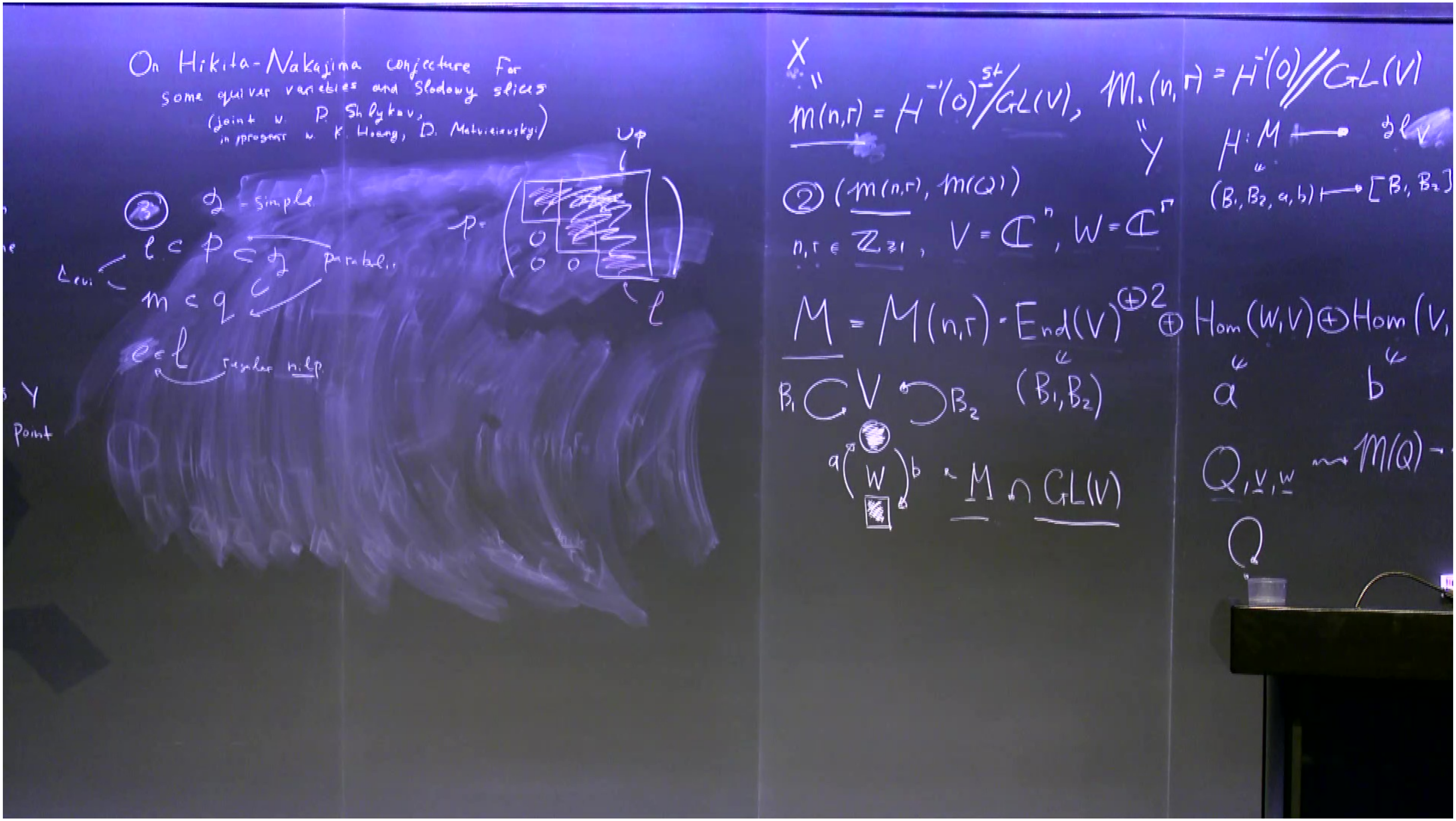
Zoom link: https://pitp.zoom.us/j/98651907502?pwd=ODA1K3NKVHFLdkp6TEtaSnJXdThVZz09

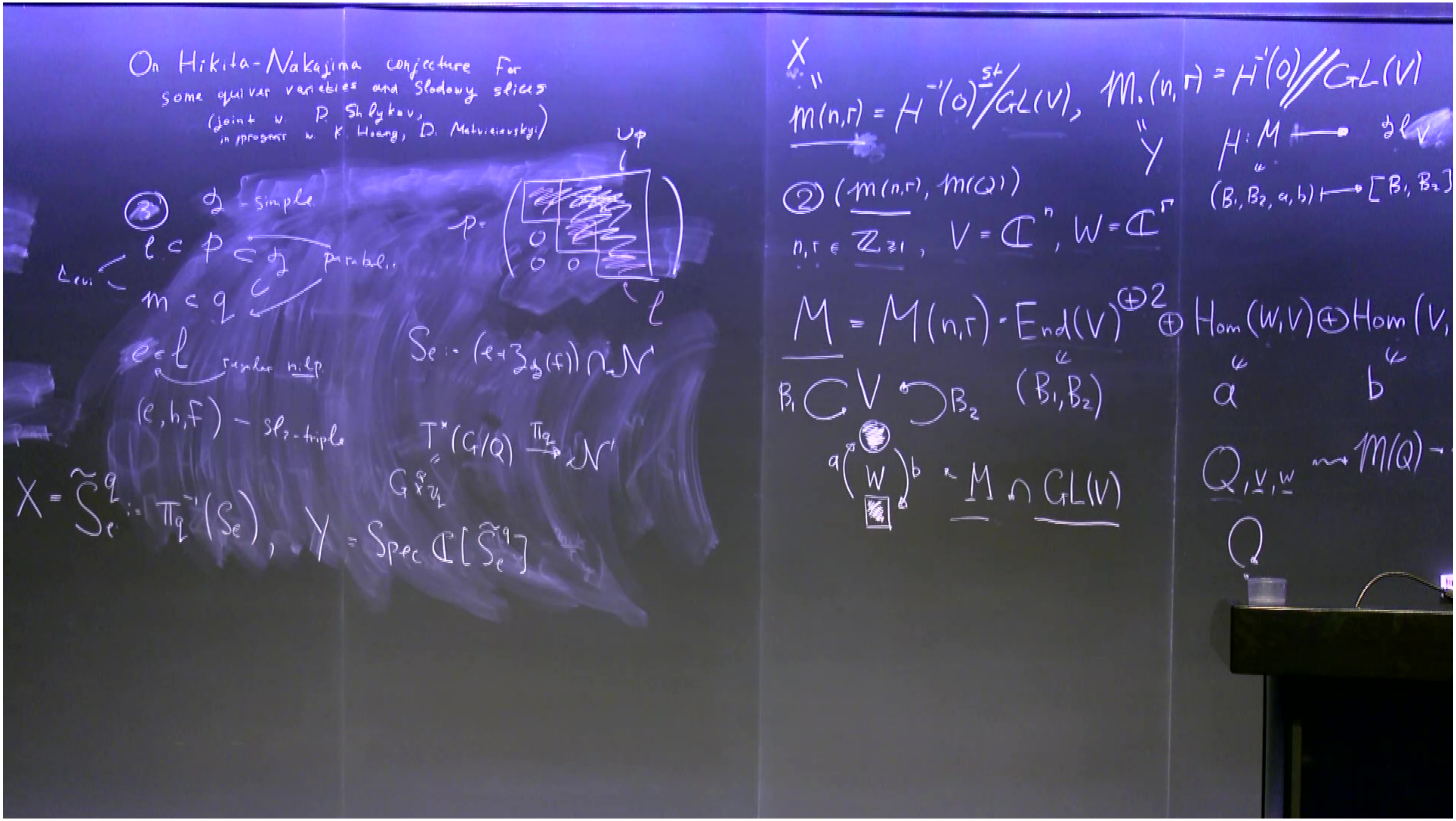

Conical symplectic resolutions
smooth symplectic
resol. of sing.
Poisson affine
π : X → Y
ℂˣ
ℂˣ
contracts Y to one point
On Hikita-Nakajima conjecture for some quiver varieties and Slodowy slices
(joint w. P. Shlykov,
in progress w. K. Hoang, D. Matviievskyi)

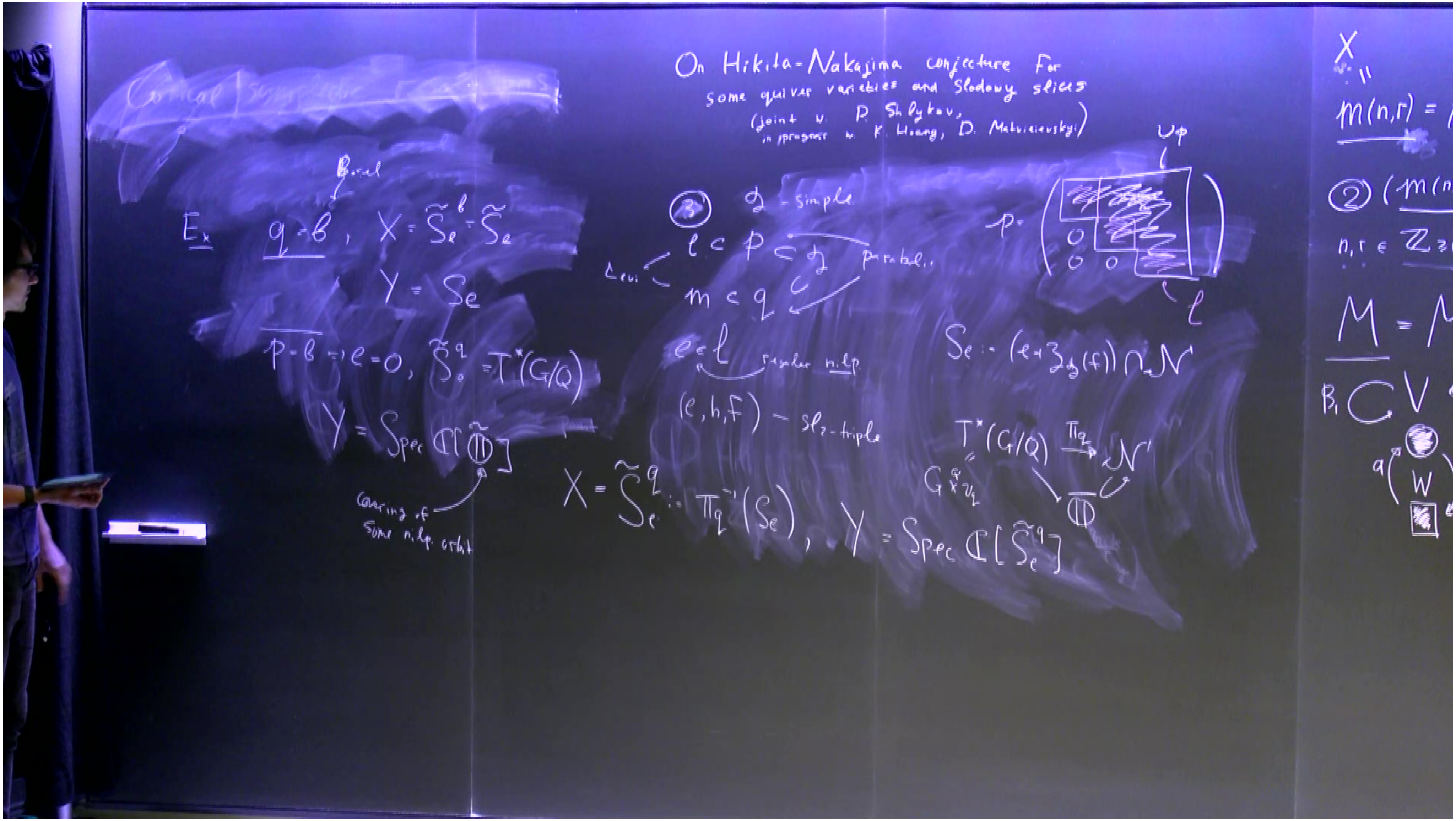

Conical symplectic resolutions
smooth symplectic
resol. of sing
Poisson affine
π: X → Y
contracts Y to one point
On Hikita-Nakajima conjecture for some quiver varieties and Slodowy slices
{(A, ℓ) | A(ℂ²) ⊂ ℓ, A(ℓ) = 0}
π: T*ℙ¹ → N_sl2 = {A = (a b; c -a), a²+bc = 0}
(A, ℓ) ↦ A
X
m(n,r) = H⁻¹(0)^st/GL(V),
Y
(m(n,r), m(ℂ))
n, r ∈ ℤ≥1, V = ℂⁿ, W = ℂʳ
M = M(n,r) = End(V)^⊕2 ⊕ Hom(W,V) ⊕ Hom
(B₁, B₂) a b
B₁ ⊂ V ⊃ B₂
W
M ∩ GL(V)
(B₁, B₂, a, b) ↦ [B₁,

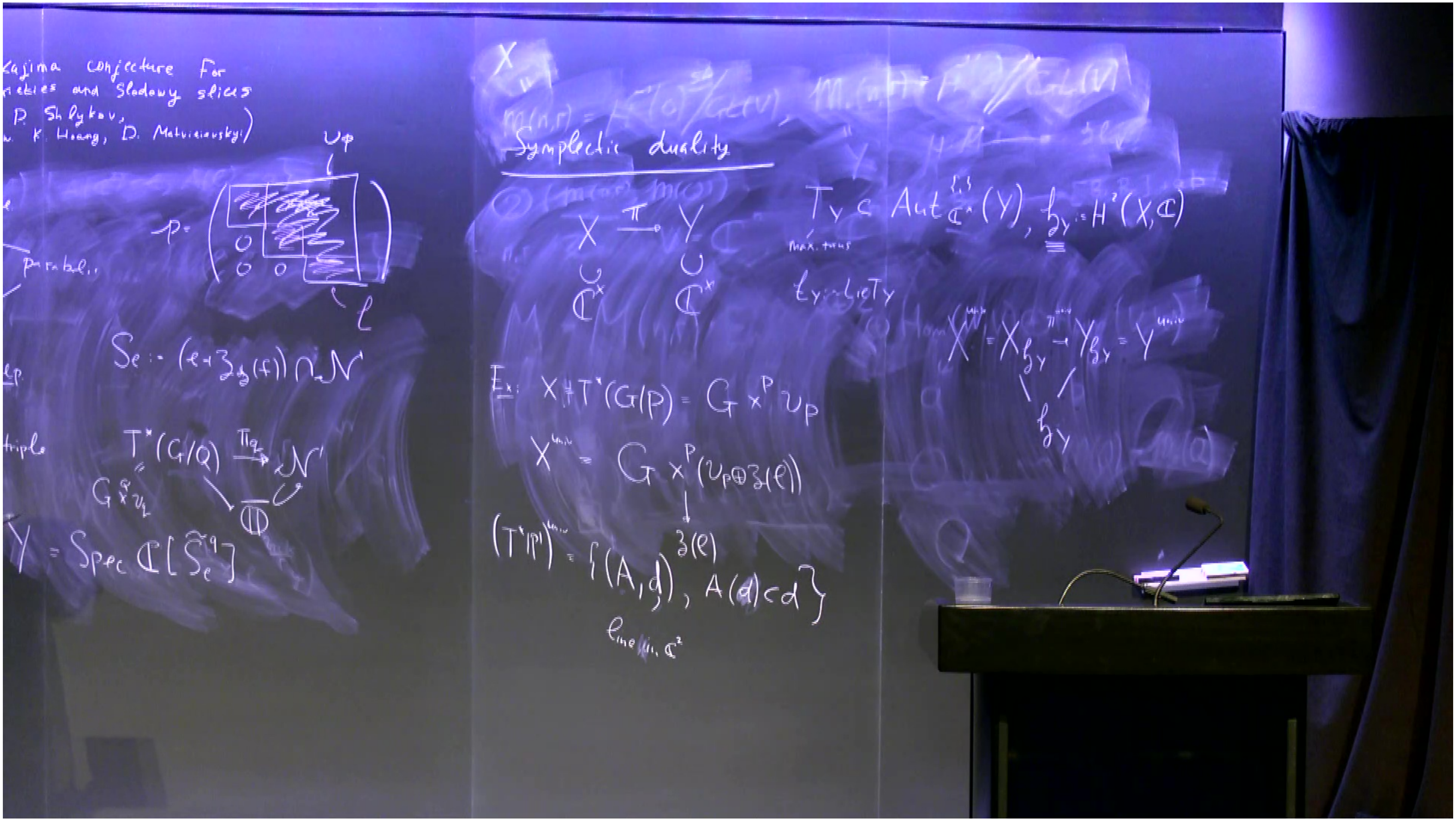


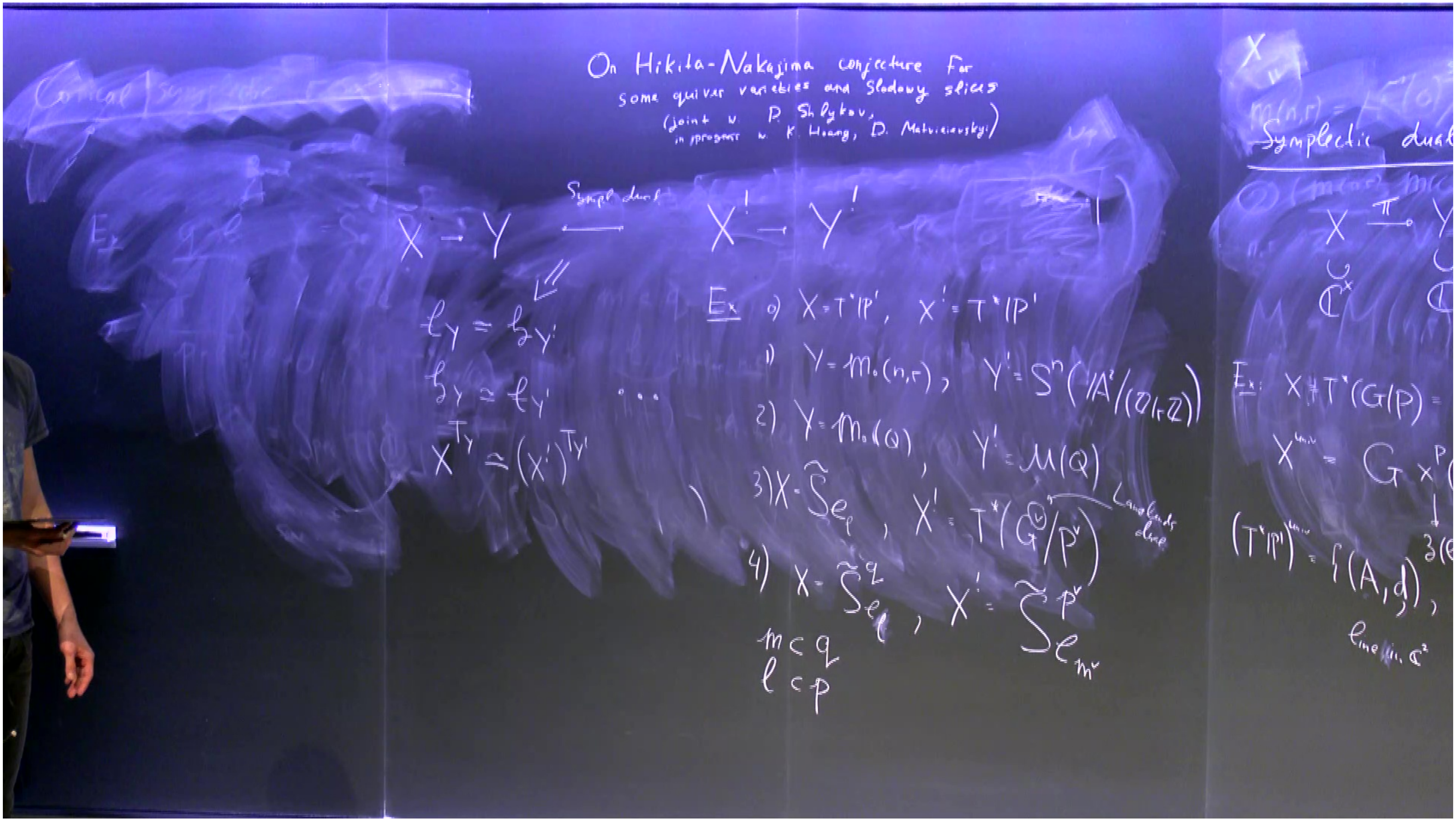
Conical symplectic resolutions
smooth symplectic
resol. of sing
Poisson affine
π : X → Y
contracts Y to one point
On Hikita-Nakajima conjecture for some quiver varieties and Slodowy slices
(joint w P. Shlykov, in progress w K. Huang, D. Matvieievskyi)
Springer res
M = M(n,r) · End(V)^⊕2 ⊕ Hom(W,V) ⊕ Hom
M ∩ GL(V)

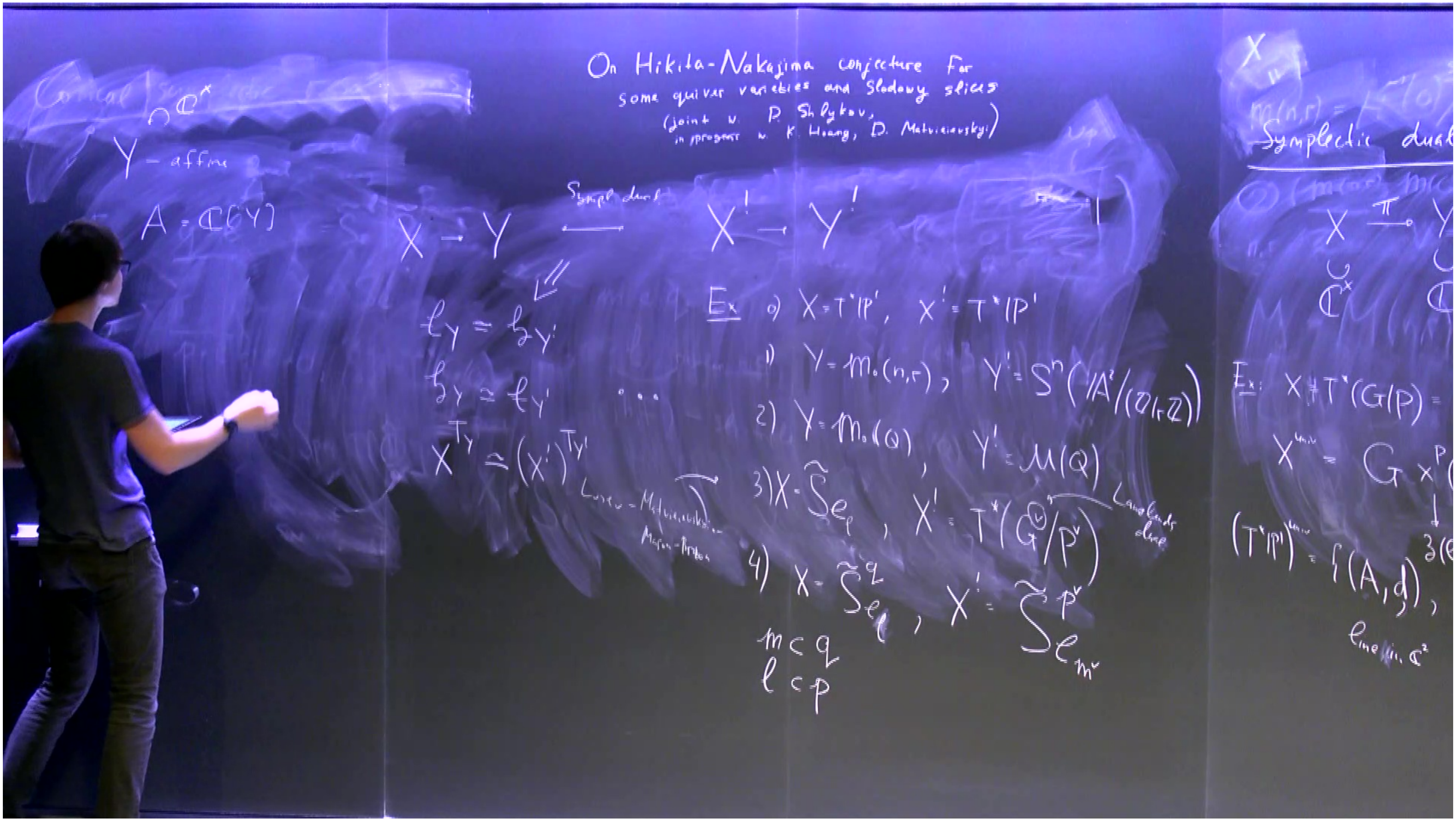

On Hikita-Nakajima conjecture for
some quiver varieties and Slodowy slices
(joint w. P. Shlykov,
in progress w. K. Hoang, D. Matvieievskyi)
g - simple
l ⊂ p ⊂ g parabol.
Levi
m ⊂ q
e ∈ l regular nilp.
Point
Up
l
M(n,r) = μ⁻¹(0)^st / GL(V), M₀(n,r) = μ⁻¹(0)//GL(V)
μ: M ⟶ gl_V
(B₁, B₂, a, b) ↦ [B₁, B₂]
(M(n,r), M(Q))
n,r ∈ ℤ≥1, V = ℂⁿ, W = ℂʳ
M = M(n,r) = End(V)^⊕2 ⊕ Hom(W,V) ⊕ Hom(V,
(B₁, B₂)
a
b
B₁ V B₂
a W b
M ∩ GL(V)
Q, v, w ⤳ M(Q)

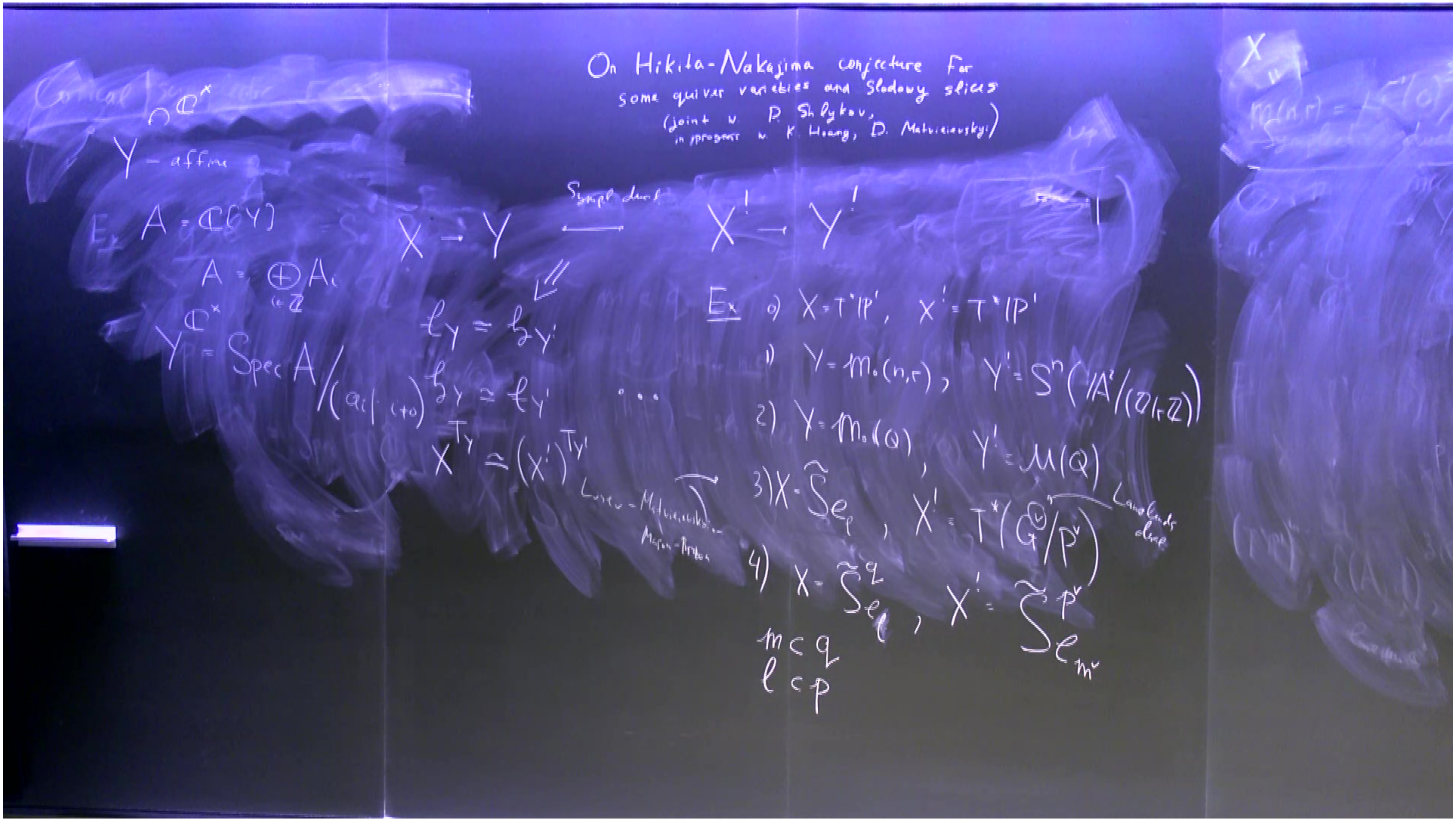

On Hikita-Nakajima conjecture for
some quiver varieties and Slodowy slices
(joint w. P. Shlykov,
in progress w. K. Hoang, D. Matvieievskyi)
g - simple
l ⊂ p ⊂ g parabol.
Levi
m ⊂ q
e ∈ l regular nilp.
(e,h,f) - sl2-triple
Se := (e + z_g(f)) ∩ N
T*(G/Q) → N
X = S̃e^q := π_q^{-1}(Se), Y = Spec C[S̃e^q]
m(n,r) = μ^{-1}(0)^{st}/GL(V), M0(n,r) = μ^{-1}(0)//GL(V)
μ: M → gl_V
(B1, B2, a, b) ↦ [B1, B2]
(2) (m(n,r), m(Q))
n, r ∈ Z≥1, V = C^n, W = C^r
M = M(n,r) = End(V)^{⊕2} ⊕ Hom(W,V) ⊕ Hom(V,
(B1,B2)
a
b
B1 ⟳ V ⟲ B2
W
a
b
⤳ M ∩ GL(V)
Q, v, w ⤳ m(Q)

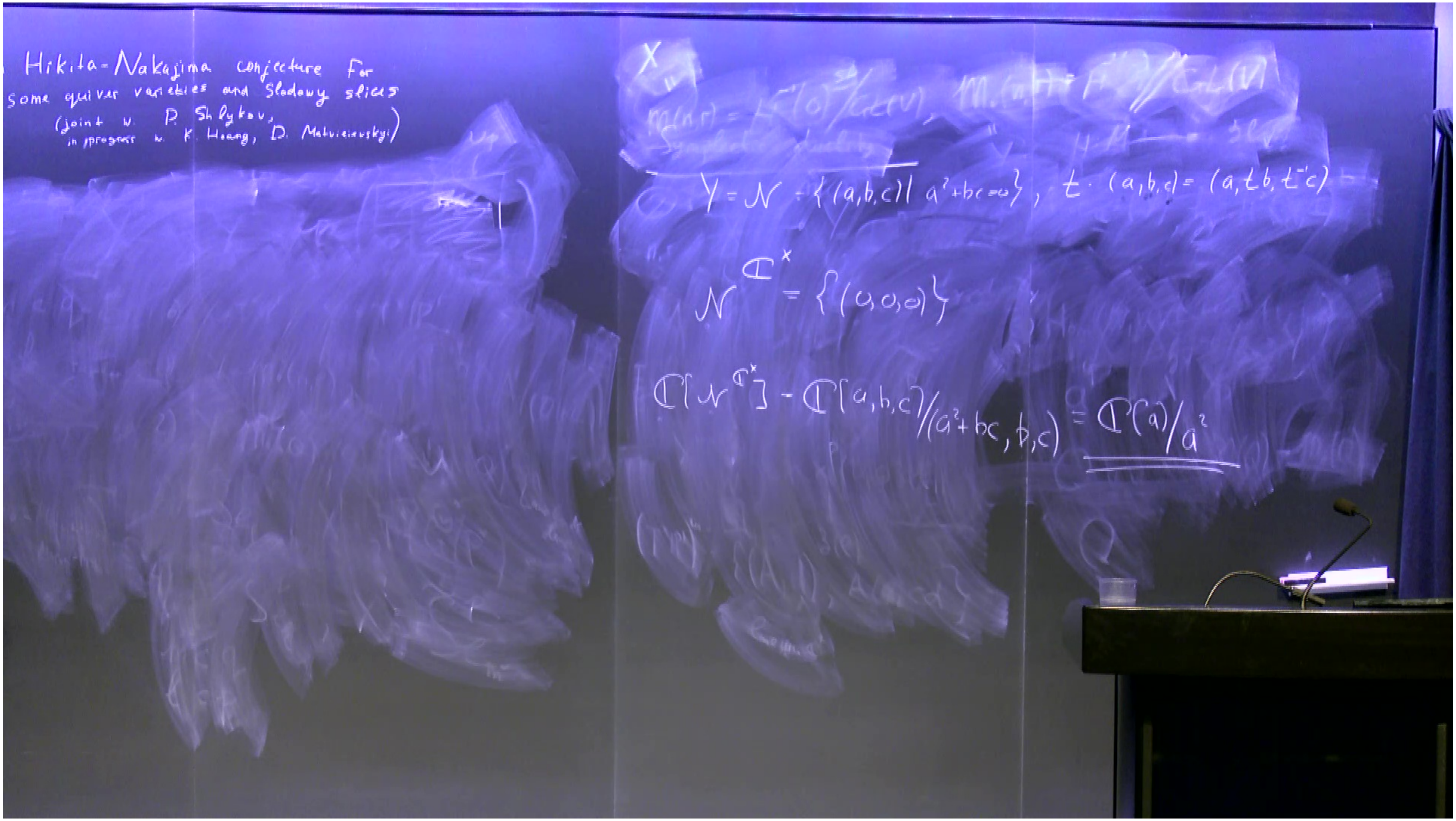

On Hikita-Nakajima conjecture for
some quiver varieties and Slodowy slices
(joint w. P. Shlykov,
in progress w. K. Hoang, D. Matviievskyi)
Ex
Borel
q = b, X = S̃_e^b = S̃_e
Y = S_e
p = b ⇒ e = 0, S̃_0^q = T*(G/Q)
Y = Spec ℂ[𝕆̃]
Covering of
some nilp. orbit
g - simple
l ⊂ p ⊂ g parabolic
Levi
m ⊂ q
e ∈ l regular nilp
(e, h, f) - sl_2-triple
p =
Up
l
S_e := (e + z_g(f)) ∩ N
T*(G/Q) → N
G ×^Q u_q
X = S̃_e^q := π_q^{-1}(S_e), Y = Spec ℂ[S̃_e^q]
m(n, r) =
n, r ∈ ℤ
M =
W

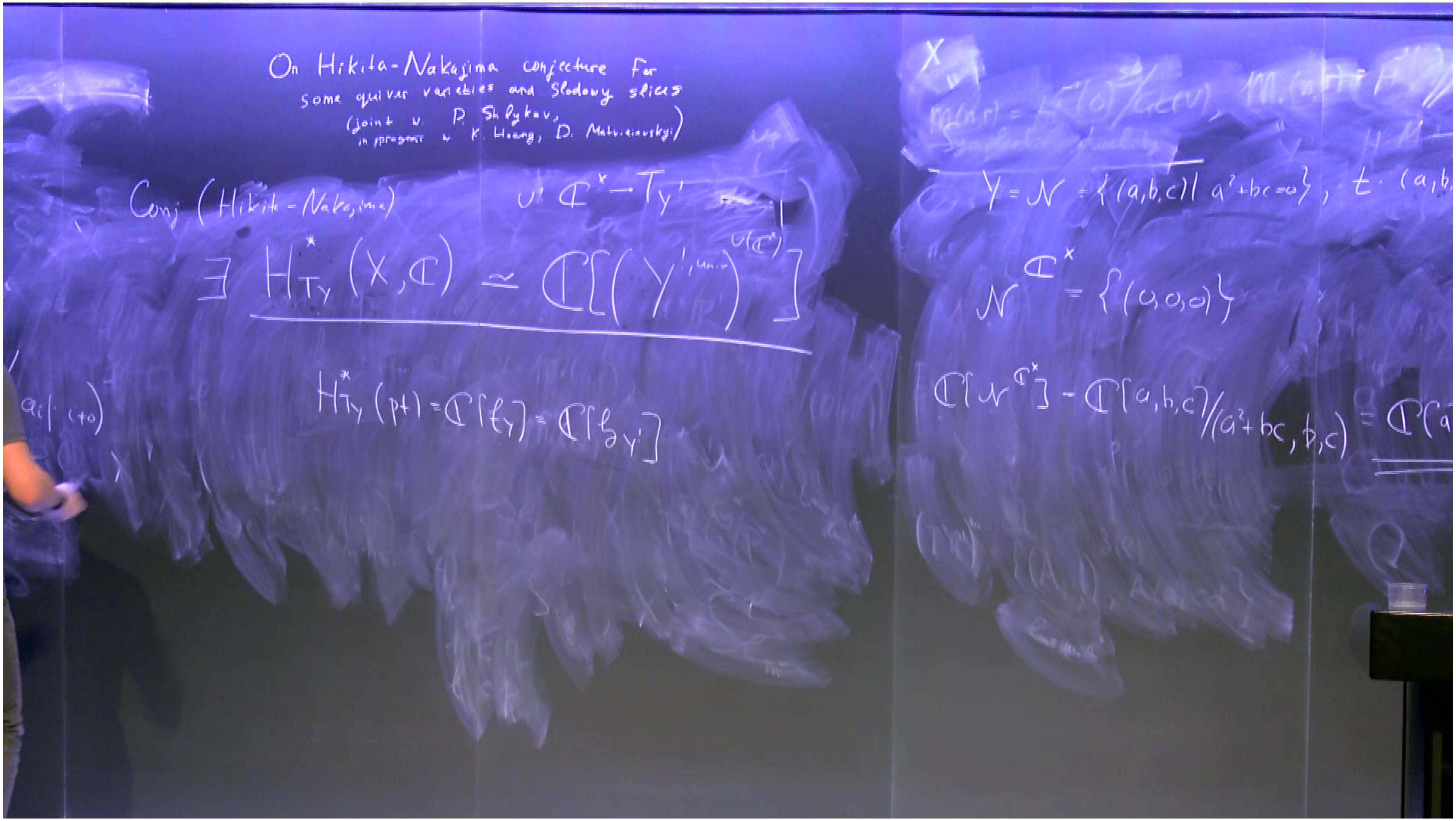

conjecture For
and Slodowy slices
P. Shlykov,
K. Hoang, D. Matviievskyi)
Symplectic duality
parabolic
triple
Max. torus
Ex:


On Hikita-Nakajima conjecture for some quiver varieties and Slodowy slices
(joint w. P. Shlykov,
in progress w. K. Hoang, D. Matviichuk)
Sympl dual
X → Y
X! → Y!
Ex 0) X = T*P1, X! = T*P1
1) Y = M0(n,r), Y! = Sn(A2/(Z/rZ))
2) Y = M0(Q), Y! = M(Q)
3) X = Se, X! = T*(G/P)
Langlands dual
4) X = Se, X! = Se
m ⊂ q
l ⊂ p
Symplectic dual
X → Y
Ex: X = T*(G/P)
line in C2

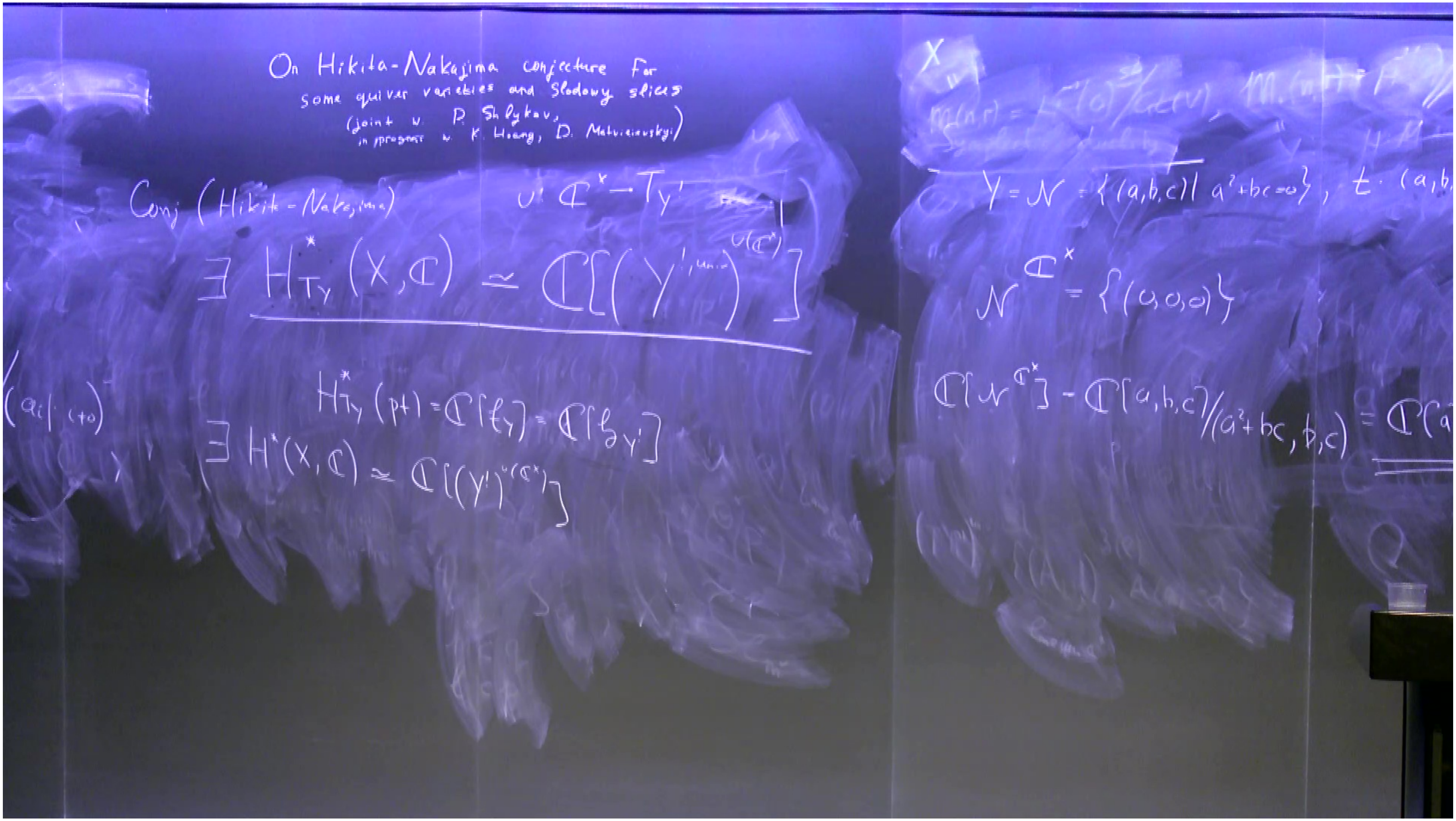

On Hikita-Nakajima conjecture for some quiver varieties and Slodowy slices
(joint w. P. Shlykov, in progress w. K. Hoang, D. Matviievskyi)
Y - affine
A = ℂ[Y]
X → Y
Sympl. dual
X! → Y!
Ex 0) X = T*ℙ¹, X! = T*ℙ¹
1) Y = M₀(n,r), Y! = Sⁿ(𝔸²/(ℤ/rℤ))
2) Y = M₀(Q), Y! = M(Q)
3) X = S̃e_l, X! = T*(G∨/P∨)
4) X = S̃e_l^q, X! = S̃e_m∨^p∨
m ⊂ q
l ⊂ p
Langlands dual
Losev - Matviievskyi
Mirkovic - Pirchon?
Symplectic dual
Ex: X = T*(G/P)
(T*ℙ¹)^univ = {(A, d),

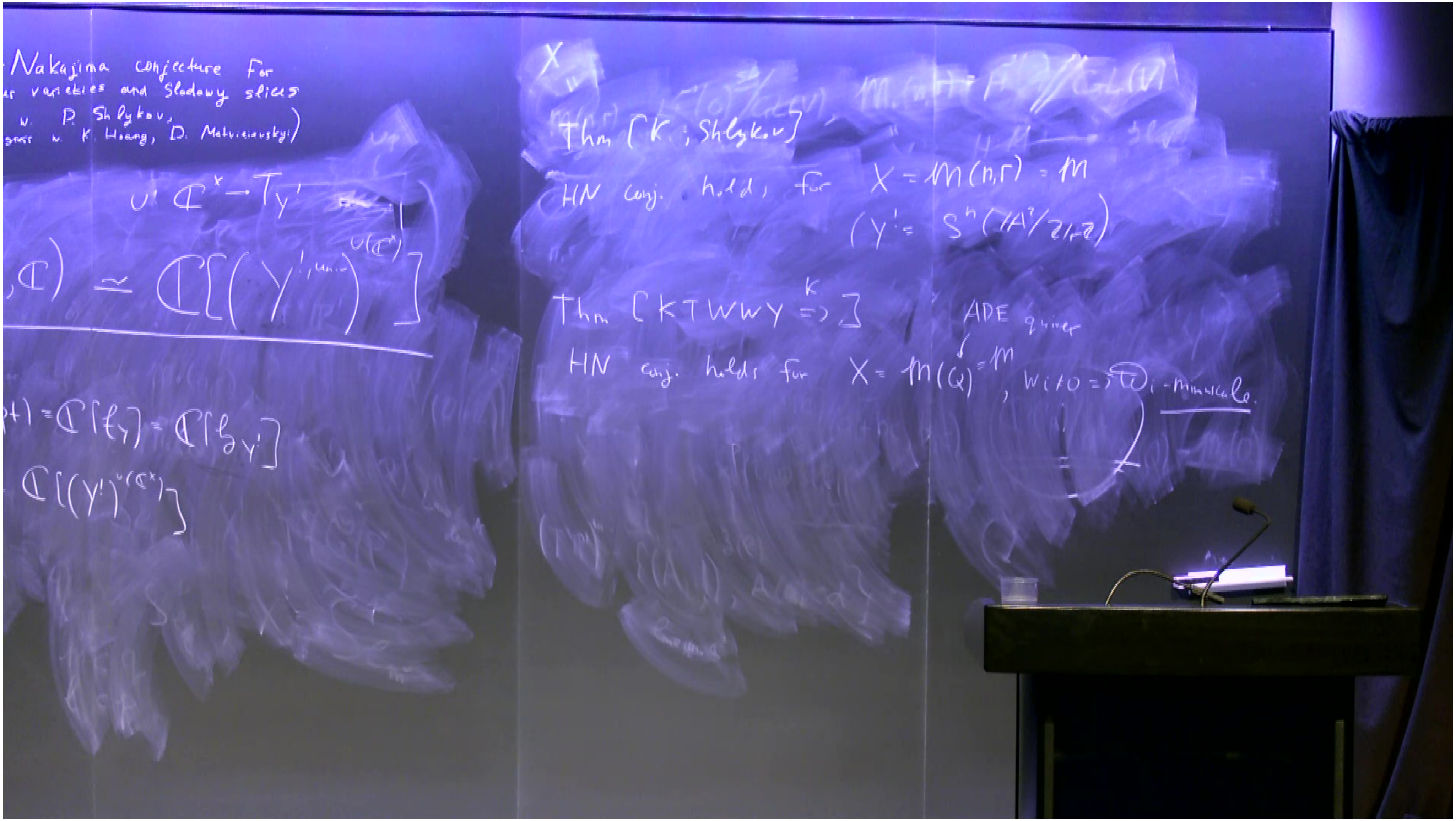

On Hikita-Nakajima conjecture for some quiver varieties and Slodowy slices
(joint w. P. Shlykov, in progress w. K. Hoang, D. Matviievskyi)
Y - affine
Ex A = ℂ[Y]
A = ⊕ A_i
Y/ℂ^× = Spec A/(a_i | i≠0)
X → Y
Sympl dual
X^! → Y^!
ℓ_Y ≃ 𝔥_{Y^!}
𝔥_Y ≃ ℓ_{Y^!}
X^{T_Y} ≃ (X^!)^{T_{Y^!}}
Ex 0) X = T*ℙ^1, X^! = T*ℙ^1
1) Y = 𝔐_0(n,r), Y^! = S^n(𝔸^2/(ℤ/rℤ))
2) Y = 𝔐_0(Q), Y^! = ℳ(Q)
3) X = S̃_{e_ℓ}, X^! = T*(G^∨/P^∨)
Langlands dual
4) X = S̃^q_{e_ℓ}, X^! = S̃^{p^∨}_{e_{m^∨}}
m ⊂ q
ℓ ⊂ p

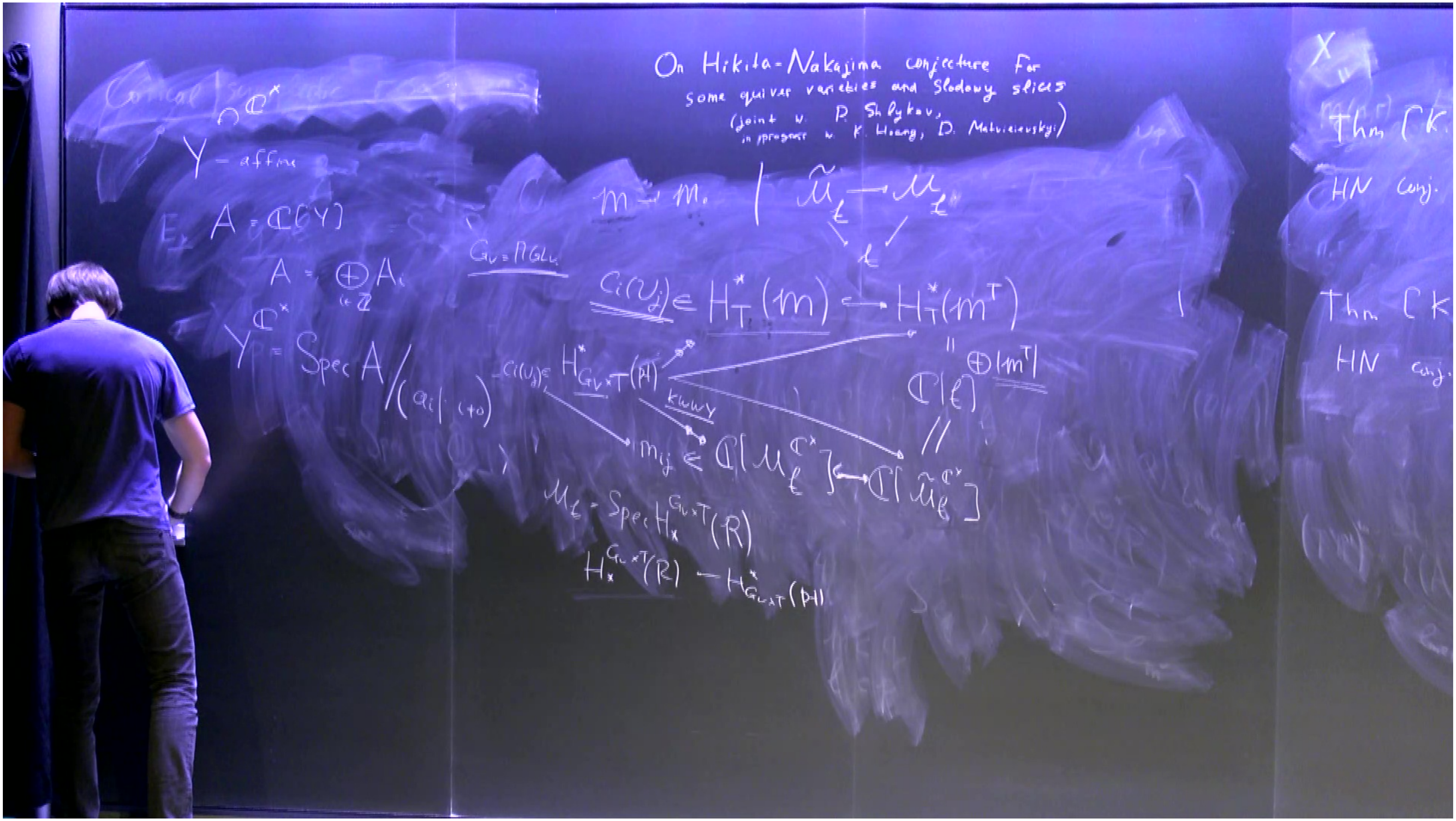

Hikita-Nakajima conjecture for
some quiver varieties and Slodowy slices
(joint w. P. Shlykov,
in progress w. K. Hoang, D. Matvieievskyi)
Y = N = {(a,b,c) | a²+bc=0}, t·(a,b,c) = (a, tb, t⁻¹c)
N^ℂ^× = {(0,0,0)}
ℂ[N^ℂ^×] = ℂ[a,b,c]/(a²+bc, b, c) = ℂ[a]/a²

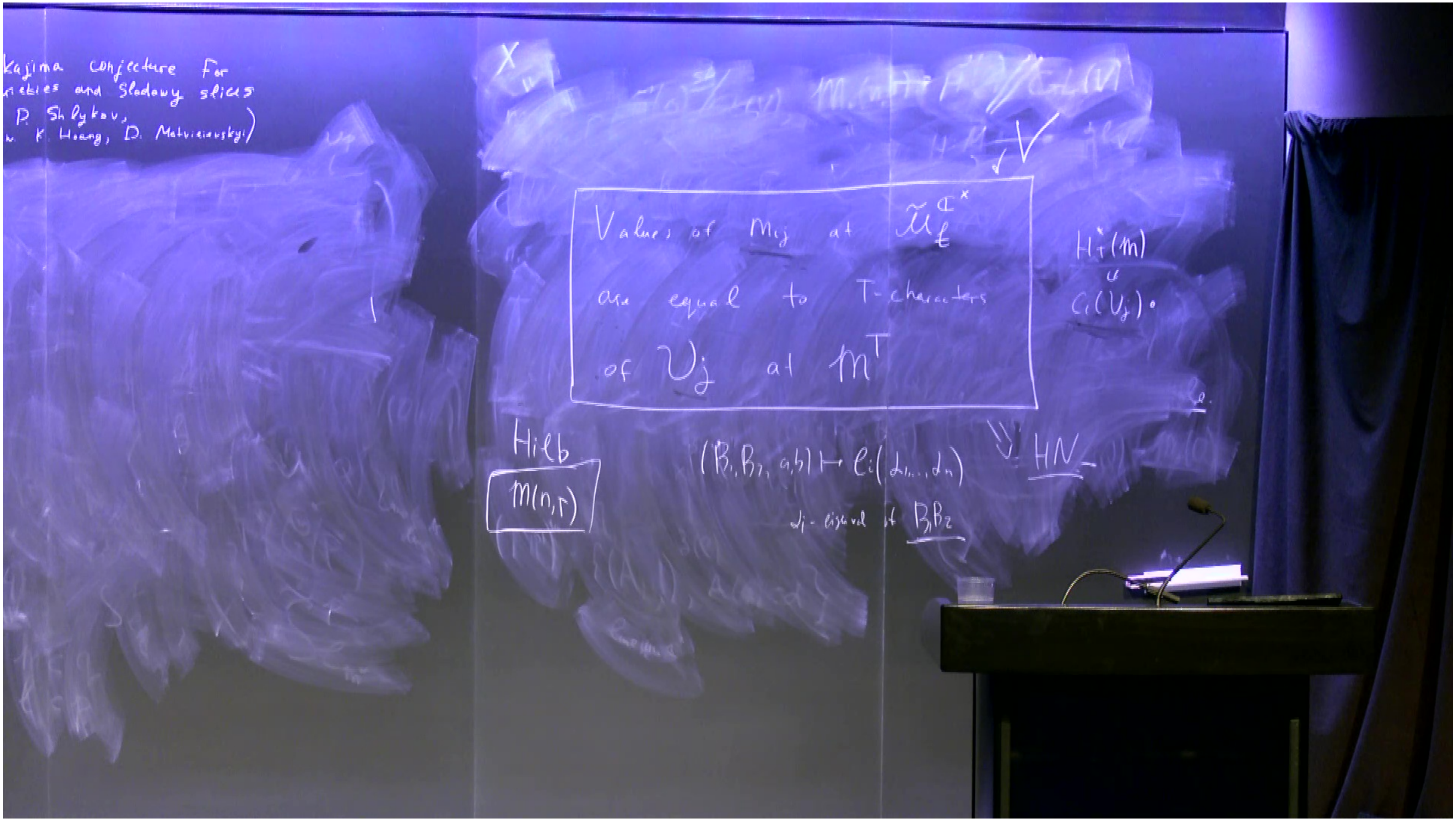


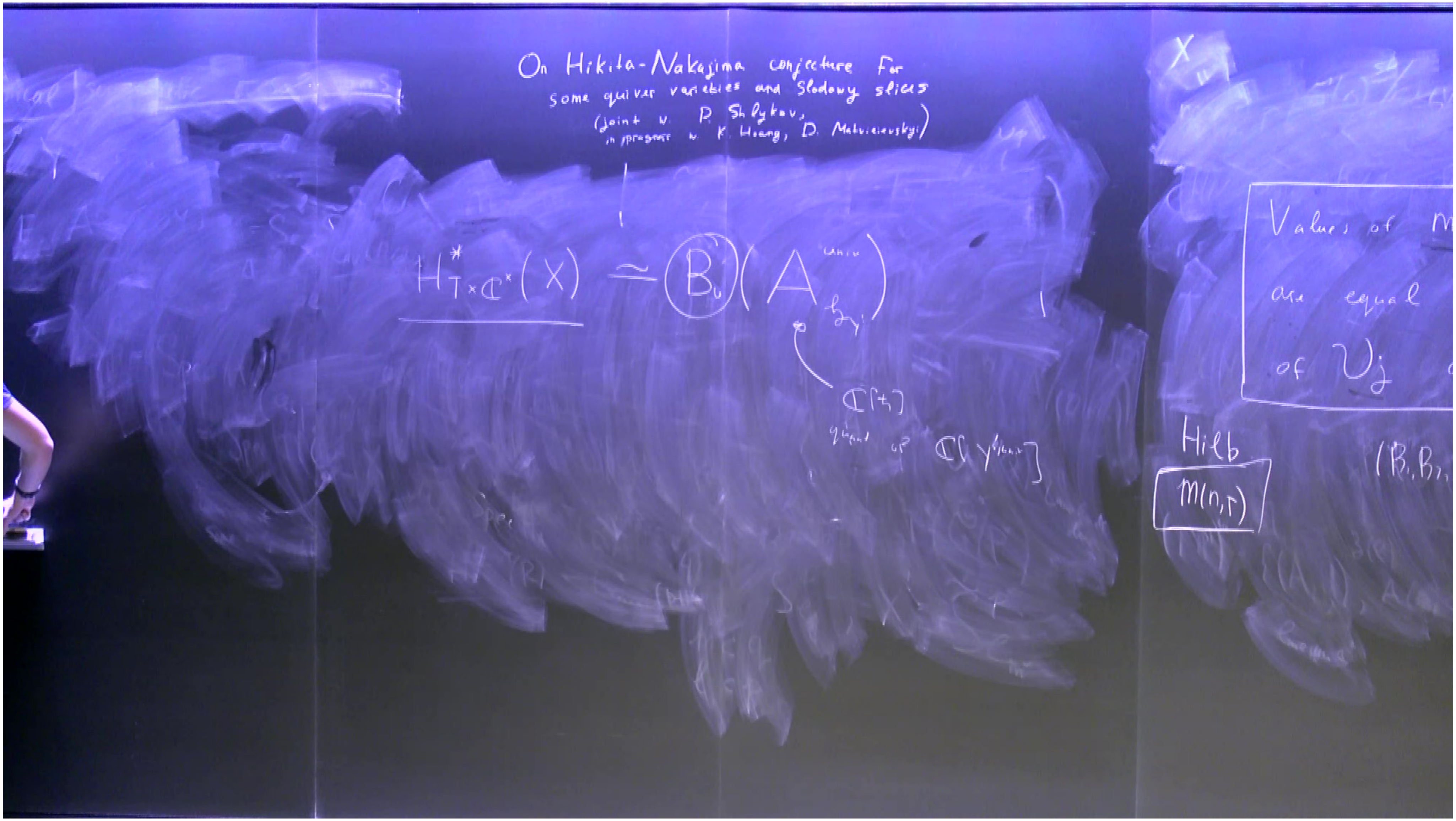
On Hikita-Nakajima conjecture for
some quiver varieties and Slodowy slices
(joint w. P. Shlykov,
in progress w. K. Hoang, D. Matviievskyi)
Conj (Hikita-Nakajima)
ν: ℂ* → T_{Y^!}
∃ H*_{T_Y}(X, ℂ) ≃ ℂ[(Y^{!,univ})^{ν(ℂ*)}]
H*_{T_Y}(pt) = ℂ[𝔱_Y] = ℂ[𝔤_{Y^!}]
Y = 𝒩 = {(a,b,c) | a²+bc=0}
𝒩^{ℂ*} = {(0,0,0)}
ℂ[𝒩^{ℂ*}] = ℂ[a,b,c]/(a²+bc, b, c)

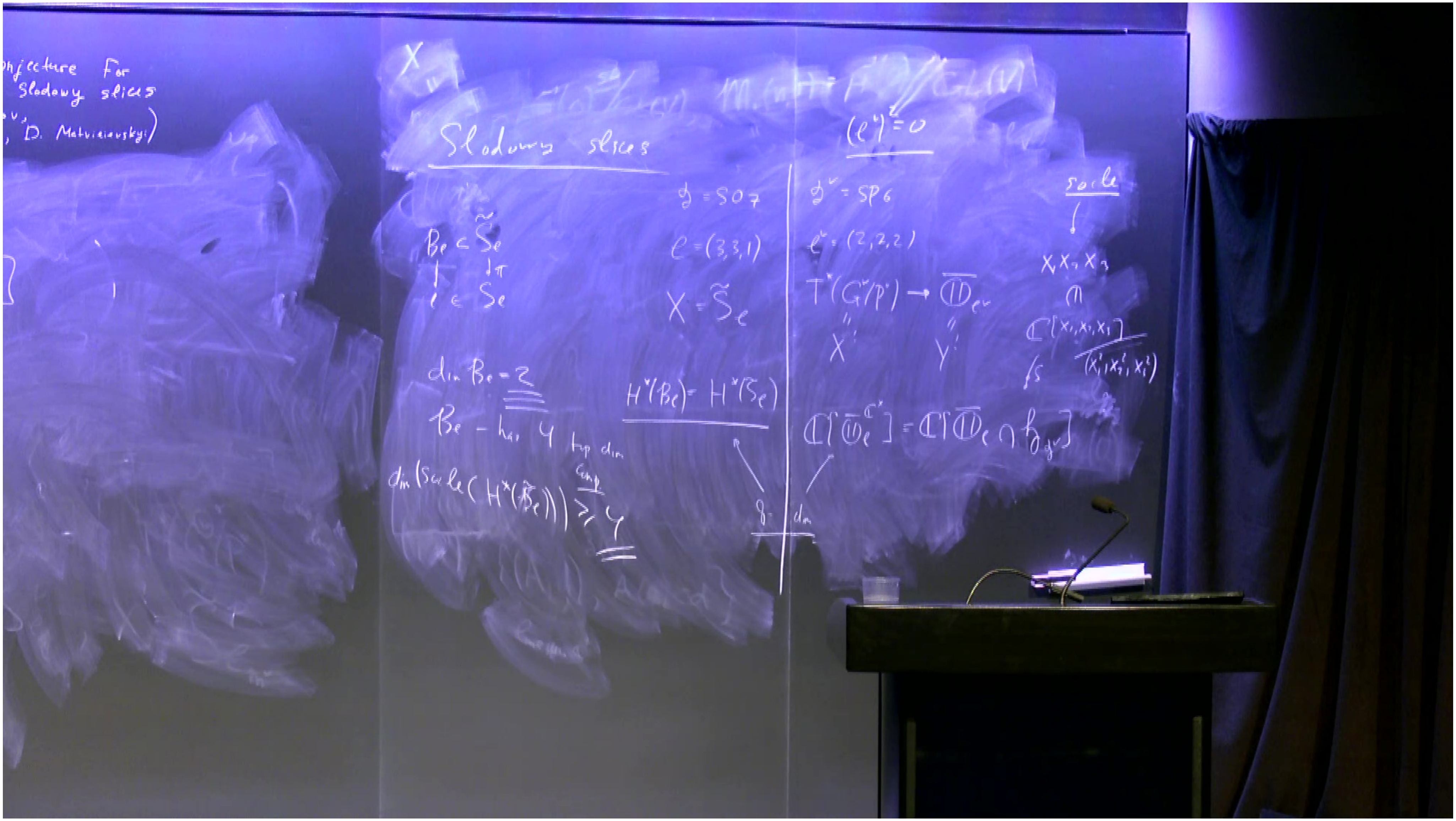

On Hikita-Nakajima conjecture for
some quiver varieties and Slodowy slices
(joint w. P. Shlykov,
in progress w. K. Hoang, D. Matviesievskyi)
Conj (Hikita-Nakajima)
$\nu: \mathbb{C}^\times \to T_{Y^!}$
$\exists\, H^*_{T_Y}(X,\mathbb{C}) \simeq \mathbb{C}[(Y^{!,\text{univ}})^{\nu(\mathbb{C}^\times)}]$
$H^*_{T_Y}(pt) = \mathbb{C}[\mathfrak{t}_Y] = \mathbb{C}[\mathfrak{z}_{Y^!}]$
$\exists\, H^*(X,\mathbb{C}) \simeq \mathbb{C}[(Y^!)^{\nu(\mathbb{C}^\times)}]$
$Y = \mathcal{N} = \{(a,b,c) \mid a^2+bc=0\}$
$\mathcal{N}^{\mathbb{C}^\times} = \{(0,0,0)\}$
$\mathbb{C}[\mathcal{N}^{\mathbb{C}^\times}] = \mathbb{C}[a,b,c]/(a^2+bc,\, b,\, c)$


Nakajima conjecture for
varieties and Slodowy slices
w. R. Shlykov,
w. K. Hoang, D. Matviievskyi)
Thm [K., Shlykov]
HN conj. holds for X = M(n,r) = M
Thm [KTWWY ⇒]
ADE quiver
HN conj. holds for X = M(Q) = M


On Hikita-Nakajima conjecture for
some quiver varieties and Slodowy slices
(joint w. P. Shlykov,
in progress w. K. Hoang, D. Matviievskyi)
Y - affine
Thm
HN conj.
Thm
HN conj.


kajima conjecture for
ieties and Slodowy slices
P. Shlykov,
w. K. Hoang, D. Matviievskyi)
Values of m_{ij} at $\tilde{\mathcal{U}}_\ell^{\mathbb{C}^\times}$
are equal to T-characters
of V_j at $\mathcal{M}^T$
$H_T^*(\mathcal{M})$
$c_i(V_j)$
Hilb
$\mathcal{M}(n,r)$
$(B_1, B_2, a, b) \mapsto e_i(d_1, \dots, d_n)$
d_i - eigenval of $B_1 B_2$
HN


On Hikita-Nakajima conjecture for
some quiver varieties and Slodowy slices
(joint w. P. Shlykov,
in progress w. K. Hoang, D. Matvieievskyi)
Values of
are equal
Hilb
M(n,r)


Slodowy slices
$\mathfrak{g} = SO_7$
$\mathfrak{g}^\vee = Sp_6$
$e = (3,3,1)$
$e^\vee = (2,2,2)$
$X = \tilde{S}_e$
$T^*(G^\vee/P^\vee) \to \overline{\mathbb{O}}_{e^\vee}$
$H^*(B_e) = H^*(\tilde{S}_e)$
socle
$\mathbb{C}[x_1,x_2,x_3]/(x_1^2,x_2^2,x_3^2)$
$\dim B_e = 2$
B_e — has 4 top dim comp
$\dim(\mathrm{Socle}(H^*(B_e))) \geq 4$

On Hikita-Nakajima conjecture for some quiver varieties and Slodowy slices
(joint w. P. Shlykov, in progress w. K. Huang, D. Matvieievskyi)

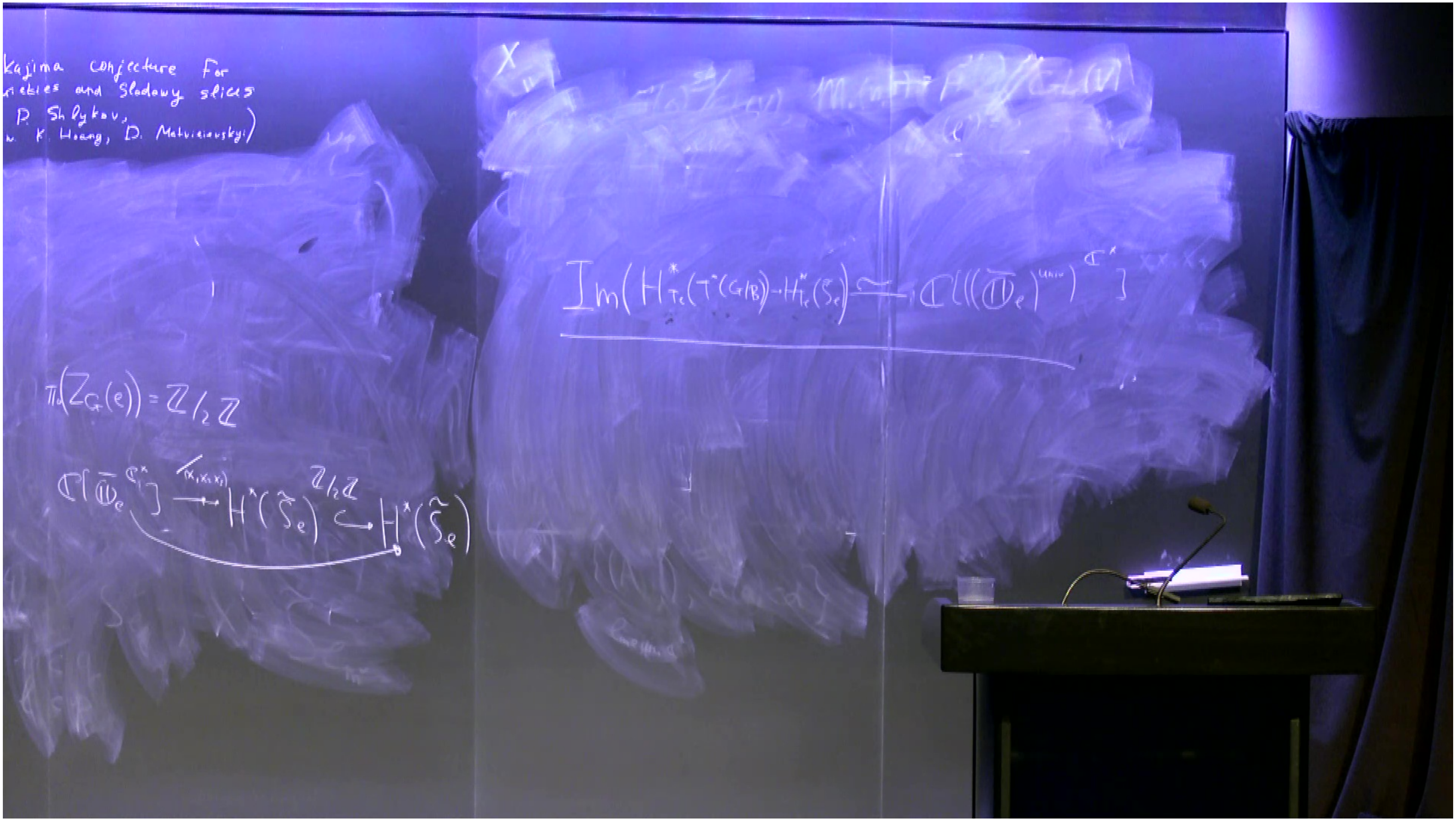
(1) $\mathbb{C}[\mathfrak{h}_{\mathfrak{g}^\vee}] \longrightarrow \mathbb{C}[\overline{\mathbb{O}}_e^{\mathbb{C}^\times}]$

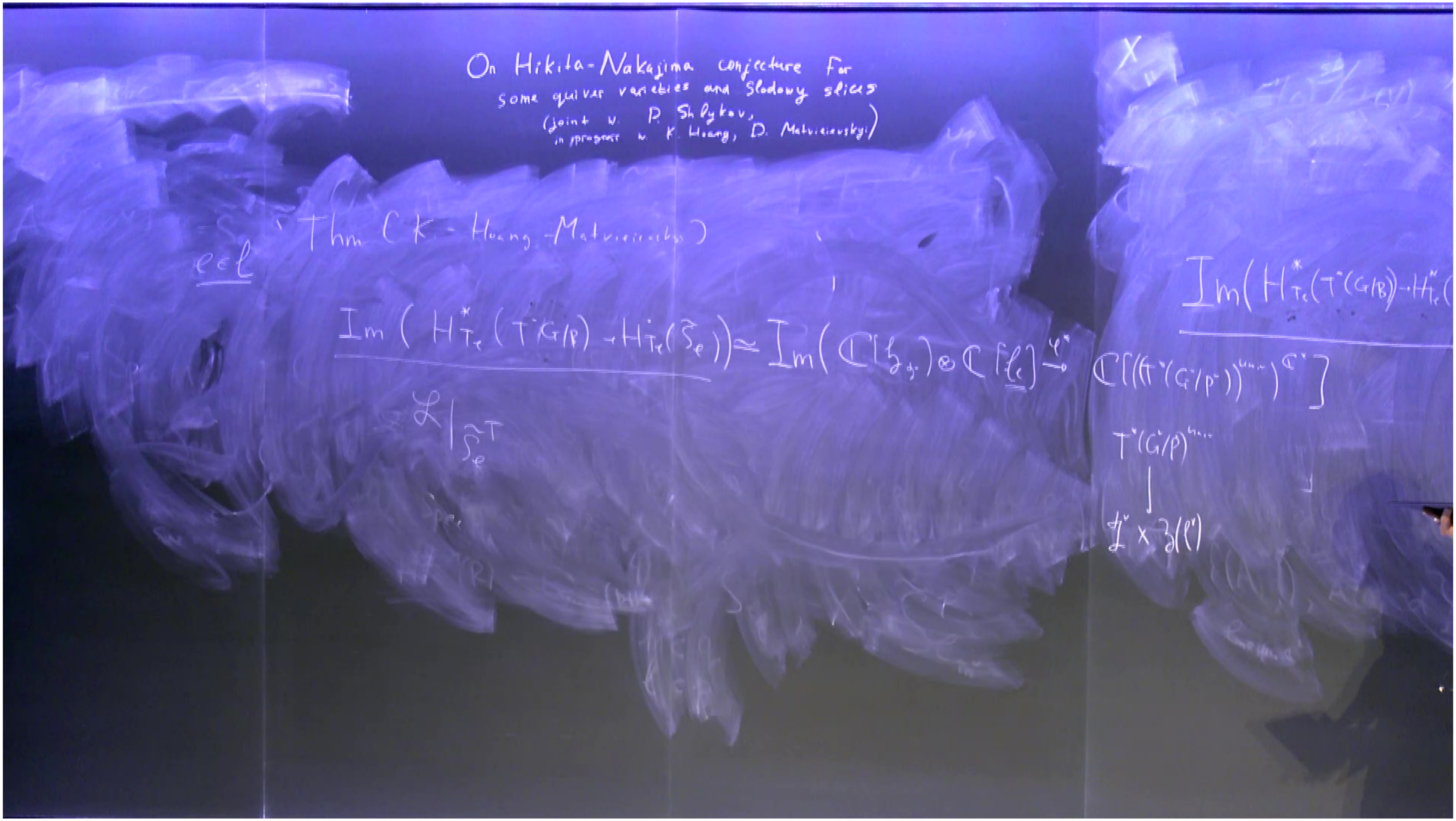
NOT surjective

(2) $S^\bullet(H^2(\tilde{S}_e)) \longrightarrow H^*(\tilde{S}_e) \curvearrowleft \pi_0(Z_G(e)) = \mathbb{Z}/2\mathbb{Z}$

$\mathfrak{h}_{\mathfrak{g}^\vee} \searrow H^*(T^*(G/B)) \nearrow$

$\mathbb{C}[\overline{\mathbb{O}}_e^{\mathbb{C}^\times}] \xrightarrow{(x_1x_2x_3)} H^*(\tilde{S}_e)^{\mathbb{Z}/2\mathbb{Z}} \hookrightarrow H^*(\tilde{S}_e)$


kajima conjecture for
ieties and Slodowy slices
P. Shlykov,
w. K. Hoang, D. Matviiavskyi)
Im(H*Te(T*(G/B)) → H*Te(S̃e)) ≃ ℂ[((Ψe)univ)ℂ*]
π0(ZG(e)) = ℤ/2ℤ
ℤ/2ℤ


On Hikita-Nakajima conjecture for
some quiver varieties and Slodowy slices
(joint w. P. Shlykov,
in progress w. K. Huang, D. Matvieievskyi)
Thm (K.-Huang-Matvieievskyi)
$\operatorname{Im}(H^*_{T_e}(T^*G/P) \to H^*_{T_e}(\tilde{S}_e)) \simeq \operatorname{Im}(\mathbb{C}[\mathfrak{h}_{\cdot}] \otimes \mathbb{C}[\mathfrak{t}_e] \xrightarrow{\varphi^*} \mathbb{C}[((T^*(G^\vee/P^\vee))^{\ldots})^{\mathbb{C}^*}]$
$\mathcal{L}|_{\tilde{S}_e^T}$
$T^*(G^\vee/P^\vee)^{\ldots}$
$\mathfrak{t}^\vee \times \mathfrak{z}(\mathfrak{l}^\vee)$