

Title: How to solve quantum many-body problems.

Speakers: Annabelle Bohrdt

Series: Colloquium

Date: March 27, 2023 - 1:30 PM

URL: <https://pirsa.org/23030116>

Abstract: In quantum many-body physics, we aim to understand and predict the exciting phenomena emerging from the interactions of many quantum particles. In this talk I will use the paradigmatic Fermi-Hubbard model, a conceptually simple model of interacting fermionic particles, as an example to highlight different approaches to gain insights into quantum many-body problems. I will describe a semi-analytical theory, established numerical methods, machine learning techniques, as well as quantum simulation experiments, which provide detailed information about the quantum state of the system. A particular focus will be on how to combine these different approaches to learn as much as possible about the system of interest, for example through new observables and modified microscopic models.

Zoom link: <https://pitp.zoom.us/j/94368447256?pwd=clhQcUJNaTR0UjVydnBHelo4N3JIQT09>

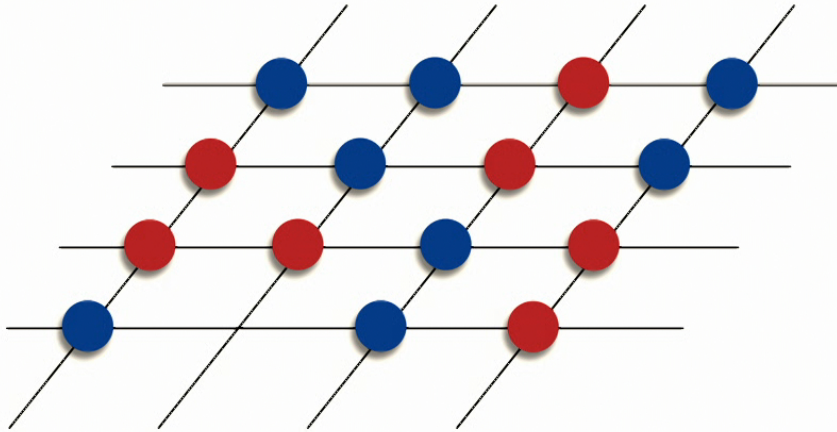


Simple set of rules:

- avoid crowding your neighbors
- align with your neighbors
- steer towards the average position of your neighbors.

Perimeter, March 2023

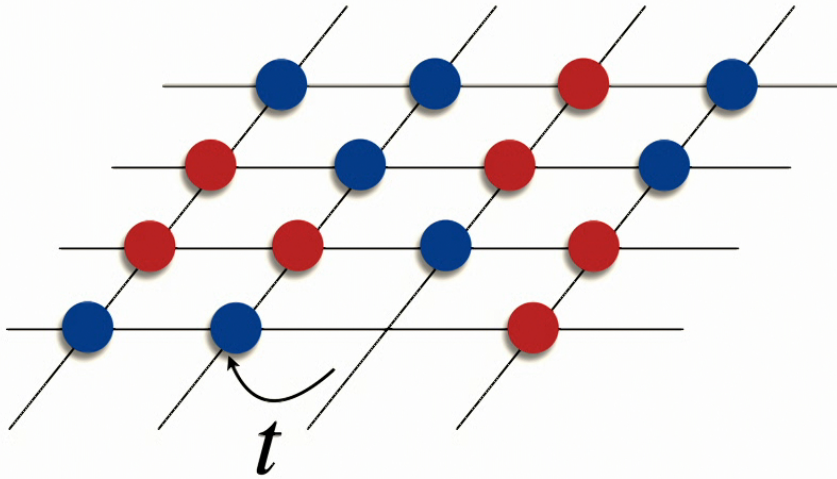
Quantum many-body systems



Simple set of rules:

- Hop around as much as possible
- Never sit on the same site as fermion with equal spin
- Try to avoid sitting on same site as fermion with opposite spin

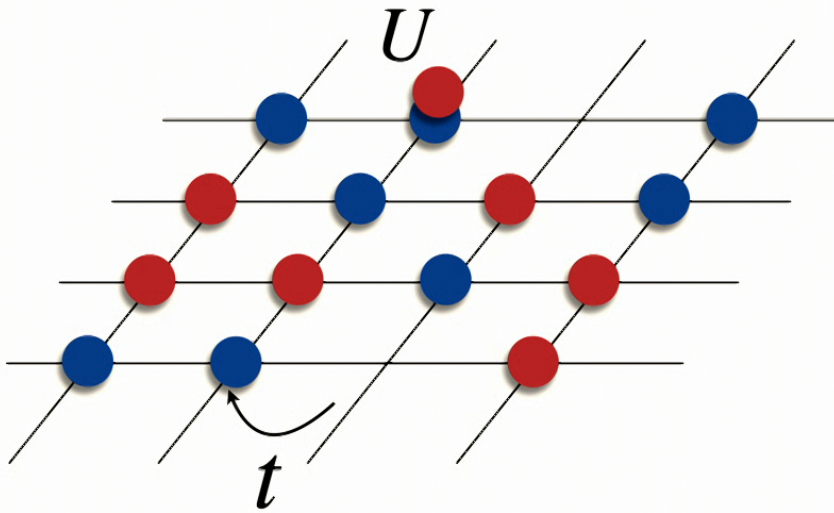
Quantum many-body systems



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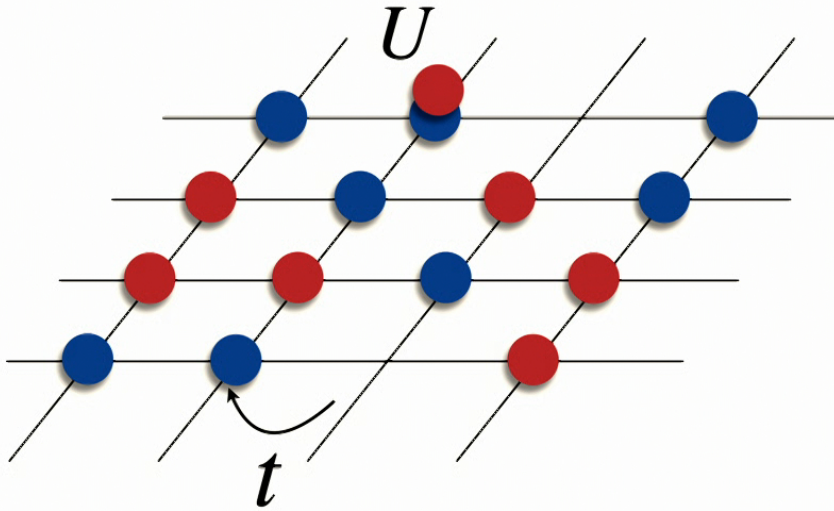
Quantum many-body systems



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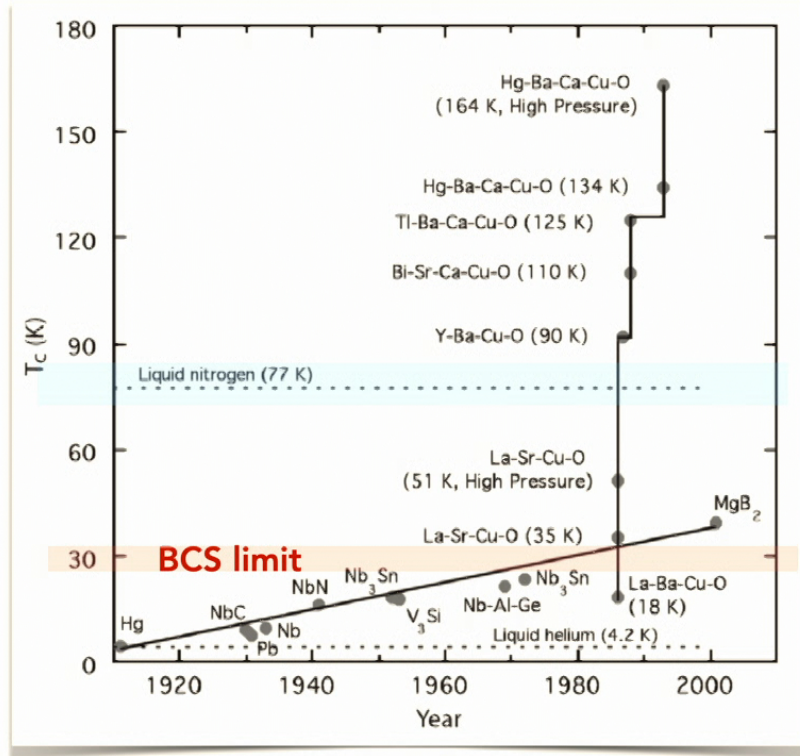
$$\hat{H} = -t \sum_{\langle i,j \rangle, \sigma} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + h.c.) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

↑
tunneling

↑
interaction

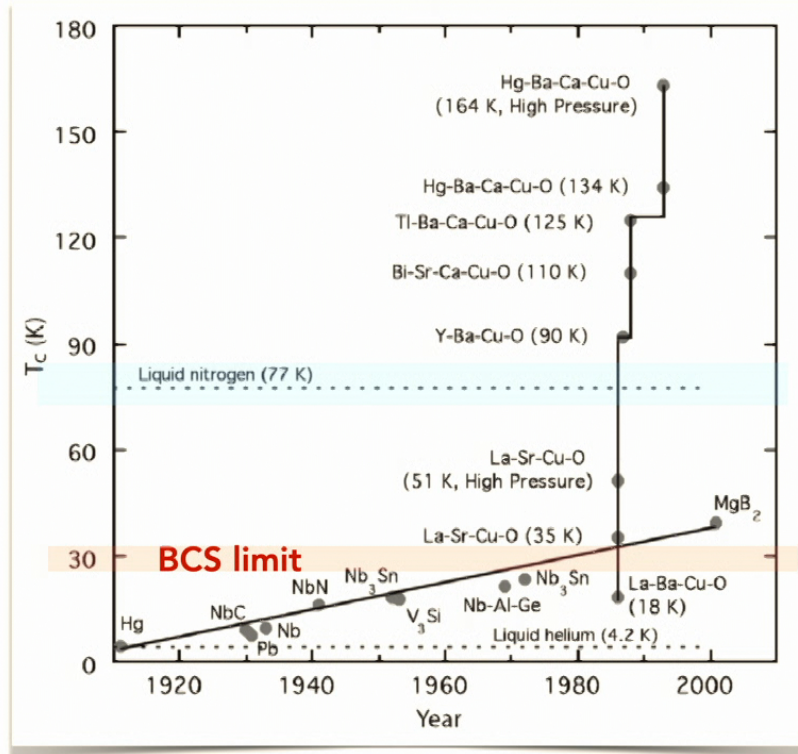
Why should you care

High-temperature superconductivity:



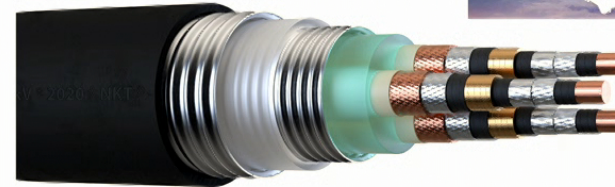
Why should you care

High-temperature superconductivity:



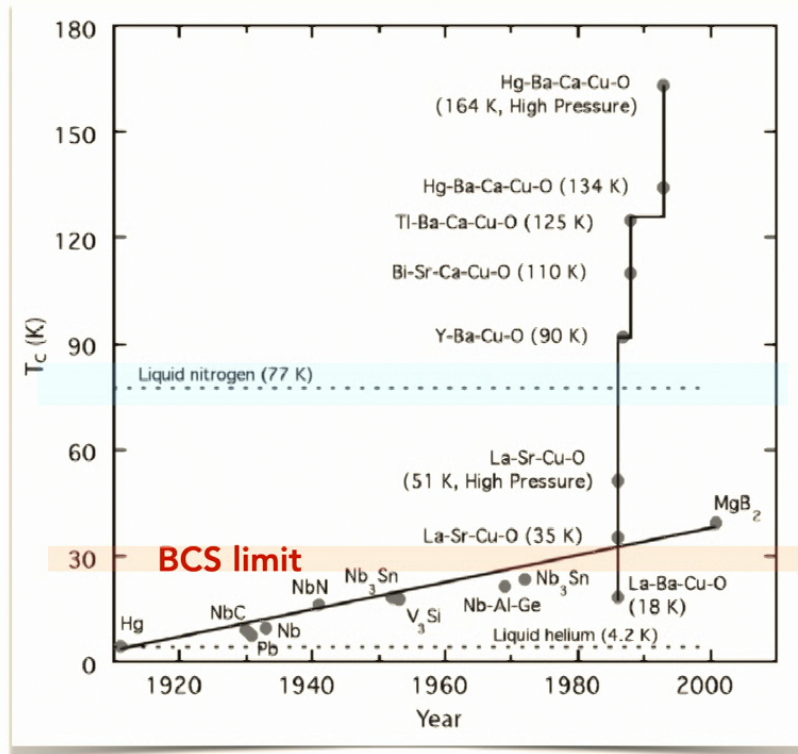
* Latest applications:

Munich's SuperLink SC cable (12 km)



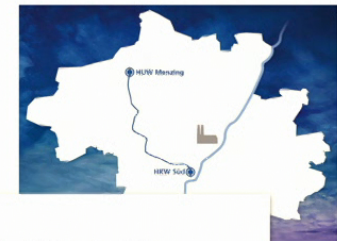
Why should you care

High-temperature superconductivity:



* **Latest applications:**

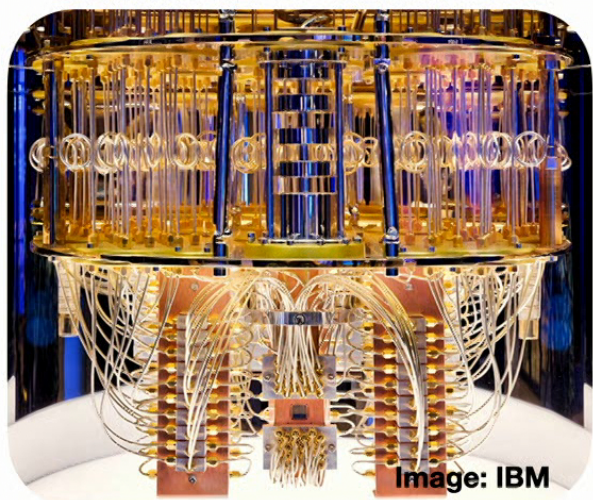
Munich's SuperLink
SC cable (12 km)



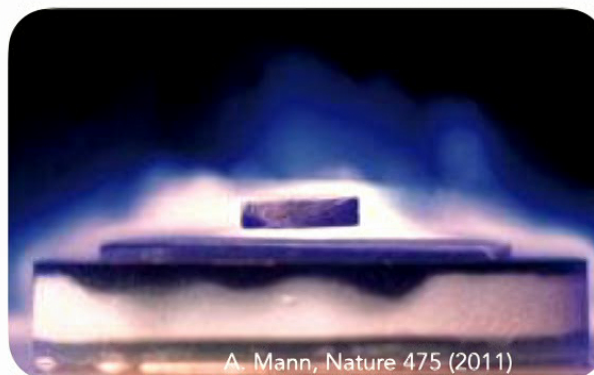
* **High-Tc
wind
turbine!**

Why should you care

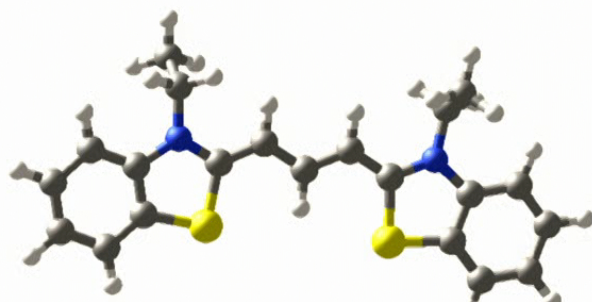
Quantum computing



Materials research

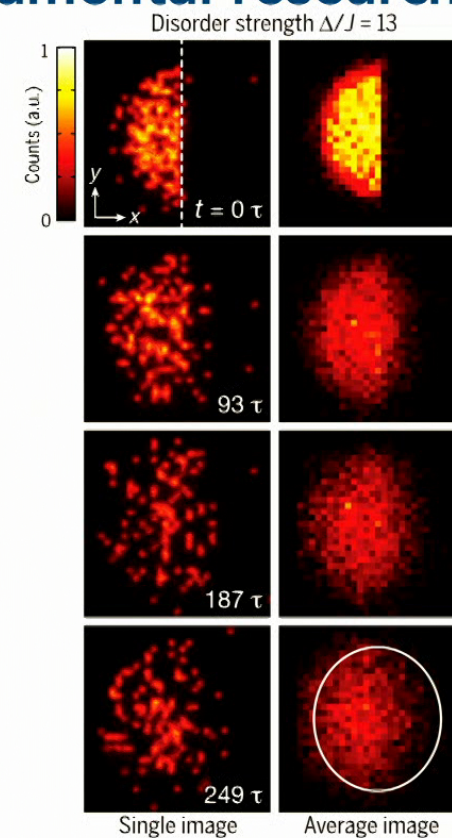


Quantum chemistry



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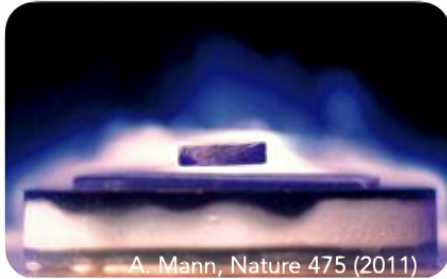
Fundamental research



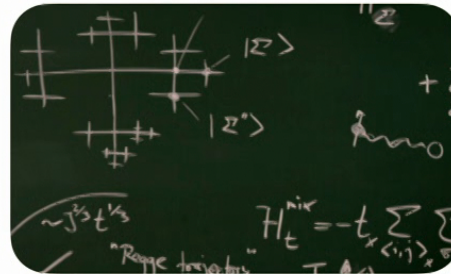
Choi et al., Science (2016)

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What can we do?



**Experiments on
real materials
(messy)**

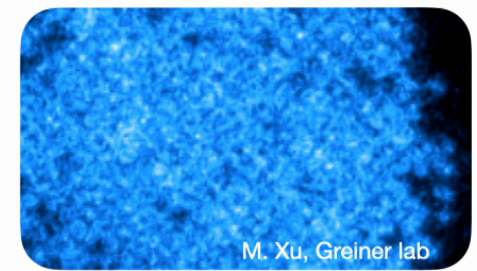


**Analytical
approximations**



**Numerical
simulations**

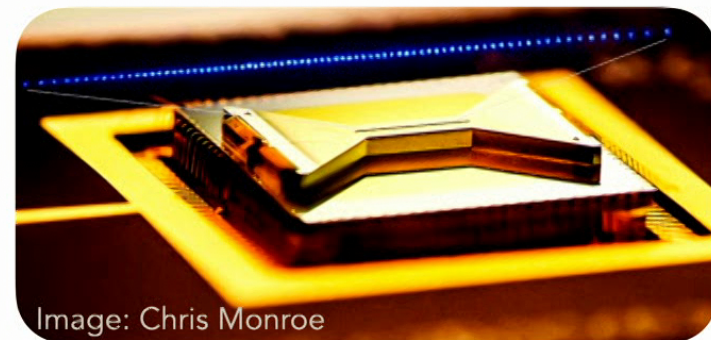
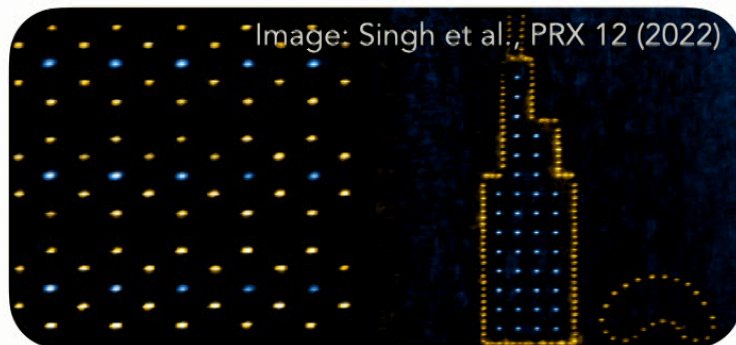
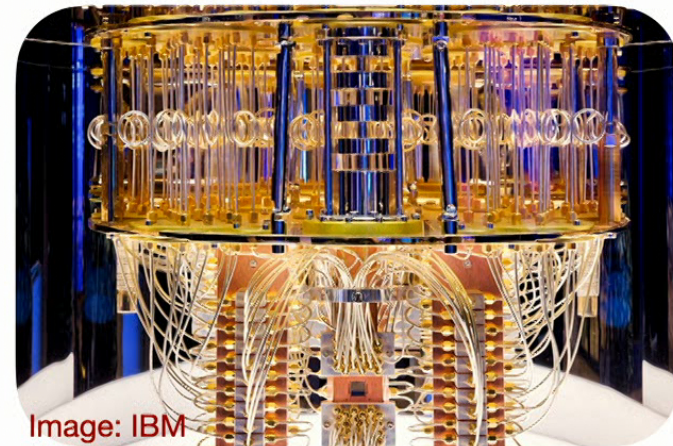
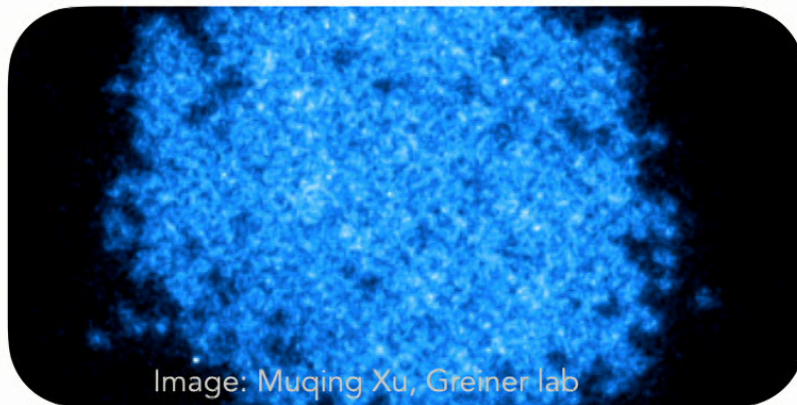
- Exact for small systems
- Approximations for large systems: what to throw away?



**Quantum
simulation**

- What should we look for?

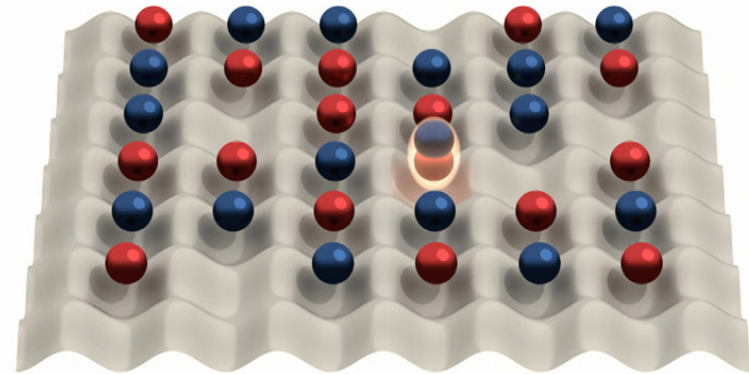
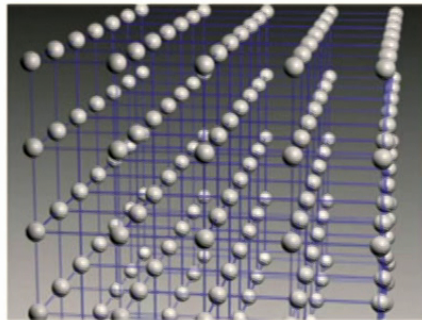
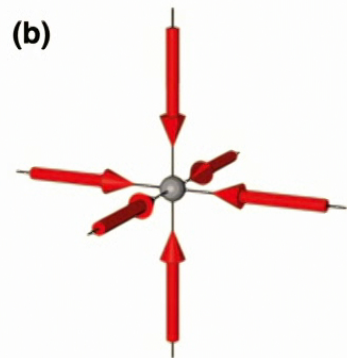
Quantum simulation



Cold atoms in optical lattices

Solid crystal: describe electrons moving in a periodic potential created by ions

Neutral atoms in a periodic potential created by interfering laser beams



Early review: Bloch & Zwerger, Rev Mod Phys. 80 (2008)

More recent review: Gross & Bloch, Science 357 (2017)

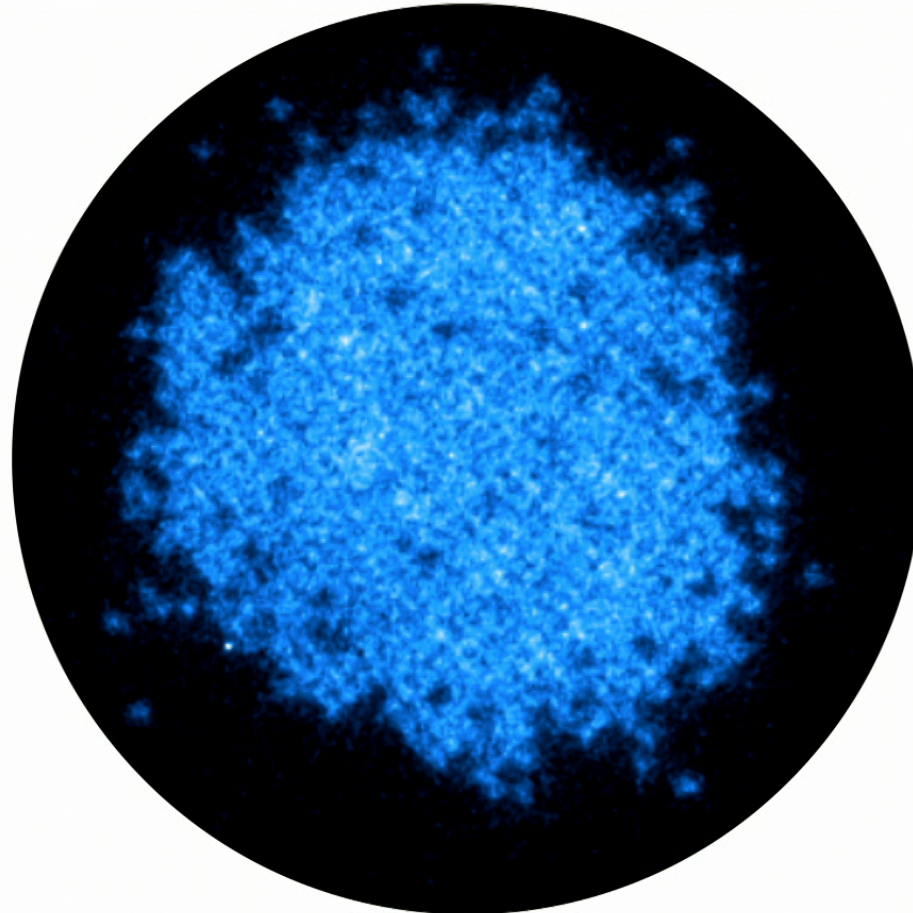


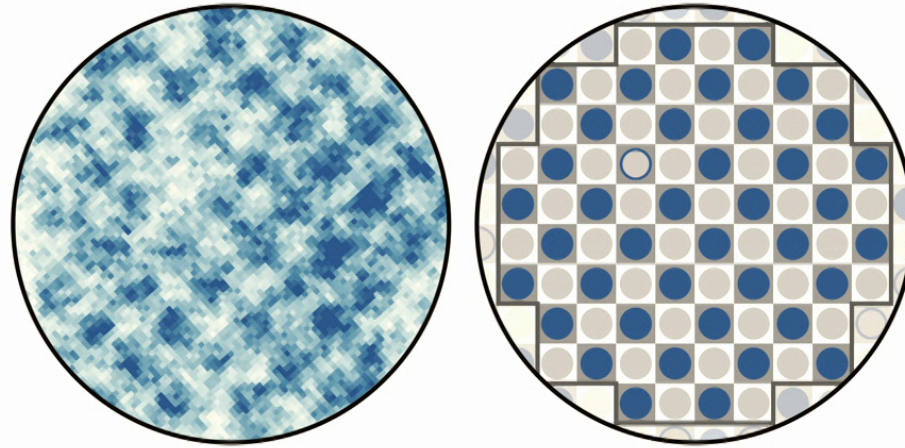
Image: Muqing Xu, Greiner lab

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Perimeter, March 2023

Quantum projective measurements

$$|\psi\rangle = \begin{matrix} \uparrow & \downarrow & \uparrow \\ \downarrow & \uparrow & \downarrow \end{matrix} \rangle + \begin{matrix} \uparrow & \downarrow & \uparrow \\ \uparrow & \downarrow & \downarrow \end{matrix} \rangle + \begin{matrix} \uparrow & \uparrow & \downarrow \\ \uparrow & \downarrow & \downarrow \end{matrix} \rangle + \begin{matrix} \downarrow & \uparrow & \downarrow \\ \uparrow & \downarrow & \uparrow \end{matrix} \rangle + \dots$$



Mazurenko et al., Nature 545 (2017)



Getting a feeling for the Fermi-Hubbard model

A single hole — with a string attached

One, two, many holes?

(Interpretable) machine learning quantum snapshots

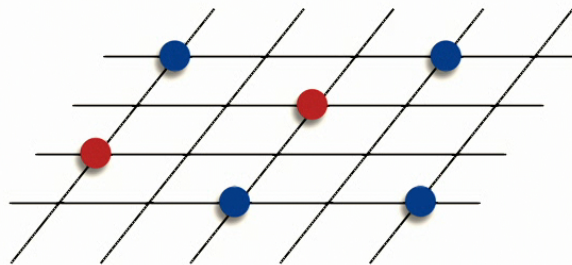
Fermi-Hubbard model

$$\hat{H} = -t \sum_{\langle i,j \rangle, \sigma} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + h.c.) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

tunneling

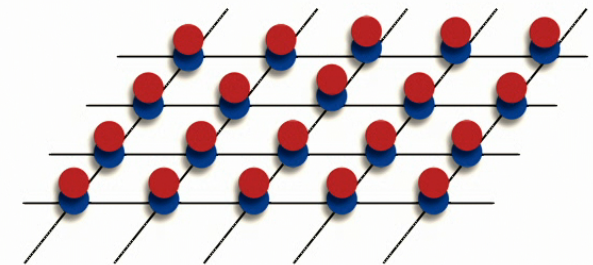
interaction

High temperatures:



metal

Increasing particle number



Band insulator

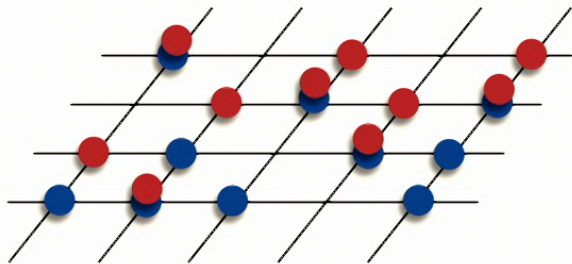
Fermi-Hubbard model

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↑
| tunneling

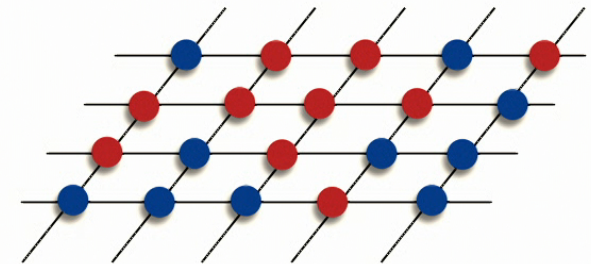
↑
| interaction

Half-filling



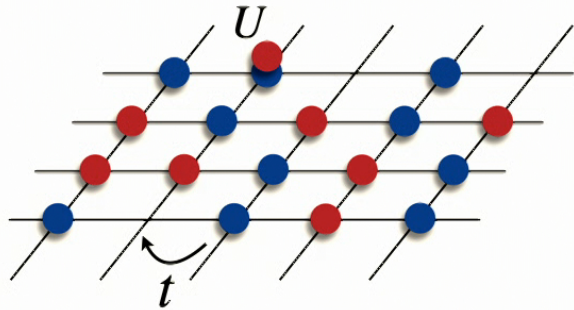
metal

Decreasing temperature $T < U$



Mott insulator

Fermi-Hubbard model



$$\hat{H} = -t \sum_{\langle i,j \rangle, \sigma} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + h.c.) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

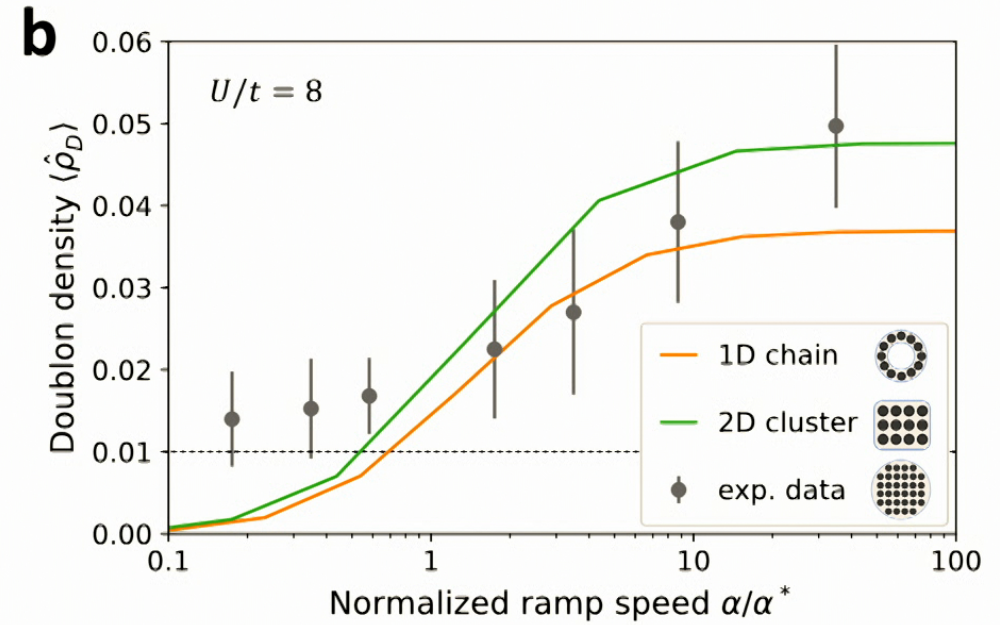
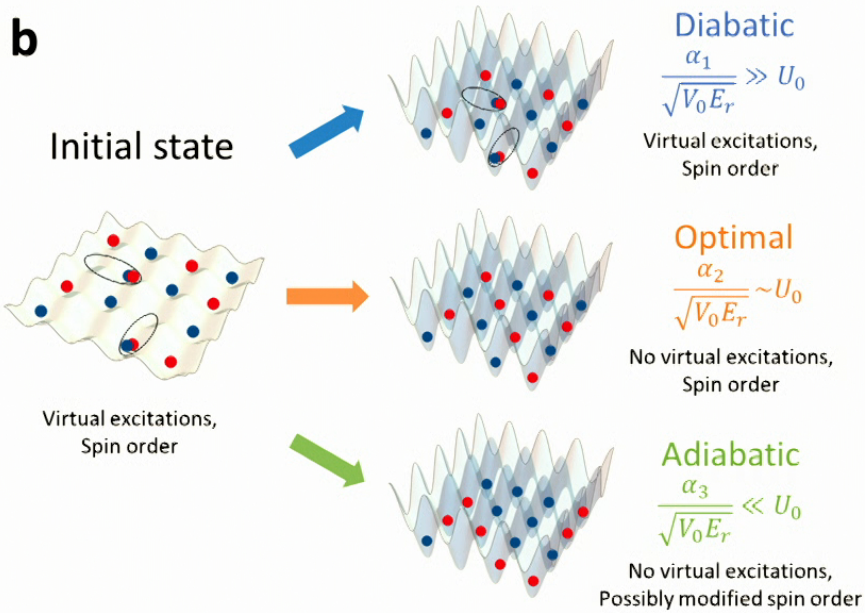
| tunneling
| interaction

Large U limit: $t - J$ model

$$\hat{H}_{t-J} = -t \hat{P} \left[\sum_{\langle \mathbf{i}, \mathbf{j} \rangle, \sigma} \hat{c}_{\mathbf{i}, \sigma}^\dagger \hat{c}_{\mathbf{j}, \sigma} + h.c. \right] \hat{P} + J \sum_{\langle \mathbf{i}, \mathbf{j} \rangle} \left(\hat{\mathbf{S}}_{\mathbf{i}} \cdot \hat{\mathbf{S}}_{\mathbf{j}} - \frac{1}{4} \hat{n}_{\mathbf{i}} \hat{n}_{\mathbf{j}} \right)$$

| no double-occupancies
| tunneling
| spin-exchange

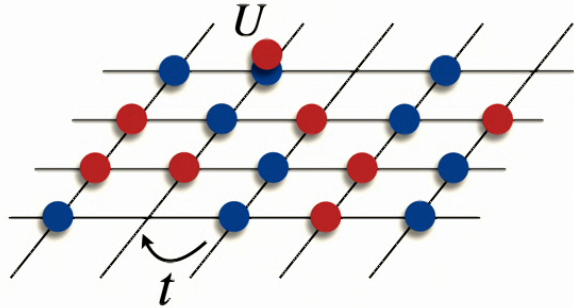
Experimental Schrieffer Wolff transformation



Kale et al., PRA 106 (2022)

see also: active learning for adaptive basis choices: Lange et al., arXiv:2203.15719

Fermi-Hubbard model



$$\hat{H} = -t \sum_{\langle i,j \rangle, \sigma} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + h.c.) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

tunneling

interaction

temperature



Large U limit: $t - J$ model

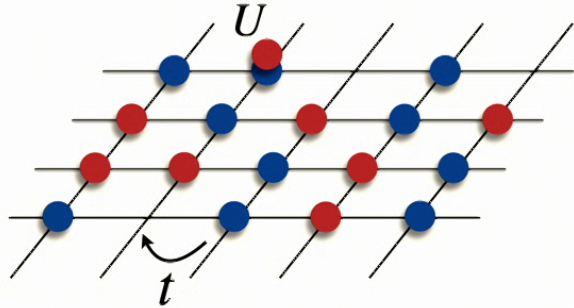
$$\hat{H}_{t-J} = -t \hat{P} \left[\sum_{\langle \mathbf{i}, \mathbf{j} \rangle, \sigma} \hat{c}_{\mathbf{i},\sigma}^\dagger \hat{c}_{\mathbf{j},\sigma} + h.c. \right] \hat{P} + J \sum_{\langle \mathbf{i}, \mathbf{j} \rangle} \left(\hat{\mathbf{S}}_{\mathbf{i}} \cdot \hat{\mathbf{S}}_{\mathbf{j}} - \frac{1}{4} \hat{n}_{\mathbf{i}} \hat{n}_{\mathbf{j}} \right)$$

no double-occupancies

tunneling

spin-exchange

Fermi-Hubbard model



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tunneling

interaction

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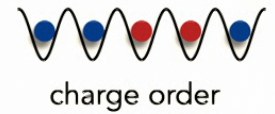
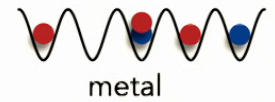
no double-occupancies

tunneling

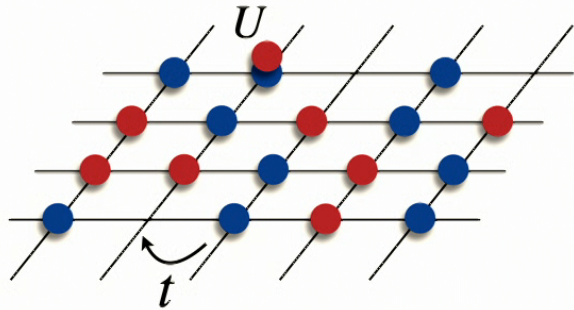
spin-exchange

$U \gg t$

temperature



Fermi-Hubbard model



$$\hat{H} = -t \sum_{\langle i,j \rangle, \sigma} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + h.c.) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

tunneling

interaction

Large U limit: $t - J$ model

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no double-occupancies

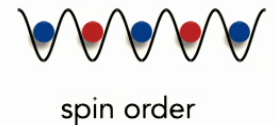
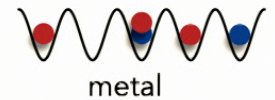
tunneling

spin-exchange

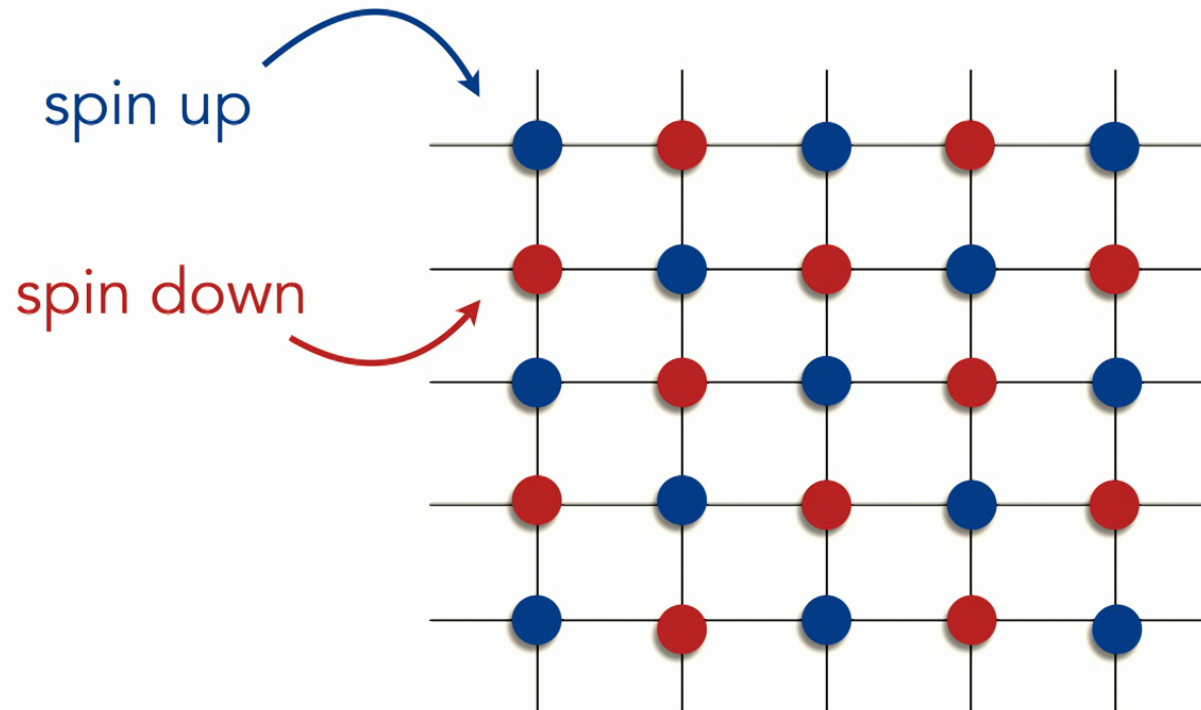
$$U \gg t$$

$$J = \frac{4t^2}{U}$$

temperature

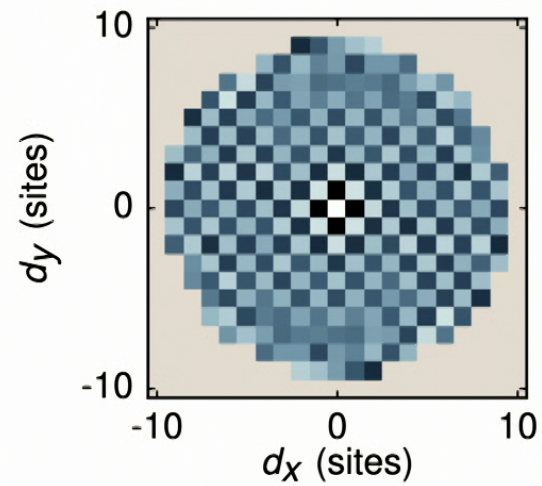


Antiferromagnetism



Antiferromagnet

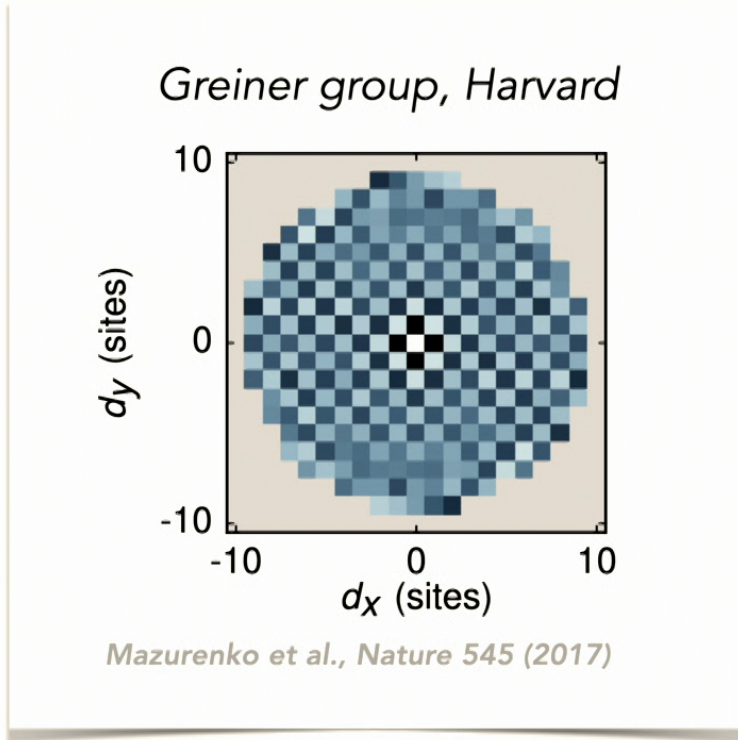
Greiner group, Harvard



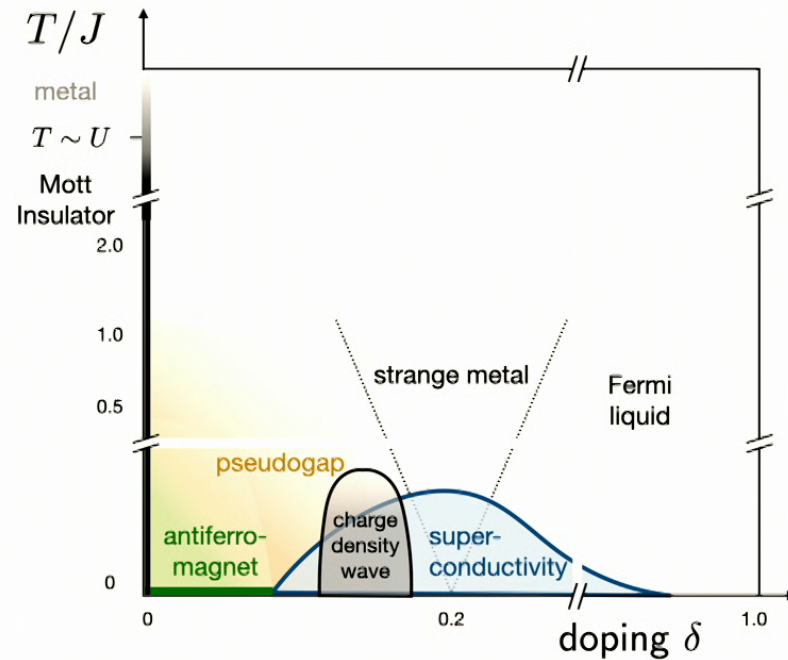
Mazurenko et al., Nature 545 (2017)

Fermi-Hubbard model

Antiferromagnet



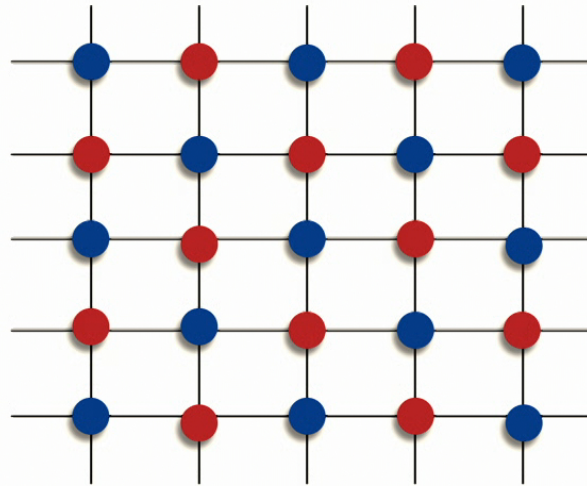
Review article: Fermi-Hubbard with cold atoms
 Bohrdt et al., Annals of Physics (2021)





A single hole

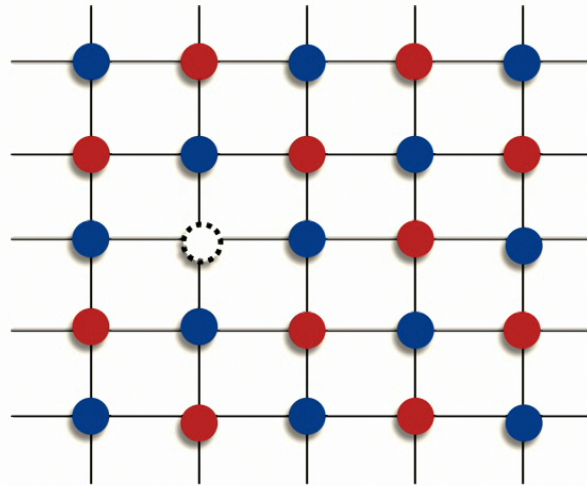
Geometric string theory



Grusdt et al., PRX 8 (2018), Grusdt et al., PRB 99 (2019), Bohrdt et al., NJP 22 (2020)

Early work: Bulaevskii et al., JETP 27 (1968), Trugman, PRB 37 (1988), Manousakis, PRB 75 (2007)

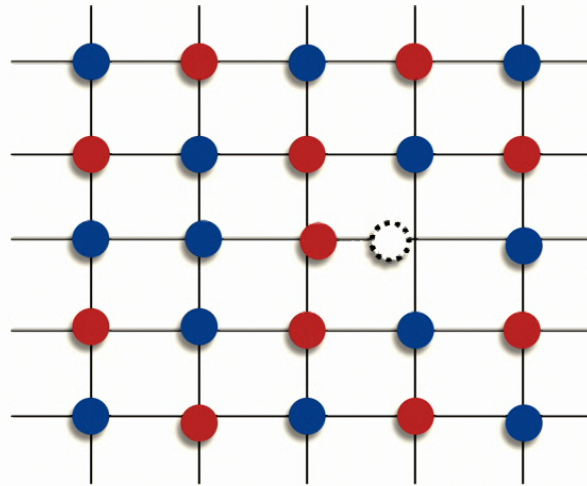
Geometric string theory



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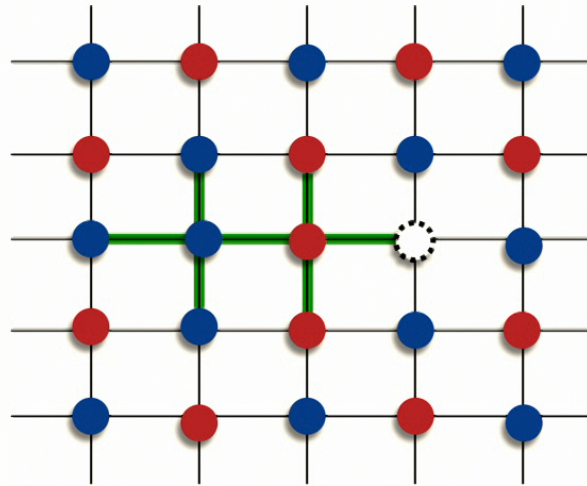
Geometric string theory



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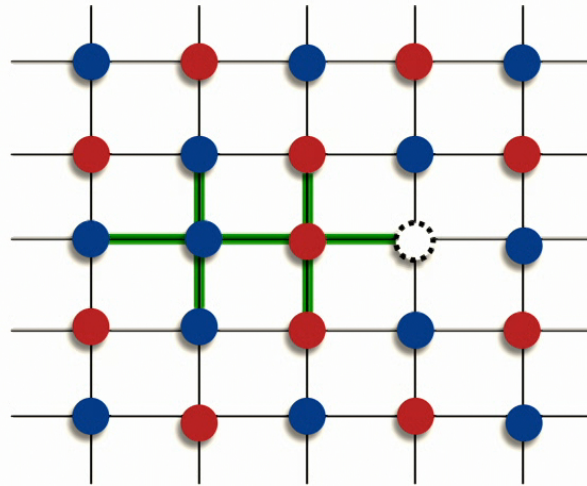
Geometric string theory



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Geometric string theory

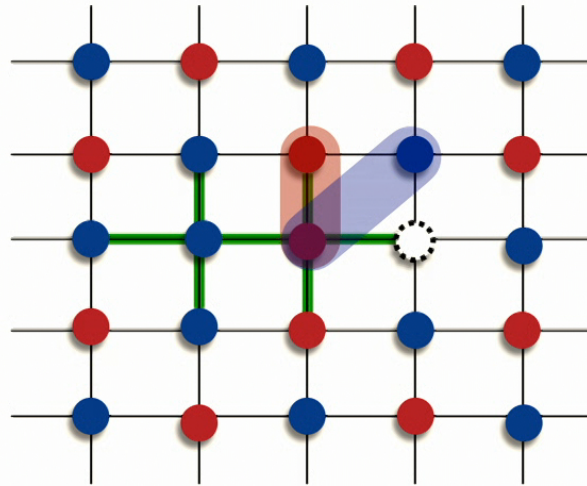


$$\frac{dE}{dl} = 2J (C_{\mathbf{e}_x + \mathbf{e}_y} - C_{\mathbf{e}_x})$$

Grusdt et al., PRX 8 (2018), Grusdt et al., PRB 99 (2019), Bohrdt et al., NJP 22 (2020)

Early work: Bulaevskii et al., JETP 27 (1968), Trugman, PRB 37 (1988), Manousakis, PRB 75 (2007)

Geometric string theory



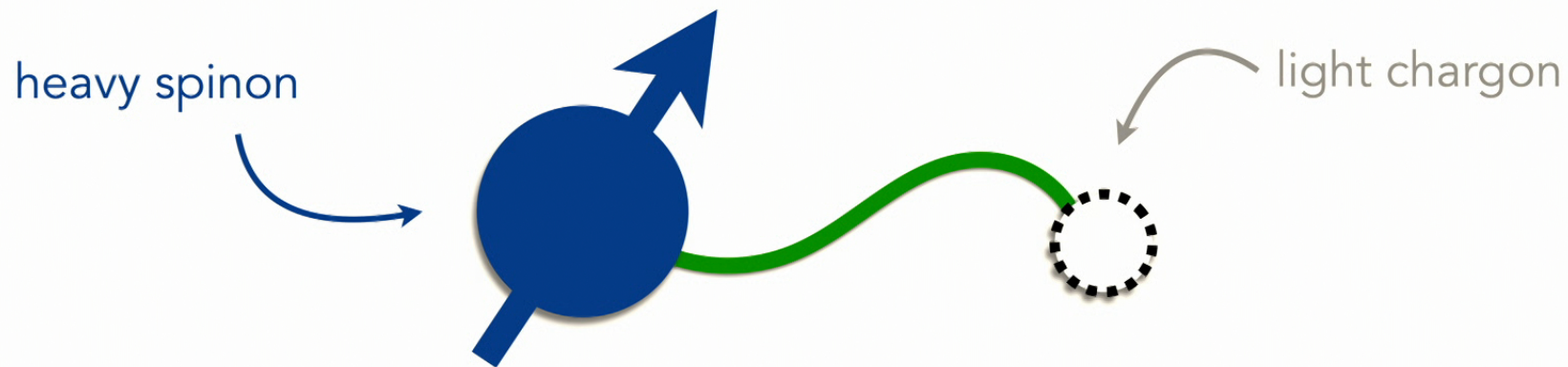
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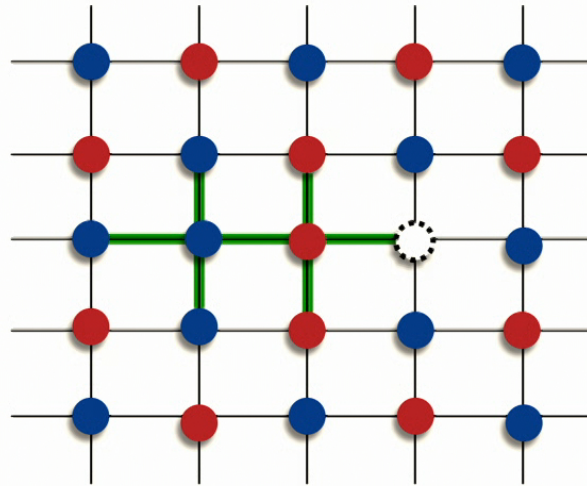
Early work: Bulaevskii et al., JETP 27 (1968), Trugman, PRB 37 (1988), Manousakis, PRB 75 (2007)

Geometric string theory

- Born-Oppenheimer approximation: $|\psi\rangle \simeq |\psi_{\text{spinon}}\rangle \otimes |\psi_{\text{string}}\rangle$

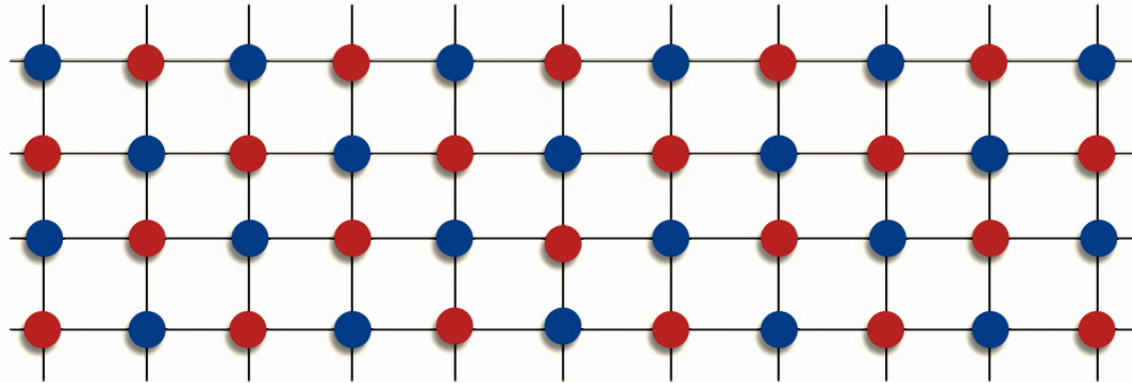


Geometric string theory

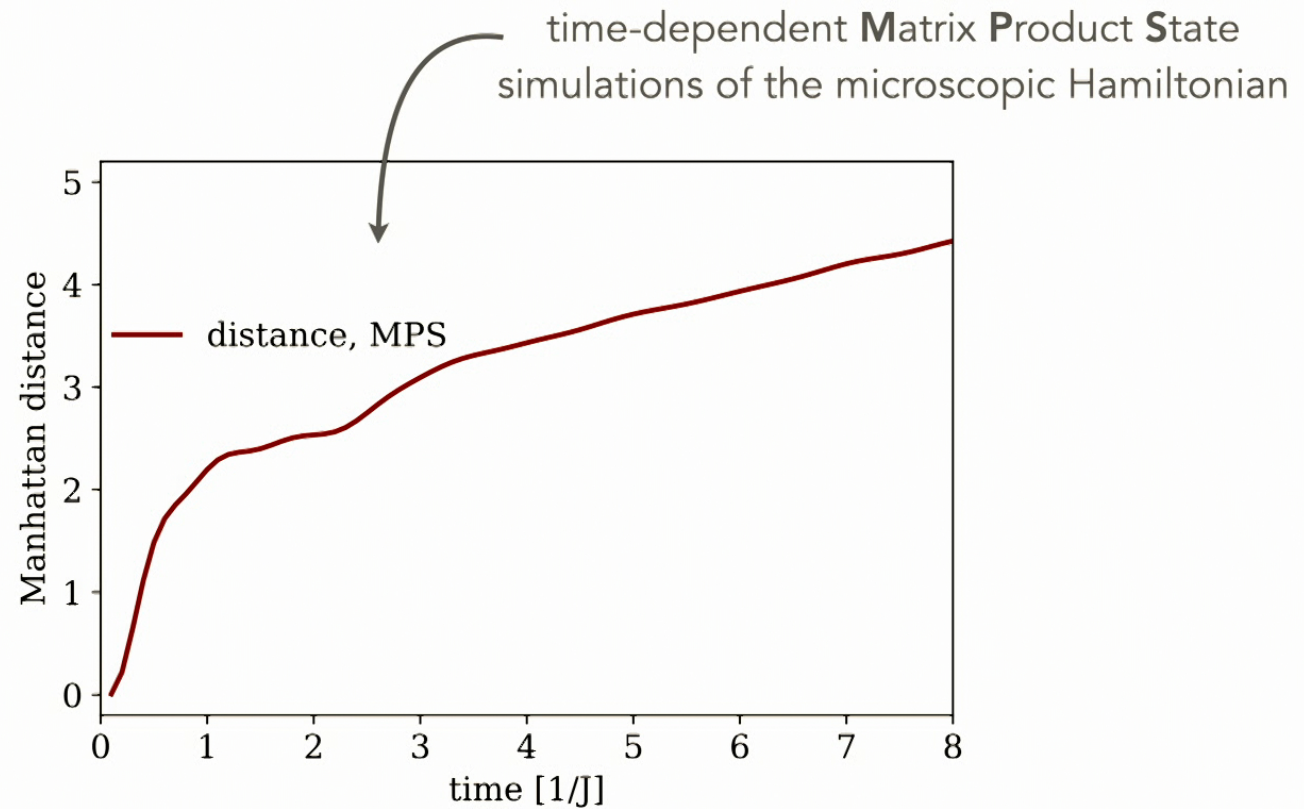


Grusdt et al., PRX 8 (2018), Grusdt et al., PRB 99 (2019), Bohrdt et al., NJP 22 (2020)

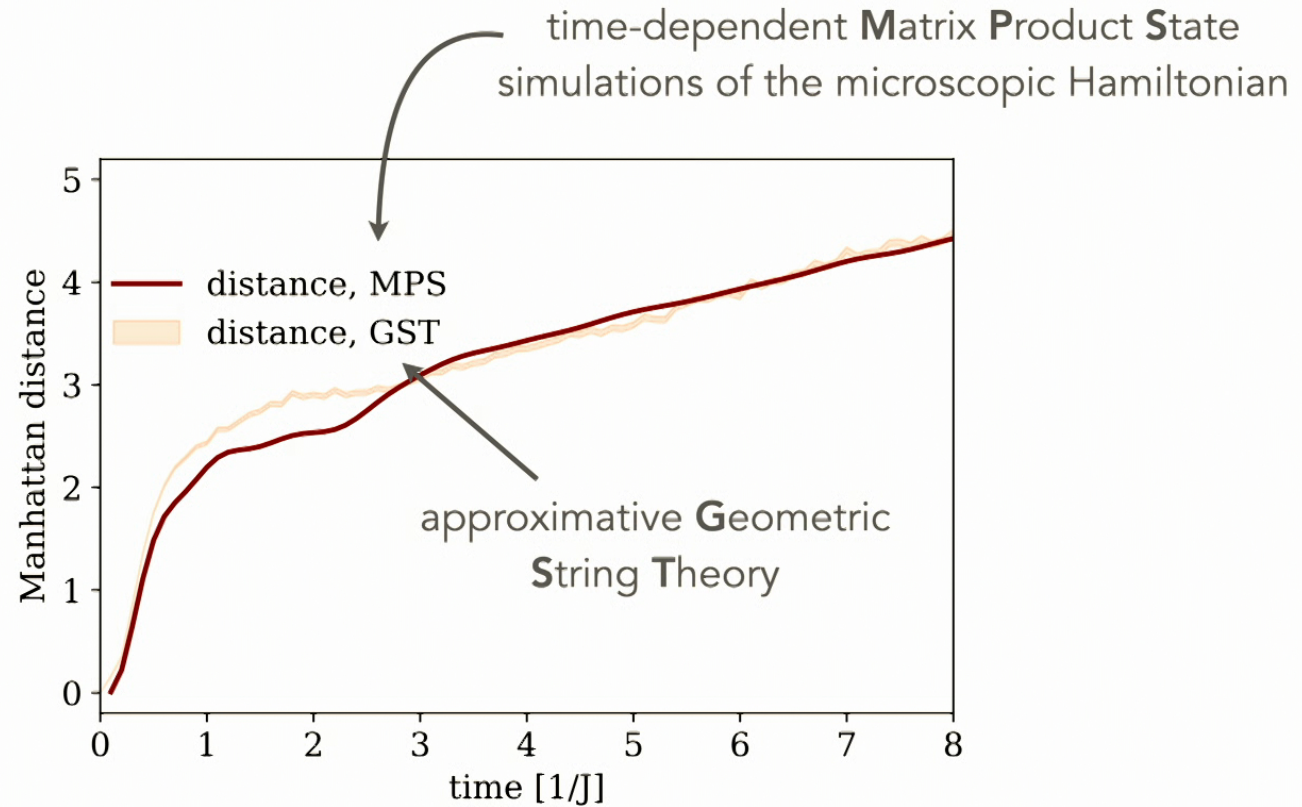
Early work: Bulaevskii et al., JETP 27 (1968), Trugman, PRB 37 (1988), Manousakis, PRB 75 (2007)



Hole dynamics



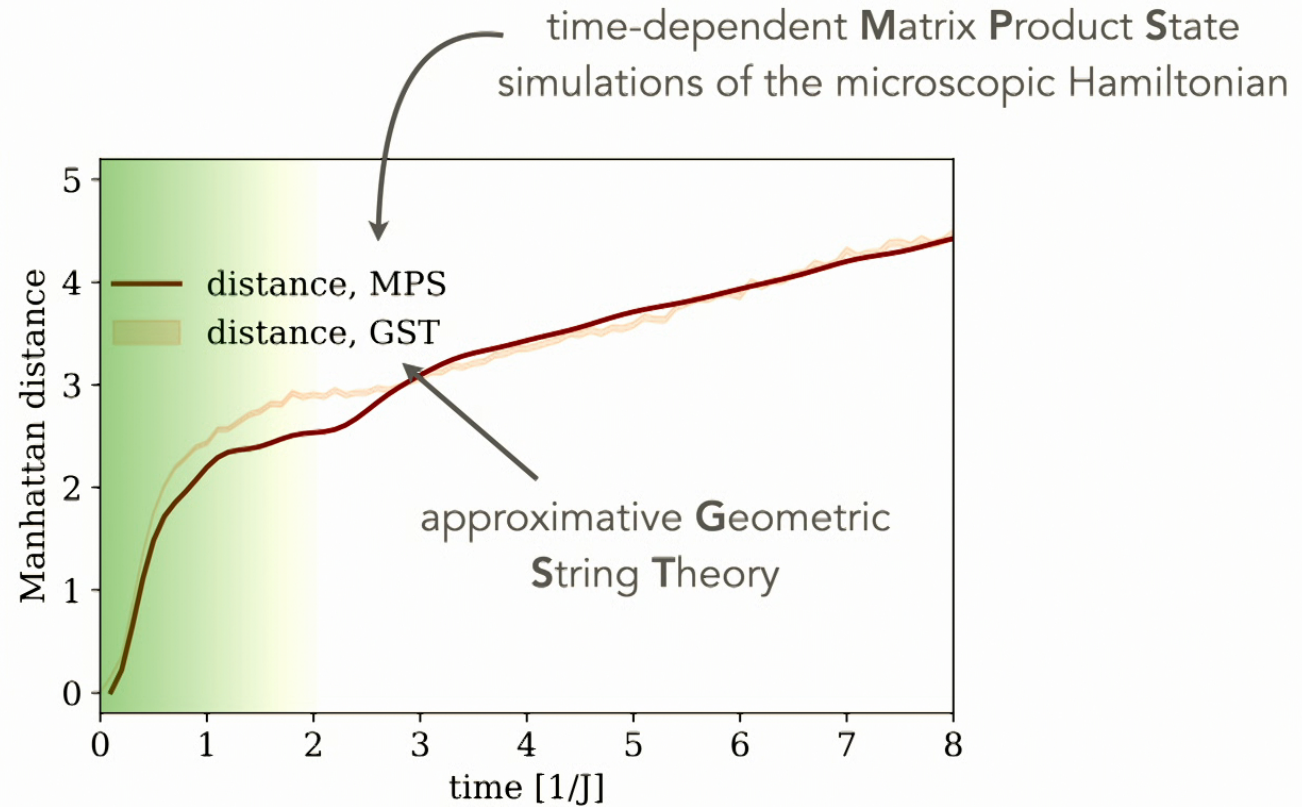
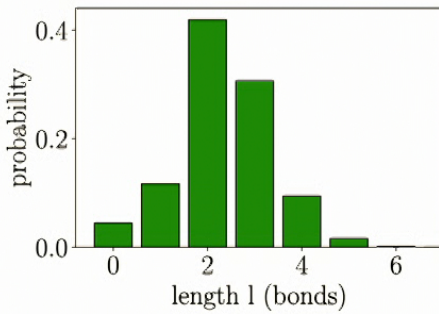
Hole dynamics



Hole dynamics



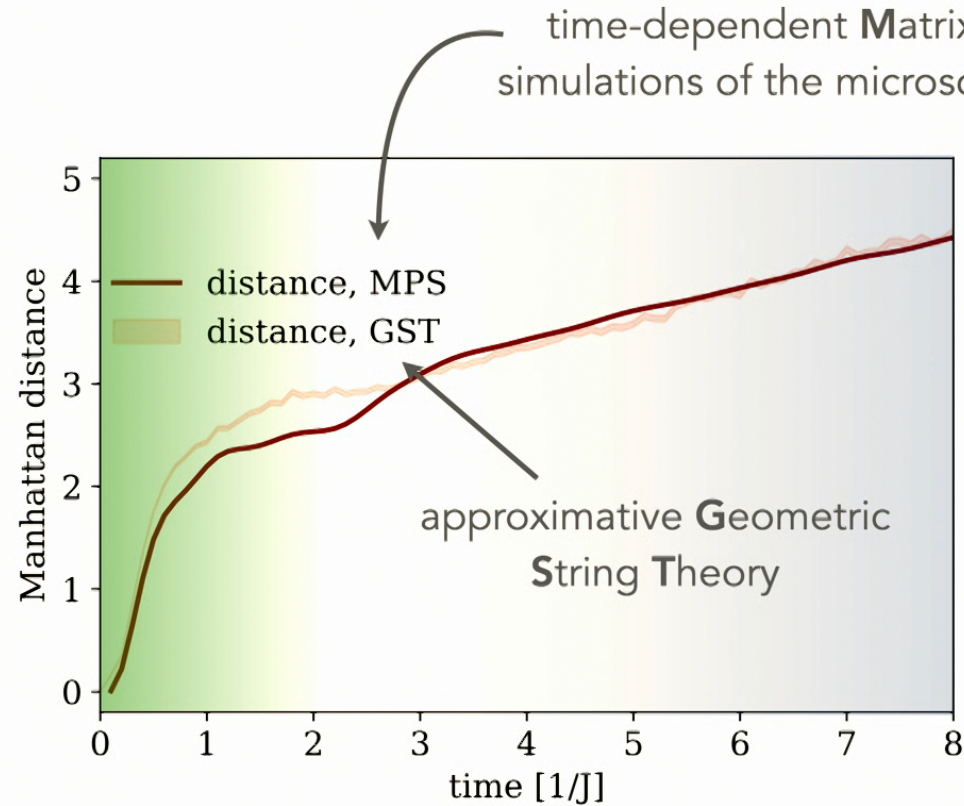
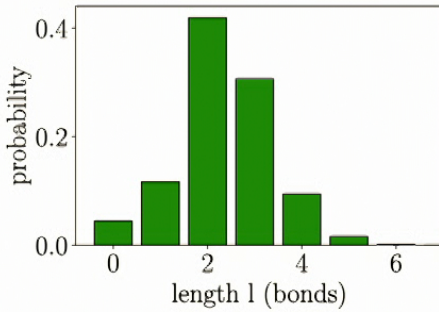
String formation



Hole dynamics



String formation



Spinon dynamics

$$\hat{H}_{\text{sp}} = \sum_{\mathbf{k}, \sigma} \omega_{\text{sp}} \hat{s}_{\mathbf{k}, \sigma}^{\dagger} \hat{s}_{\mathbf{k}, \sigma}$$

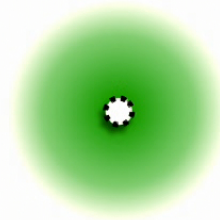
Quantum projective measurements

$$|\psi\rangle = |\begin{array}{c} \bullet \\ | \\ \hline \bullet \end{array}\rangle + |\begin{array}{c} \bullet \\ \hline \bullet \end{array}\rangle + |\begin{array}{c} \bullet \\ \hline \bullet \\ | \\ \bullet \end{array}\rangle + |\begin{array}{c} \bullet \\ | \\ \bullet \end{array}\rangle + \dots$$

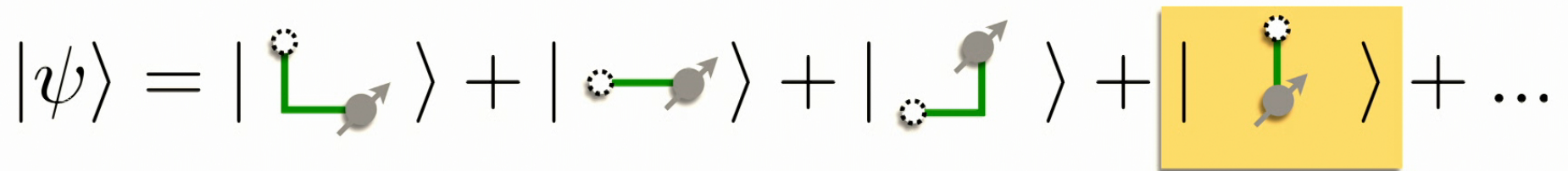
Quantum projective measurements

$$|\psi\rangle = |\begin{array}{c} \bullet \\ | \\ \hline \bullet \end{array}\rangle + |\begin{array}{c} \bullet \\ \hline \bullet \end{array}\rangle + |\begin{array}{c} \bullet \\ \hline \bullet \end{array}\rangle + |\begin{array}{c} \bullet \\ | \\ \bullet \end{array}\rangle + \dots$$

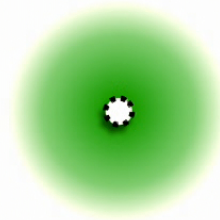
averaging
→



Quantum projective measurements

$$|\psi\rangle = |\text{L-shaped}\rangle + |\text{horizontal}\rangle + |\text{vertical}\rangle + |\text{vertical}\rangle + \dots$$


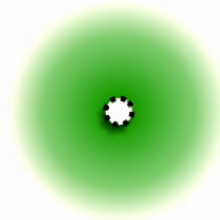
averaging
→



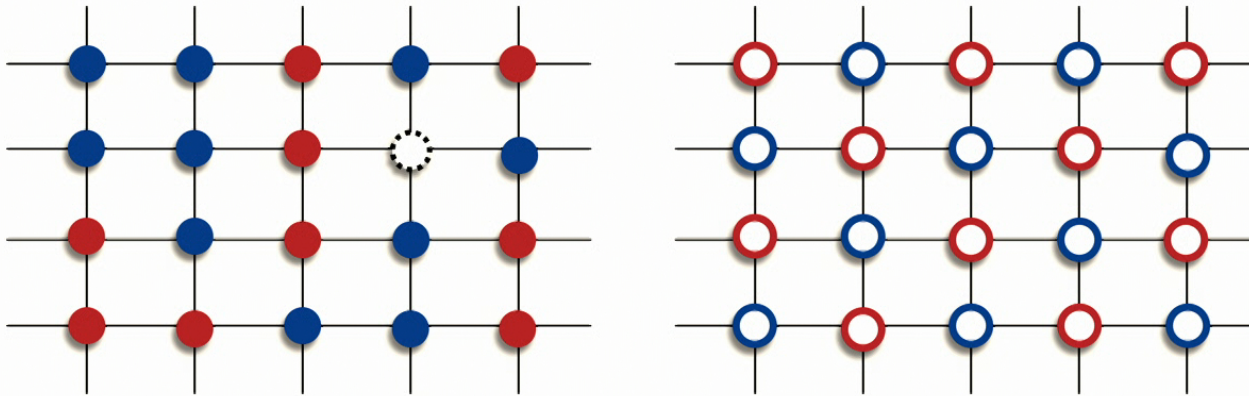
Quantum projective measurements

$$|\psi\rangle = |\text{state 1}\rangle + |\text{state 2}\rangle + |\text{state 3}\rangle + |\text{state 4}\rangle + \dots$$

averaging
→



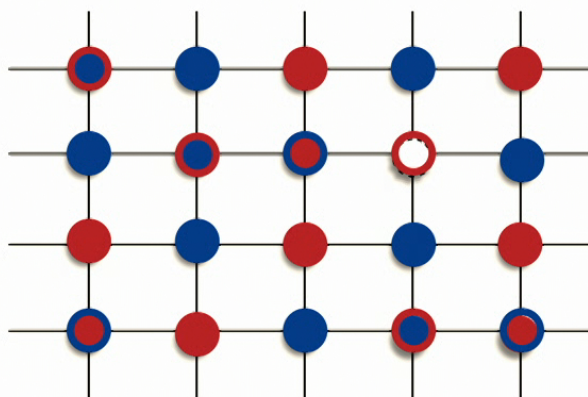
String patterns



Grusdt et al., PRX 8 (2018), Grusdt et al., PRB 99 (2019), Chiu et al., Science 365 (2019), Bohrdt et al., NJP 22 (2020)



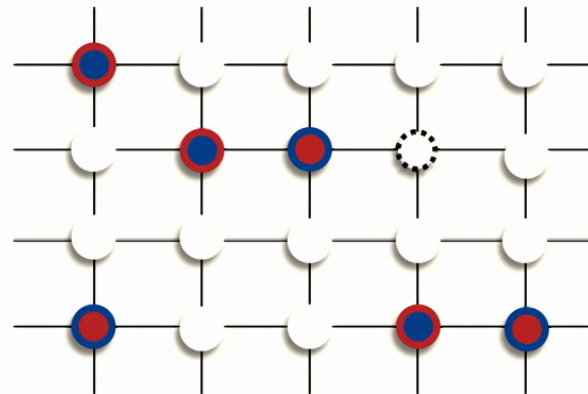
String patterns



Grusdt et al., PRX 8 (2018), Grusdt et al., PRB 99 (2019), Chiu et al., Science 365 (2019), Bohrdt et al., NJP 22 (2020)



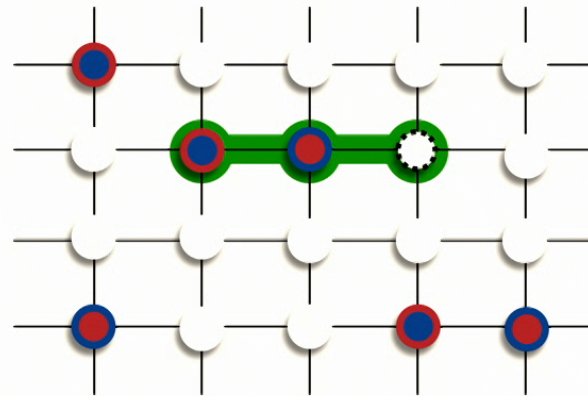
String patterns



Grusdt et al., PRX 8 (2018), Grusdt et al., PRB 99 (2019), Chiu et al., Science 365 (2019), Bohrdt et al., NJP 22 (2020)



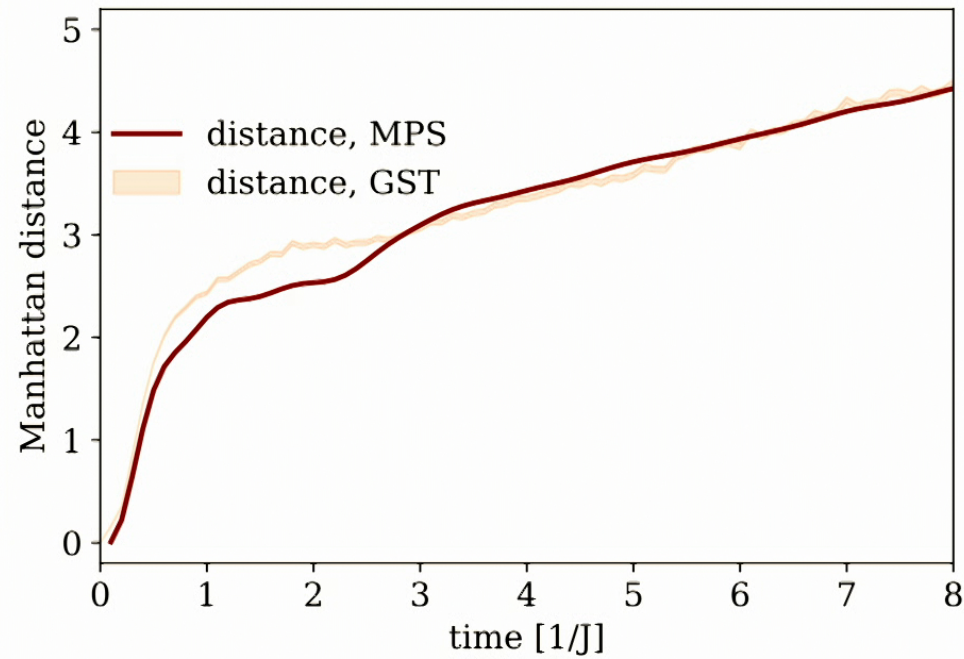
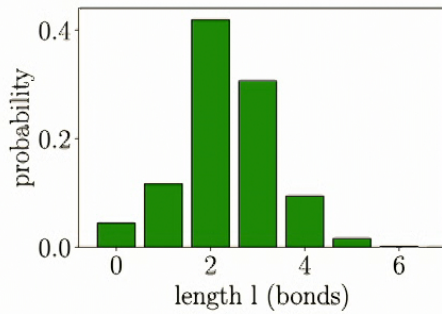
String patterns



Grusdt et al., PRX 8 (2018), Grusdt et al., PRB 99 (2019), Chiu et al., Science 365 (2019), Bohrdt et al., NJP 22 (2020)



String formation



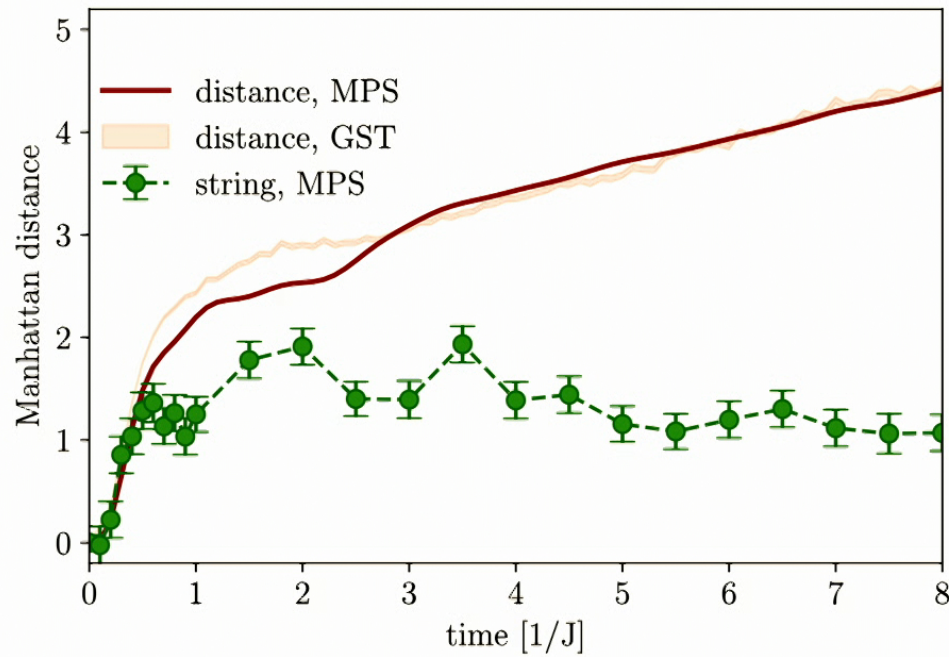
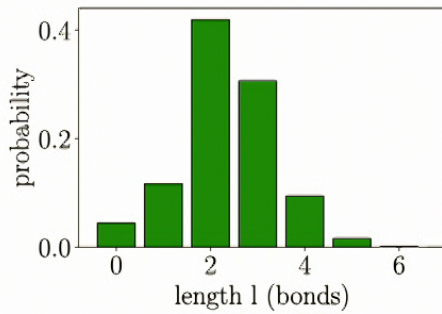
Spinon dynamics

$$\hat{H}_{\text{sp}} = \sum_{\mathbf{k}, \sigma} \omega_{\text{sp}} \hat{s}_{\mathbf{k}, \sigma}^{\dagger} \hat{s}_{\mathbf{k}, \sigma}$$

String patterns



String formation



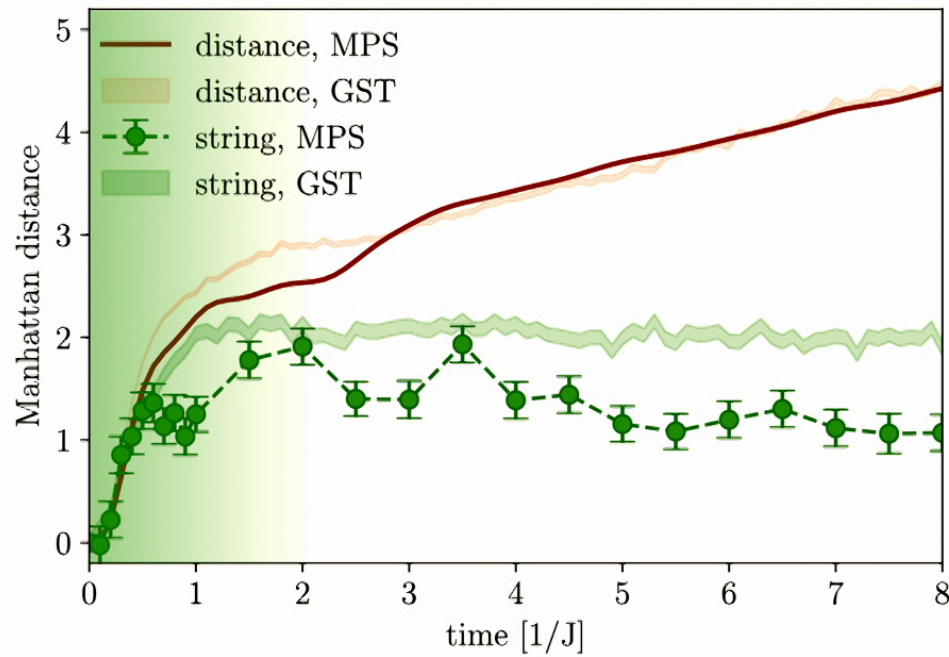
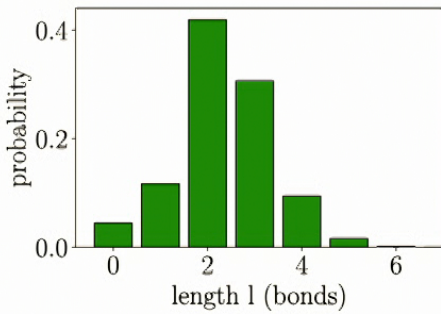
Spinon dynamics

$$\hat{H}_{\text{sp}} = \sum_{\mathbf{k}, \sigma} \omega_{\text{sp}} \hat{s}_{\mathbf{k}, \sigma}^{\dagger} \hat{s}_{\mathbf{k}, \sigma}$$

String patterns



String formation



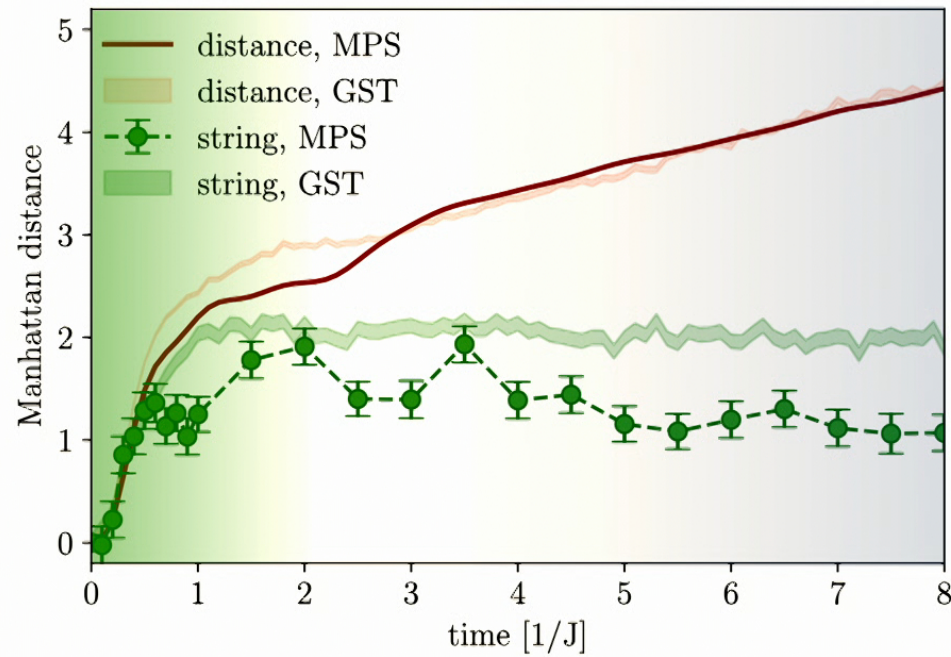
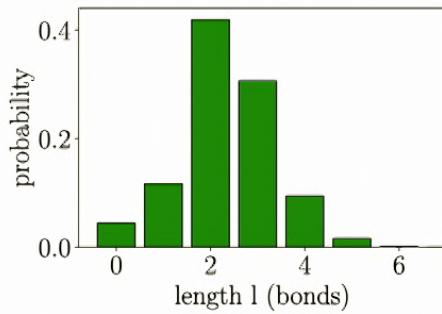
Spinon dynamics

$$\hat{H}_{\text{sp}} = \sum_{\mathbf{k}, \sigma} \omega_{\text{sp}} \hat{s}_{\mathbf{k}, \sigma}^{\dagger} \hat{s}_{\mathbf{k}, \sigma}$$

String patterns

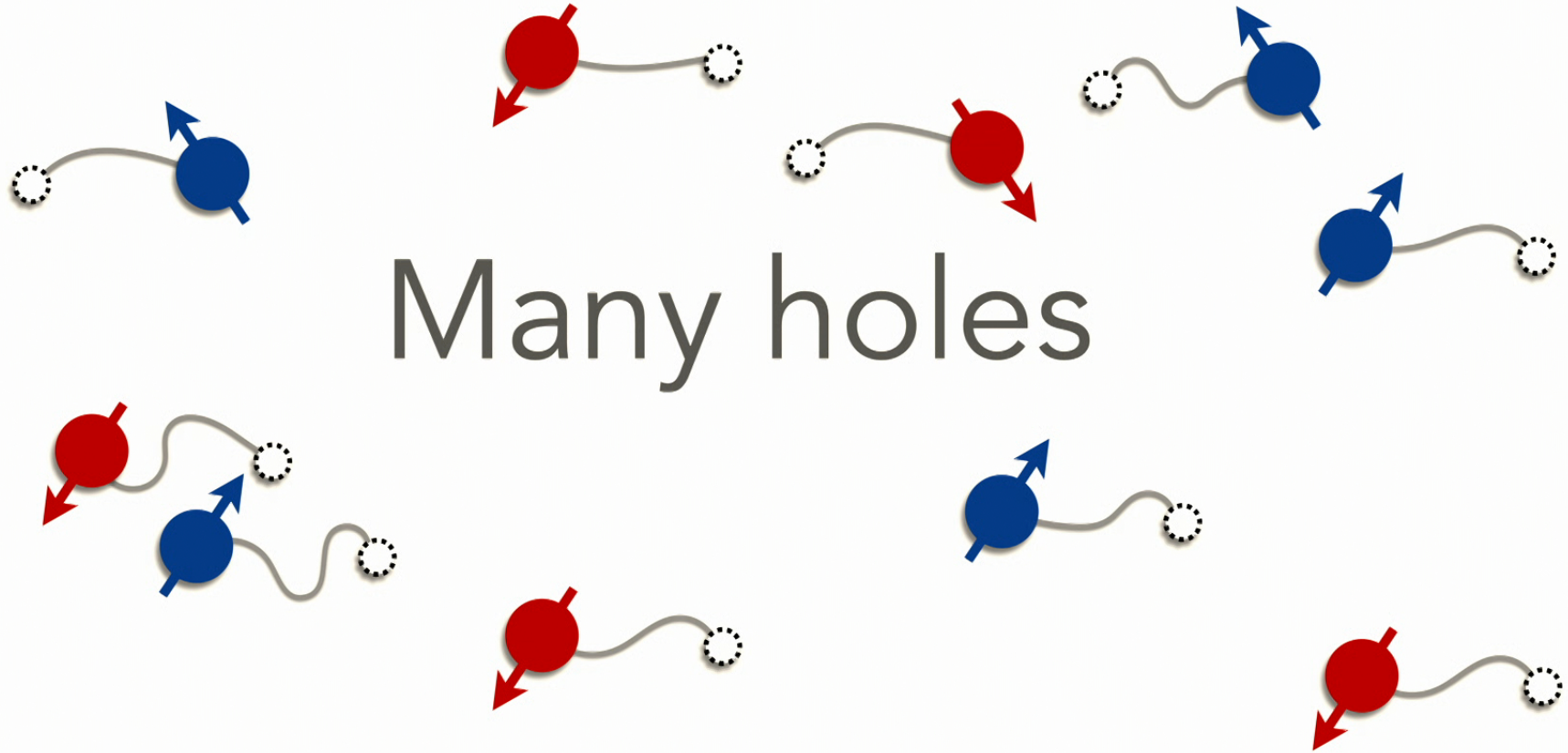


String formation



Spinon dynamics

$$\hat{H}_{\text{sp}} = \sum_{\mathbf{k}, \sigma} \omega_{\text{sp}} \hat{s}_{\mathbf{k}, \sigma}^{\dagger} \hat{s}_{\mathbf{k}, \sigma}$$



Numerics Review from last year:

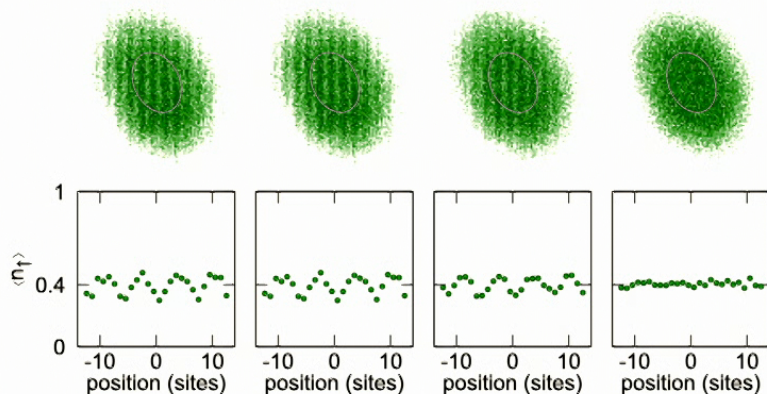
2. Phase diagram and phase boundaries

Not much is known about the precise phase diagram of the Hubbard model in two dimensions. While the ground-state phase diagram of the model at weak coupling is fairly well understood (49, 266), the phase diagram in the intermediate-to-strong interaction limit is hotly debated. Determining precise phase boundaries is hindered by the fact that the diver-

What should we look for?

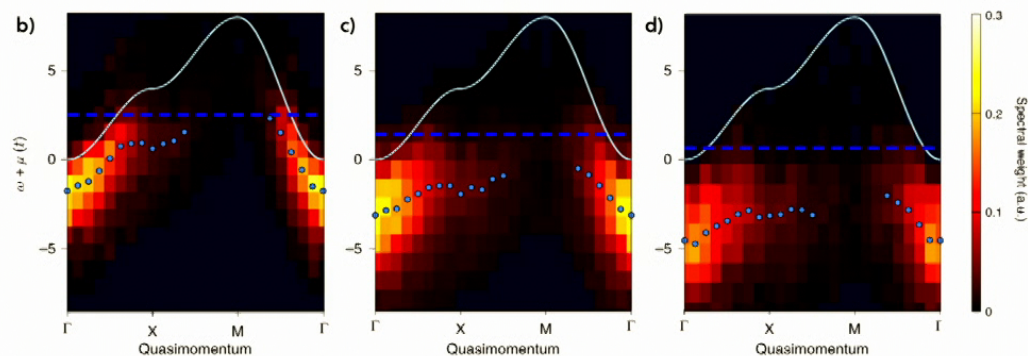
Option 1: Emulate solid state experiments

Transport properties



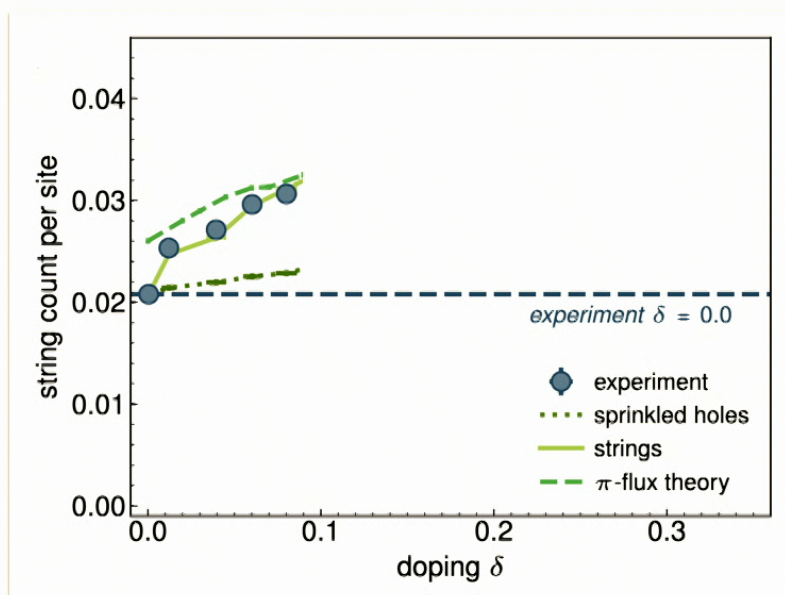
Brown et al., Science 363 (2019)

Angle resolved photoemission spectroscopy

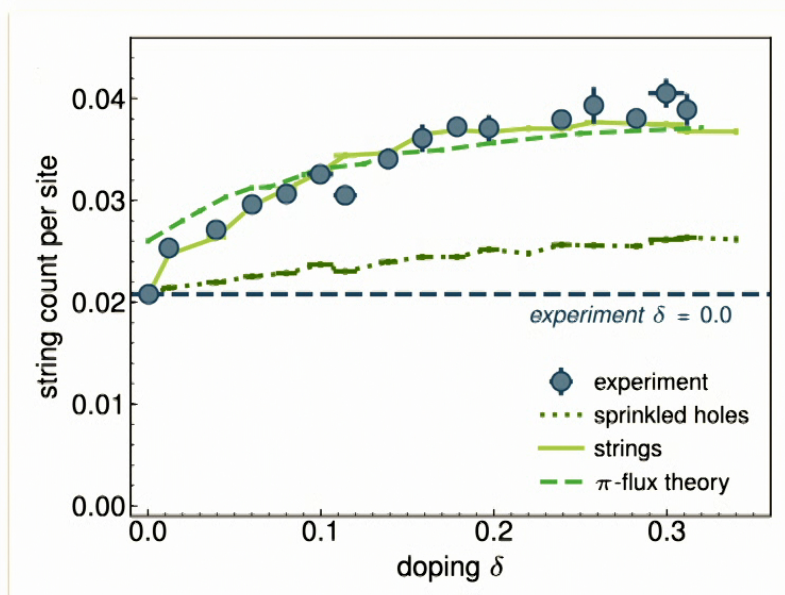


Brown et al., Nature Physics 16 (2020)

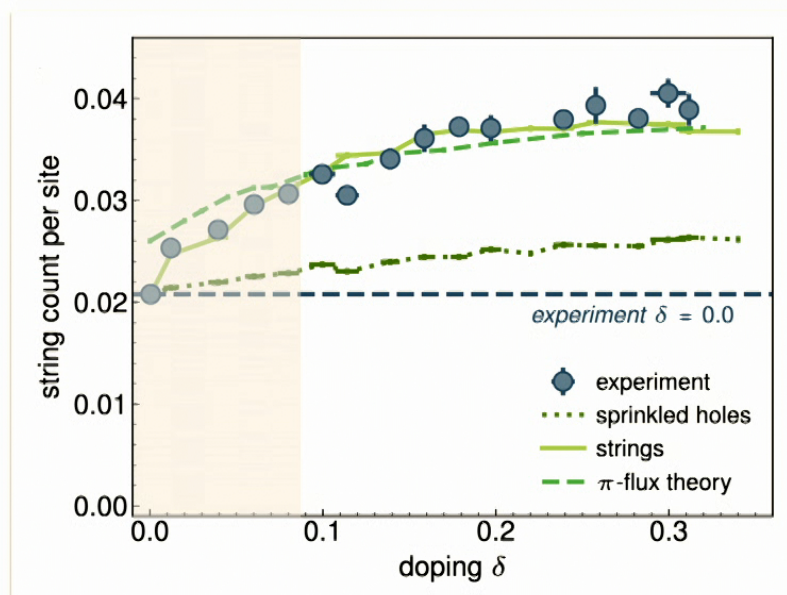
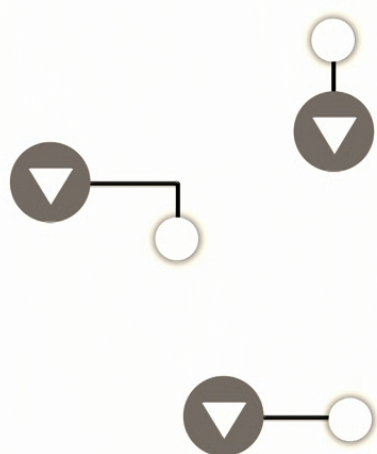
Option 2: Make use of microscopic real-space information



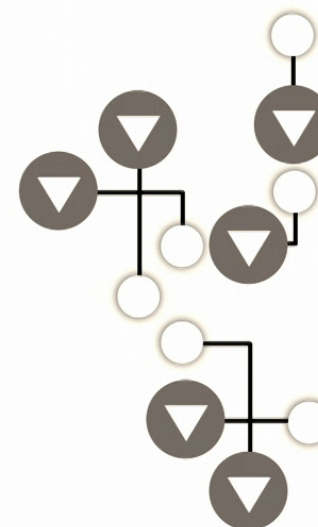
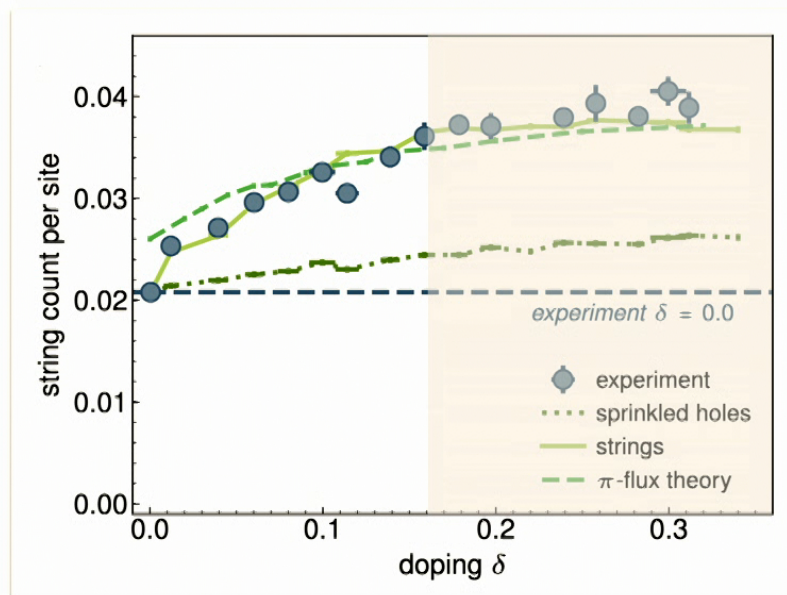
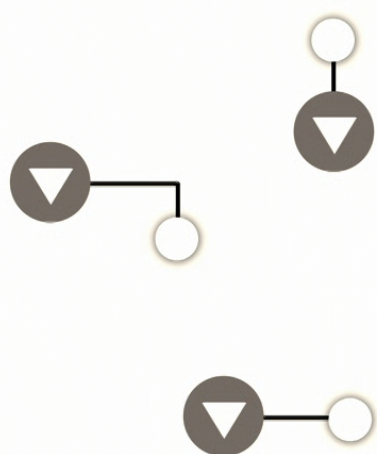
Option 2: Make use of microscopic real-space information



Option 2: Make use of microscopic real-space information

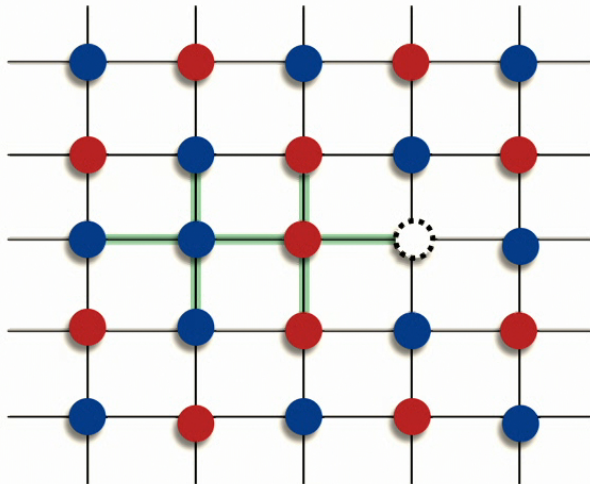


Option 2: Make use of microscopic real-space information



Candidate theories

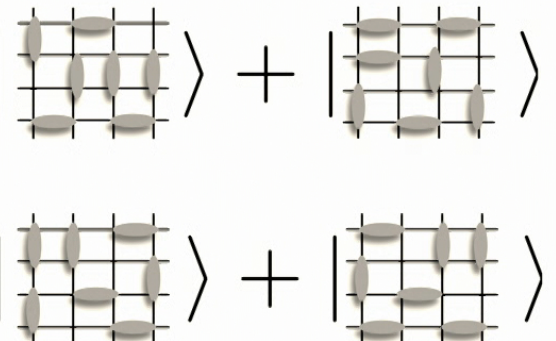
Geometric string theory



Grusdt et al., PRX 8 (2018), Grusdt et al., PRB 99 (2019),
Bohrdt et al., NJP 22 (2020)

Early work: Bulaevskii et al., JETP 27 (1968),
Trugman, PRB 37 (1988), Manousakis, PRB 75 (2007)

Resonating valence bond theory (π -flux theory)

$$|\psi\rangle = |\text{state 1}\rangle + |\text{state 2}\rangle + |\text{state 3}\rangle + |\text{state 4}\rangle + \dots$$


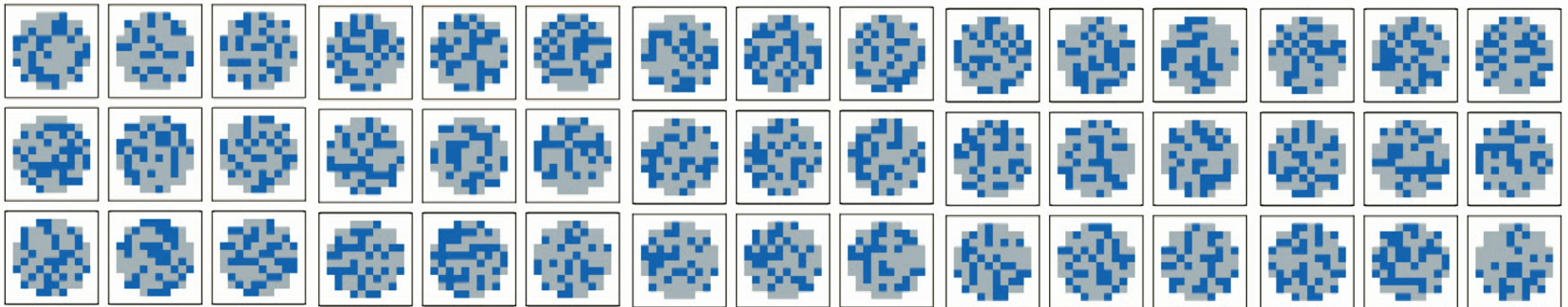
P. W. Anderson, Science 235, 1196 (1987)



“The theoretical problem is so hard that
there isn't an obvious criterion for right.”
— Steven Kivelson, Stanford University
Science 314 (2006)

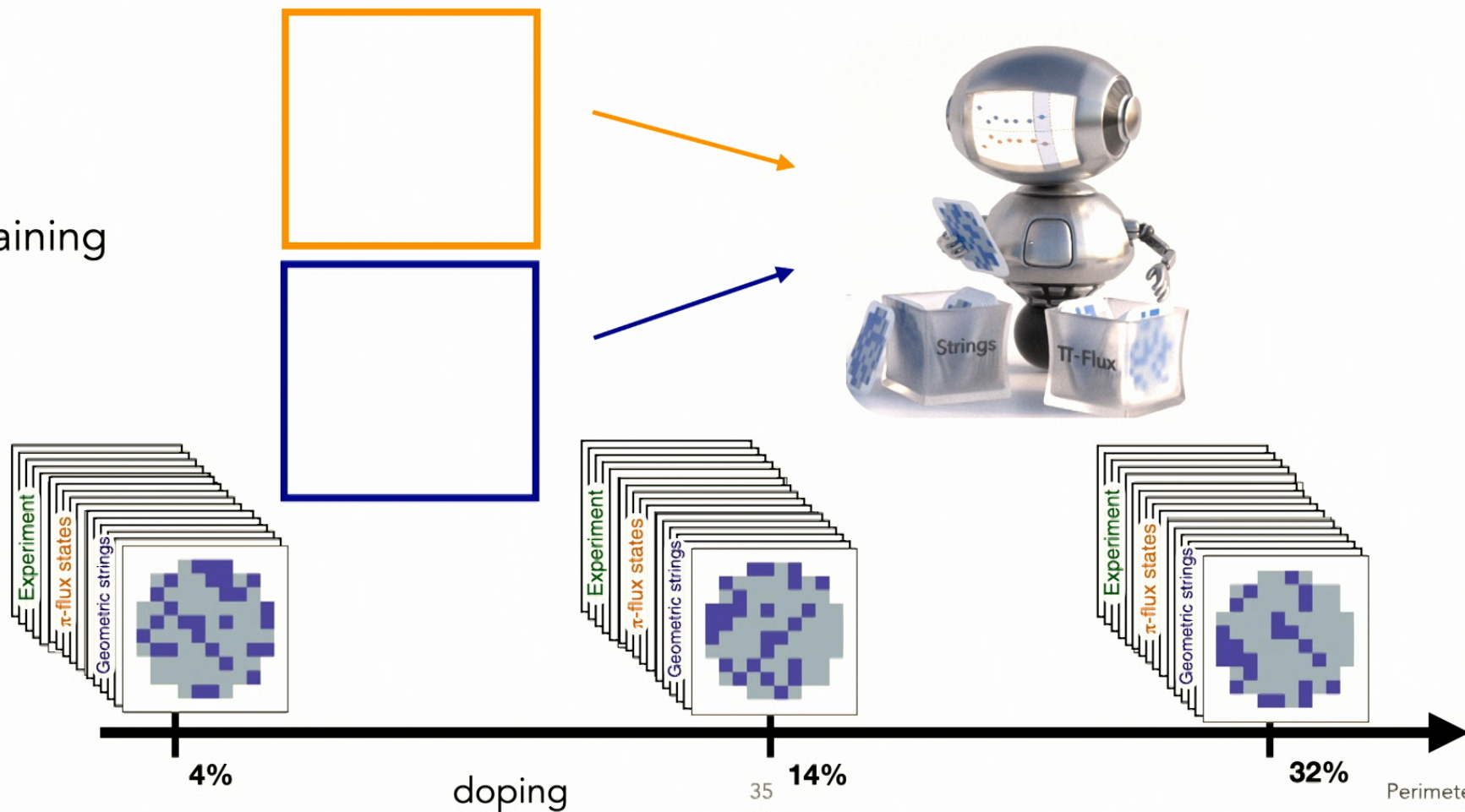
“The theoretical problem is so hard that
there isn't an obvious criterion for right.”
— Steven Kivelson, Stanford University

Science 314 (2006)



Machine learning snapshots

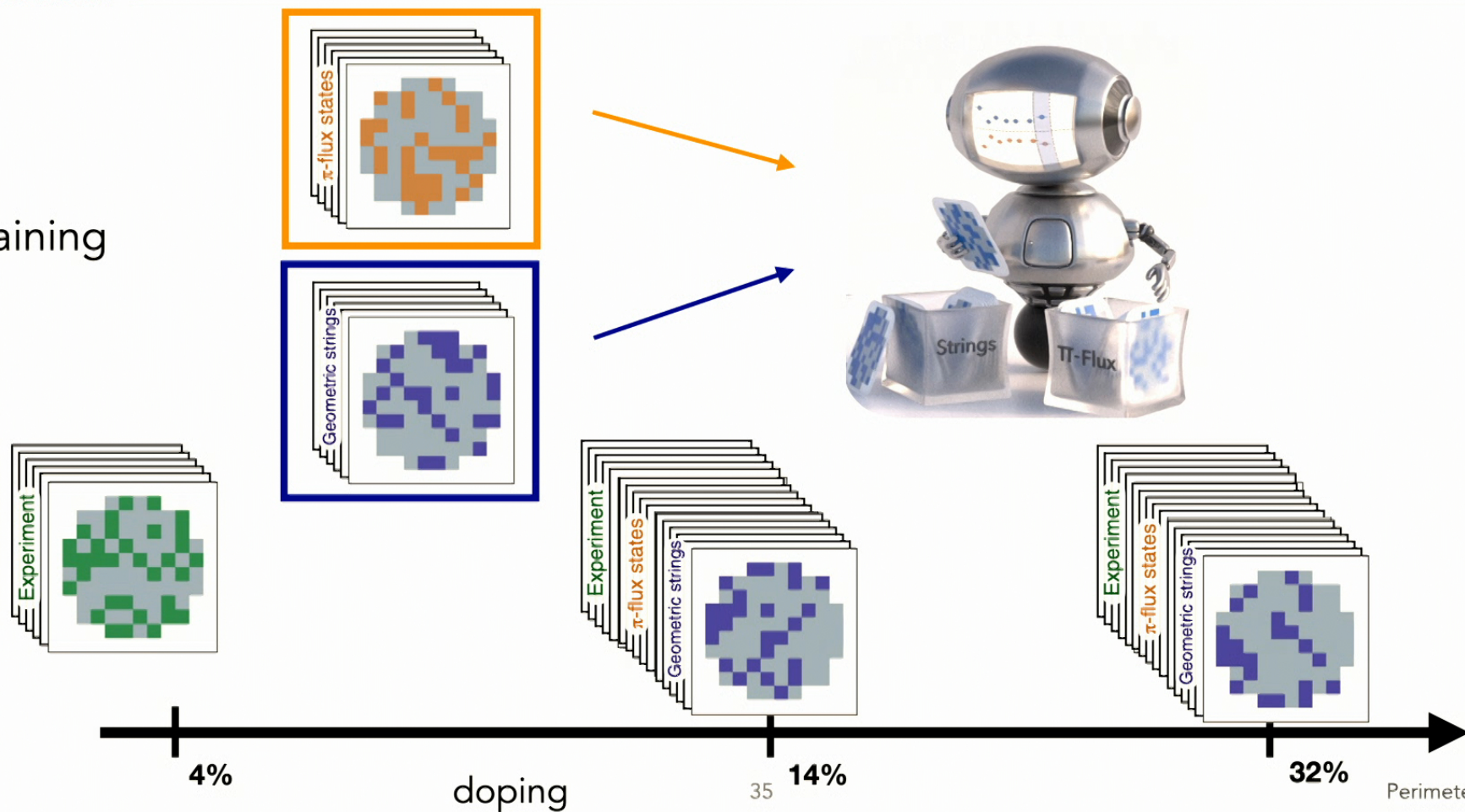
Training



Perimeter, March 2023

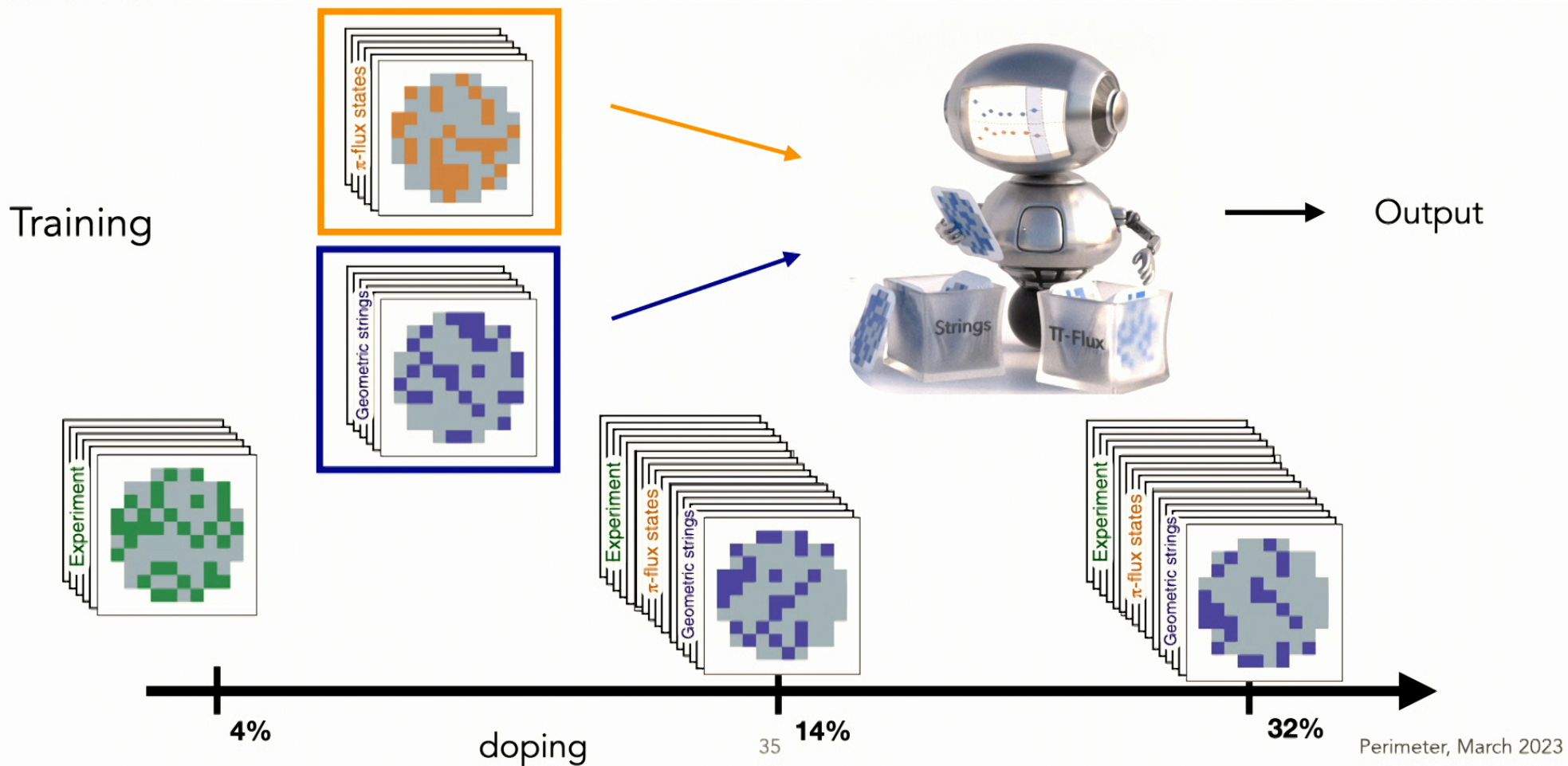
Machine learning snapshots

Training



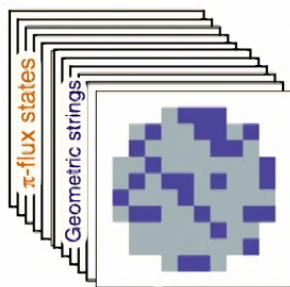
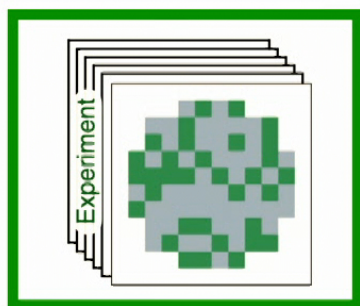
Perimeter, March 2023

Machine learning snapshots



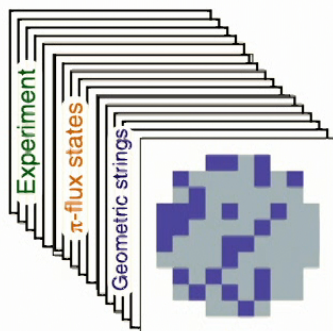
Machine learning snapshots

Classify

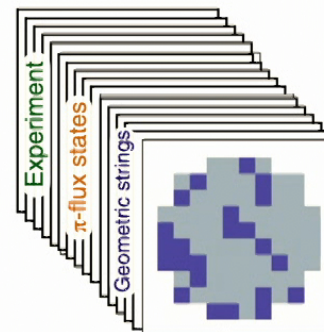


4%

doping



35 14%

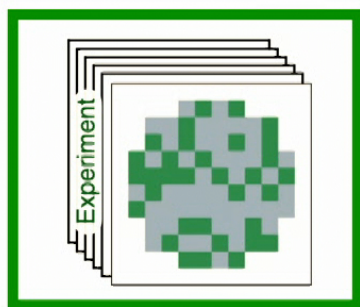


32%

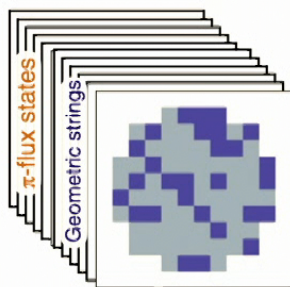
Perimeter, March 2023

Machine learning snapshots

Classify

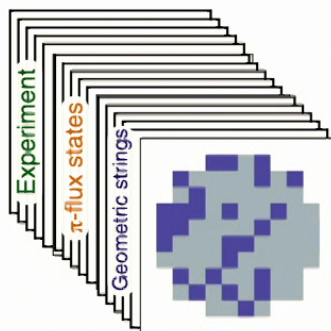


Output

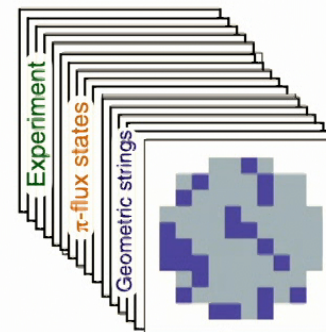


4%

doping



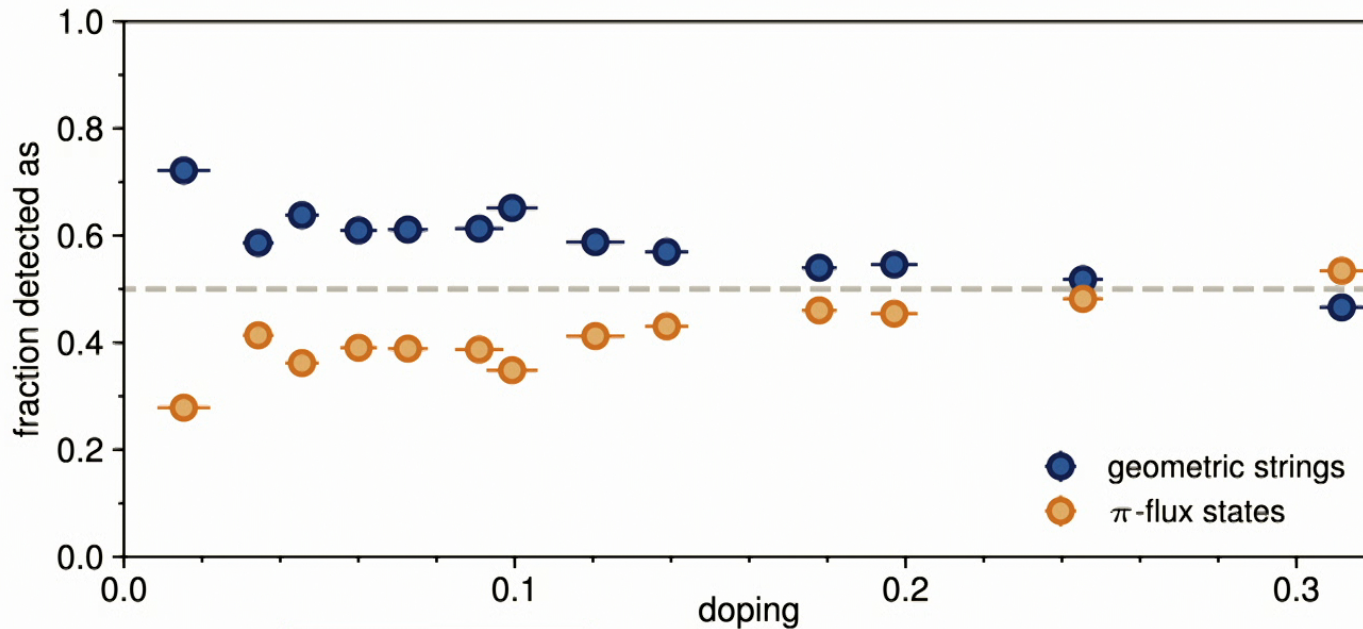
35 14%



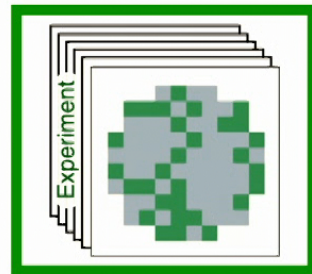
32%

Perimeter, March 2023

Machine learning snapshots



Classify



Output

Bohrdt et al., Nature Physics 15 (2019)

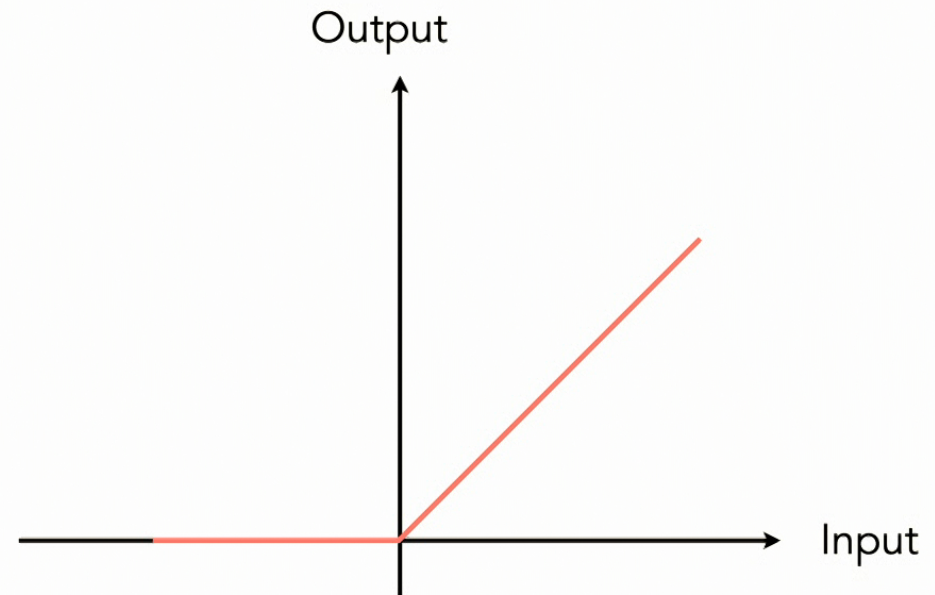
36

Perimeter, March 2023

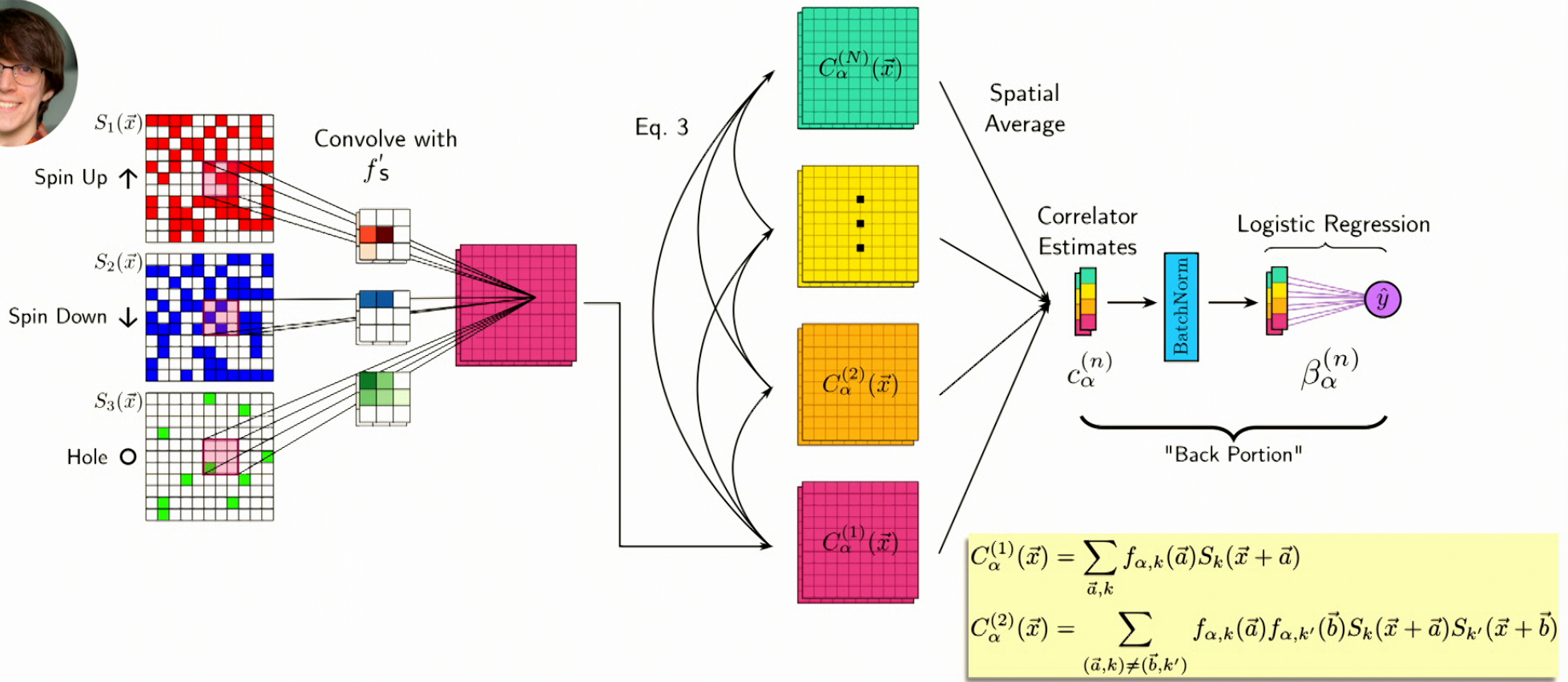
Interpretability?

- Looking at filters difficult: non-linearity mixes all orders of correlations
- Non-linearity necessary: otherwise, network corresponds to function that multiplies the input by a weight matrix and adds to it an additional bias vector.

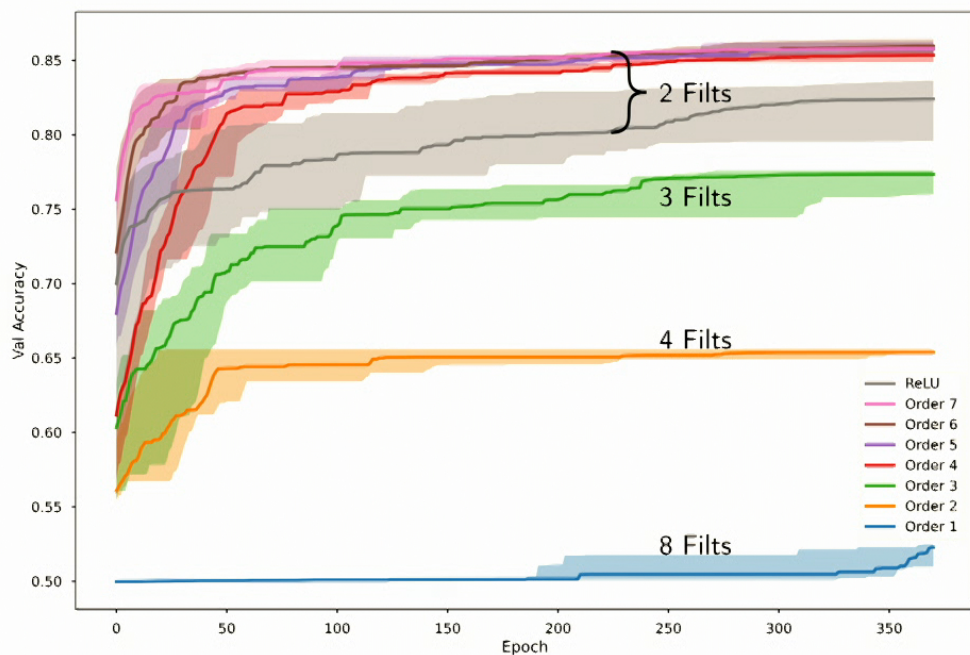
Typical activation function:
rectified linear unit



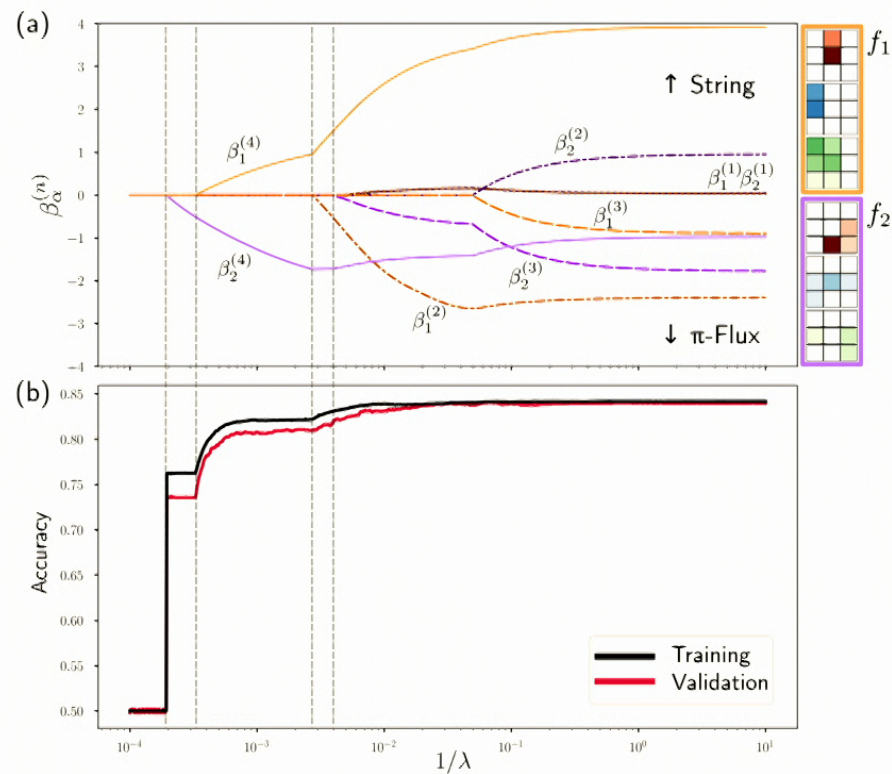
Correlator convolutional neural network



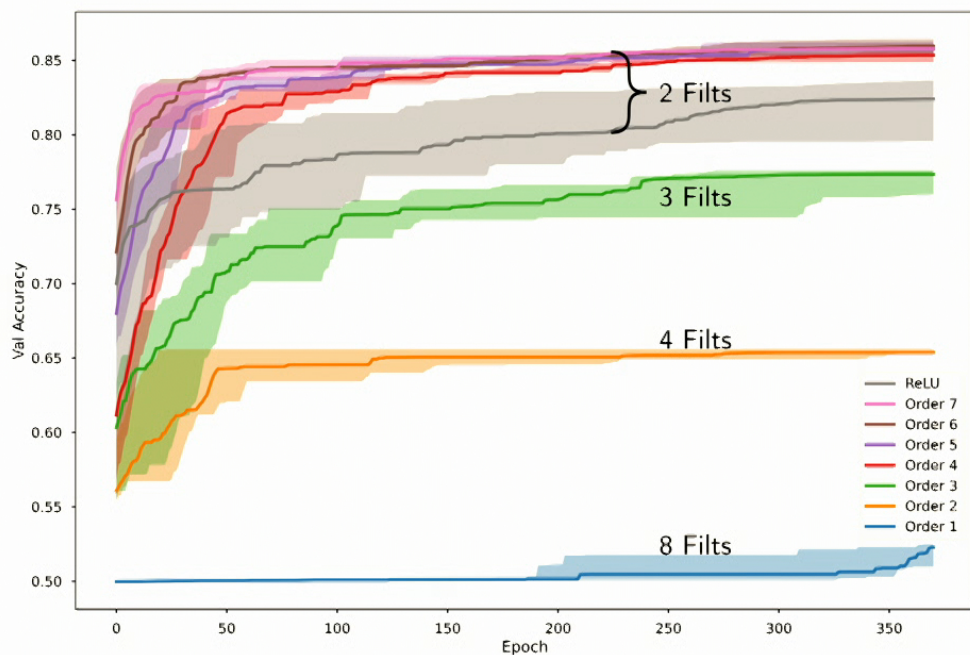
Regularization path analysis



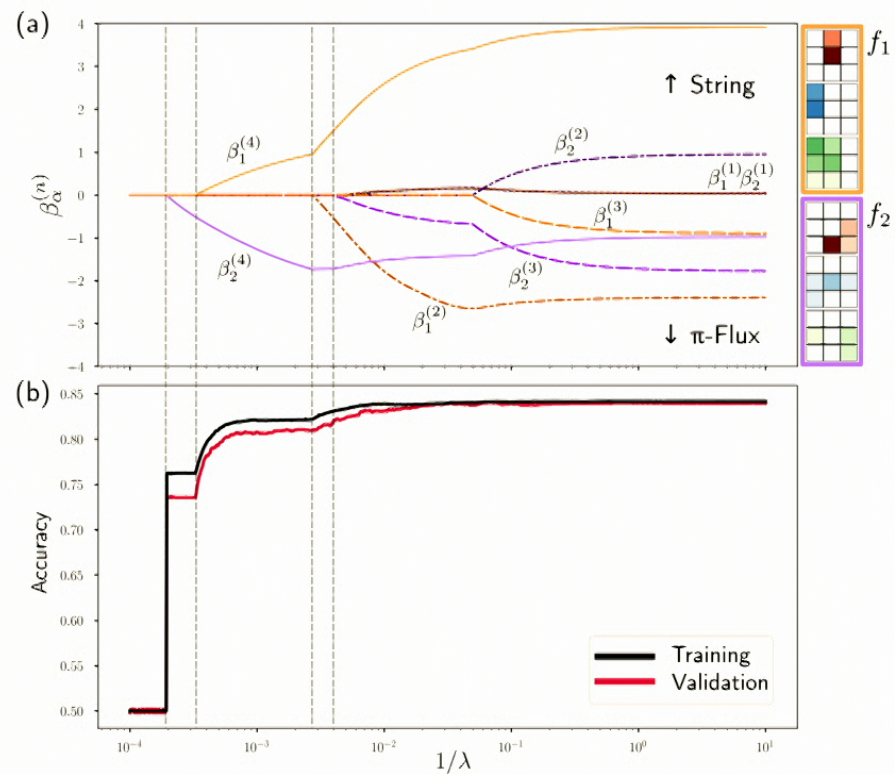
C. Miles, A. Bohrdt et al., Nature Communications 12, 3905 (2021)



Regularization path analysis

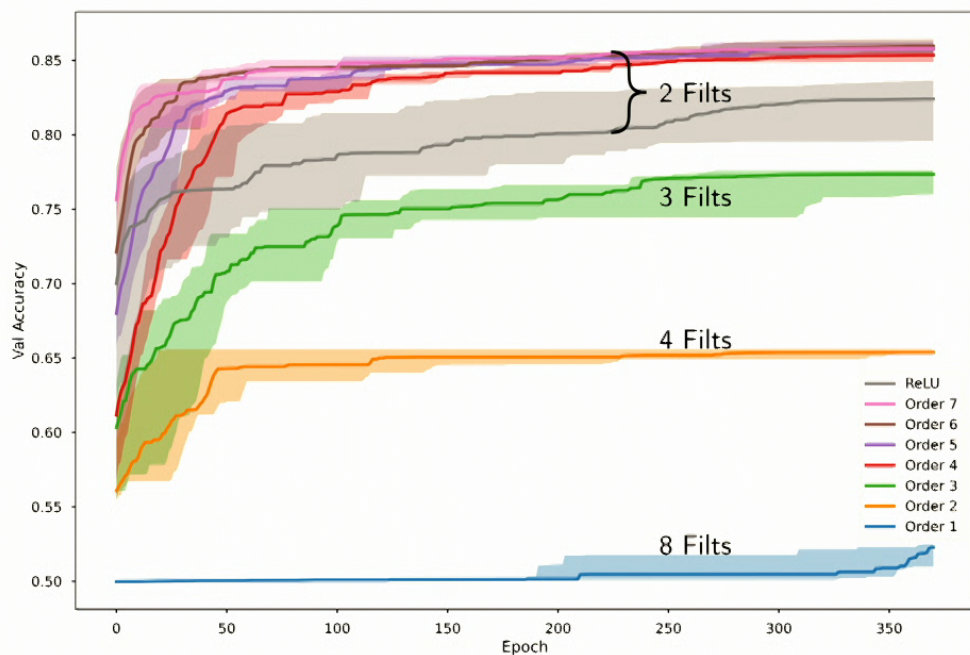


C. Miles, A. Bohrdt et al., Nature Communications 12, 3905 (2021)

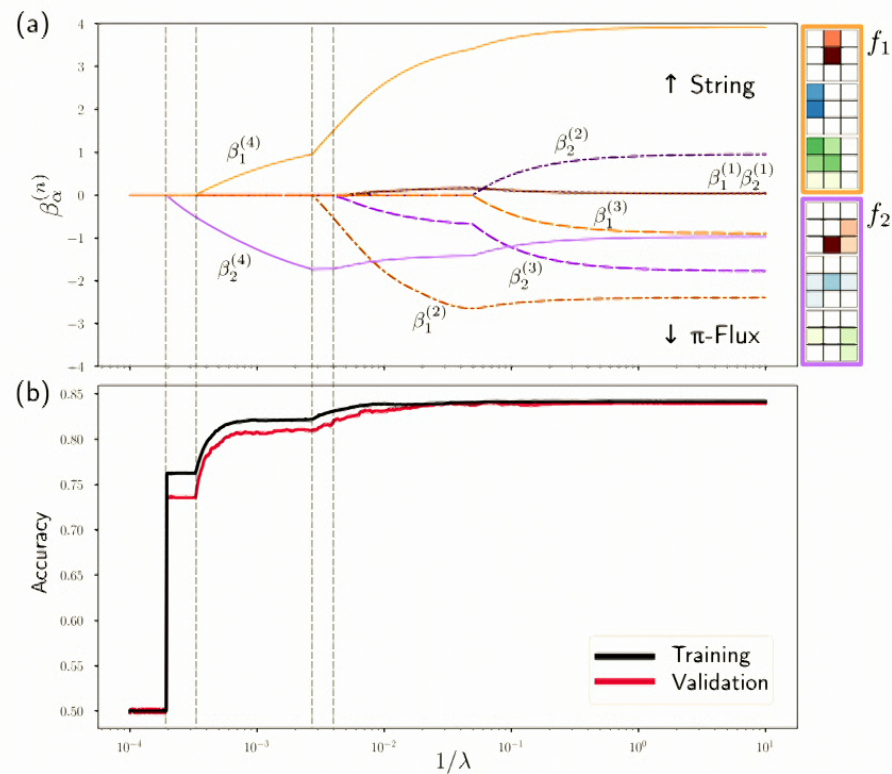




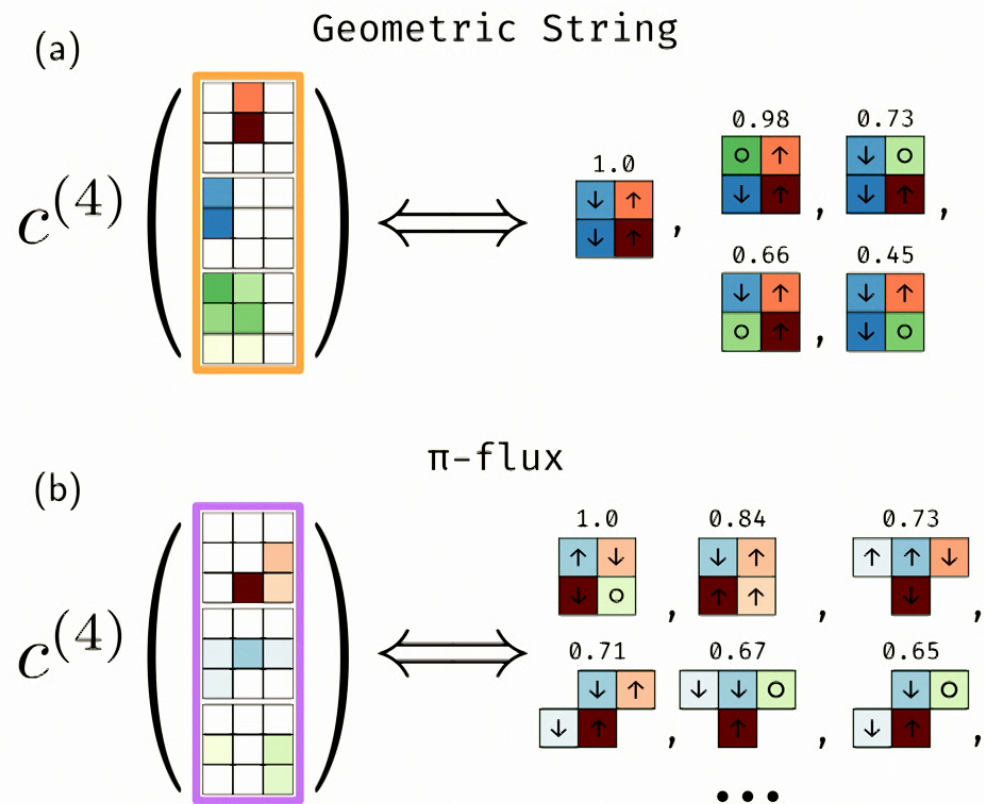
Regularization path analysis

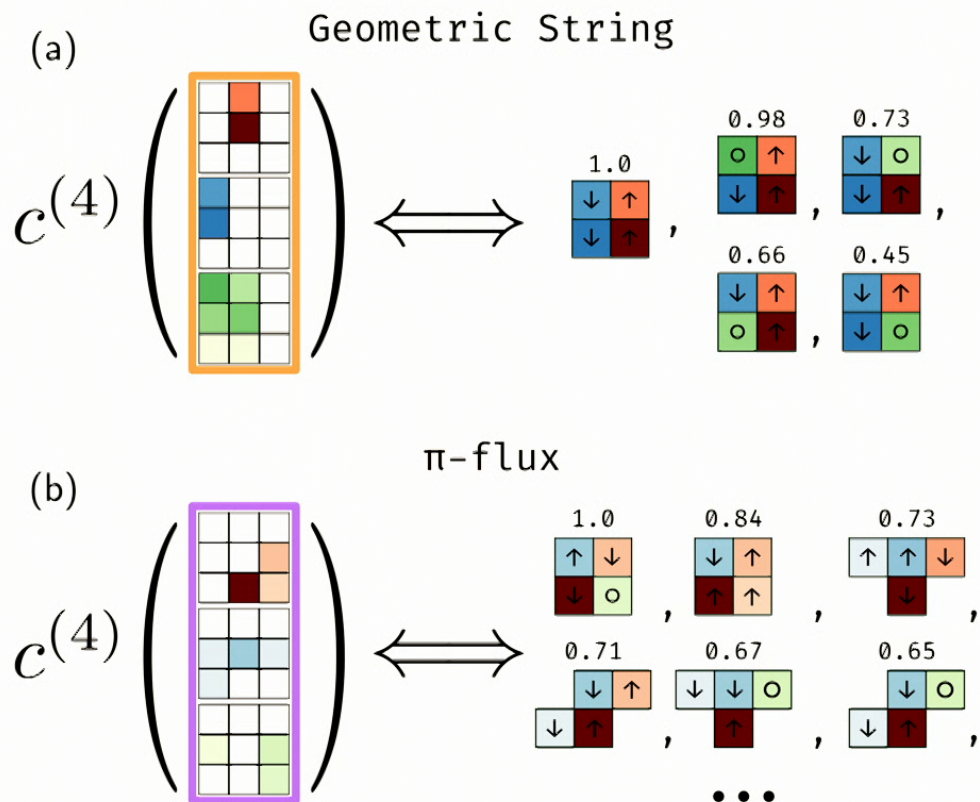
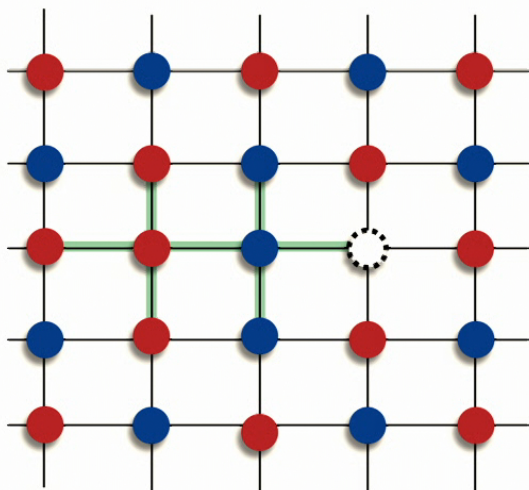


C. Miles, A. Bohrdt et al., Nature Communications 12, 3905 (2021)

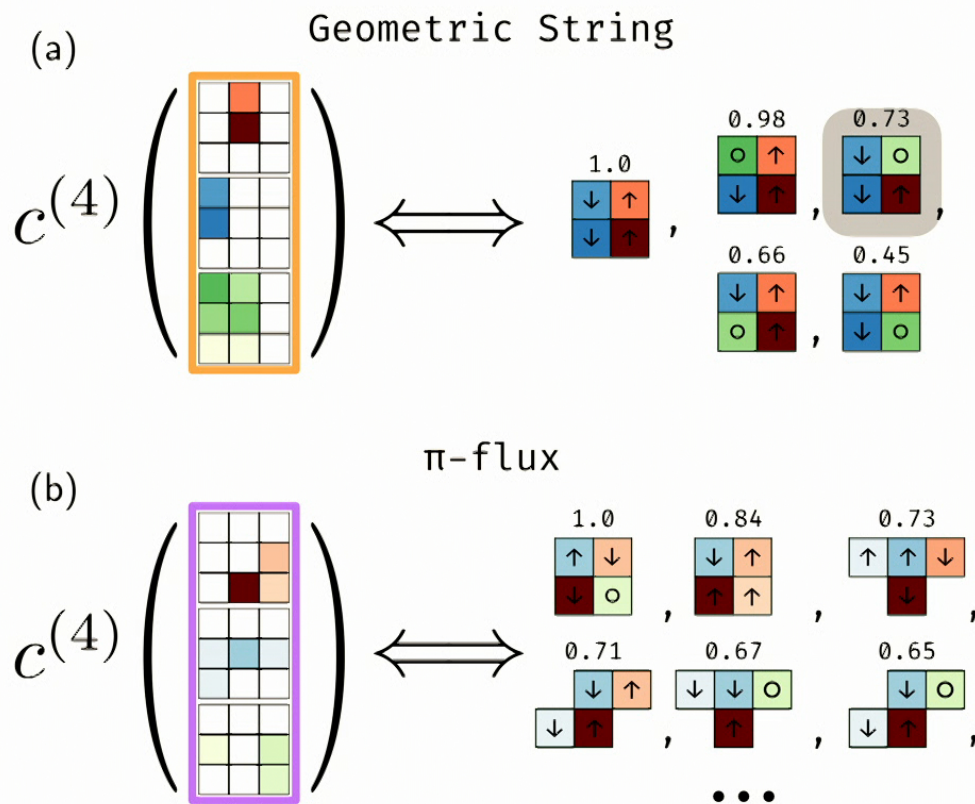
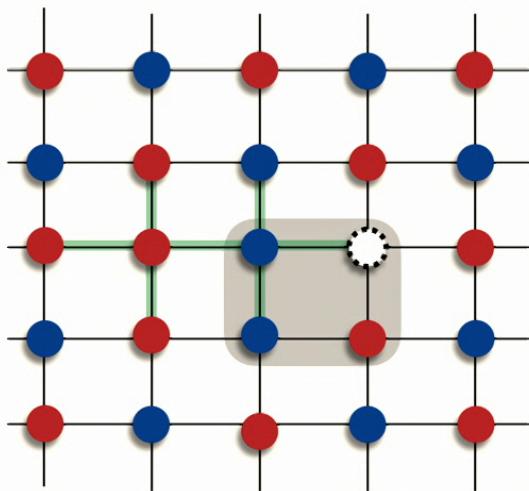


Interpretability

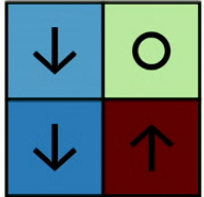




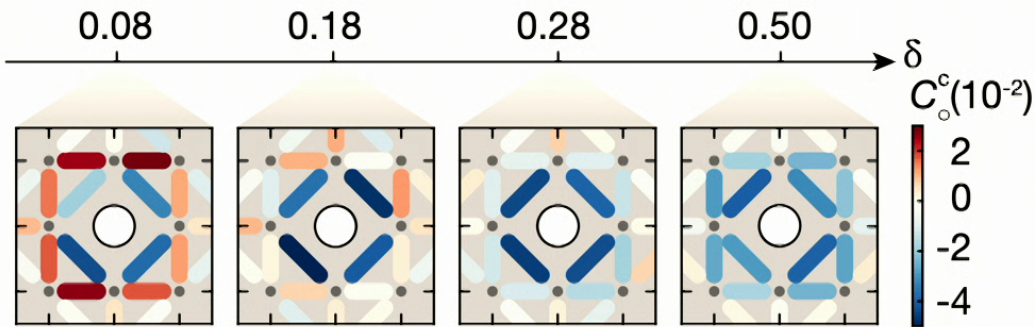
Interpretability



Higher-order correlations

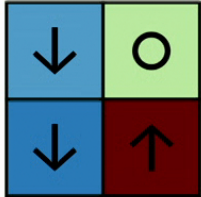


$$c_o^c \text{ (orange) } = \text{Bare (blue) } - c^c \text{ (blue) }$$



↖ Magnetic polaron correlations

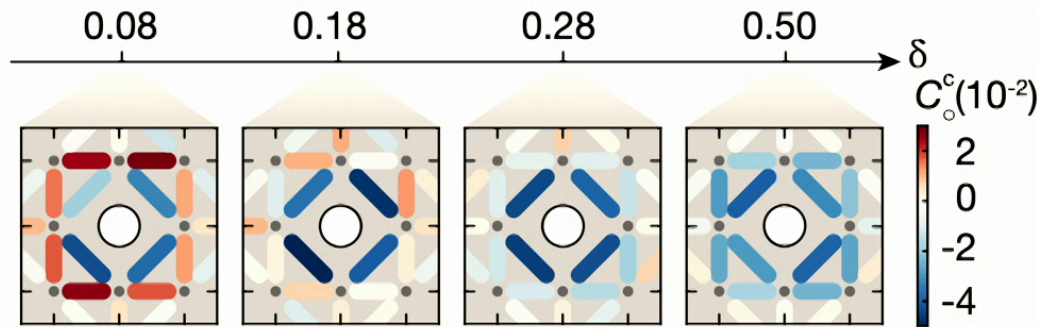
Higher-order correlations



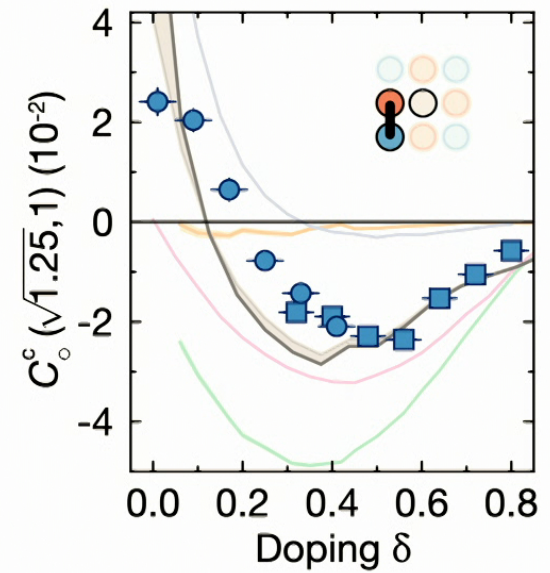
Bare

$$C_o^c = \text{Bare} - C^c$$

ED Free uRVB π -flux String

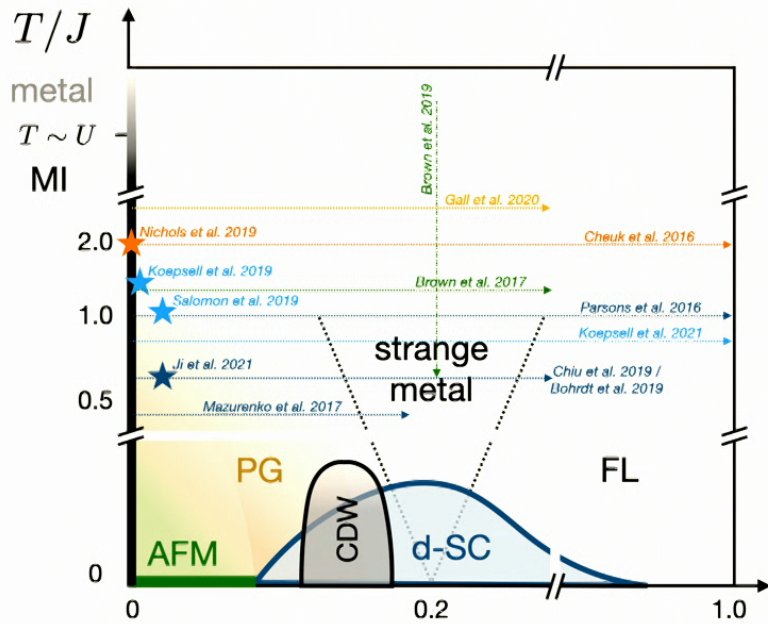


Magnetic polaron correlations

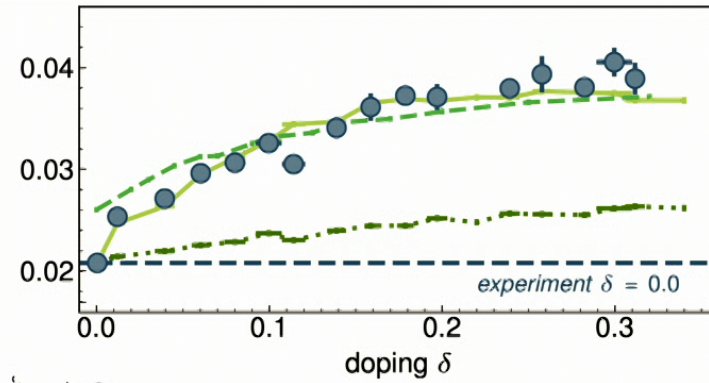


Koepsell et al., Science 374 (2021)

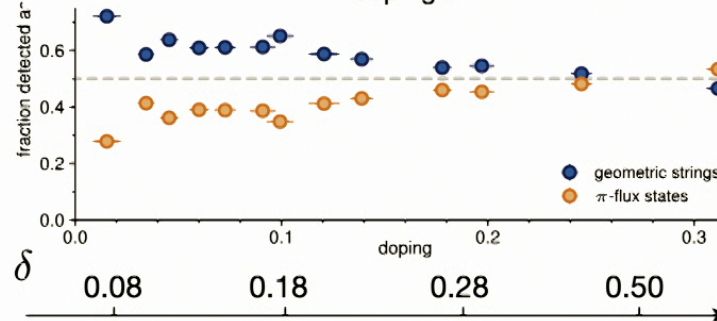
Phase diagram



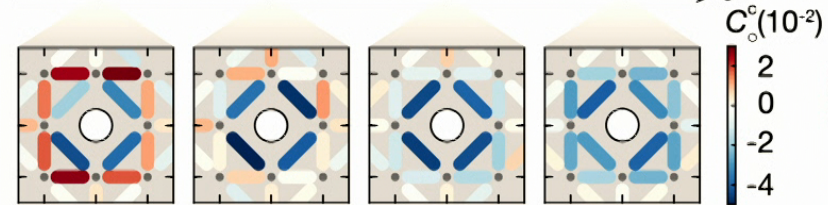
Bohrdt et al., *Annals of Physics* (2021)



C. Chiu et al., *Science* 365 (2019)



Bohrdt et al., *Nature Physics* 15 (2019)

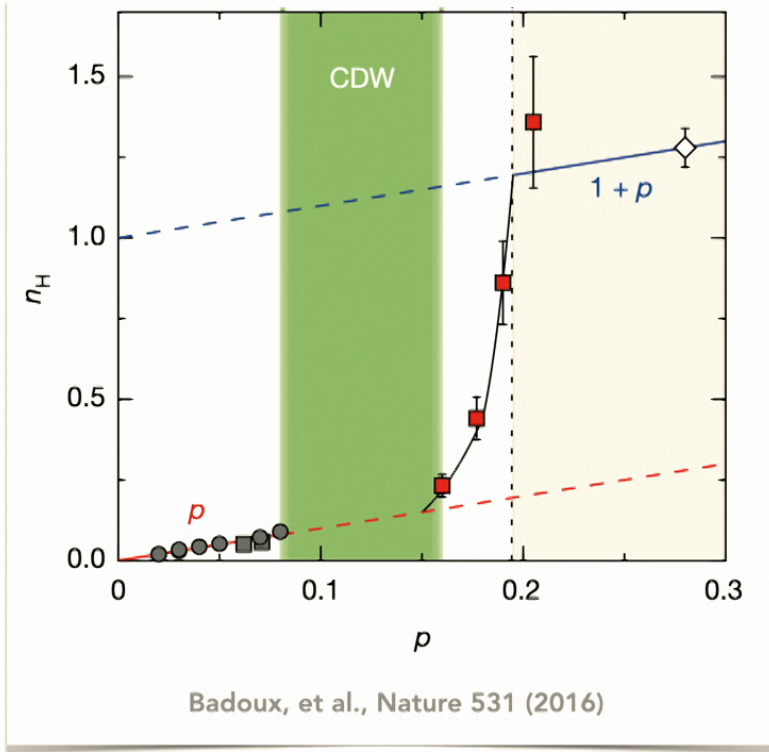


Koepsell et al., *Science* 374 (2021)

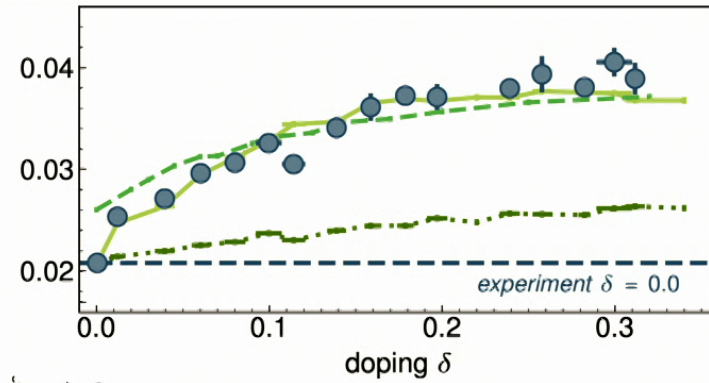
43

Perimeter, March 2023

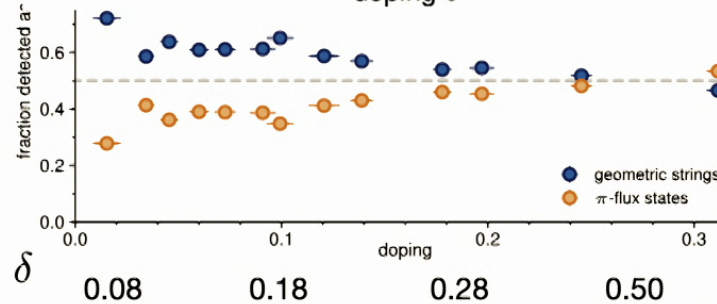
Phase diagram



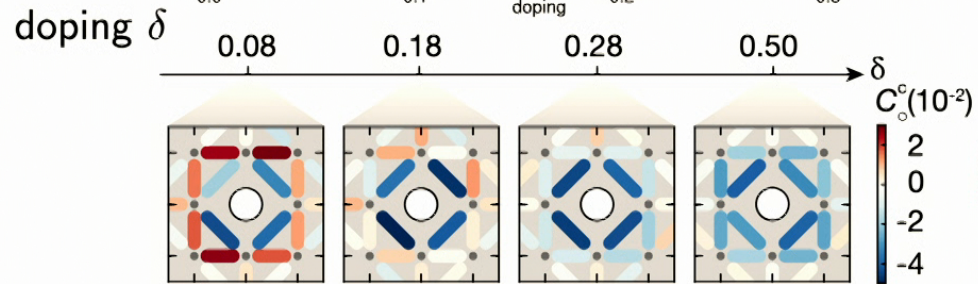
Bohrdt et al., Annals of Physics (2021)



C. Chiu et al., Science 365 (2019)



Bohrdt et al., Nature Physics 15 (2019)

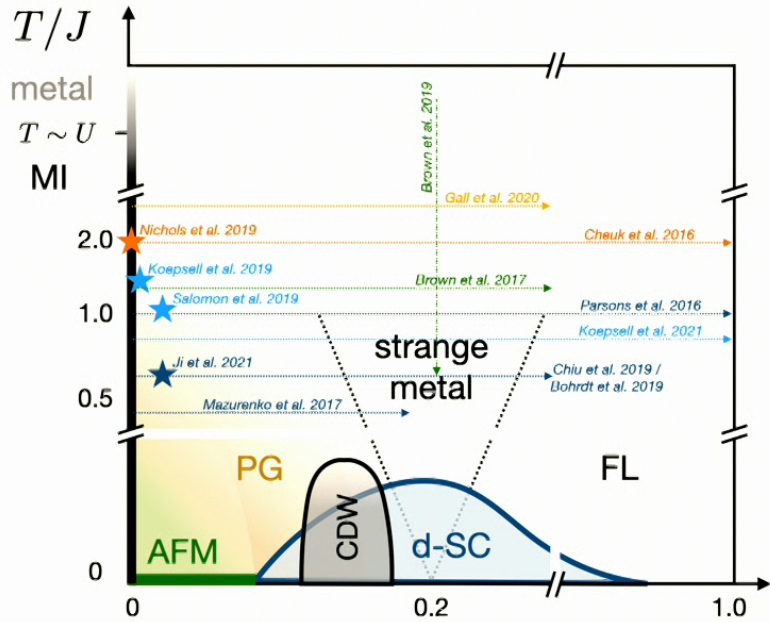


Koepsell et al., Science 374 (2021)

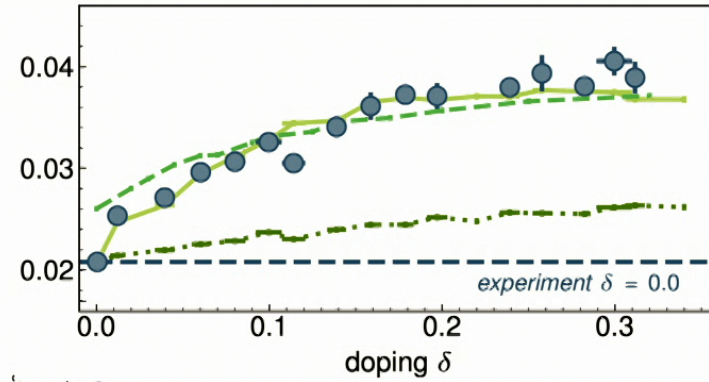
43

Perimeter, March 2023

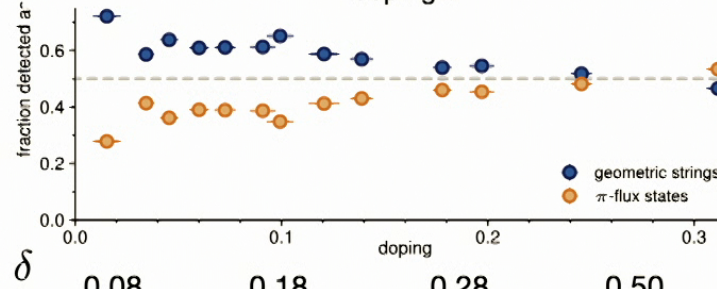
Phase diagram



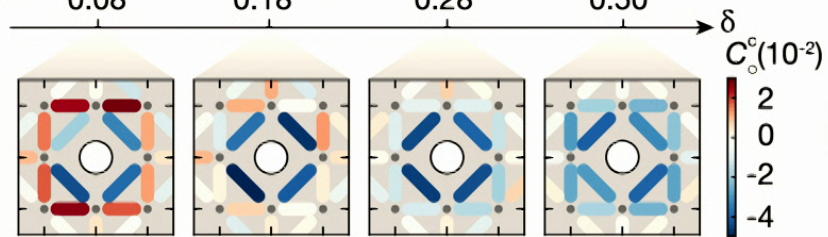
Bohrdt et al., *Annals of Physics* (2021)



C. Chiu et al., *Science* 365 (2019)



Bohrdt et al., *Nature Physics* 15 (2019)



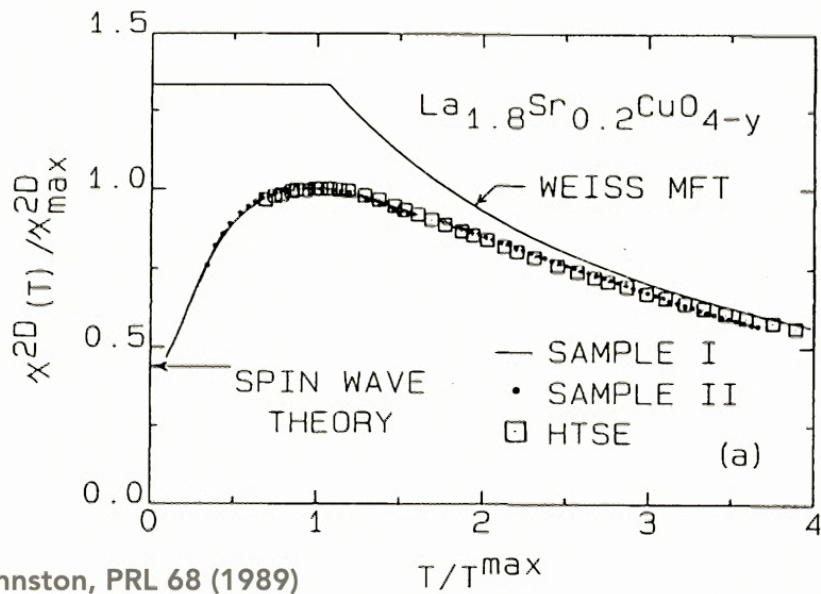
Koepsell et al., *Science* 374 (2021)

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Perimeter, March 2023

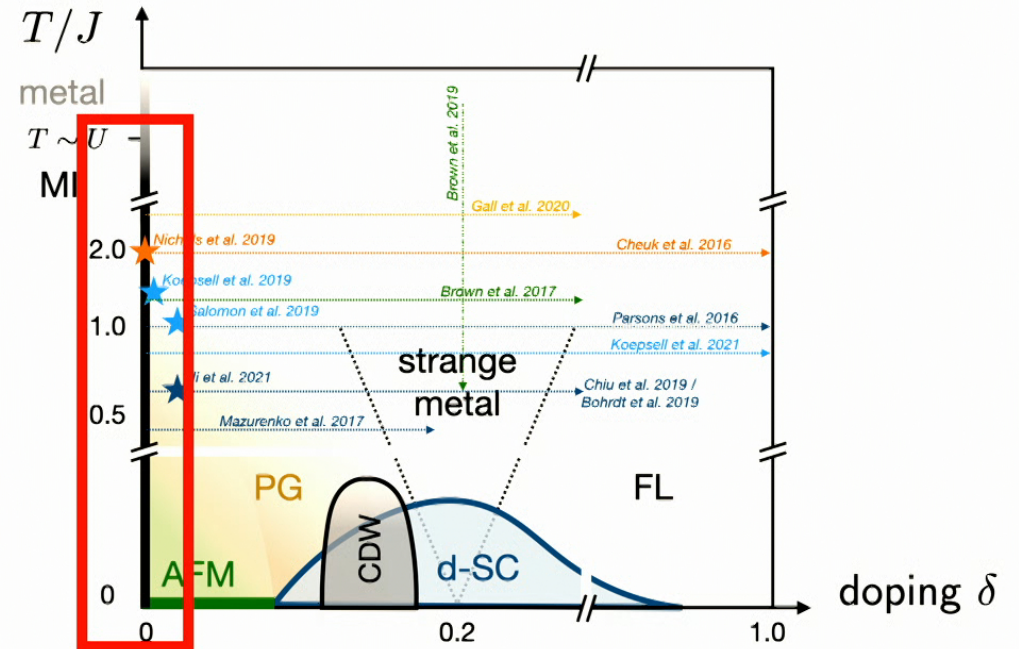
Unsupervised learning of snapshots

Measuring magnetic susceptibility



Johnston, PRL 68 (1989)

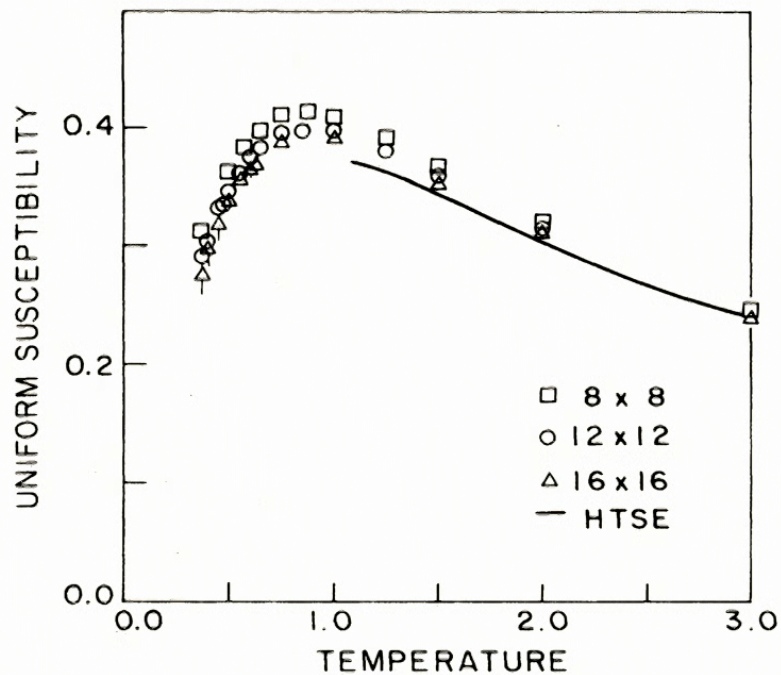
- pair state is spin singlet, suppression of spin excitations below T_c
- Preformed pairs???



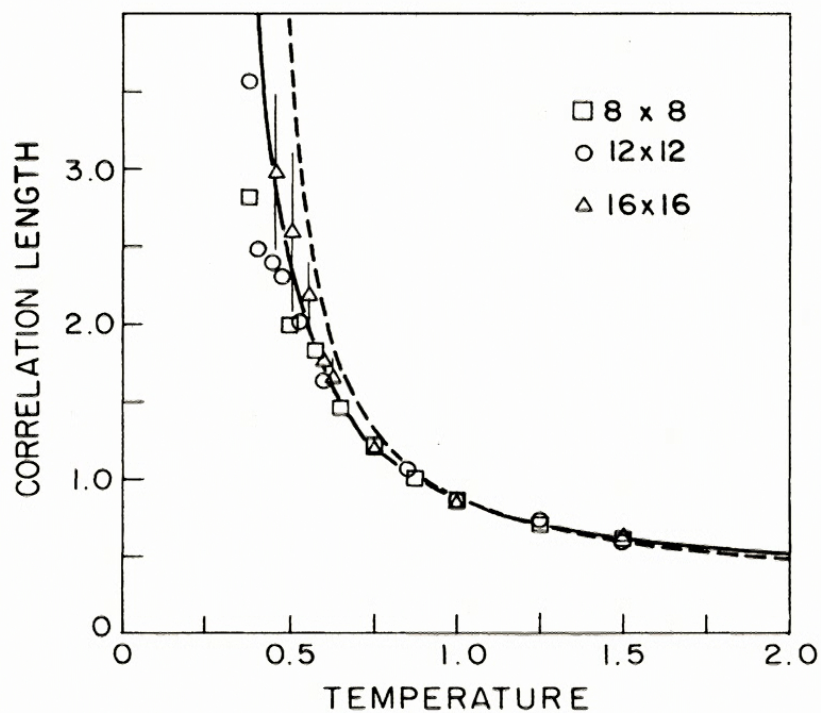
Approximate at half-filling:

Heisenberg model
$$\hat{H} = J \sum_{\langle i,j \rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$

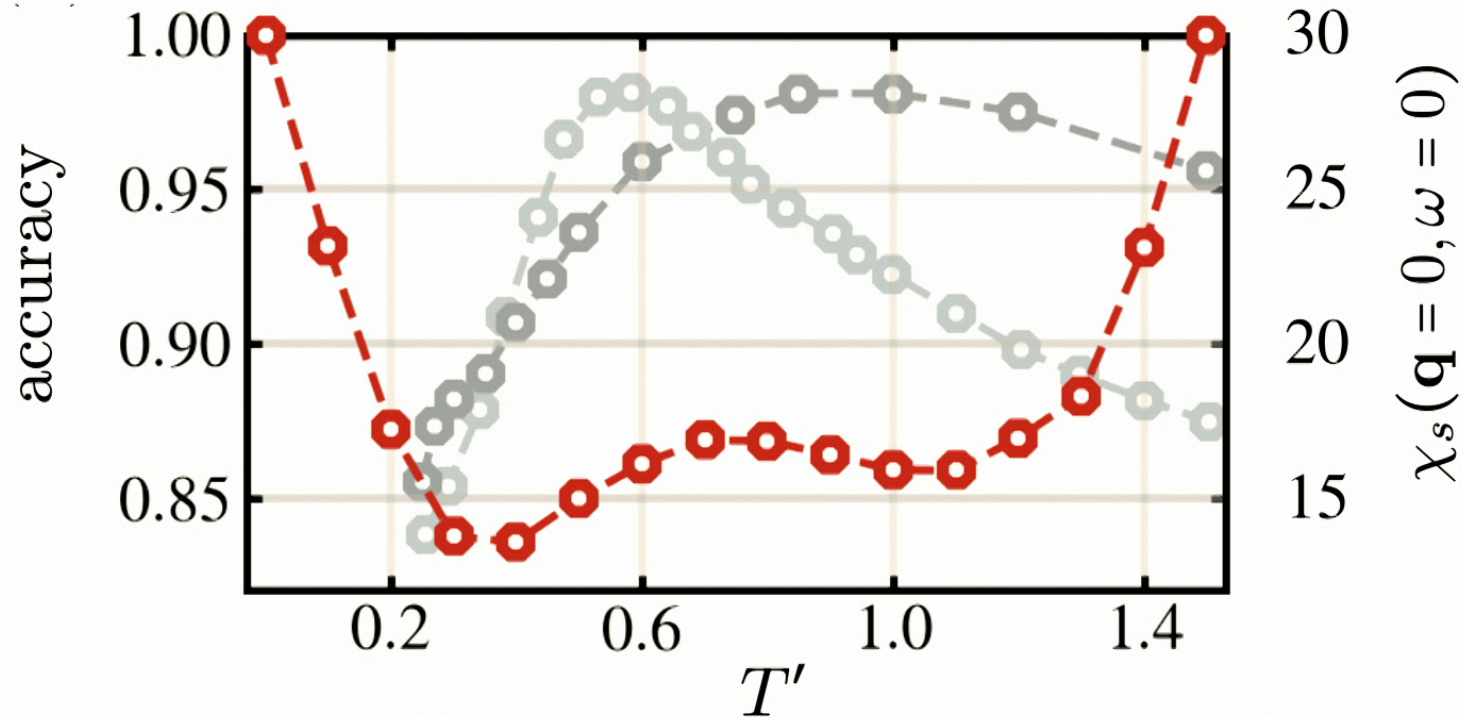
QMC for the Heisenberg model



Gomez-Santos et al., PRB 39 (1989)



Confusion learning Heisenberg snapshots

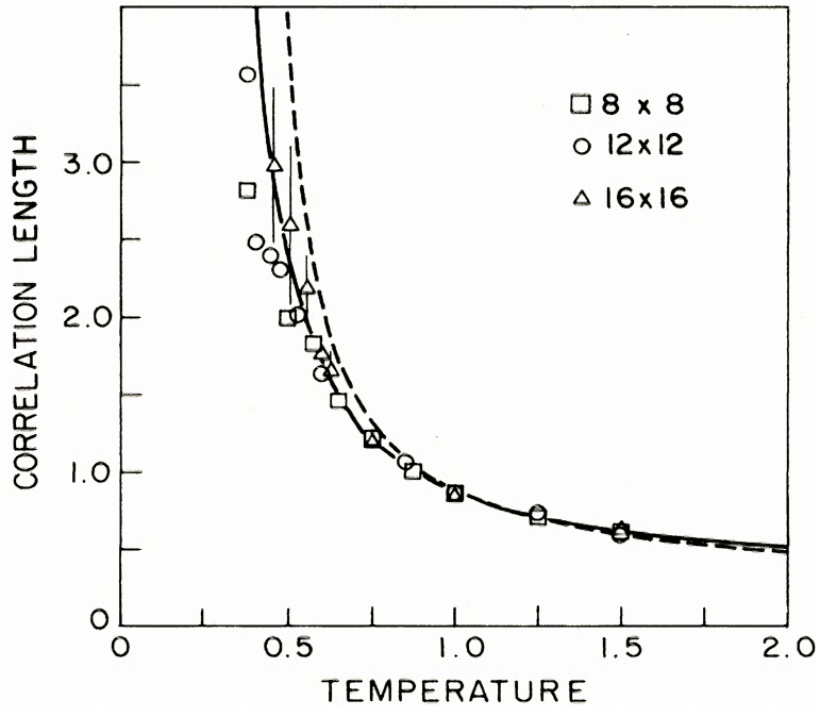


Schlömer et al., arXiv2303.XXXX

Introduced in van Nieuwenburg et al., Nature Physics, 13 (2017)

46

Perimeter, March 2023

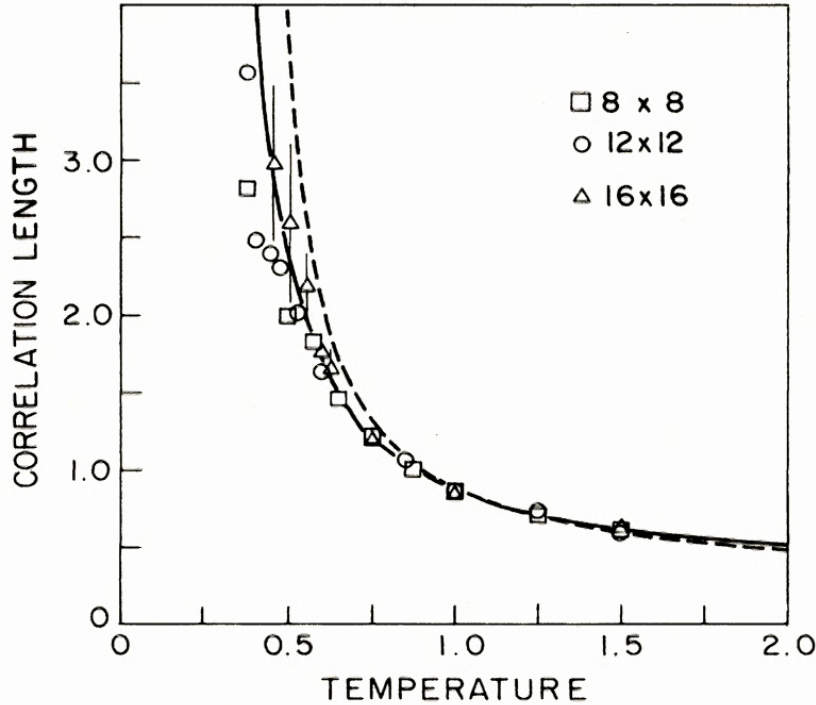


Gomez-Santos et al., PRB 39 (1989)

$$\begin{aligned} \sigma_{c_1}^2 &= \left\langle \left(\sum_{i,e} \hat{S}_i^z \hat{S}_{e_i}^z \right)^2 \right\rangle - \left\langle \sum_{i,e} \hat{S}_i^z \hat{S}_{e_i}^z \right\rangle^2 \\ &= \sum_{i,j,e_i,e_j} \left\langle \hat{S}_i^z \hat{S}_{e_i}^z \hat{S}_j^z \hat{S}_{e_j}^z \right\rangle - \left\langle \sum_{i,e} \hat{S}_i^z \hat{S}_e^z \right\rangle^2 \end{aligned}$$



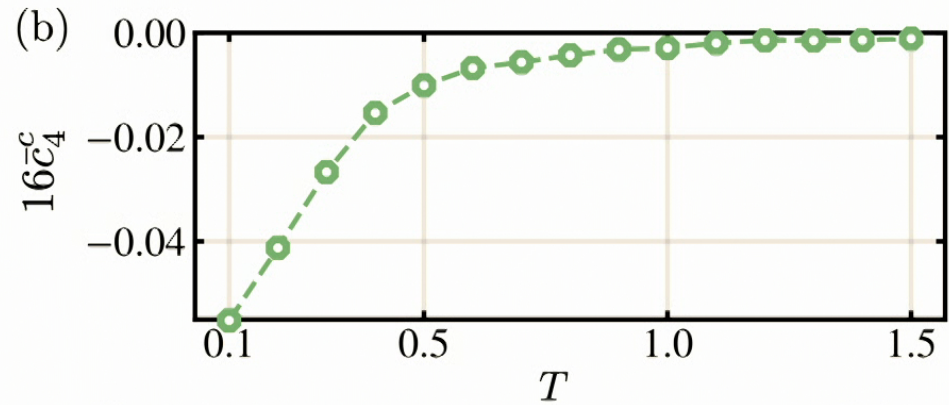
Interpretability



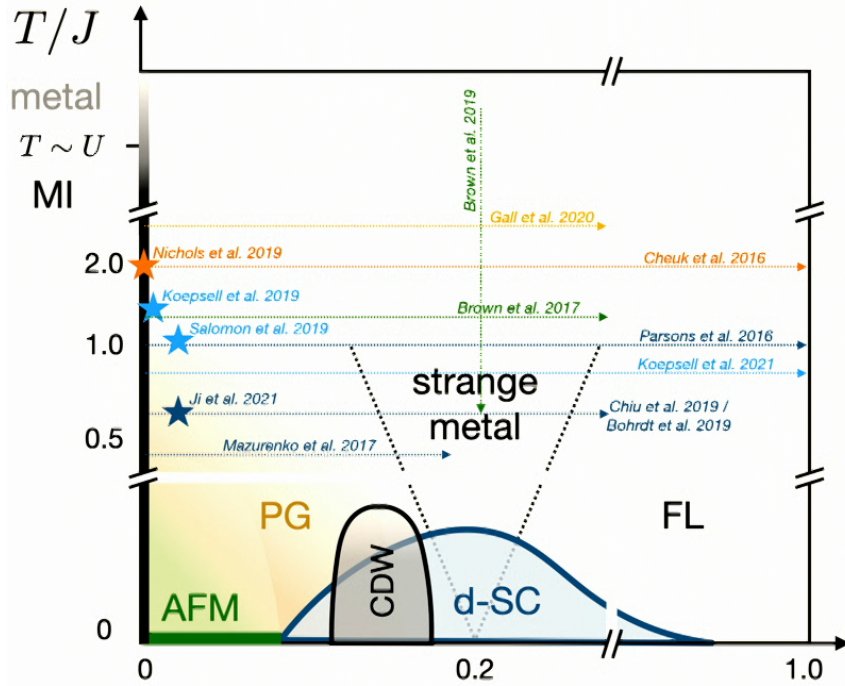
Gomez-Santos et al., PRB 39 (1989)

$$\begin{aligned} \sigma_{c_1}^2 &= \left\langle \left(\sum_{i,e} \hat{S}_i^z \hat{S}_{e_i}^z \right)^2 \right\rangle - \left\langle \sum_{i,e} \hat{S}_i^z \hat{S}_{e_i}^z \right\rangle^2 \\ &= \sum_{i,j,e_i,e_j} \left\langle \hat{S}_i^z \hat{S}_{e_i}^z \hat{S}_j^z \hat{S}_{e_j}^z \right\rangle - \left\langle \sum_{i,e} \hat{S}_i^z \hat{S}_e^z \right\rangle^2 \end{aligned}$$

Connected 4 point c1-c1 correlator



Interpretability

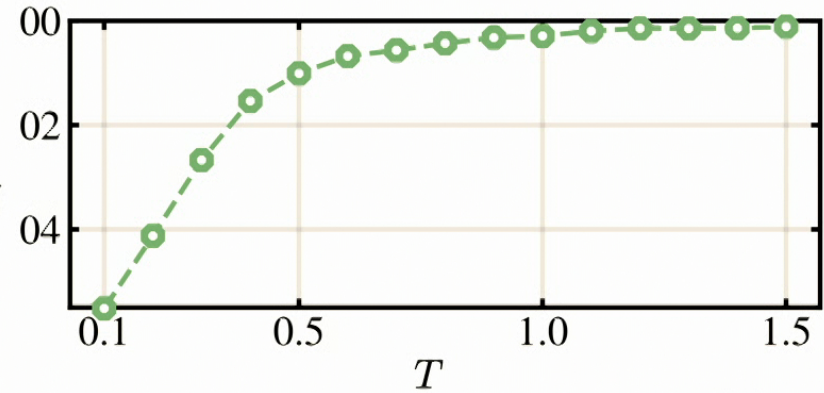


Bohrdt et al., *Annals of Physics* (2021)

$$\sigma^2 = \left\langle \left(\sum_{i,e} \hat{S}_i^z \hat{S}_{e_i}^z \right)^2 \right\rangle - \left\langle \sum_{i,e} \hat{S}_i^z \hat{S}_{e_i}^z \right\rangle^2$$

$$= \sum_{i,j,e_i,e_j} \left\langle \hat{S}_i^z \hat{S}_{e_i}^z \hat{S}_j^z \hat{S}_{e_j}^z \right\rangle - \left\langle \sum_{i,e} \hat{S}_i^z \hat{S}_e^z \right\rangle^2$$

Connected 4 point c1-c1 correlator



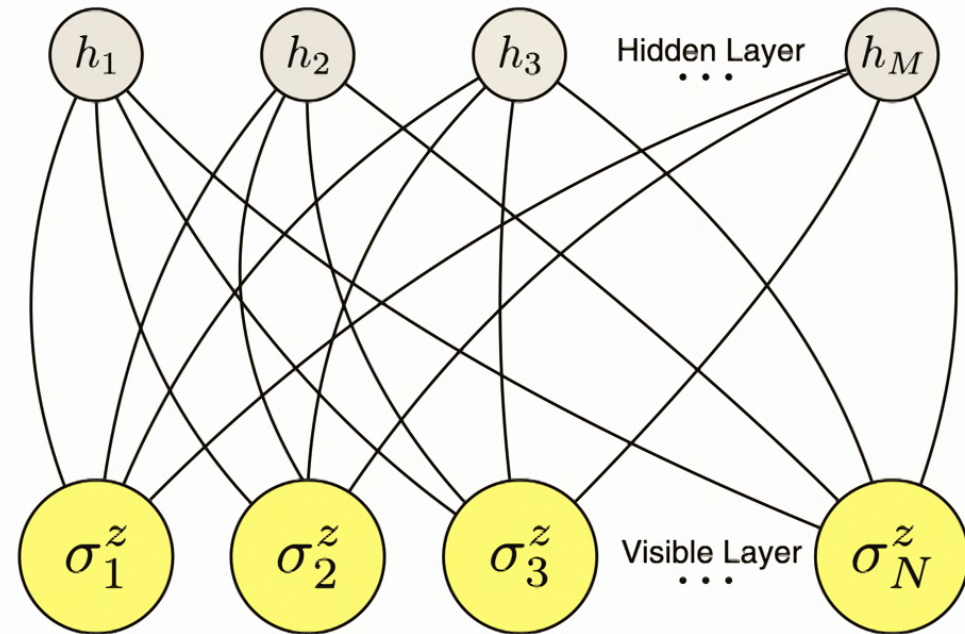
Numerics Review from last year:

2. Phase diagram and phase boundaries

Not much is known about the precise phase diagram of the Hubbard model in two dimensions. While the ground-state phase diagram of the model at weak coupling is fairly well understood (49, 266), the phase diagram in the intermediate-to-strong interaction limit is hotly debated. Determining precise phase boundaries is hindered by the fact that the diver-

Represent (complex) quantum states with neural networks:

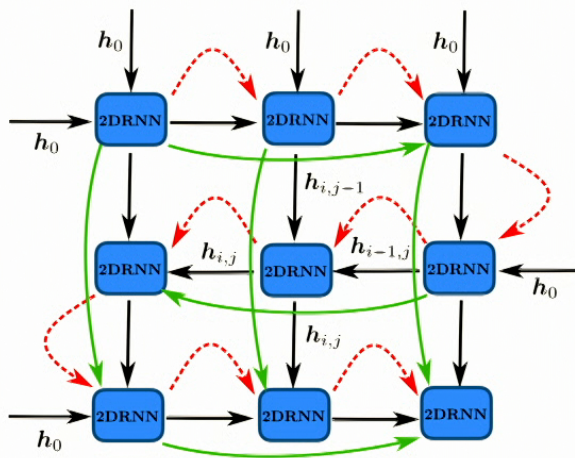
Carleo & Troyer, 2017
Hibat-Allah et al., 2020
Roth et al., 2020
Hibat-Allah et al. 2022
...



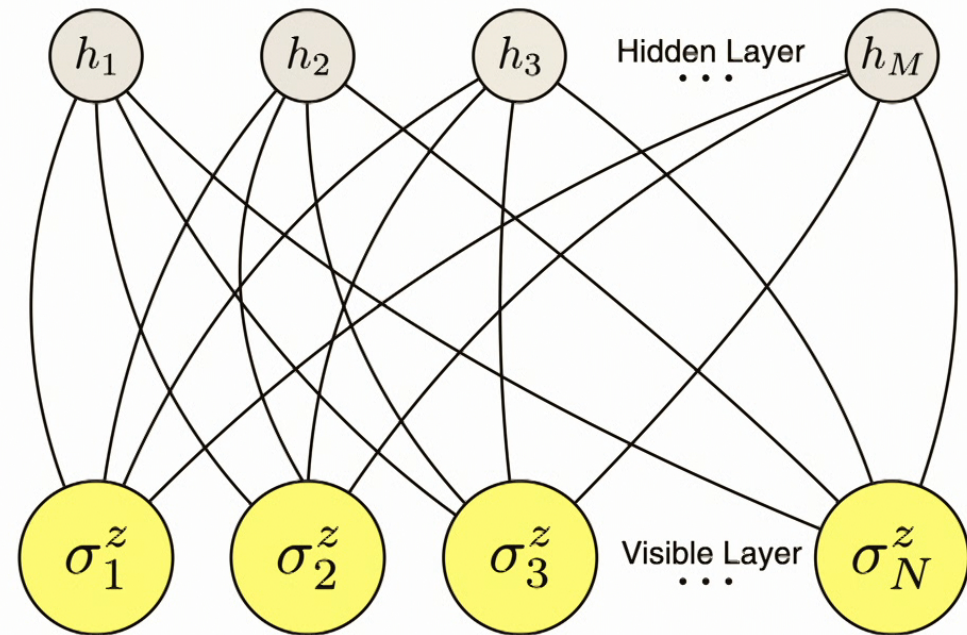
Carleo & Troyer, Science 355 (2017)

Represent (complex) quantum states with neural networks:

Carleo & Troyer, 2017
 Hibat-Allah et al., 2020
 Roth et al., 2020
 Hibat-Allah et al. 2022
 ...



Hibat-Allah et al., arXiv:2303.11207

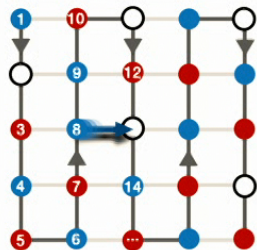
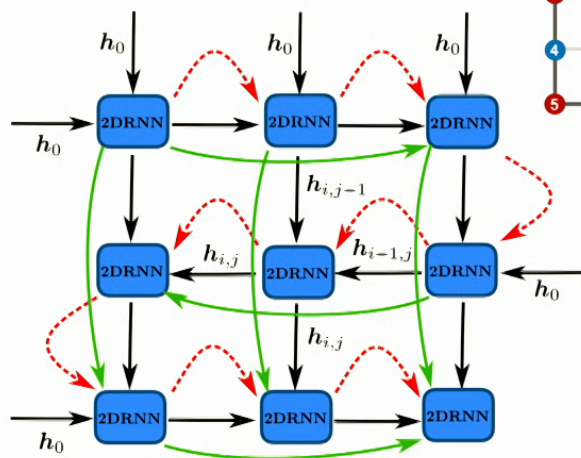


Carleo & Troyer, Science 355 (2017)

Neural network quantum states for fermions

Represent (complex) quantum states with neural networks:

Carleo & Troyer, 2017
 Hibat-Allah et al., 2020
 Roth et al., 2020
 Hibat-Allah et al. 2022
 ...

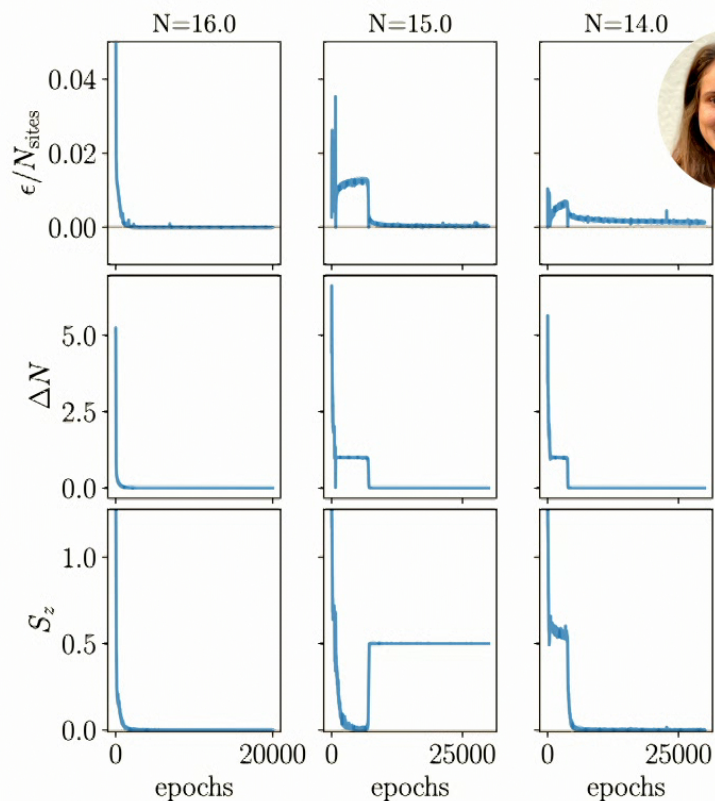


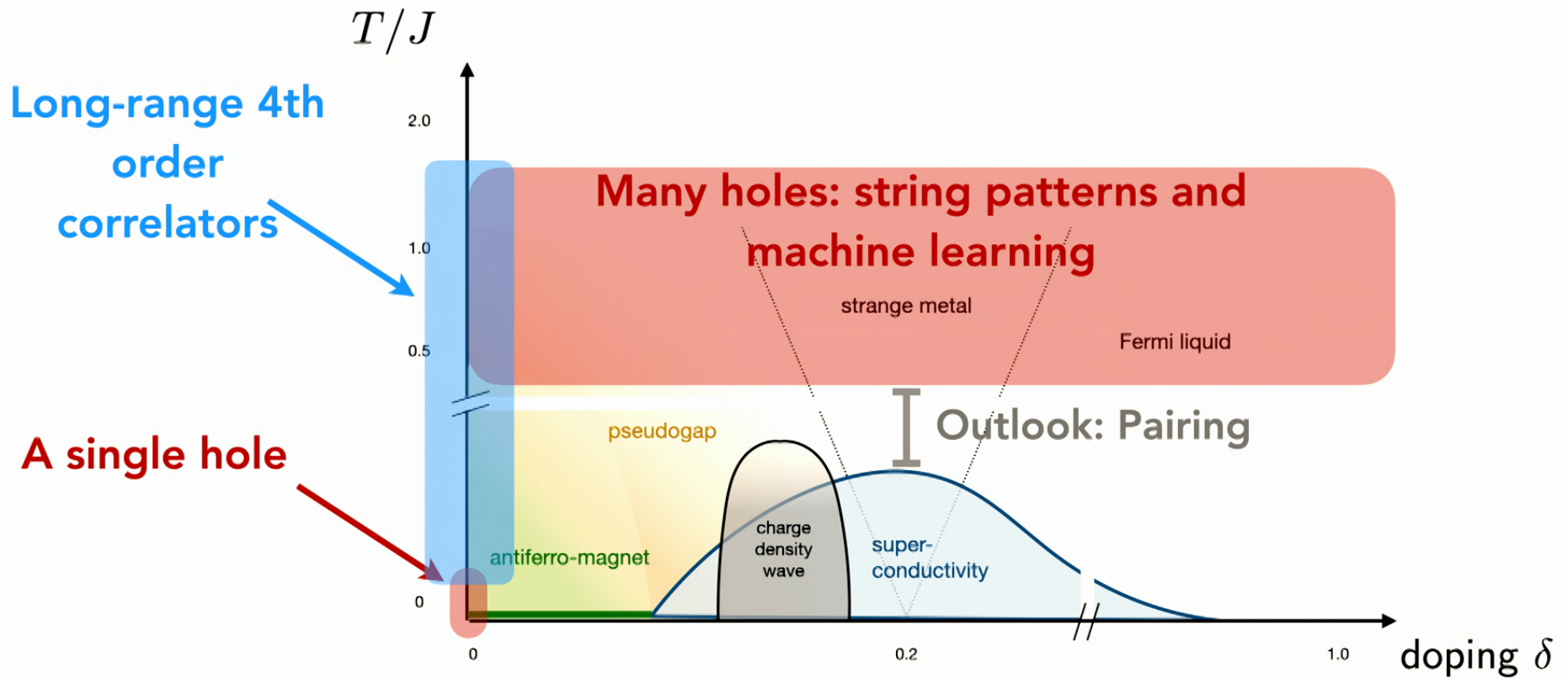
Energies

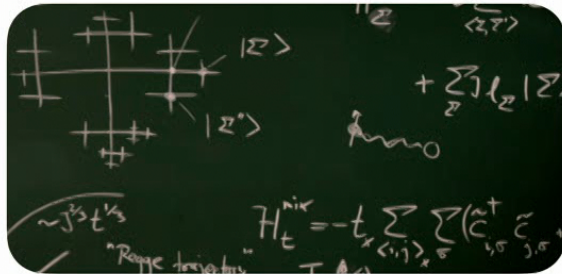
$$\epsilon = \frac{|E_{ED} - E_{RNN}|}{|E_{ED}|}$$

Particle Number

Total Magnetization







- Microscopic models with strong pairing or robust stripes

Bohrdt et al., *Nature Physics* (2022)
 Schlömer et al., *arXiv 2208.07366*
 Hirthe et al., *Nature* (2023)



- Pairing symmetry & signatures of pairs

Bohrdt et al., *arXiv:2210.02322*
 Grusdt et al., *arXiv:2210.02321*

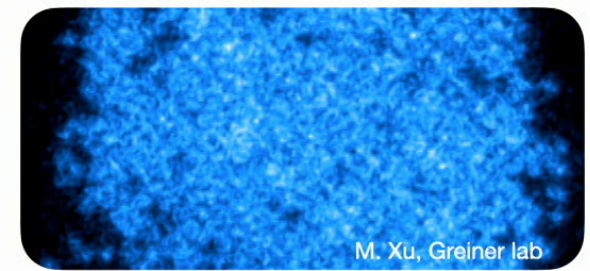


- Neural network states for fermionic systems

Lange et al., *arXiv2304.XXXX*



- Large 2D systems, Fermi-Hubbard model, other geometries



- Pseudogap signatures from snapshots

Schlömer et al., *arXiv2303.XX*



- Application at finite doping, experimental data

Thank you for your attention!

Hannah Lange

Fabian Döschl

Tizian Blatz

Christian Reinmoser

Henning Schlömer

Fabian Grusdt

Matjaz Kebric

Felix A. Palm

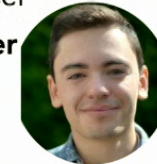
Maximilian Buser

Lukas Homeier

Jad Halimeh

Ulrich Schollwöck

Lode Pollet



Immanuel Bloch

Christian Gross

Joannis Koepsell

Jayadev Vijayan

Pimonpan Sompet

Guillaume Salomon

Dominik Bourgund

Petar Bojovic

Timon Hilker

Thomas Chalopin

Sarah Hirthe

Eugene Demler

Yao Wang

Michael Knap

Frank Pollmann

Eun-Ah Kim

Cole Miles

Yanjun Liu

Kaarthik Varma

...



Markus Greiner

Daniel Greif

Christie Chiu

Geoffrey Ji

Muqing Xu

Martin Lebrat

Lev Kendrick

Justus Brüggjenjürgen

Anant Kale

Julian Léonard

Sooshin Kim

Matthew Rispoli

Alex Lukin

Robert Schittko