Title: Quantum Bergmann-Komar group, U(1)^3 quantum gravity and loop quantum gravity

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Abstract: In any approach to quantum gravity, the quantum representation theory of the "algebra" of Cauchy hypersurface deformations plays a crucial role. Its faithful implementation is a key step towards constructing a valid theory of quantum gravity as it ensures quantum spacetime diffeomorphism covariance. Bergmann and Komar were the first to consider the possibilty of a corresponding quantum "group". Its construction is mathematically challenging in more than 1+1 spacetime dimensions because one leaves the realm of Lie algebras and Lie groups. After an introduction to these concepts, we show that the Bergmann Komar "group" can indeed be faithfully implemented in a weakly self-interacting truncation of 3+1 quantum gravity with two propagating polarisations. We then discuss possible implications for the actual, untruncated theory.

Zoom link: https://pitp.zoom.us/j/91488383205?pwd=bkxGS0huMGNEYXc3Y2FJSGZHQ0pqQT09

Quantum Bergmann-Komar group, $U(1)^3$ quantum gravity and LQG

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arXiv: 2207.08299 (strategy); 2207.08302, 2207.08290 (models); 2207.08294, 2207.08291 (tools)

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ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS

Classical formulation

- Canonical approach to classical & quantum GR has long tradition
- Classical: Initial value formulation, numerical integration of Einstein equations, BH merger simulations ... [Arnowitt,Deser,Misner,..]
- Quantum: QG as a constrained QFT [Bergmann, DeWitt, Dirac, Komar,Wheeler,..]
- Key assumption: Globally hyperbolic spacetimes (M, g)
- $\Rightarrow M \cong \mathbb{R} \times \sigma$ [Geroch, Sanchez & Bilal]
- M can be foliated by leaves $t \mapsto \Sigma_t \cong \sigma$ ("3+1" split: space+time)
- Legendre transform for constrained systems (Dirac algorithm): $(g, \dot{g}) \mapsto (q, P)$ (3-metric on σ, conj. momentum), Poisson brackets $\{.,.\}$.
- plus: spatial diffeomorphism and Hamiltonian constraints D_a; $a = 1, 2, 3,\quad C$ (temporal-spatial, temporal-temporal components of Einstein eqns)

Classical Hypersurface Deformation Algebra (HDA)

- Let $Q := \sqrt{\det(q)}$, smeared constraints:
  $D[u] = \int_\sigma d^3x\ u^a\ C_a; \quad C[f] = \int_\sigma d^3x\ f\ C$
- Classical HDA $\mathfrak{h}$ [Hojman, Kuchar, Teitelboim]

$$\{D[u], D[v]\} = -D[[u,v]], \quad \{D[u], C[f]\} = -C[u[f]],$$

$$\{C[f],\ C[g]\} = -D[q^{-1}\ (M\ dN - N\ dM)]$$

- Observations:
  - encodes local spacetime diffeomorphism covariance
  - Ill-defined for degenerate metrics ($Q = 0$) due to q^{-1}
  - $\mathfrak{h}$ not a Lie algebra due to q^{-1}: “open algebra, algebroid”
  - Formal exponentiation $\mathfrak{H} = \exp(\mathfrak{h})$ (“BK group”) not a (Lie) group.
  - Mathematically more precise: $\mathfrak{h}'$: Lie algebra generated by $\mathfrak{h}$, $\mathfrak{H}' = \exp(\mathfrak{h}')$ (drop $'$ again)
- Anomaly-free implementation of $\mathfrak{h}$ or $\mathfrak{H}$ major challenge in QG

Vacuum representations

- To allow for spinorial matter: use densitised triad E^a_j; $\delta^{jk} E^a_j E^b_k = Q\, q^{ab}$, conj. momentum A^j_a (canonical transf.) [Ashtekar,Barbero]
- Every single term in C (vacuum, cosm. const., matter) couples to E^a_j
- Motivates q'ion choice: in order that C be densely defined, pick vacuum s.t. $E^a_j\, \Omega = 0$
- $\Rightarrow$ quantum degenerate vacuum $Q\, \Omega = 0$
- Proposition: This already fixes a rep. of CCR and AR of Narnhofer-Thirring type via GNS (e.g. rep. used in LQG)
- Corollary: At least one type of Weyl operators $W[G] := \exp(-i\, E[G])$, $w[F] := \exp(-i\, A[F])$ weakly discont., HS not sep.
- Since D_a, C depend on A, not $w[F]$, expect that at best $\mathfrak{H}$ implementable in this rep. but not $\mathfrak{h}$
- Quantum Non-degeneracy: Smearing functions F need to have support everywhere in order that quantum $\mathfrak{H}$ well defined (domain of $\mathfrak{H}$)

Further plan of talk

- Consider $U(1)^3$ model in 3+1: close relative of GR, technically simpler, still full complexity of $\mathfrak{h}$
- Quantum integrability: Q'ion programme can be completed
- non-anomalous rep. of $\mathfrak{H}$
- extension to full GR: additional steps (e.g. renormalisation) necessary (outlook)

Definition of classical $U(1)^3$ model

- Hamiltonian definition [Smolin]:
  Take Euclid. vac. GR in Ashtekar-Barbero variables, drop A^2 terms from $C[f]$ (weak Newtonian constant limit)
- Lagrangian definition [Bakhoda, TT]:
  Take Euclid. vac. GR in self-dual variables [Ashtekar], drop A^2 terms from L
- Almost Euclidian vacuum GR, but Abelian structure group
- Consistent deformation of Euclidian vacuum GR [Barbero]: 2 phys. d.o.f.
- Possible connection to twistor string theory [Abou-Zeid, Hull, Mason]
- Vacuum constraints (curvature: $R^j_{ab} = 2\partial_{[a}A^j_{b]}$):

$$Z_j = \partial_a E^a_j, \;\; D_a = R^j_{ab}\, E^b_j, \;\; Q\; C = \epsilon^{jkl}\, \delta_{ji}\, R^i_{ab}\, E^a_k\, E^b_l$$

- Classical hypersurf. def. alg. $\mathfrak{h}$ unchanged
- in particular: still non-trivial, non-polynomial struct. fns.
- ideal test laboratory for many technical/conceptual issues of QG [Varadarajan et al] both canonical and covariant

Construction of q'um non-deg. $U(1)^3$ QG

- Narnhofer-Thirring type of rep.

$$< \Omega, w[F]\Omega >= \delta_{F,0},\ w[F] = \exp(-i\,A[F]),\ E[G]\Omega = 0,\ A[F] := \int d^3x\, F^a_j\, A^j_a$$

- F : form factor, generalised "holonomies" $w[F]$ discont., "fluxes" $E[G]$ cont.
- Geometrical ops. diagonal, e.g. volume

$$V(R)\ w[F]\ \Omega = \ell_P^3\ [\int_R d^3x\ \sqrt{|\det(F)|}]\ w[F]\ \Omega$$

- q'um non-deg dense domain: $\det(F) \neq 0$
- solution of Gauss constraint: $\partial_a F^a_j = 0$
- spatial diffeo $D[u]$, Ham. constr. $C[f]$: ill-defined as $A\ \nexists$
- No rep. of $\mathfrak{h}$ on $\mathcal{H}$. But: can exponentiate $\mathfrak{H} := \exp(\mathfrak{h})$ on $\mathcal{H}$
- $U(u,f)\ w[F]\ \Omega := \exp(iD[u] + iC[f])\ w[F]\ \Omega = w[(e^{X_{u,f}} \cdot K)(0,F)]\ \Omega$
- $X_{u,f}$: HVF of $D[u] + C[M]$, $K(G,F) := F$ momentum coordinate fn.

Properties of $U(1)^3$ QG

- to best of knowledge: first q'um realisation of Bergann-Komar "group" in 3+1
- derived using standard point splitting reg.
- $U(u,f)$ densely defined, in fact unitary, reduces to spatial diffeo group $\varphi^u_{t=1}$ for $f=0$
- anomaly freeness realised: q'um algebra encoded by Hamiltonian flow of classical constraints on non-deg. form factors

$$U(sf)\;U(tg)\;U(-sf)\;U(-tg)w[F]=w[F_{s,t}],\quad e^{ist\{C(f),C(g)\}}\;w[F]=w[\hat{F}_{s,t}],$$

$$(\frac{d^2}{ds\;dt}[F_{s,t}-\hat{F}_{s,t}])_{s=t=0}=0$$

- q'um non-degeneracy crucial: HVF otherwise ill-defined

Properties of $U(1)^3$ QG

- Abelian q'um electr. shift/gauge cov. diffeo [Giesel, TT 06; Ashtekar, Varadarajan 21] all orders
- Highly non-linear Ham. flow $[e^{X_{u,f}} \cdot K](0, F)$ computable at N-th order wrt u, f:
- E.g. for $u = 0$, mod. $\det(F)^{-N}$ factors get with $[B_f(F, G)]^a_j := \epsilon_{jkl}\ [f\ F_k^{[b} G_l^{a]}]_{,b}$

$$X_f \cdot F = B_f(F, F),\ X_f^2 \cdot F = 2\ B_f(F, B_f(F, F)),$$

$$X_f^3 \cdot F = 2\ B_f(B_f(F, F), B_f(F, F)) + 4\ B_f(F, B_f(F, B_f(F, F))),\ \ldots$$

- Ham. constr. action: Mollify CNW-FF $F^a_j(x) = \sum_e n^j_e \int_e dy^a\ \delta(x, y)$, then:
  1. action along whole graph (not only vertices), no abrupt loop attachment,
  2. action on charges non-polynomial
- C can be extended by "potential" $V[E]$ (e.g. cosm. const.)
- perfect match: op. constr. q'ion vs. red. phase sp. q'ion (relational observables)
- Physical HS and Hamiltonian: non-linear, self-interacting electrodynamics: N-point Wightman fns. not determined by 2-pt fn.
- e.g. gauge fix/solve for $E^a_\alpha, A^\alpha_a,\ \alpha = 1, 2$, keep $Z_3 = \partial_a E^a_3,\ B^a_3 = \epsilon^{abc} A^3_{c,b}$

$$H = \int_\sigma d^3x\ B^a_3\ Pol^{(2)}_a(E)\ [E^z_3]^{-1}$$

- non-relational weak Dirac observables of CDJ type [Capovilla, Dell, Jacobson]
- Technically simpler Abelian Spin Foam Model

Lessons and Outlook

- $U(1)^3$ QG (almost) q'um integrable in Narnhofer-Thirring type of rep.
- Convergence of ideas: canonical, covariant, relational observables, ...
- can be considered paradigm model or "harmonic oscillator" of (L)QG in 4D
- importance of q'um non-degeneracy: part of definition of HDA
- current LQG HS rep. for full GR: all states in domain of C quantum degenerate as F supported on graphs ==> HDA implementation difficult. Why?
- $U(1)^3$: HVFs preserve momentum polarisation of phase space
- full QG: C not polarisation preserving ==> graphs in LQG vital s.t. C has dense inv. domain
- Need A. q'um non-degeneracy (domain of $\mathfrak{h}$) and B. dense inv. domain (of C)
  - 1st approach: pert. theory around integrable model = consistent deformation of (Euclidian) GR [Barbero]
  - 2nd approach: Non-pert., CQFT method = Hamiltonian Renormalisation [TT]:
    i. Define family of q'um non-deg., dens. def. theories at finite resolutions
    ii. construct infinite resolution theory using renormalisation flow
    iii. ==> "transport" quantum non-degeneracy into new rep.

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