Title: 3d gravity and gravitational entanglement entropy

Speakers: Gabriel Wong

Series: Quantum Gravity

Date: March 30, 2023 - 2:30 PM

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Abstract: Recent progress in AdS/CFT has provided a good understanding of how the bulk spacetime is encoded in the entanglement structure of the boundary CFT. However, little is known about how spacetime emerges directly from the bulk quantum theory. We address this question in an effective 3d quantum theory of pure gravity, which describes the high temperature regime of a holographic CFT. This theory can be viewed as a q-deformation and dimensional uplift of JT gravity. Using this model, we show that the Bekenstein-Hawking entropy of a two-sided black hole equals the bulk entanglement entropy of gravitational edge modes. These edge modes transform under a quantum group, which defines the data associated to an extended topological quantum field theory Our calculation suggests an effective description of bulk microstates in terms of collective, anyonic degrees of freedom whose entanglement leads to the emergence of the bulk spacetime. Finally, we give a proposal for obtaining the Ryu Takayanagi formula using the same quantum group edge mode

Zoom link: https://pitp.zoom.us/j/98275430953?pwd=TzdTUXIvVWU4Ym1jcWRWbkgxZnhMdz09

The holographic principle

In QG, deg. of freedom in a spatial region resides on its boundary

Bekenstein-Hawking

$$S_{BH} = \frac{A}{4G}$$

Generalized entropy

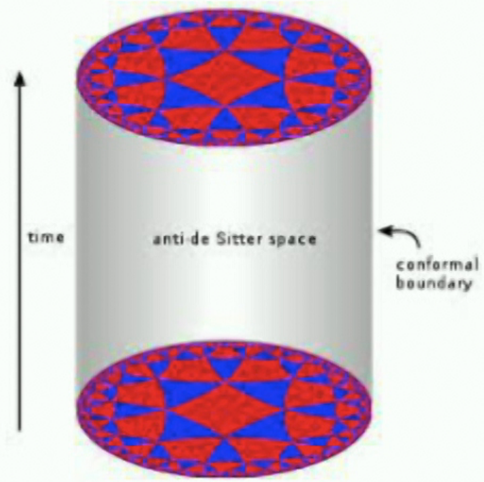
$$S_{\text{gen}} = \frac{A}{4G} + S_{\text{out}}$$

AdS holography

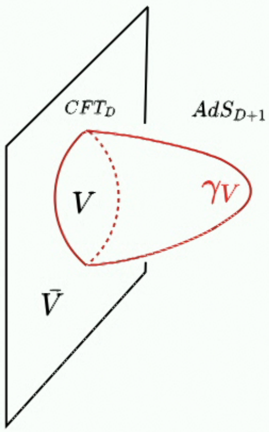
A lot of progress in building a dictionary relating bulk and boundary quantities.

Main lesson: the bulk spacetime geometry is encoded in the entanglement structure of the boundary QM

Can we understand the emergence of spacetime directly from the bulk? Are bulk quantum information quantities like entanglement entropy well defined?



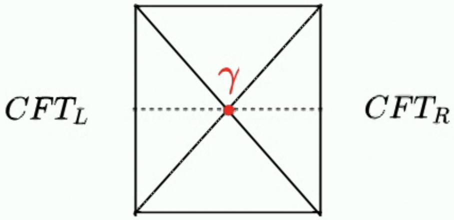
Quantum Extremal Surface (QES) Prescription



$$S_{\text{CFT}} = S_{\text{gen}} = \frac{A(\gamma_V)}{4G} + S_{\text{bulk}}$$

What is the **bulk** microstate interpretation of the area term?

Interesting because it measures the entanglement **that makes up spacetime** (Van Raamsdonk)

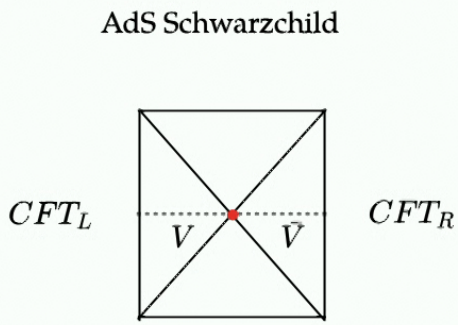


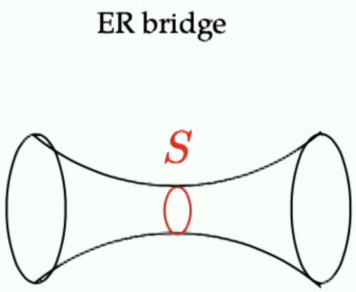
Folklore: S_{gen} = entanglement entropy of bulk quantum gravity

Bulk spacetime is fluctuating. What is entangled with what?

Diffeo invariance —> gravitational degrees of freedom are non local: how do we factorize the Hilbert space?

The factorization problem in bulk gauge theory (Harlow)





Bulk charges must be exist that split the wormhole-crossing Wilson line into gauge inv. operators.

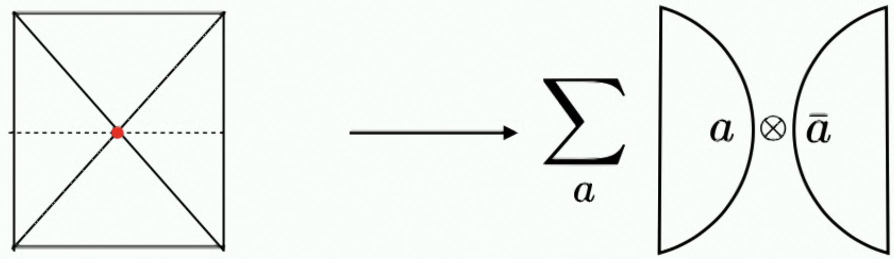
In the low energy effective gauge theory, these are **entanglement edge modes** (Donnelly-Freidel)

$$i : \mathcal{H}_{\text{bulk}} \to \mathcal{H}_V \otimes \mathcal{H}_{\bar{V}}$$

They transform under a surface symmetry G_S and contribute to the bulk entanglement entropy:

$$S_V = -\text{tr}\ \rho_V \log \rho_V = S_{\text{bulk}} + S_{\text{edge}} \qquad S_{\text{edge}} \sim \log \dim a$$

The factorization problem in bulk quantum gravity



$$\frac{A}{4G} \stackrel{?}{=} \log \dim a$$

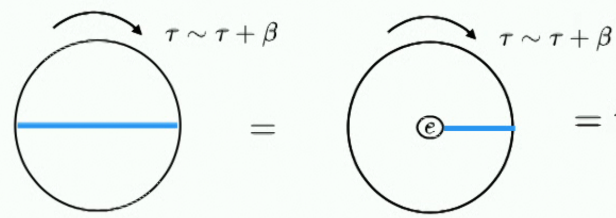
Perhaps the QES area term is the entanglement entropy of quantum gravity edge modes which glues together the space-time (J Lin, D. Harlow, Donnelly-Friedel, GW-Donnelly)

An exact description of the bulk edge modes would require solving bulk QG, i.e. solving IIB string theory…

The shrinkable boundary condition

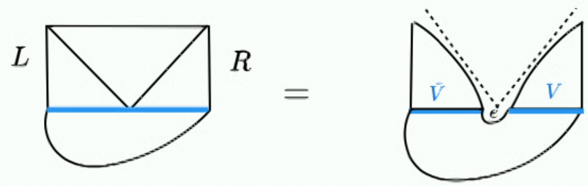
Bottom-up approach: Introduce edge modes as an extension of the EFT Hilbert space, constrained by a shrinkable boundary condition e (Hawking, Mathur, Jafferis Kolchmeyer, GW-Donnelly)

$Z(\beta) =$



$= \mathrm{tr}_V \, e^{-\beta H}$

$$S_{\mathrm{gen}} = (1 - \beta\partial_\beta) \log Z(\beta) = -\mathrm{tr}_V \, \rho_V \log \rho_V$$



Shrinkable BC can be incorporated into extended TQFT describing 2D gauge theory, Chern Simons theory, topological A model strings (Donnelly,Kim,Jiang, GW). Also applied to 2D JT gravity (Jafferis Kolchmeyer)

This talk: apply the same strategy for AdS3 gravity ~ PSL(2,R) x PSL(2,R) Chern Simons theory

3d gravity as a PSL(2,R) xPSL(2,R) gauge theory

$$I_{\rm EH} = \frac{1}{16\pi G}\int \sqrt{g}(R-\Lambda) = \frac{k}{4\pi}\int A\wedge dA + \frac{2}{3}A\wedge A\wedge A \qquad \text{(Witten)}$$

Dynamical spacetime geometry is encoded into the field space:

$$A = e + \omega \qquad \bar{A} = e - \omega$$

Vielbein (e)

Spin connection (ω)

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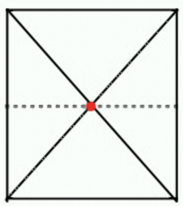
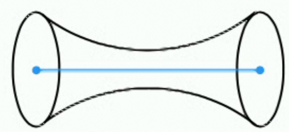
Spin connection

BTZ black holes can be identified with Wilson line in CS theory:

(M, J)



CS theory description



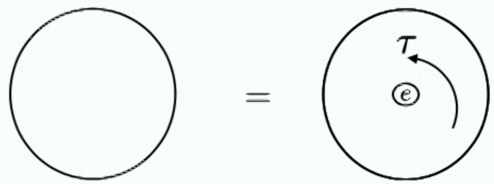
$a = (M, J)$ Representation label

What are the bulk edge modes that will factorize these gravitational Wilson lines?

The naive gauge theory answer - PSL(2,R)xPSL(2,R) edge modes- is NOT correct because the shrinkable BC for gauge theory and gravity is different

Shrinkable boundary condition for gauge theory vs gravity

Shrinkable B.C. for gauge theory is local and generically leads to an infinite EE



$$A_\tau = 0, \quad P\exp\oint A = 1$$

Shrinkable B.C. for gravity is non-local due to the Gauss Bonnet theorem (Jafferis Kolchmeyer), and should give finite EE

$$I_{\text{gravity}} \supset \int_{\text{disk}} \sqrt{g}R = 1 - \int_{\partial\text{disk}} \sqrt{\gamma}K$$

$$\oint_{\partial\text{disk}} \omega = 2\pi$$ —> no conical singularity

To describe gravity, gauge theory has to modified

e.g For 2D JT gravity = PSL(2,R) BF gauge theory+ defect

3d gravity as a topological phase

The question of edge modes in 3d gravity is directly related to a proposal by Jackson-McGough-Verlinde:

3d gravity is a topological phase in which BH entropy = topological entanglement entropy

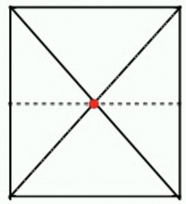
= EE of anyon edge modes

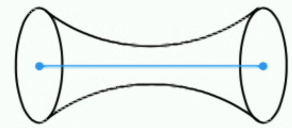
Anyons are collective degrees of freedom describing a topological phase. They are described by a TQFT defined by a modular tensor category Rep(LG) or Rep ($U_q(G)$)

This proposal suggests bulk edge modes are described by gravitational anyons. What is the bulk TQFT and the associated modular tensor category?

BH entropy as topological entanglement entropy?

(M, J)



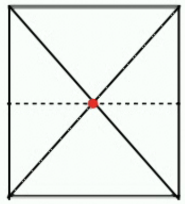


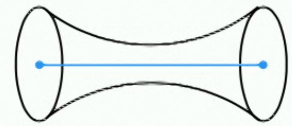
$a = (M, J)$

Verlinde-McGough-Jackson observed:

$$\frac{A(M,J)}{4G} = \log S_0^a \longleftarrow \text{Virasoro modular S-matrix}$$

BH entropy as topological entanglement entropy?





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Puzzle: The edge modes in CS theory with shrinkable BC $A_\tau = 0$ gives:

$$S_{EE} = \frac{\text{``}Area\text{''}}{\epsilon} + \log S_a^0 \longleftarrow S_{\text{edge}} = \text{Topological entanglement entropy}$$

The boundary partition function

We define an effective theory by truncating to the vacuum block in the dual channel

$$Z(\tau,\bar{\tau}) \equiv |\chi_0(-1/\tau)|^2$$

To go back to the original channel , write in terms of modular transformed Virasoro characters
We use Liouville notation (but the boundary theory is NOT the Liouville CFT):

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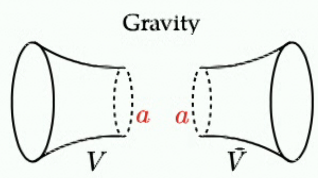
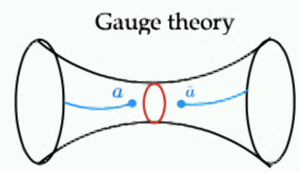
Summary of our work:

Define an effective AdS3 gravity theory describing the high temperature limit of a holographic CFT. (Jackson-McGough-Verlinde, Ghosh-Maxfield-Turiaci, Cotler-Jensen).

We find QES formula = bulk entanglement entropy

$$\frac{A}{4G} = \log \dim a$$





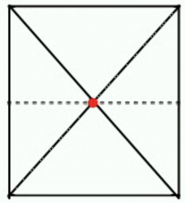
$a \in \mathrm{Rep}(\mathrm{SL}_q^+(2,\mathbb{R}) \otimes \mathrm{SL}_q^+(2,\mathbb{R}))$ $\quad$ $a \in \mathrm{Rep}(\mathrm{PSL}_q(2,\mathbb{R}) \otimes \mathrm{PSL}_q(2,\mathbb{R}))$

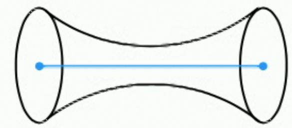
Suggest that the bulk theory is an extended topological quantum field theory associated to the representation category $\mathrm{Rep}(\mathrm{SL}_q^+(2,\mathbb{R}) \otimes \mathrm{SL}_q^+(2,\mathbb{R}))$

The TQFT gives subregion wave functions, bulk factorization maps, and a bulk entanglement entropy that agrees the single interval RT formula

Main message: The gauge theory TEE arises from cutting a CS Wilson line inserted on a fixed background. The gravitational entropy arises from cutting a Wilson line that ``makes up the spacetime itself".

BH entropy as topological entanglement entropy?

(M, J) 

 $a = (M, J)$

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$$S_{EE} = \frac{\text{``}Area\text{''}}{\epsilon} + \log S_a^0 \longleftarrow S_{\text{edge}} = \text{Topological entanglement entropy}$$

But $S_a^0 = 0$ for the Virasoro S-matrix, and BH entropy is finite. So gravity modifies the usual CS computation

Summary of our work:

Define an effective AdS3 gravity theory describing the high temperature limit of a holographic CFT. (Jackson-McGough-Verlinde, Ghosh-Maxfield-Turiaci, Cotler-Jensen).

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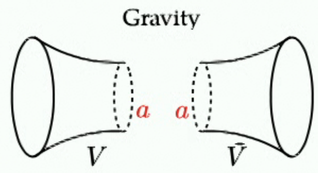

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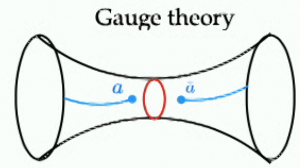

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Outline

Part 1:Definition of 3d gravity

- Boundary partition function and its thermal entropy
- Bulk path integral and shrinkable boundary condition

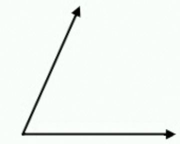
Part 2: Bulk factorization

- $\mathrm{SL}_q^+(2,\mathbb{R})$ and the co product
- Bulk entanglement entropy in 3d gravity

Conclusions

The boundary partition function

$$\tau = \frac{\beta}{l}(i\mu - 1)$$



Consider a modular invariant, holographic CFT (gap + sparse spectrum)

The torus partition function with temperature $\frac{\beta}{l}$ and chemical potential μ can be written in terms of Virasoro characters $\chi_h(\tau)$:

$$Z(\tau,\bar{\tau})_{\text{micro}} = \sum_{h,\bar{h}} M_{h,\bar{h}}\, \chi_h(\tau)\chi_{\bar{h}}(\bar{\tau}) = \sum_{h,\bar{h}} M_{h,\bar{h}}\, \chi_h(-1/\tau)\chi_{\bar{h}}(-1/\bar{\tau})$$

Discrete spectrum

$$\mathfrak{q} \equiv e^{2\pi i\tau}, \quad \chi_0(\tau) = \frac{(1-\mathfrak{q})}{\eta(\tau)}\mathfrak{q}^{-\frac{c-1}{24}}, \qquad \chi_h(\tau) = \frac{1}{\eta(\tau)}\mathfrak{q}^{h-\frac{c-1}{24}}, \qquad \eta(\tau) \equiv \mathfrak{q}^{1/24}\prod_{m=1}^{+\infty}(1-\mathfrak{q}^m)$$

The boundary partition function

$$Z(\tau,\bar{\tau})_{\text{micro}} = \sum_{h,\bar{h}} M_{h,\bar{h}}\, \chi_h(\tau)\chi_{\bar{h}}(\bar{\tau}) = \sum_{h,\bar{h}} M_{h,\bar{h}}\, \chi_h(-1/\tau)\chi_{\bar{h}}(-1/\bar{\tau})$$

In the high temperature limit where $\frac{\beta}{\ell} << \Delta_{\text{gap}} \equiv h + \bar{h}$

$$Z(\tau,\bar{\tau})_{\text{micro}} \sim |\chi_0(-1/\tau)|^2$$

In the **dual channel ,** the vacuum block dominates

The boundary partition function

We define an effective theory by truncating to the vacuum block in the dual channel

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The boundary partition function

Our effective 3d gravity theory is defined by the grand canonical partition function

$$Z(\beta,\mu) \equiv \mathrm{Tr}[e^{-\beta H + i\mu\frac{\beta}{l}J}] = \int_0^{+\infty}\int_0^{+\infty} dp\, d\bar{p}\ S_0^p S_0^{\bar{p}}\ \frac{e^{-\frac{\beta}{\ell}(p^2+\bar{p}^2)}\, e^{i\mu\frac{\beta}{\ell}(p^2-\bar{p}^2)}}{|\eta(\tau)|^2}$$

$(p,\bar{p})$ are Virasoro primaries with energy and angular momentum $H = \dfrac{p^2+\bar{p}^2}{\ell}$ $J = p^2 - \bar{p}^2$

$S_0^p, S_0^{\bar{p}}$ is a density of states for the primaries ~ black holes microstates

$\eta(\tau) \sim$ descendants =boundary gravitons

Black hole entropy

In the ultra high temperature limit, $\beta/\ell << 1$ the partition function is dominated by $p, \bar{p}$

Moreover when $c >> 1 \to b >> 1$ $\quad S_0^p \sim \exp(2\pi b p) = \exp(\sqrt{\frac{cL_0}{6}})$ Cardy density of states

$$S = (1 - \beta\partial_\beta)\log Z(\beta,\mu) \to \log S_0^{p*} S_0^{\bar{p}^*} = \frac{\text{Area}(M^*, J^*)}{4G_N}$$

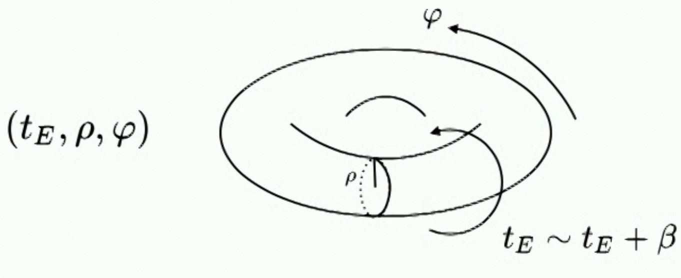
$$M^* l_{AdS} = p^{*2} + \bar{p}^{*2}$$

$$J^* = p^{*2} - \bar{p}^{*2}$$

$c = \frac{3 l_{\text{AdS}}}{2G_N}$ Brown-Hennauex

Explains Verlinde's observation, but does not have manifest entanglement interpretation

The bulk path integral



PSL(2,R)x PSL(2,R) CS theory path integral on the solid torus with AdS3 B.C. is equal to the vacuum character. (Cotler-Jensen, Freidel)

$$Z(\tau, \bar{\tau}) = |\chi_0(-1/\tau)|^2 = \int d[A]d[\bar{A}]e^{-S[A,\bar{A}]}$$

Usual bulk interpretation: Vacuum module=perturbative fluctuations around a single Euclidean BTZ saddle:

$$Z(\tau, \bar{\tau}) = |\exp(\frac{2\pi i}{\tau}\frac{c}{24})\prod_{n=2}^{\infty}\frac{1}{1-\exp(\frac{2\pi i n}{\tau})}|^2 \neq \mathrm{Tr}_{\mathrm{bulk}}\, e^{-\beta H}$$

Euclidean BTZ

Fluctuations about BTZ=boundary gravitons
1-loop exact (Maloney -Witten)

Generalized entropy:

$$S_{\mathrm{gen}} = (1-\beta\partial_\beta)\log Z(\beta) \sim (1-\beta\partial_\beta)e^{-I_{EH}[g_{BTZ}(\beta)]}$$

No stat. mech. interpretation

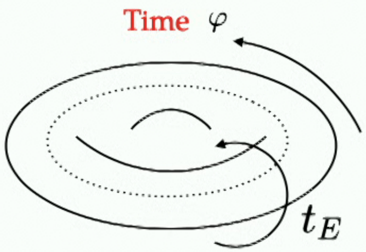
The shrinkable boundary condition

In the boundary theory, there is a natural canonical interpretation in both channels

$$Z(\tau, \bar{\tau}) = |\chi_0(-1/\tau)|^2$$

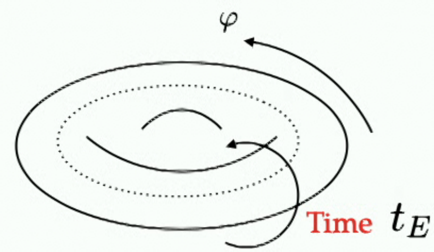
$$Z(\tau, \bar{\tau}) = \sum_{p,\bar{p}} S_0{}^p S_0{}^{\bar{p}} \chi_p(\tau) \chi_{\bar{p}}(\bar{\tau})$$

(t_E, ρ, φ)



semi-classical channel

$t_E \sim t_E + \beta$



quantum statistical channel

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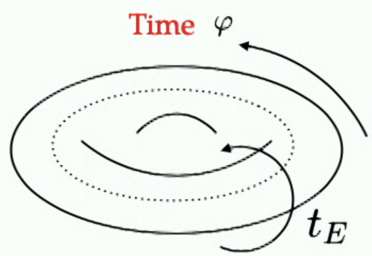
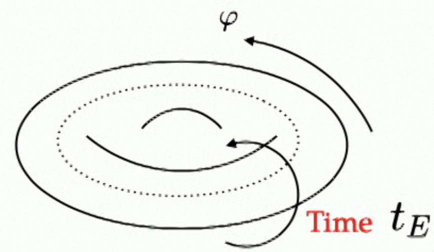


semi-classical channel

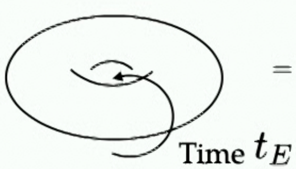
$t_E \sim t_E + \beta$



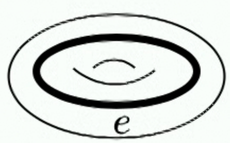
quantum statistical channel

In the bulk, defining a trace in the quantum channel requires shrinkable boundary condition e and bulk edge modes:

$Z(\beta) =$



$= \lim_{\epsilon\to 0}$



$= \mathrm{tr}_V \rho_V^{\beta/2\pi} \longrightarrow$

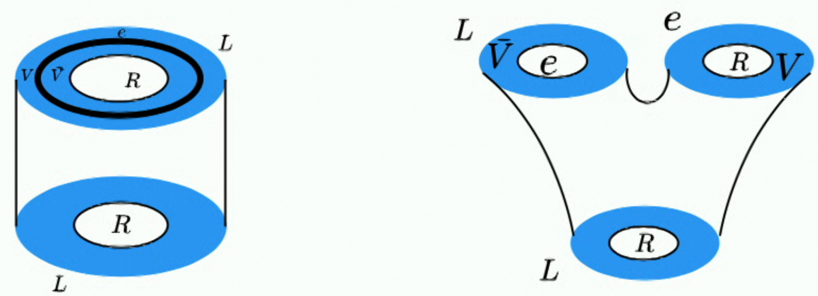
$$S_{\text{gen}} = (1-\beta\partial_\beta)\log Z(\beta) = -\mathrm{tr}_V \rho_V \log \rho_V$$

Bulk edge modes from local holography

The shrinkable boundary condition allows us to define a factorized state:

View e as abstract boundary condition.   We propose an associated factorization map incorporating quantum group edge modes :

$$\mathcal{H}_V \otimes \mathcal{H}_{\bar{V}} \xleftarrow{\ i\ } \mathcal{H}_{\text{bulk}}$$



$e \longrightarrow \mathrm{SL}_q^+(2,\mathbb{R})$ edge modes

$L, R \longrightarrow$ Virasoro edge modes

We identify i with the co product on $\mathrm{SL}_q^+(2,\mathbb{R})$, which satisfies the shrinkablility constraint.

What is $SL_q^+(2,\mathbb{R})$

Definition (Teschner)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad a, b, c, d = \text{operators on } L^2(\mathbb{R}\otimes\mathbb{R}) \text{ with positive spectrum}$$

$$ab = q^{1/2}ba, \quad ac = q^{1/2}ca \quad bd = q^{1/2}db, \quad cd = q^{1/2}dc$$

$$bc = cb, \quad ad - da = (q^{1/2} - q^{-1/2})bc$$

A more useful Characterization

A quantum (semi) group G is defined by the algebra of $L^2(G)$ functions on G. This algebra has a product and co-product:

$$(f_1(g), f_2(g)) \to f_1(g)\cdot f_2(g) \quad \text{Product}$$

$$f(g) \to f(g_1, g_2) = f(g_1 \cdot g_2) \quad \text{Coproduct}$$

A basis for this non commutative algebra is given by products of matrix elements $g_{i_1 j_1} \cdots g_{i_n j_n} \qquad n = 0, 1, \cdots \infty$

Peter Weyl theorem

A group G acts on $L^2(G)$ via the regular representation:

$$f(g) \to f(h_L g h_R^{-1})$$

Peter Weyl Theorem: the regular rep decomposes into representations V_R of G:

$$L^2(G) = \oplus_R V_R \otimes V_{R^*}$$

Basis and completeness

$$R_{ab}(g) \qquad a, b = 1, \ldots \dim R \qquad \delta(g, g') = \sum_{R,a,b} R_{ab}(g) R^*_{ab}(g')$$

This means we can define a symmetry G by Rep(G)

Peter Weyl theorem

A group G acts on $L^2(G)$ via the regular representation:

$$f(g) \to f(h_L g h_R^{-1})$$

Peter Weyl Theorem: the regular rep decomposes as

$$L^2(G) = \int d\mu(R) V_R \otimes V_R^*$$

Plancherel measure

Basis and completeness

$$R_{ab}(g) \qquad \mathrm{tr}_R(1) = \dim R \equiv d\mu(R) \qquad \delta(g - g') = \sum_{ab} \int d\mu(R)\; R_{ab}(g) R^*_{ab}(g')$$

This means we can define a symmetry G by Rep(G)

Peter Weyl theorem for Rep($SL_q^+(2,\mathbb{R})$)

$L^2(SL_q(2,\mathbb{R})^+)$ can be defined via the continuous series reps V_p of $SL_q(2,\mathbb{R})$

$$L^2(SL_q^+(2,\mathbb{R})) = \int_{\oplus_{p\geq 0}} \dim_q(p)\, V_p \otimes V_{p^*} \qquad \dim_q p = \sqrt{2}\sinh(2\pi b p)\sinh(2\pi b^{-1} p) \qquad q = e^{i\pi b^2}$$

The Plancherel measure distinguishes this from the spectral decomposition of $L^2(SL_q(2,\mathbb{R}))$

This equation says V_p should be viewed as a complete set of representations of $SL_q^+(2,\mathbb{R})$

The **representation matrices** $R^p_{ab}(g)$ with the measure $\dim_q(p)$ have been computed by Ip

Why does Rep($SL_q^+(2,\mathbb{R})$) show up in 3d gravity?

Ponsot/Teschner showed that Rep ($SL_q^+(2,\mathbb{R})$) solves the Virasoro modular bootstrap ~ Liouville theory

For $q = e^{i\pi b^2}$, $c = 1 + 6(b + b^{-1})^2$ there is a one to one map (a ``functor")

$$\mathrm{Rep}(SL_q^+(2,\mathbb{R})) \longleftrightarrow \mathrm{Rep}_c(\mathrm{Vir})$$

$$V_p^{SL_q^+(2,\mathbb{R})} \longleftrightarrow V_p^{\mathrm{Vir}}$$

Representation ring $\longleftrightarrow$ Fusion Algebra

$$\dim_q p \longleftrightarrow S_0^p$$

Co-product as a factorization map

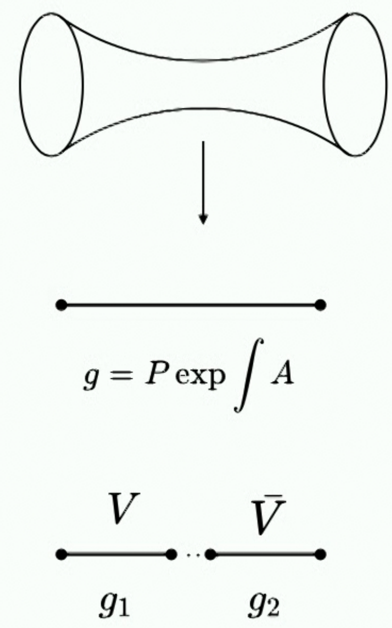
$L^2(G)$ has a natural factorization map given by the co product

$$i : L^2(G) \to L^2(G) \otimes L^2(G)$$

$$R_{ab}(g) \to R_{ab}(g_1 \cdot g_2) = \sum_{c=1}^{\dim R} R_{ac}(g_1) R_{cb}(g_2)$$

The c indices labels edge modes which form a singlet under the diagonal action of G

Each basis state has an entanglement entropy of $\log \dim R$



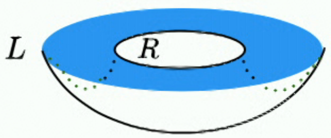
``subregion'' variables

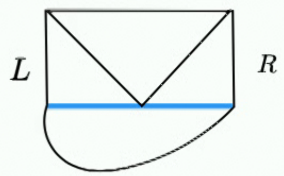
We will identify $L^2(\mathrm{SL}_q^+(2,\mathbb{R}))$ as the zero mode subspace of the black hole Hilbert space

—> each black hole state in the representation $(p, \bar{p})$ has an entanglement entropy

$$S_V = \log \left(\dim_q p \dim_q \bar{p} \right)$$

The two-sided bulk phase space (Hennaux,Woux,Ranjbar)





Asymptotic gauge theory BC—>WZW edge modes

$$A_t - A_\varphi = 0$$

AdS3 asymptotic BC —>Virasoro edge modes

$$A_t = A_\varphi = \begin{pmatrix} 0 & \mathcal{L}(\varphi,\tau) \\ 1 & 0 \end{pmatrix}$$

The phase space of two-sided geometries is parameterized by 4 stress tensor components $\mathcal{L}_{L/R}(t,\varphi), \bar{\mathcal{L}}_{L/R}(t,\varphi)$ (Banados). They are components of the gauge fields $A_\varphi, \bar{A}_\varphi$ at the L/R boundaries. The L/R stress tensor zero modes are linked because of the ER bridge

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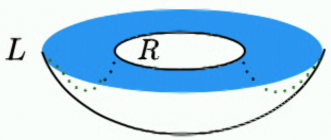

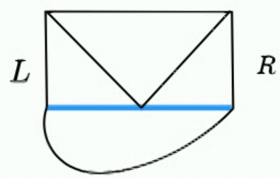

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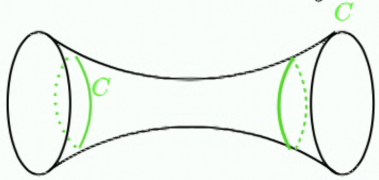
Phase space zero modes = Virasoro Primaries

$$2\cosh(p/2) = \mathrm{tr} P \exp\left(\oint_C d\varphi A_\varphi \right)$$

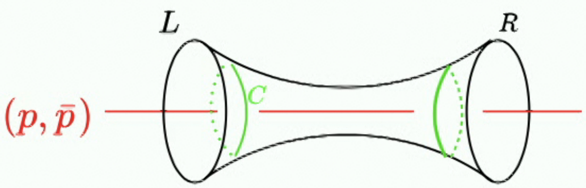


$$(p, \bar{p}) \leftrightarrow (M, J)$$

$$M\ell_{AdS} = p^2 + p^2$$

$$J = p^2 - p^2$$

The two-sided Bulk Hilbert space



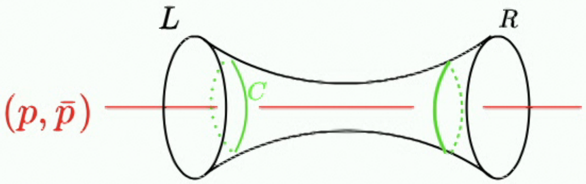
Left-right entanglement creates an ``entanglement'' Wilson line
Czech-Lamprous-Susskind

The bulk Hilbert space $\mathcal{H}_{\text{bulk}}$ is built out of Virasoro Reps $\widetilde{V}_p$

$$\widetilde{V}_p \otimes \widetilde{V}_p^* = \text{span}\{\prod_i L^L_{-n_i} \prod_j \bar{L}^R_{-n_j} |p\rangle\}$$

$$\mathcal{H} = \int d\mu(p)\ \widetilde{V}_p \otimes \widetilde{V}_p^* , \quad \mathcal{H}_{\text{bulk}} = \mathcal{H} \otimes \bar{\mathcal{H}}$$

The two-sided Bulk Hilbert space



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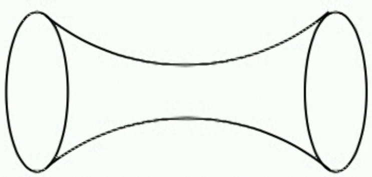
Left-right entanglement creates an ``entanglement" Wilson line

Czech-Lamprous-Susskind

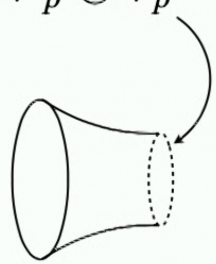
$$\boxed{\mathcal{H} = \int d\mu(p) \; \widetilde{V}_p \otimes \widetilde{V}_p^*}$$

Using the relation between Rep(Vir) and Rep ($\text{SL}_q^+(2, \mathbb{R})$),we define $\mathcal{H}$ as the fusion of one sided Hilbert spaces:

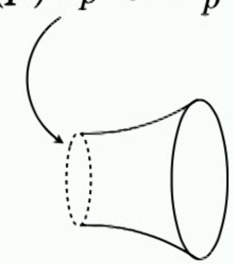
$$\mathcal{H} = \mathcal{H}_L \otimes_{\text{SL}_q^+(2,\mathbb{R})} \mathcal{H}_R$$



$$\mathcal{H}_L = \int d\mu(p) \widetilde{V}_p \otimes V_p^*$$



$$\mathcal{H}_R = \int d\mu(p) V_p \otimes \widetilde{V}_p^*$$

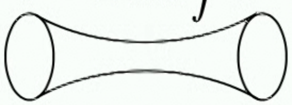


Explicit realization of the zero mode Hilbert space (Drinfeld-Sokolov)

Concretely, we identify the normalized zero mode wavefunction with a representation matrix of element of $\mathrm{SL}_q^+(2,\mathbb{R})$

$$|p\rangle \to |p\ \mathrm{i}_L\ \mathrm{i}_R\rangle$$ Indices are frozen due to AdS B.C.

$g =$ "$P \exp \int A$"



$$\langle g|p,\mathrm{i}_L,\mathrm{i}_R\rangle = \sqrt{\dim_q p}\ R^p_{\mathrm{i}_L,\mathrm{i}_R}(g) \in \mathrm{L}^2(\mathrm{SL}_q^+(2,\mathbb{R}))$$

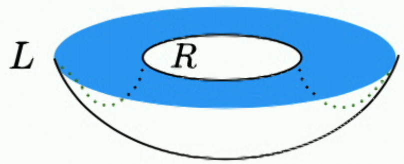
g_L

$$\langle \underset{L}{g}|p\ \mathrm{i}_L\, s\rangle = \sqrt{\dim_q p}\ R^p_{\mathrm{i}_L\, s}(g)$$ subregion Wavefunctions

Co product
$$i : |p\ \mathrm{i}_L \mathrm{i}_R\rangle \to \frac{1}{\sqrt{\dim_q p}} \int_{-\infty}^{\infty} ds |p\, \mathrm{i}_L\, s\rangle \otimes |p\, s\, \mathrm{i}_R\rangle$$

Bulk Entanglement entropy=BH entropy

Bulk Hartle Hawking state :



$$= \quad |\mathrm{HH}_{\beta,\mu}\rangle \otimes \overline{|\mathrm{HH}_{\beta,\mu}\rangle}$$

$\mathrm{SL}_q^+(2,\mathbb{R})$ measure

$$|\mathrm{HH}\rangle_{\beta,\mu} = \int_0^\infty dp \sqrt{\dim_q(p)} e^{-\frac{\beta}{l}p^2(1-i\mu)} \sum_{m_L=m_R} \mathfrak{q}^{N/2} |p\, \mathfrak{i}_L \mathfrak{i}_R\rangle \otimes |m_L m_R\rangle$$

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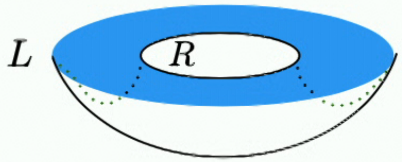

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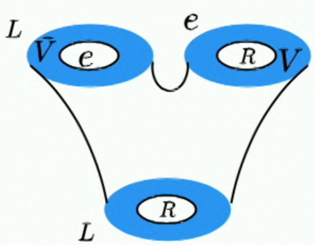
Co-product is a factorization map satisfying shrinkability.

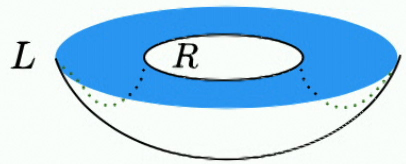


$$|\mathrm{HH}\rangle_{\beta,\mu} \to \int_0^\infty dp \int_{-\infty}^\infty ds \;\; e^{-\frac{\beta}{l}(1-i\mu)} |p \mathfrak{i}_L s\rangle \otimes |p s \mathfrak{i}_R\rangle (\dots)$$

$$\rho_V = \int_0^\infty dp \int_{-\infty}^\infty ds \;\; e^{-\frac{\beta}{l}(1-i\mu)} |p \mathfrak{i}_L s\rangle \otimes \langle p s \mathfrak{i}_R| (\dots)$$

Bulk Entanglement entropy=BH entropy

Bulk Hartle Hawking state :



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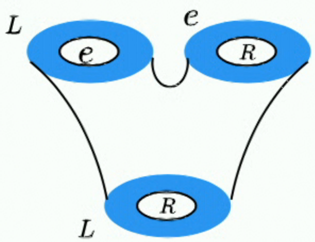

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$$S \equiv -\mathrm{tr}\rho_V \log \rho_V \;=\; S_{\mathrm{gen}} \;\longrightarrow\; \log \dim_q p^* \dim_q \bar{p}^* = \frac{A}{4G}$$

Shrinkability and
1 loop exactness

Semi-classical
limit

Comments on the bulk edge modes

Asymptotic AdS 3 boundary conditions fixes metric at infinity. This gives Virasoro edge modes, which arise from a Drinfeld reduction that freezes the zero mode degrees of freedom

At the bulk entangling surface , there is no reason to fix the metric. So the zero mode in the bulk is liberated and corresponds to the sum over matrix indices.

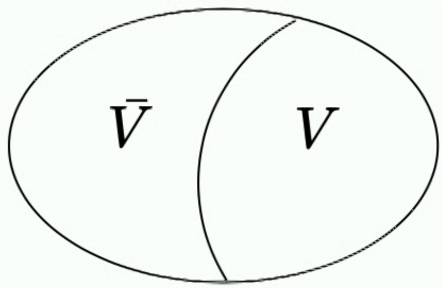
Bulk quantum group edge modes lead to a finite entanglement entropy. In contrast, bulk Virasoro edge modes would give an infinite entanglement entropy in the shrinking limit, due to the tower of descendants at infinite temperature

Rep $(\mathrm{SL}_q^+(2,\mathbb{R}))$ is known to be the modular tensor category associated to Teichmuller TQFT. Recently this TQFT has been used to compute 3d gravity amplitudes using surgery (Eberhardt…et al). Our work suggests that this TQFT also computes bulk entanglement entropy.

RT from bulk entanglement entropy

Consider the holographic CFT calculation of single interval entanglement entropy in the global vacuum state. Near the entangling surface, the (AdS) Rindler observer experiences high temperatures, where our bulk theory is valid.

The situation is completely analogous to the black hole case. We will find that after a conformal map, the single interval reduced density matrix maps to chiral copy of the black hole reduced density matrix on one side.



$$\log \dim_q p^* \to \frac{c}{3} \log \frac{L}{\epsilon}$$