Title: Entropy versus the Action in Causal Set Theory

Speakers: Sumati Surya

Series: Quantum Gravity

Date: March 09, 2023 - 10:30 AM

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Abstract: The Lorentzian path sum in causal set theory(CST) can be defined using the discrete EInstein-Hilbert or Benincasa-Dowker-Glaser(BDG) action. It has been a long standing question in CST whether the path sum is dominated by a class of non-continuum like layered posets -- this would make it much harder to find a dynamically generated continuum approximation -- or whether the BDG action suppresses this contribution. In this talk I will discuss a series of results that show that to leading order in the saddle point approximation, this dominating class of layered posets is strongly suppressed. Moreover this is true for a more general class of actions which include the BDG action. We conclude with some remarks on the interpretation of these results and related open questions.

Zoom link: https://pitp.zoom.us/j/91794153633?pwd=TTFnS0hpQ2s5eFVDM3k5ZDkxcndkUT09

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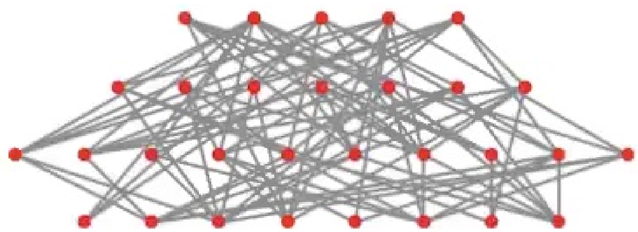
Entropy versus the Action in Causal Set Theory



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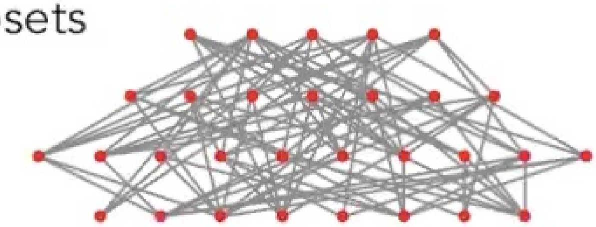
March 9th, 2023
Perimeter Institute

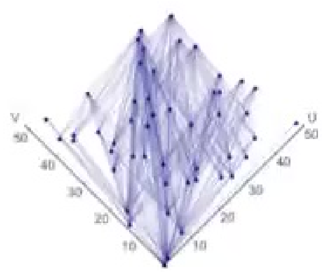
Outline

- The Causal Set Partition Function
- Entropy: The Kleitmann-Rothschild and Dhar posets
- Action: The BDG discrete actions
- Suppression of Entropy:
  - Bilayers *- Loomis & Carlip, 2017*
  - Link action and Layered Posets *- Anand Singh, Mathur & Surya, 2020*
  - Link versus BDG Action on Layered Posets



- Carlip, Carlip & Surya, 2022, ongoing

- Open Questions

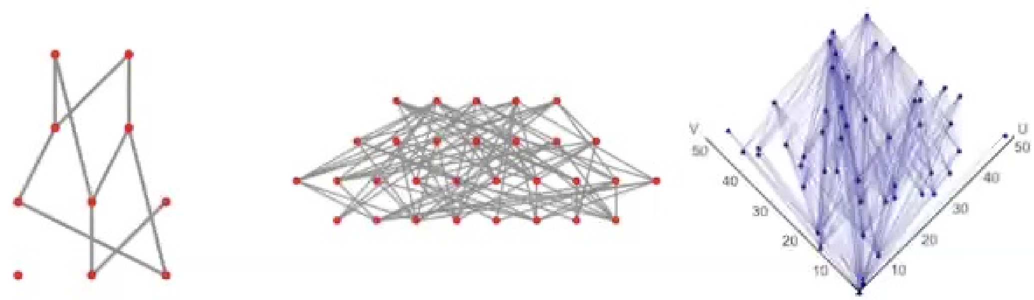


- Work done in collaboration with Anup A. Singh, Abhishek Mathur, Peter Carlip, Steve Carlip,

Causal Set Paradigm

Bombelli, Lee, Meyer and Sorkin, 1987

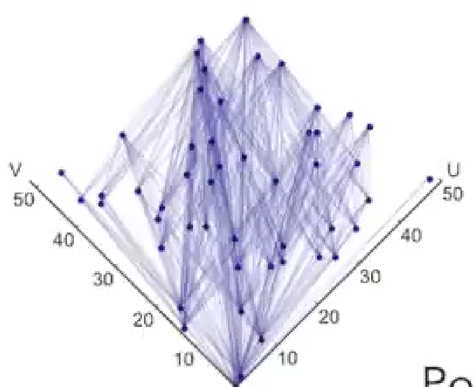
1. Causal Sets are the fine grained structure of spacetime



Acyclic: $x \prec y \Rightarrow y \nprec x$
Transitive:
$x \prec y, y \prec z \Rightarrow x \prec z$
Locally Finite:
$|\mathrm{Fut}(x) \cap \mathrm{Past}(y)| < \infty$

2. Continuum Approximation:

Order + Number ~ Spacetime, $C \sim (M, g)$
* Order ↔ Causal Structure
* Number ↔ Volume



$$P_V(n) = \frac{(\rho V)^n}{n!} e^{-\rho V}$$

$$\langle N \rangle = \rho V, \quad \Delta N = \sqrt{\rho V}$$

Poisson Sprinkling

Causal Set Dynamics

Ω : Sample Space of Causal Sets

Lorentzian Path Sum:

$$Z_n = \sum_{c \in \Omega_n} \exp(iS_{BDG}(c))$$

Ω_n : Fixed cardinality causal sets,

$$Z = \lim_{n \to \infty} Z_n$$

State Sum models:

$$Z_\beta = \sum_{c \in \Omega} \exp(-\beta S_{BDG}(c))$$

$\Omega_{2d,3d}$: Dimensionally restricted causal sets

— *Surya, 2011*
— *Surya & Glaser, 2016*
— *O'Connor, Glaser & Surya 2017*
— *Cunningham & Surya, 2020*

Sequential Growth models:

$$D(c, c'), \quad \mu_{\bar{\nabla}}(c) \in \mathscr{H}_{hist}$$

Generates
Ω_{pf} : Past finite causal sets

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Lorentzian Path Sum

$$Z_n = \sum_{c \in \Omega_n} \exp(iS_{BDG}(c))$$

Ω_n : set of all n element posets

S_{BDG}: Discrete Benincasa-Dowker-Glaser Action

$$Z = \lim_{n \to \infty} Z_n$$

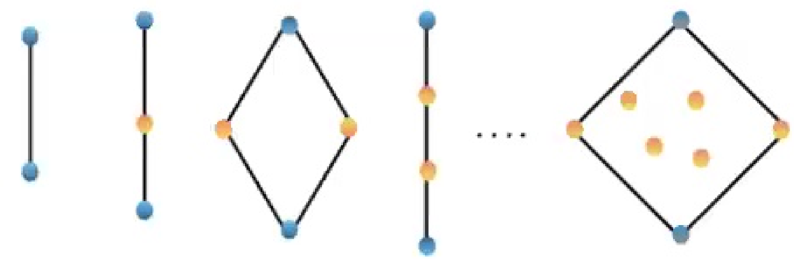
Large n ~ Thermodynamic limit

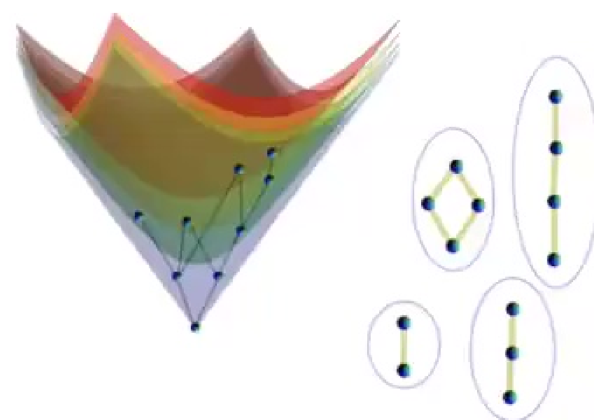
Discrete Einstein-Hilbert Action

— *Benincasa & Dowker, 2010,*
— *Dowker & Glaser, 2012,*
— *Glaser, 2014*

The Benincasa-Dowker-Glaser Action(s)

N_i = # of i-element intervals





$$S^{(d)}_{BDG}(C) = \mu\left(n + \sum_{j=0}^{j_{max}} \lambda_j N_j\right)$$

$$S^{(4)}_{BDG} = \frac{4}{\sqrt{6}}\left(n - N_0 + 9N_1 - 16N_2 + 8N_3\right)$$

$$\lim_{\rho_c \to \infty} \hbar \frac{l_c^2}{l_p^2} \langle S_{BDG} \rangle = S_{EH} + \text{bdry terms}$$

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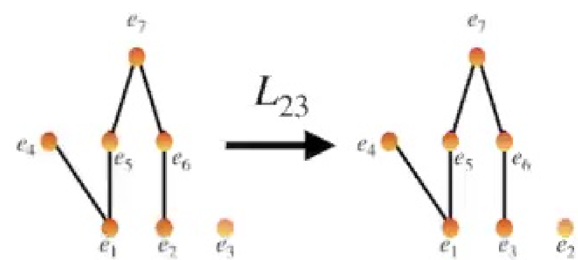
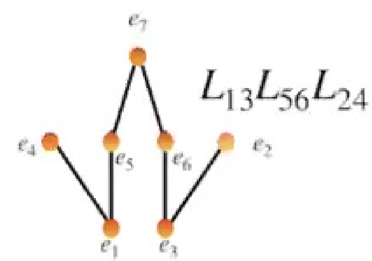
Discrete Covariance = Label Invariance

$\widetilde{\Omega}$: space of labelled causal sets



A relabelling L generates a "gauge" group, $\mathscr{G}_L : \widetilde{\Omega} \to \widetilde{\Omega}$

Orbits of the group action $L \circ \widetilde{c}$, $[\widetilde{c}\,] \sim c$



$\mathscr{G}_L$ has little group $Aut(c)$: Labelling n elements: there are $n!$ ways modulo $Aut(c)$

Swapping labels doesn't produce a new labelled causal set

At worst overcounting of $n! \sim 2^{n \ln n}$ (sub-leading for our analysis)

Typical posets in Ω_n: Entropy

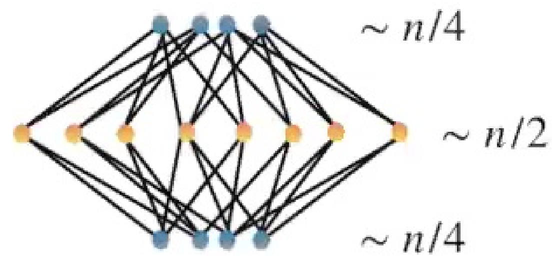
- Kleitman and Rothschild, Trans AMS, 1975

Ω_n : sample space of all n-element causal sets

$$|\Omega_n| \sim 2^{\frac{n^2}{4}+\frac{3n}{2}+o(n)}$$

Typical causal sets are Kleitmann-Rothschild (KR):

- 3 layers: $\mathbb{L}_k, \quad k = 1,2,3, \quad |\mathbb{L}_{1,3}| \sim \frac{n}{4}, |\mathbb{L}_2| \sim \frac{n}{2}$
- elements of $\mathbb{L}_k$ form an **antichain**
- $\forall e \in \mathbb{L}_1, \exists \sim \frac{n}{4}$ no . of $e' \in \mathbb{L}_2$ such that $e \prec_* e'$,
- $\forall e \in \mathbb{L}_3, \exists \sim \frac{n}{4}$ no . of $e' \in \mathbb{L}_2$ such that $e' \prec_* e$
- $\forall e \in \mathbb{L}_1, e' \in \mathbb{L}_3, \quad e' \prec e$

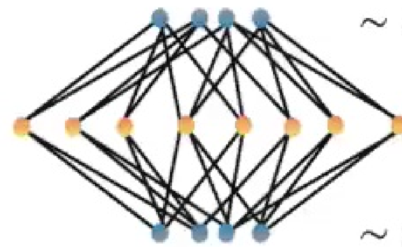


Onset of asymptotic regime $n \sim 100$

$$|\Omega_{KR}| \sim 2^{\frac{n^2}{4}+\frac{3n}{2}+o(n)}$$

- J. Henson, D. Rideout, R. Sorkin and S.Surya, JEM, 2015

A KR poset is not continuum-like

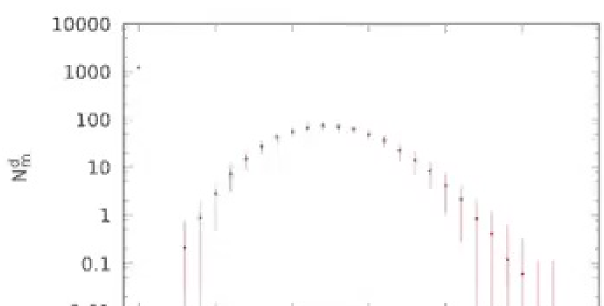


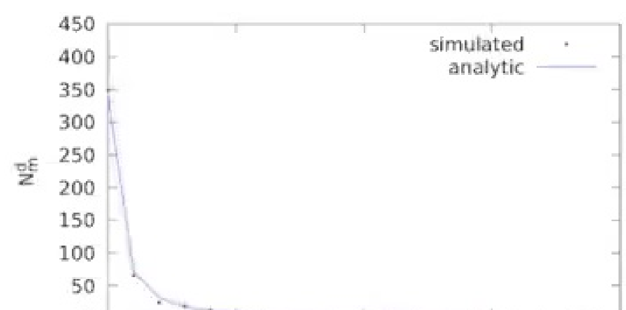
$$\langle R \rangle \sim \frac{3}{16} n^2$$

- Does not arise from a typical Poisson sprinkling into any continuum (M, g)
  - Myrheim-Myer Continuum Dimension is fractional :

$$\frac{\langle R \rangle}{n^2} = \frac{\Gamma(d+1)\Gamma(d/2)}{4\Gamma(3d/2)} \Rightarrow \frac{\Gamma(d_{KR}+1)\Gamma(d_{KR}/2)}{4\Gamma(3d_{KR}/2)} = \frac{3}{16} \Rightarrow d_{KR} \sim 2.5$$

  - Maximal time-like distance $H_{KR} = 3$
  - Interval Abundances are not like the continuum:





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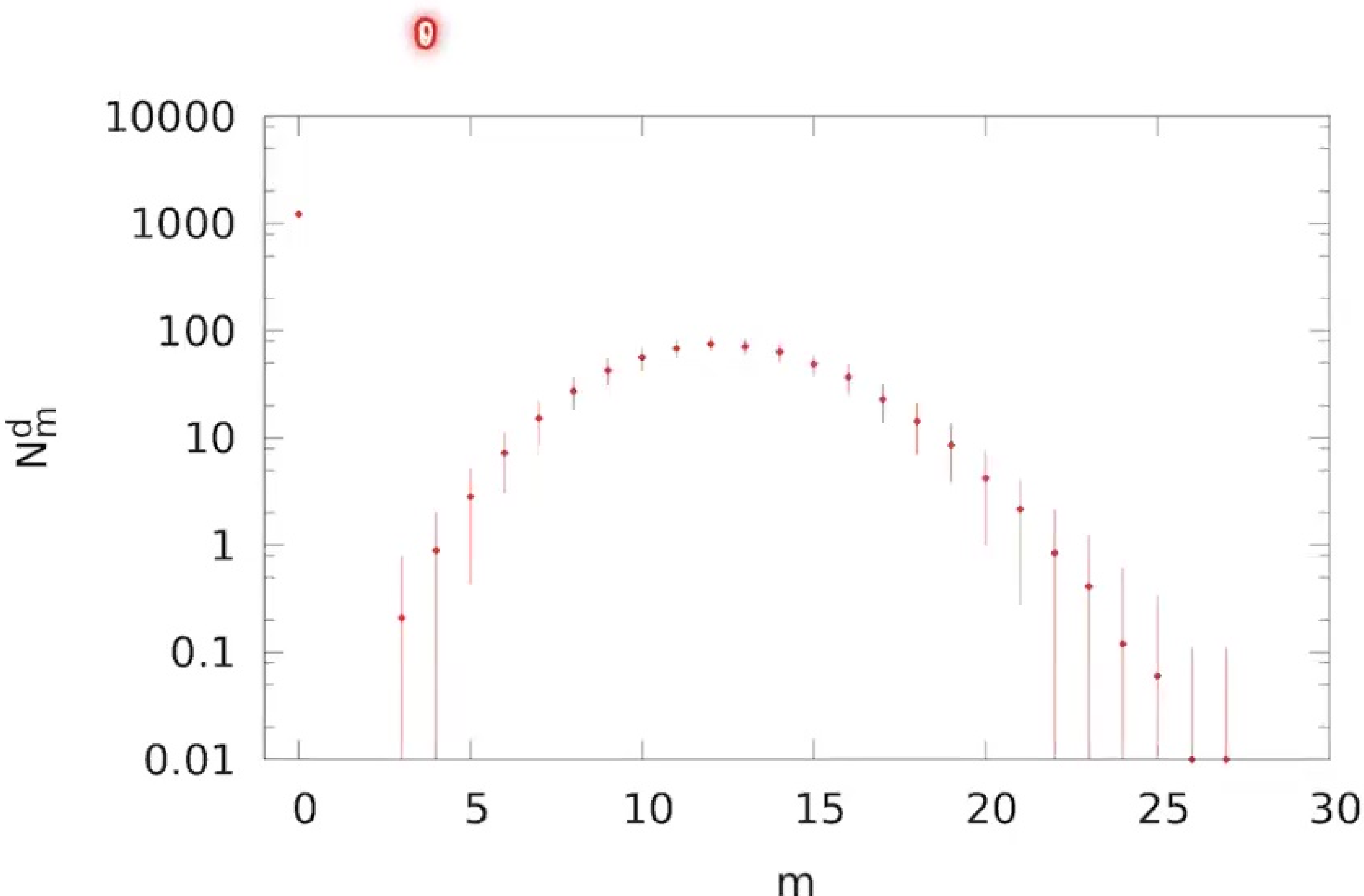
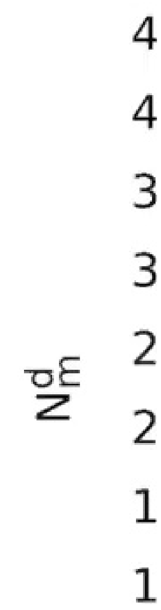




Entropy versus action

- Equivalence class of iso-action causal sets $c \sim c' \Rightarrow S_{BDG}[c] = S_{BDG}[c']$

$$Z_n = \sum_{[c]} |[c]| \exp(iS_{BDG}([c]))$$

Entropy

$$|\Omega_{KR}| \sim 2^{n^2/4+3n/2+o(n)}$$

Action

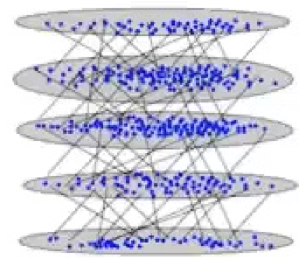
$$S_{BDG}(c_{KR}) \sim n^2$$

Which one wins?

If Entropy wins then KR posets dominate Z_n

$\Rightarrow$ Obstruction to the continuum approximation of the theory

The layered hierarchy



-D. Dhar, JMP, 1978
- Promel, Steger, Taraz 2001

- K-layered poset: $C = \mathbb{L}_1 \sqcup \mathbb{L}_2 \ldots \mathbb{L}_K : e \prec e', e \in \mathbb{L}_k, e' \in \mathbb{L}_{k'} \Rightarrow k < k'$

- $|\Omega_n^{(K)}| \sim 2^{c(d)n^2 + o(n^2)}, \quad c(d) \leq 1/4, \quad d = \text{ordering fraction},$

- Dominant hierarchy: $|\Omega_n^{(3)}| > |\Omega_n^{(2)}| > |\Omega_n^{(4)}| > |\Omega_n^{(5)}| \ldots$

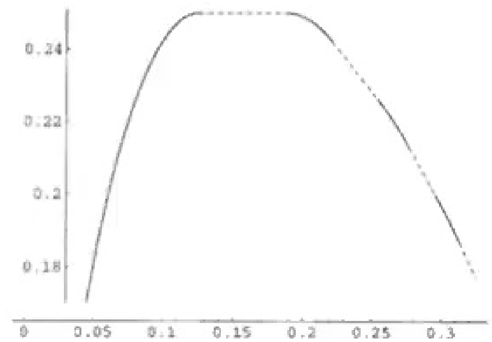


FIG. 5. c(d) in the range [0.05, 0.32].

- Action would have to win over ALL layered posets!

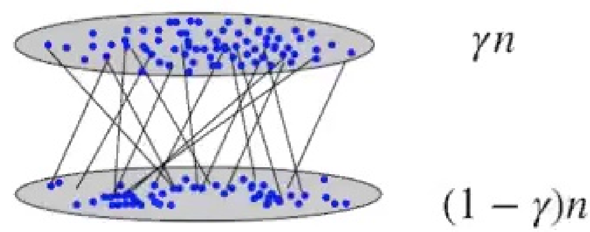
- Is $\Omega_n^{(K)}$ an obstruction to the continuum approximation of the theory??

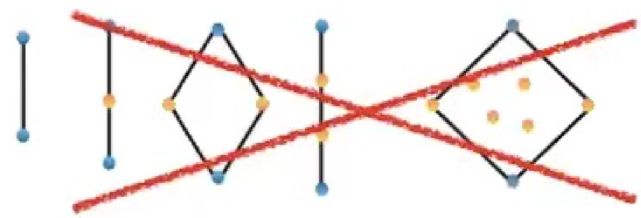
A Small Bi-te: the Bilayer Posets

—*Loomis and Carlip, 2017*

Link Action

- $Z = Z_{bilayer} + Z_{rest}$
- $\mathcal{S}(C) = \mu\left(n + \sum_{j=0}^{j_{max}} \lambda_j N_j\right) = \mu(n + \lambda_0 N_0) = \mathcal{S}_L$
- Iso-action reduces to iso-N_0
- $N_0^{max} = n(n-1)/2 \sim n^2/2,$
- $N_0 = pn^2, \quad p \in [0,1/2]$
- $\mathscr{C}_{p,n}$: set of of bilayers with pn^2 links
- $\mathscr{C}_{p,\gamma,n} \subset \mathscr{C}_{p,n}$: set of bilayers with filling fraction γ and pn^2 links
- $|\mathscr{C}_{p,\gamma,n}| \sim \binom{\gamma(1-\gamma)n^2}{pn^2}$ is maximised by $\gamma = 1/2$





A Small Bi-te: the Bilayer Posets

—Loomis and Carlip, 2017

- $Z = Z_{bilayer} + Z_{rest}$

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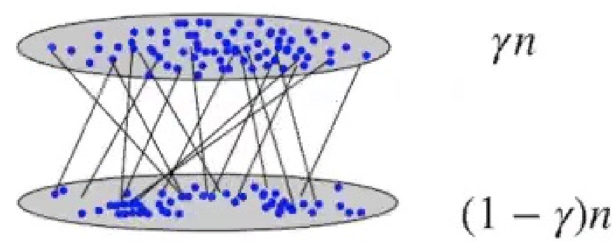

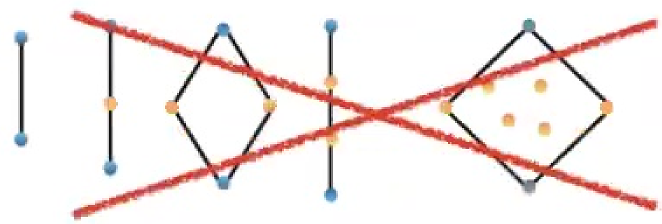

- Iso-action reduces to iso-N_0
- $N_0^{max} = n(n-1)/2 \sim n^2/2,$
- $N_0 = pn^2, \quad p \in [0,1/2]$
- $\mathscr{C}_{p,n}$: set of of bilayers with pn^2 links
- $\mathscr{C}_{p,\gamma,n} \subset \mathscr{C}_{p,n}$: set of bilayers with filling fraction γ and pn^2 links
- $|\mathscr{C}_{p,\gamma,n}| \sim \binom{\gamma(1-\gamma)n^2}{pn^2}$ is maximised by $\gamma = 1/2$

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A Clever Bounding Argument

- $\ln|\mathscr{C}_{p,\gamma,n}| = \left[\gamma(1-\gamma)\ln(\gamma(1-\gamma)) - \frac{1}{2}p\ln\left(\frac{1}{2}p\right) - \left((\gamma(1-\gamma) - \frac{1}{2}p\right)\ln\left((\gamma(1-\gamma) - \frac{1}{2}p\right)\right]n^2 + o(n^2)$
- $|\mathscr{C}_{p,\frac{1}{2},n}| \leq |\mathscr{C}_{p,n}| \leq \sum_{\gamma} |\mathscr{C}_{p,\gamma,n}|$,
- $\sum_{\gamma} |\mathscr{C}_{p,\gamma,n}|$ dominated by $\gamma = \frac{1}{2}$
- $\ln|\mathscr{C}_{p,n}| = \frac{1}{4}h(2p)n^2 + o(n^2),$
- $h(p) = -p\ln p - (1-p)\ln(1-p)$ — **Dhar's Entropy function.**
- $Z_{bilayer}[\mu, \lambda_0] \sim \int_0^{1/2} \mathrm{d}p\, |\mathscr{C}_{p,n}| \exp(iS_L(p)) = e^{i\mu n}\int_0^{1/2} \mathrm{d}p \exp\left[n^2\left(i\mu\lambda_0 p/2 + h(2p)/4\right) + o(n^2)\right]$

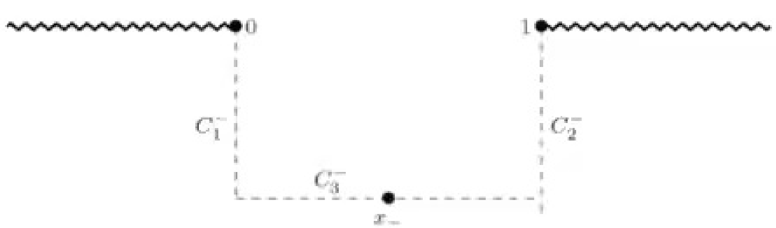
Saddle point contribution

- $Z_{bilayer}[\mu, \lambda_0] \sim e^{i\mu n} \int_0^{1/2} \mathrm{d}p \exp\left[n^2 \left(i\mu\lambda_0 p/2 + h(2p)/4\right) + o(n^2)\right]$

- $\beta = -\frac{\mu\lambda_0}{2}, \quad 2p = x, \; Z_{bilayer}[\mu, \lambda_0] \sim e^{i\mu n} \int_0^1 \mathrm{d}x \exp\left[\left(\frac{1}{4}E(x)\, n^2\right) + o(n^2)\right]$

- Saddle Point: $E'(x_0) = 0 \Rightarrow x_0 = \frac{e^{-i\beta}}{2\cos\beta}$,

- $Z_{bilayer} \sim \sqrt{\frac{\pi}{8}} \frac{e^{i\mu n}}{n\cos\beta} \exp\left(\frac{n^2}{4}[-i\beta + \ln(2\cos\beta)]\right)$



Suppression for:

$$\tan(-\frac{\mu\lambda_0}{2}) > \sqrt{3}$$

$$d = 4, \quad \mu = \left(\frac{l}{l_p}\right)^2 \Rightarrow l \sim 1.452 l_p$$

- Loomis & Carlip, 2017

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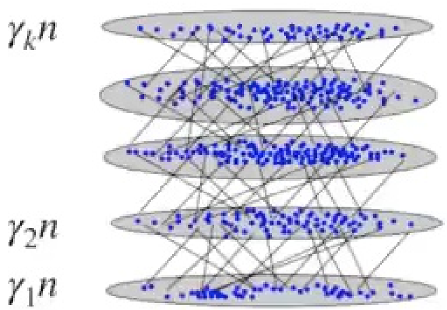
Is it possible to extend this to all layered orders?

Next Bi-te: K-layer orders with Link Action

— *A.Anand Singh, A.Mathur and Surya, 2021*



- $Z = Z_{layered} + Z_{rest}\,, \quad \mathcal{S}_L(C) \sim \mu\left(N + \lambda_0 N_0\right)$
- Filling fraction: $\vec{\gamma} = \{\gamma_1, \gamma_2, \ldots \gamma_K\}, \ \sum_{i=1}^{K} \gamma_i = 1,$
- $N^{\gamma}_{max} = \alpha(\vec{\gamma})n^2, \ \ \alpha(\vec{\gamma}) = \sum_{i=1}^{K-1} \gamma_i \gamma_{i+1}$

Includes Symmetric Bilayers and KR

- $\alpha_{max}(\vec{\gamma}) = 1/4, \ \gamma_x = (1/4 - x, 1/2, 1/4 + x), \quad x \in [-1/4, 1/2, 1/4]$
- $|P_{\vec{\gamma},p,n}| = \begin{pmatrix} \alpha(\vec{\gamma})n^2 \\ pn^2 \end{pmatrix}$
- A similar bounding argument: $|P_{\vec{\gamma}_x,p,n}| \leq |Q^K_{p,n}| \leq K^n |P_{\vec{\gamma}_x,p,n}|$

The first bi-te was good .. back to the bilayer case!

- $Z_{layered} \sim \int_0^{1/2} dp \,|\, Q^K_{p,n} \,|\, \exp(iS_L(p)) = e^{i\mu n} \int_0^1 dp \exp\left[\alpha_{\max}(\vec{\gamma}) n^2 \left(i\mu\lambda_0 p + h(p)\right) + o(n^2)\right]$

$$\tan\left(-\frac{\mu\lambda_0}{2}\right) > \sqrt{3},$$

$$\text{for } d = 4, \quad \mu = \left(\frac{l}{l_p}\right)^2 \Rightarrow l \approx 1.452 l_p$$

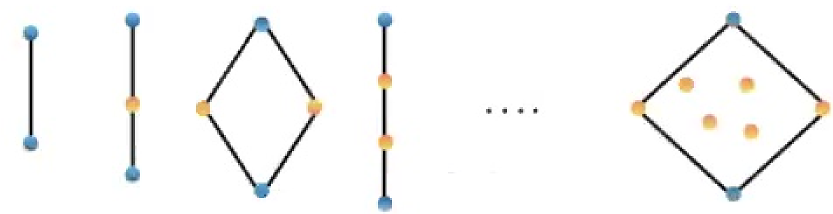
The link action suppresses all K-layer orders for $K << n$

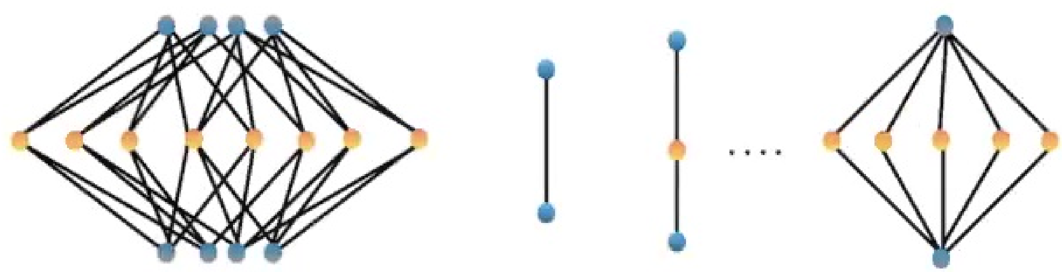
The last bi-te with the BDG Action

— P. Carlip, S. Carlip and S. Surya, 2022
— P. Carlip, S. Carlip and S. Surya, in preparation

$$S^{(d)}_{BDG}(C) = \mu\left(n + \sum_{j=0}^{j_{max}} \lambda_j N_j\right)$$

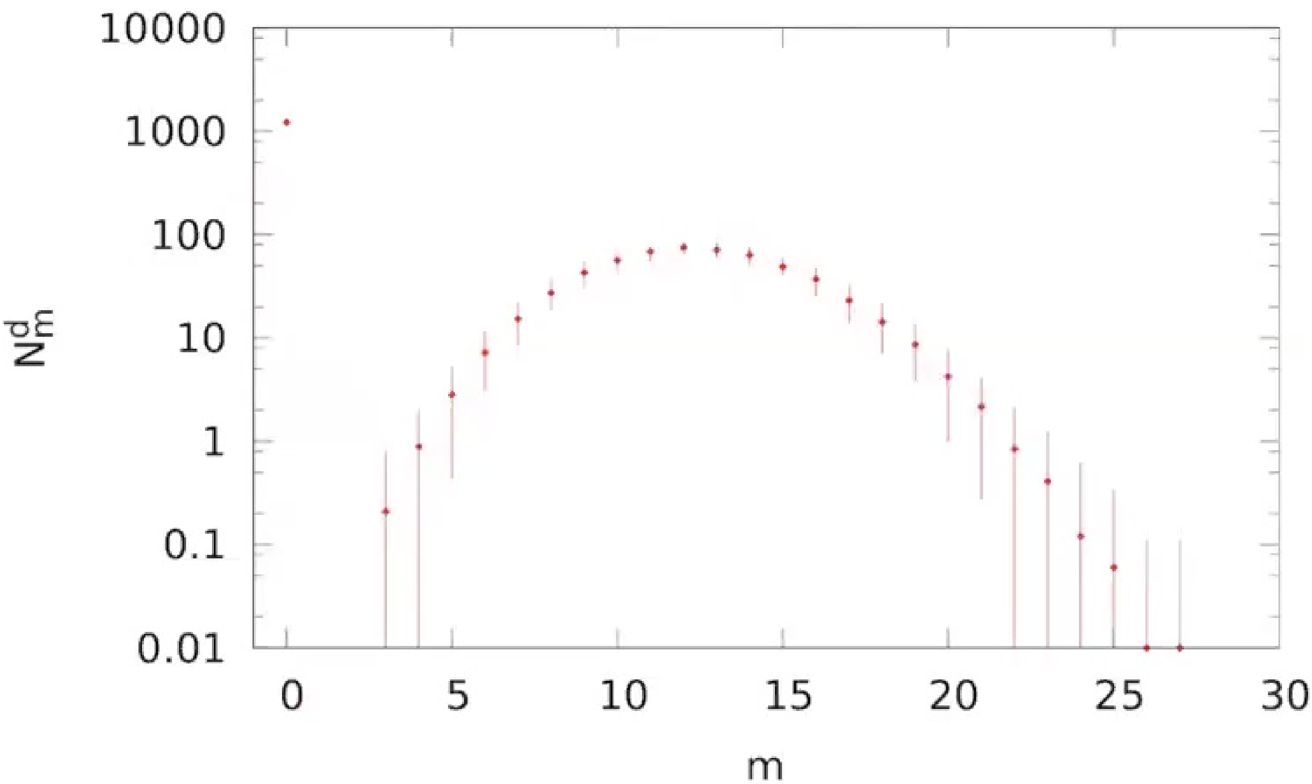
$$S^{(4)}_{BDG} = \frac{4}{\sqrt{6}}\left(n - N_0 + 9N_1 - 16N_2 + 8N_3\right)$$



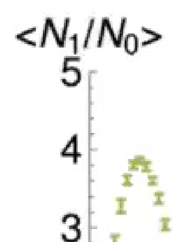


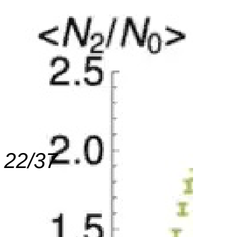
How do we count Iso-action KR posets?

How many N_i are there?



Figur
for n=





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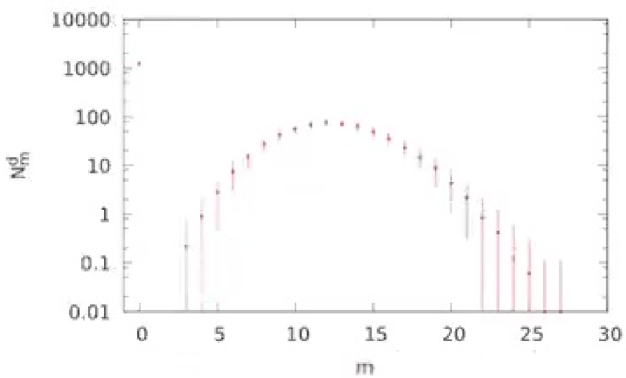


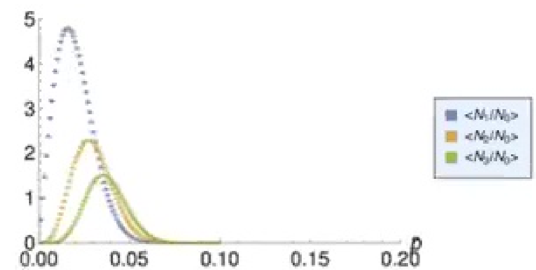


Figure 10: The ratios $\langle N_1/N_0\rangle$, $\langle N_2/N_0\rangle$ and $\langle N_3/N_0\rangle$ vs the linking fraction $\bar{p}$ for n=4000 with $\vec{q}_{kr}$. The average is taken over twenty samples.

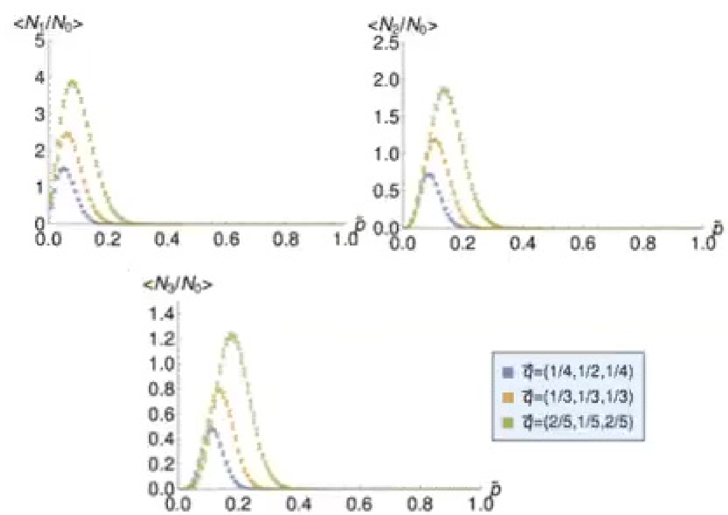
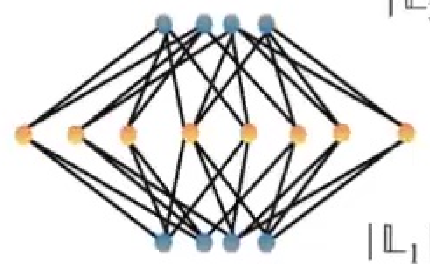


Figure 9: The ratios $\langle N_1/N_0\rangle$, $\langle N_2/N_0\rangle$ and $\langle N_3/N_0\rangle$ vs the linking fraction $\bar{p}$ for $n \approx 400$ for three different types of 3-PQL orders $\vec{q}_{kr}$, $\vec{q} = (1/3, 1/3, 1/3)$ and $\vec{q} = (2/5, 1/5, 2/5)$. The average is taken over twenty samples.

Counting the N_i for KR orders

p : linking fraction from $\mathbb{L}_1$ to $\mathbb{L}_2$

q : linking fraction from $\mathbb{L}_2$ to $\mathbb{L}_3$



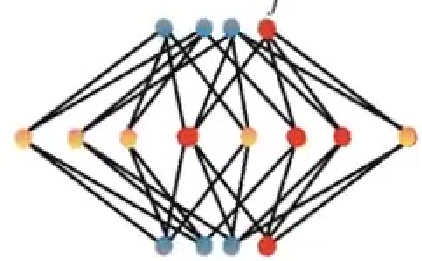
$|\mathbb{L}_3| = \gamma_3 n$

$|\mathbb{L}_2| = \gamma_2 n$

$|\mathbb{L}_1| = \gamma_1 n$

Link matrix: $\mathscr{L} = \begin{pmatrix} 0 & \mathscr{L}^{(12)} & \mathscr{L}^{(13)} \\ 0 & 0 & \mathscr{L}^{(23)} \\ 0 & 0 & 0 \end{pmatrix}$, $\mathscr{L}_{ij} = 1 \Leftrightarrow e_i \prec_L e_j$, else $\mathscr{L}_{ij} = 0$

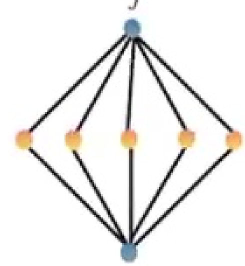
e_j



e_i

$\mathscr{L}^2 = \begin{pmatrix} 0 & 0 & \mathscr{L}^{(12)}\mathscr{L}^{(23)} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\mathscr{L}^2_{ij}$ gives the number of elements between e_i and e_j

e_j



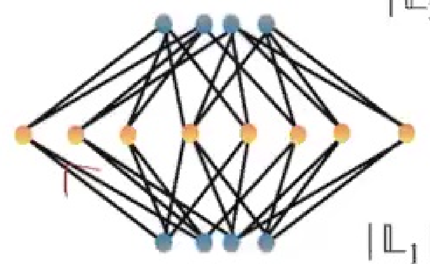
$$\mathscr{L}^2_{ij} = R \Rightarrow |[e_i, e_j]| = R - 1, \qquad \mathscr{L}^2_{ij} = \sum_{k, e_k \in \mathbb{L}_2} \mathscr{L}^{(12)}_{ik} \mathscr{L}^{(23)}_{kj} = R\,.$$

Counting the N_i for KR orders

prob. of a link

p : linking fraction from $\mathbb{L}_1$ to $\mathbb{L}_2$

q : linking fraction from $\mathbb{L}_2$ to $\mathbb{L}_3$



$|\mathbb{L}_3| = \gamma_3 n$

$|\mathbb{L}_2| = \gamma_2 n$

$|\mathbb{L}_1| = \gamma_1 n$

$e_i \prec_L e_j$

$e_j \prec_L e_k$

$e_i \prec e_k$.

Link matrix: $\mathscr{L} = \begin{pmatrix} 0 & \mathscr{L}^{(12)} & \mathscr{L}^{(13)} \\ 0 & 0 & \mathscr{L}^{(23)} \\ 0 & 0 & 0 \end{pmatrix}$, $\mathscr{L}_{ij} = 1 \Leftrightarrow e_i \prec_L e_j$, else $\mathscr{L}_{ij} = 0$

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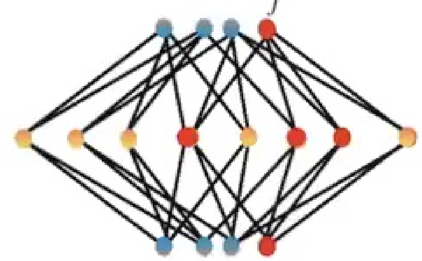

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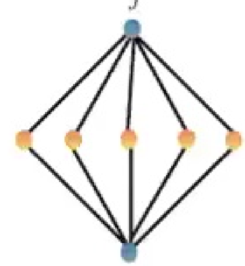

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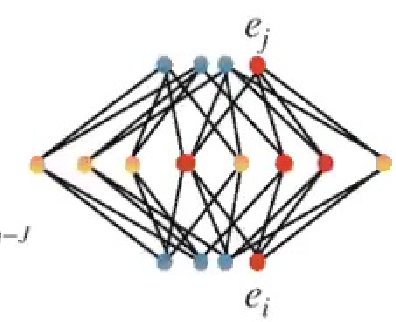


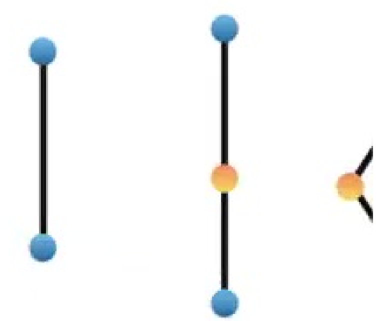
- $Prob(C \text{ has property } P)) = \dfrac{|\Omega_n(P)|}{|\Omega_n|}$
- Probability for $[e_i, e_j]$ to be a J element interval: $Q(J) = \binom{\gamma_2 n}{J}(pq)^J(1-pq)^{\gamma_2 n - J}$
- $P(N_J = R) = \binom{\gamma_1\gamma_3 n^2}{R} Q(J)^R(1-Q(J))^{\gamma_1\gamma_3 n^2 - R}$
- For large n, $P(N_J = c_J\, n) \approx A_J n^J (1-pq)^{\gamma_2 n} \approx 2^{-\gamma_2 c_J n^2 |\ln\alpha| + o(n^2)}$
- $P(N_J = sN_J^{max}) = \binom{\gamma_1\gamma_3 n^2}{s\gamma_1\gamma_3 n^2}\left(A_J\, n^J \alpha^{\gamma_2 n}\right)^{s\gamma_1\gamma_3 n^2} \sim 2^{-s\gamma_1\gamma_2\gamma_3|\ln\alpha| n^3 + o(n^2)}$, where $N_J^{max} = \gamma_1\gamma_3 n^2$
- $P(N_J > \beta_J n) \sim \int_{\beta_J}^{\gamma_1\gamma_3 n} d\beta'\ 2^{-\beta'\gamma_2|\ln\alpha| n^3} \sim 2^{-\beta_J\gamma_2|\ln\alpha| n^2}$
- Therefore the contribution of $N_{J>0}$ to the BDG action is sub-leading
- Analysis extends to all K-layer causal sets

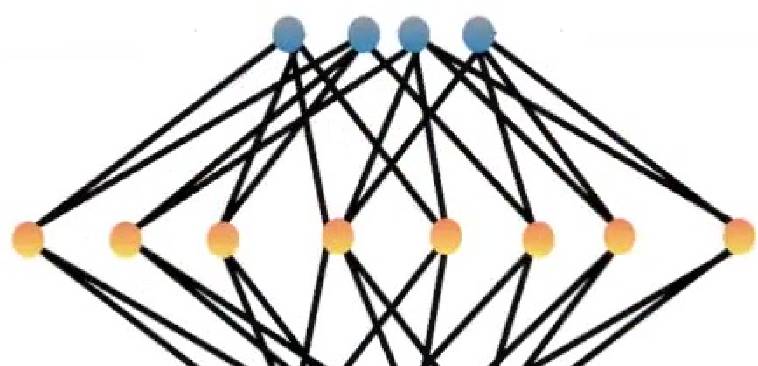
$$\boxed{\mathcal{S}_{BDG}|_{\Omega_n(K)} \sim \mathcal{S}_L}$$

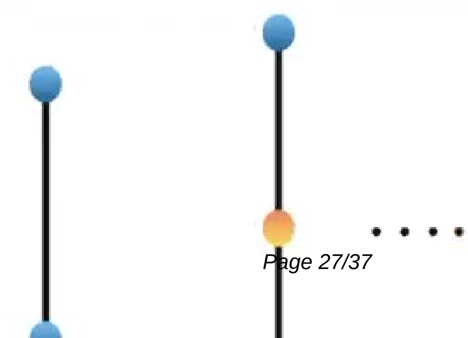
The last bi-te with the BDG Action

$$S^{(d)}_{BDG}(C) = \mu\left(n + \sum_{j=0}^{j_{max}} \lambda_j N_j\right)$$

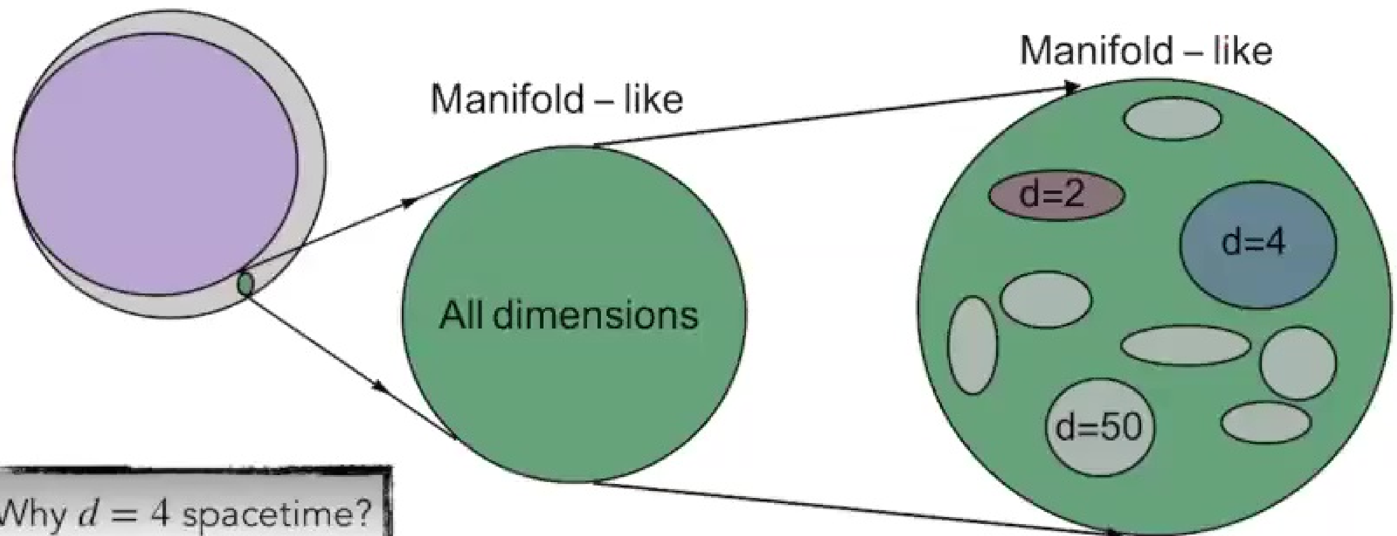
$$S^{(4)}_{BDG} = \frac{4}{\sqrt{6}}\left(n - N_0 + 9N_1 - 16N_2 + 8N_3\right)$$







What ***does*** the Action favour?



Why $d = 4$ spacetime?

Does the d dimensional discrete Einstein Hilbert action suppress $d' \neq d$?

Continuum-like contributions

Kinematic Ensemble $(M, g) \to \{c \in \Omega_n(M)\}$

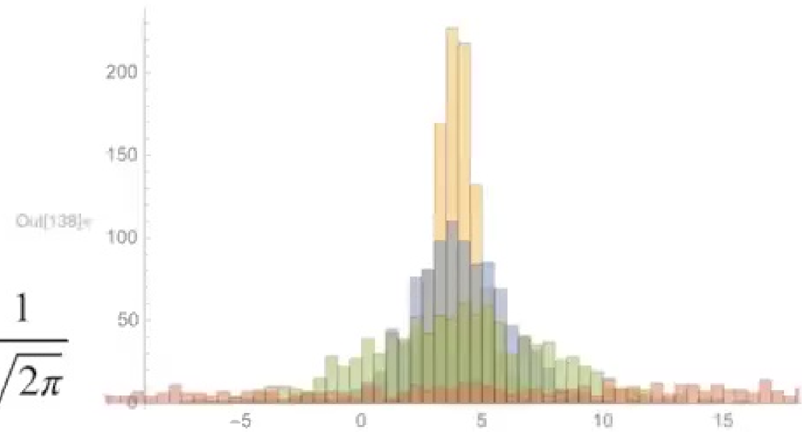
$$S(c) = \langle S(M) \rangle + \Delta S, \quad \lim_{n\to\infty} \Delta S = 0$$

$$Z_M = \sum_{c\in\Omega_n(M)} e^{i\langle S(M)\rangle + \Delta S} \sim e^{i\langle S(M)\rangle} \int_{-\infty}^{\infty} d(\Delta S)\; e^{i\Delta S} F(\Delta S)$$

$$F(\Delta S) = |\,\Omega_n(M)\,| \frac{1}{\pi}\sqrt{\frac{\alpha}{2}}\, e^{-\alpha(\Delta S)^2}$$

$$|\,Z_M\,| \sim |\,\Omega_n(M)\,| \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{4\alpha}}$$

$$\text{IF } \lim_{n\to\infty} \frac{1}{\alpha} \to 0 \Rightarrow \lim_{n\to\infty} \frac{1}{|\,\Omega_n(M)\,|} |\,Z_M\,| = \frac{1}{\sqrt{2\pi}}$$



$\langle N_J \rangle \simeq f(J)\, n^{2-\frac{2}{d}} + o(n^{2-\frac{2}{d}})$ in $\mathbb{M}^d$: $d \neq d_0$ be purely oscillatory

Spacetime-like causal sets may just have a fighting chance..

Open Questions

- How do dimension (d=4) and geometry (de Sitter) emerge dynamically?
- Can we calculate expectation values of observables like the dimension?
- Do these results extend to the double path integral/ Decoherence functional formulation?
- What is a "graviton"? Can it be a non-geometric fluctuation, for example ΔN_J?
- Interpretational questions..

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--:-- Zero KB

A Clever Bounding Argument

- $\ln|\mathscr{C}_{p,\gamma,n}| = \left[\gamma(1-\gamma)\ln(\gamma(1-\gamma)) - \frac{1}{2}p\ln\left(\frac{1}{2}p\right) - \left((\gamma(1-\gamma)-\frac{1}{2}p\right)\ln\left((\gamma(1-\gamma)-\frac{1}{2}p\right)\right]n^2 + o(n^2)$
- $|\mathscr{C}_{p,\frac{1}{2},n}| \leq |\mathscr{C}_{p,n}| \leq \sum_{\gamma} |\mathscr{C}_{p,\gamma,n}|$,
- $\sum_{\gamma} |\mathscr{C}_{p,\gamma,n}|$ dominated by $\gamma = \frac{1}{2}$
- $\ln|\mathscr{C}_{p,n}| = \frac{1}{4}h(2p)n^2 + o(n^2),$
- $h(p) = -p\ln p - (1-p)\ln(1-p)$ — **Dhar's Entropy function.**
- $Z_{bilayer}[\mu, \lambda_0] \sim \int_0^{1/2} \mathrm{d}p\, |\mathscr{C}_{p,n}| \exp(iS_L(p)) = e^{i\mu n}\int_0^{1/2} \mathrm{d}p \exp\left[n^2\left(i\mu\lambda_0 p/2 + h(2p)/4\right) + o(n^2)\right]$

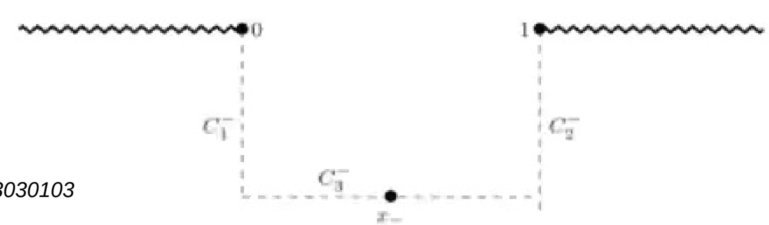
Saddle point contribution

- $Z_{bilayer}[\mu, \lambda_0] \sim e^{i\mu n} \int_0^{1/2} \mathrm{d}p \exp\left[n^2 \left(i\mu\lambda_0 p/2 + h(2p)/4\right) + o(n^2)\right]$

- $\beta = -\frac{\mu\lambda_0}{2}, \quad 2p = x, \; Z_{bilayer}[\mu, \lambda_0] \sim e^{i\mu n} \int_0^1 \mathrm{d}x \exp\left[\left(\frac{1}{4}E(x)\, n^2\right) + o(n^2)\right]$

- Saddle Point: $E'(x_0) = 0 \Rightarrow x_0 = \frac{e^{-i\beta}}{2\cos\beta}$,

- $Z_{bilayer} \sim \sqrt{\frac{\pi}{8}} \frac{e^{i\mu n}}{n\cos\beta} \exp\left(\frac{n^2}{4}[-i\beta + \ln(2\cos\beta)]\right)$



Suppression for:

$$\tan\left(-\frac{\mu\lambda_0}{2}\right) > \sqrt{3}$$

$$d = 4, \quad \mu = \left(\frac{l}{l_p}\right)^2 \Rightarrow l \sim 1.452 l_p$$

A Clever Bounding Argument

- $\ln|\mathscr{C}_{p,\gamma,n}| = \left[\gamma(1-\gamma)\ln(\gamma(1-\gamma)) - \frac{1}{2}p\ln\left(\frac{1}{2}p\right) - \left((\gamma(1-\gamma) - \frac{1}{2}p\right)\ln\left((\gamma(1-\gamma) - \frac{1}{2}p\right)\right]n^2 + o(n^2)$
- $|\mathscr{C}_{p,\frac{1}{2},n}| \le |\mathscr{C}_{p,n}| \le \sum_\gamma |\mathscr{C}_{p,\gamma,n}|$,
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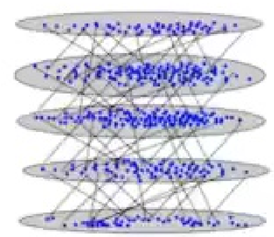
$$e^{i\mu\lambda_0 \frac{p}{2} n^2} \times e^{h(2p)n^2/4}$$

The layered hierarchy



-D. Dhar, JMP, 1978
- Promel, Steger, Taraz 2001

- K-layered poset: $C = \mathbb{L}_1 \sqcup \mathbb{L}_2 \ldots \mathbb{L}_K : e \prec e', e \in \mathbb{L}_k, e' \in \mathbb{L}_{k'} \Rightarrow k < k'$
- $|\Omega_n^{(K)}| \sim 2^{c(d)n^2 + o(n^2)}, \quad c(d) \leq 1/4, \quad d = \text{ordering fraction},$
- Dominant hierarchy: $|\Omega_n^{(3)}| > |\Omega_n^{(2)}| > |\Omega_n^{(4)}| > |\Omega_n^{(5)}| \ldots$
- Action would have to win over ALL layered posets!
- Is $\Omega_n^{(K)}$ an obstruction to the continuum approximation of the theory??

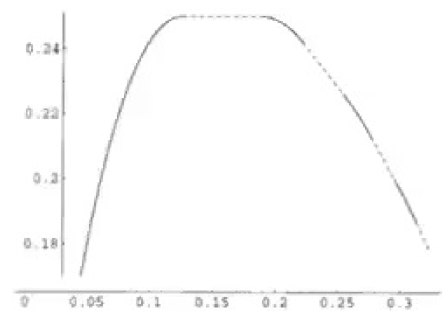


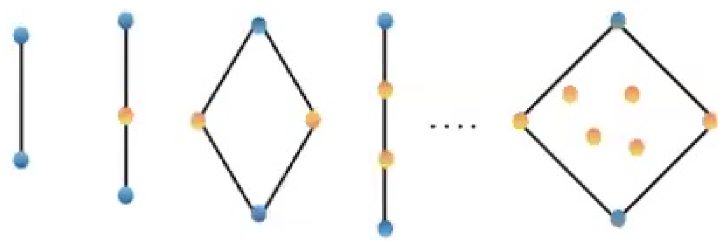
FIG. 5. $c(d)$ in the range [0.05, 0.32].

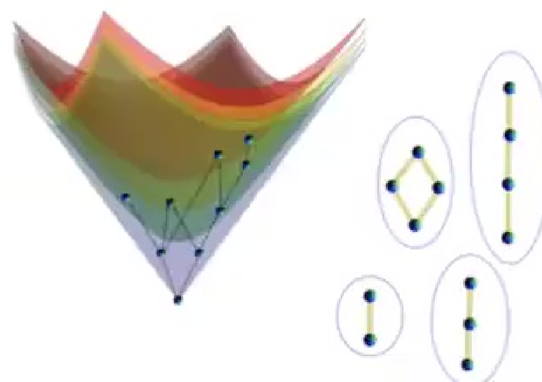
Discrete Einstein-Hilbert Action

— *Benincasa & Dowker, 2010,*
— *Dowker & Glaser, 2012,*
— *Glaser, 2014*

The Benincasa-Dowker-Glaser Action(s)

N_i = # of i-element intervals





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$$\lim_{\rho_c \to \infty} \hbar \frac{l_c^2}{l_p^2} \langle S_{BDG} \rangle = S_{EH} + \text{bdry terms}$$

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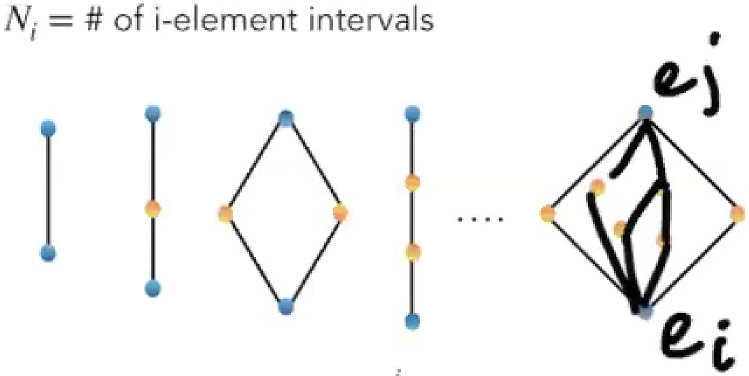

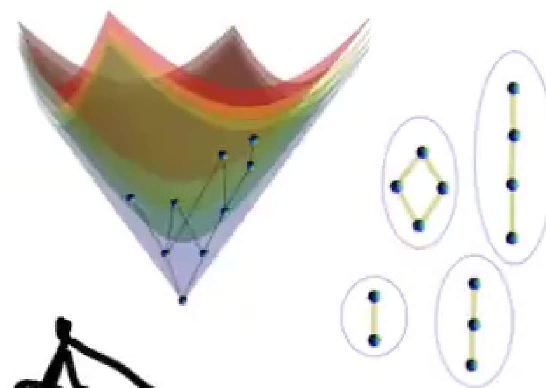

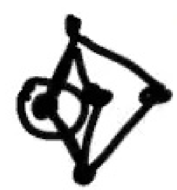

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L, L^2

$L^2_{ij} = R$