Title: Celestial Locality and the Jacobi Identity

Speakers: Adam Ball

Series: Quantum Fields and Strings

Date: March 07, 2023 - 2:00 PM

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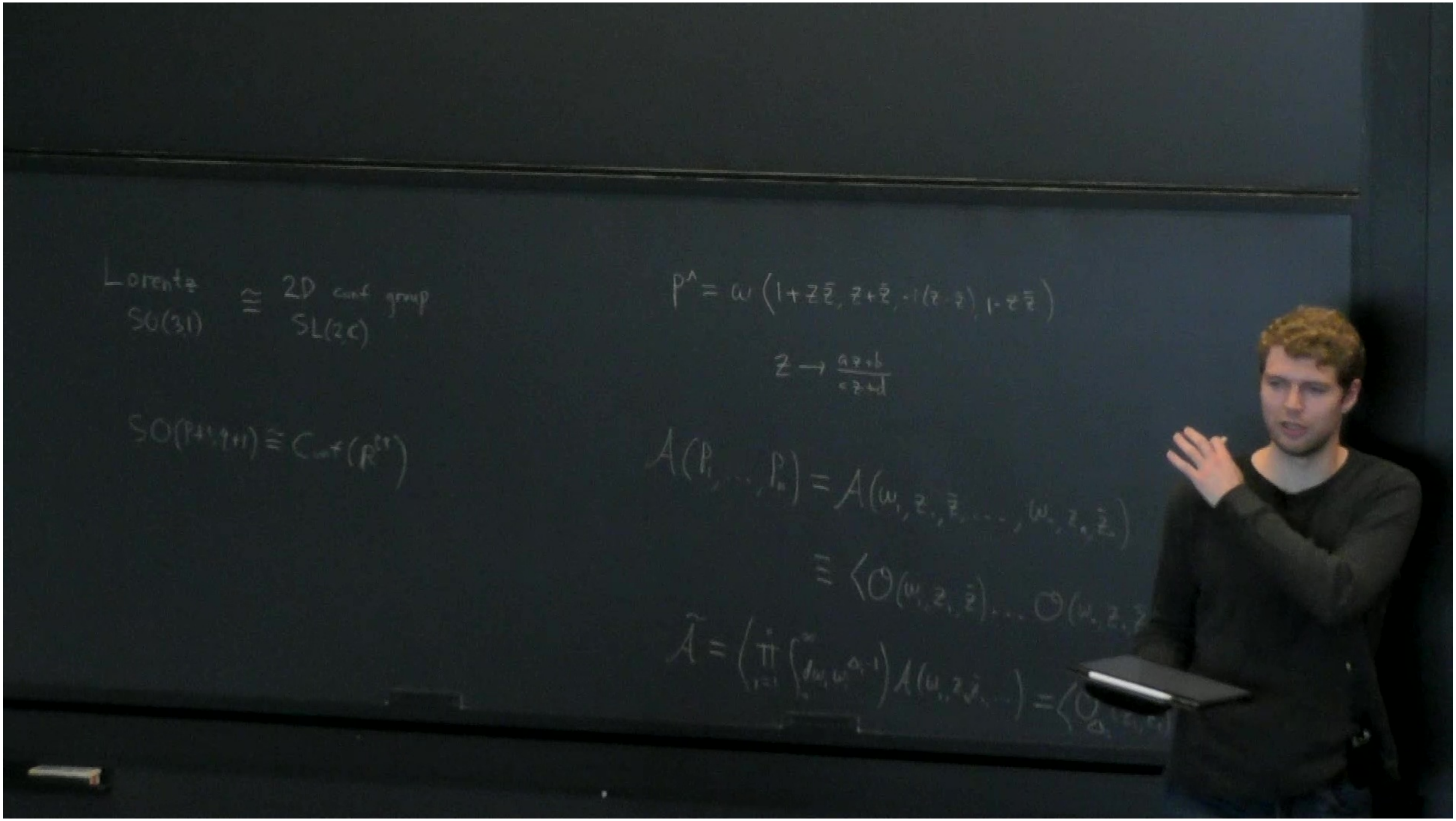
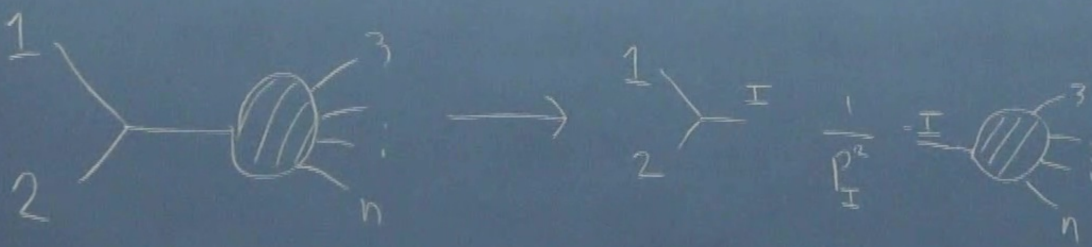
Abstract: In this talk I will show the equivalence of several different tests of the Jacobi identity for celestial currents at tree level, in particular finding a simple, practical condition on hard momentum space 4-point amplitudes in any EFT. Along the way I will clarify the role of the order of soft and collinear limits in obstructing the Jacobi identity for soft insertions and I will argue that, despite their current-algebra-like properties, soft insertions as formulated in this talk cannot be interpreted as local operators in celestial conformal field theory.

Zoom link: https://pitp.zoom.us/j/99642885302?pwd=dWZhbzZUYnBuSlJ3UzBhSHdqdzVKdz09


Lorentz $\cong$ 2D conf group
SO(3,1) SL(2,C)
$SO(p+1,q+1) \cong Conf(\mathbb{R}^{p,q})$
$p^\mu = \omega\left(1+z\bar{z}, z+\bar{z}, -i(z-\bar{z}), 1-z\bar{z}\right)$
$z \to \frac{az+b}{cz+d}$
$A(p_1,\dots,p_n) = A(\omega_1,z_1,\bar{z}_1,\dots,\omega_n,z_n,\bar{z}_n)$
$\equiv \langle O(\omega_1,z_1,\bar{z}_1)\dots O(\omega_n,z_n,\bar{z}_n)$
$\tilde{A} = \left(\prod_{i=1}^{n}\int_0^\infty d\omega_i\,\omega_i^{\Delta_i-1}\right) A(\omega_i,z_i,\bar{z}_i,\dots) = \langle O_{\Delta_1}$

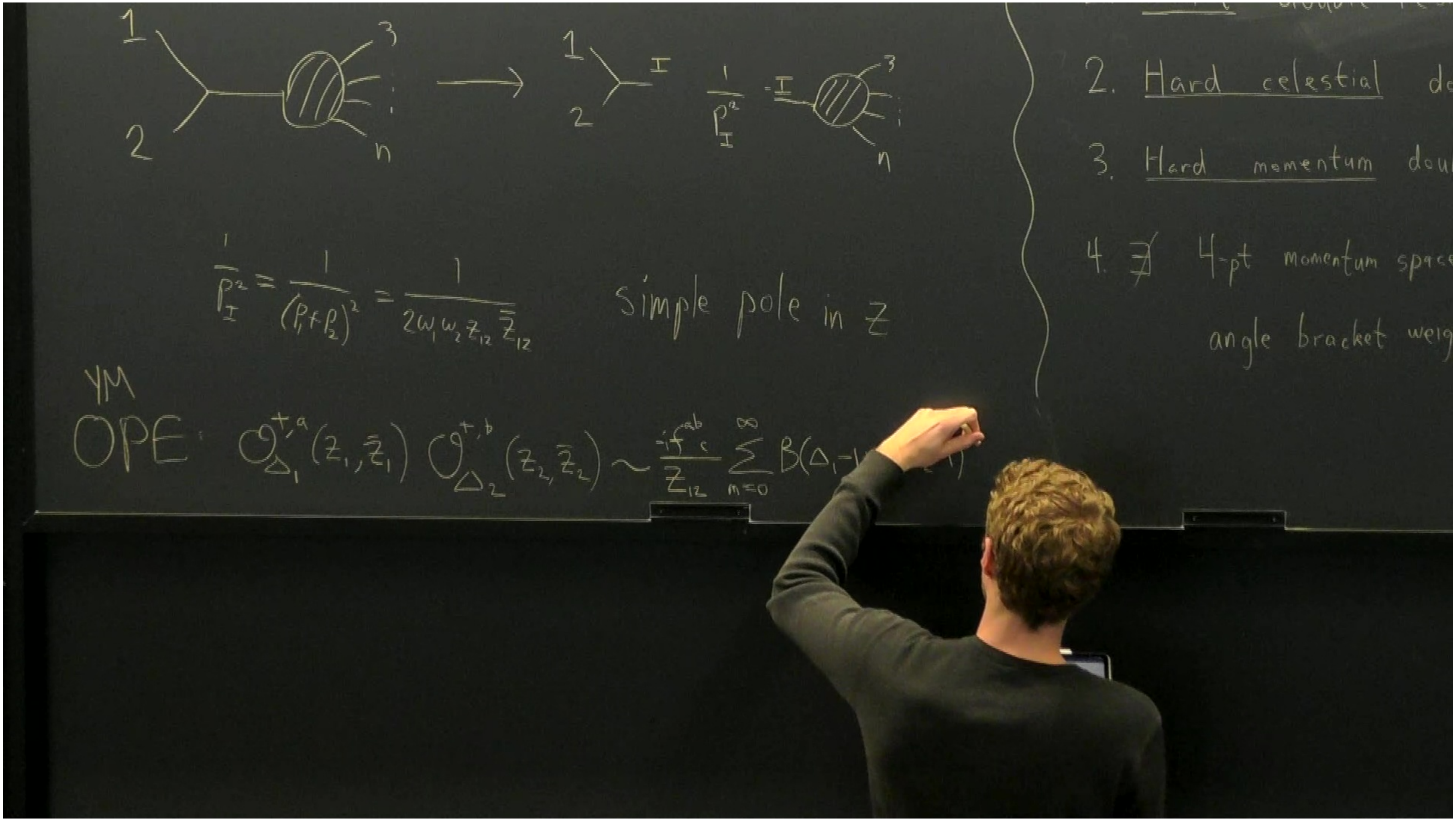
Collinear limit

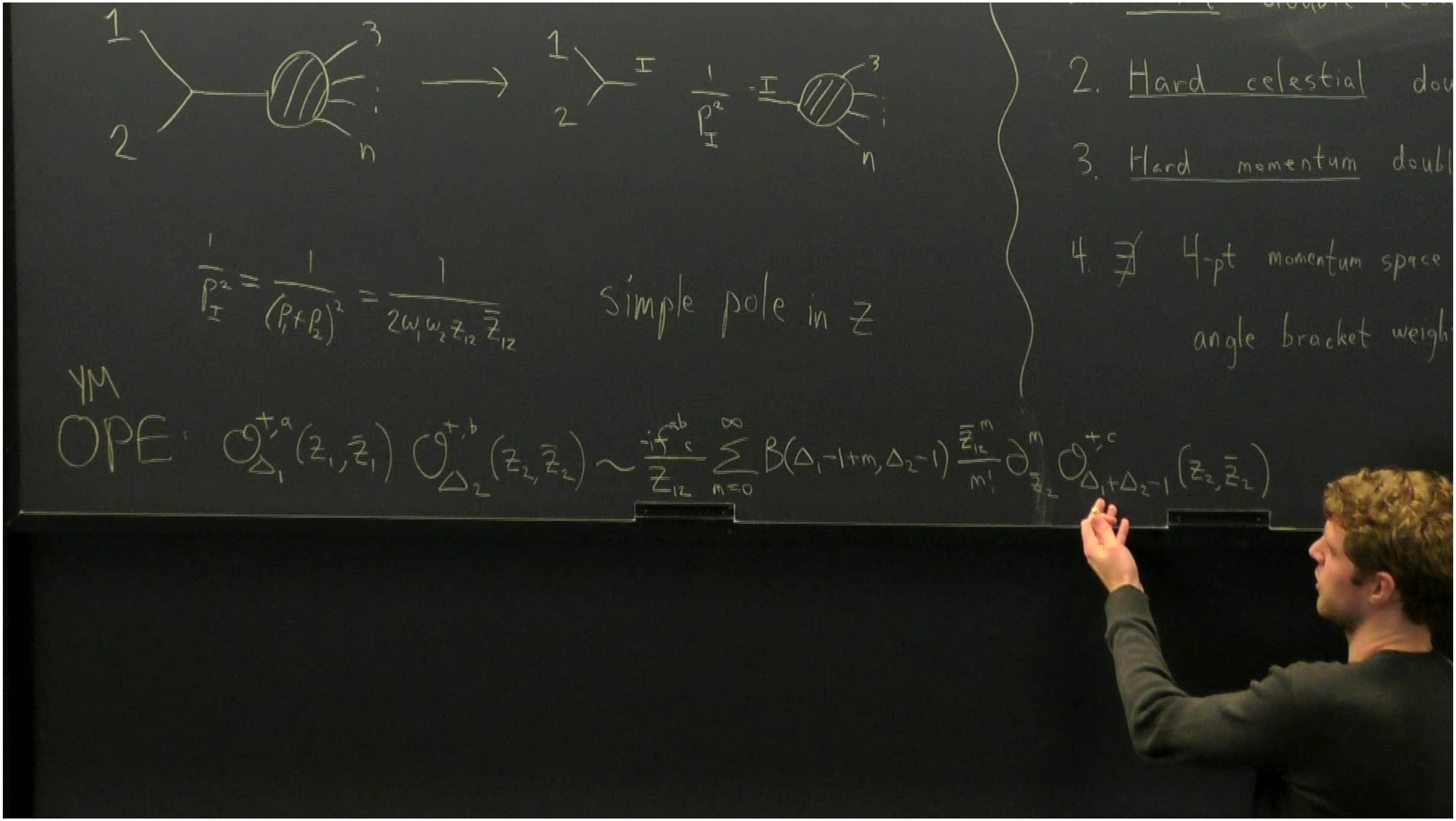


$$\frac{1}{P_I^2} = \frac{1}{(P_1+P_2)^2} = \frac{1}{2\omega_1\omega_2 z_{12}\bar{z}_{12}}$$ simple pole in z

The following are …

1. Soft double re…
2. Hard cele…
3. Hard mome…
4. $\nexists$ 4…


2. Hard celestial
3. Hard momentum
4. ∄ 4-pt momentum
angle bracket
$\frac{1}{p_I^2} = \frac{1}{(p_1+p_2)^2} = \frac{1}{2\omega_1\omega_2 z_{12}\bar{z}_{12}}$
simple pole in z
YM
OPE: $\mathcal{O}^{+,a}_{\Delta_1}(z_1,\bar{z}_1)\,\mathcal{O}^{+,b}_{\Delta_2}(z_2,\bar{z}_2) \sim \frac{-if^{ab}{}_c}{z_{12}} \sum_{m=0}^{\infty} B(\Delta_1-$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2. Hard celestial
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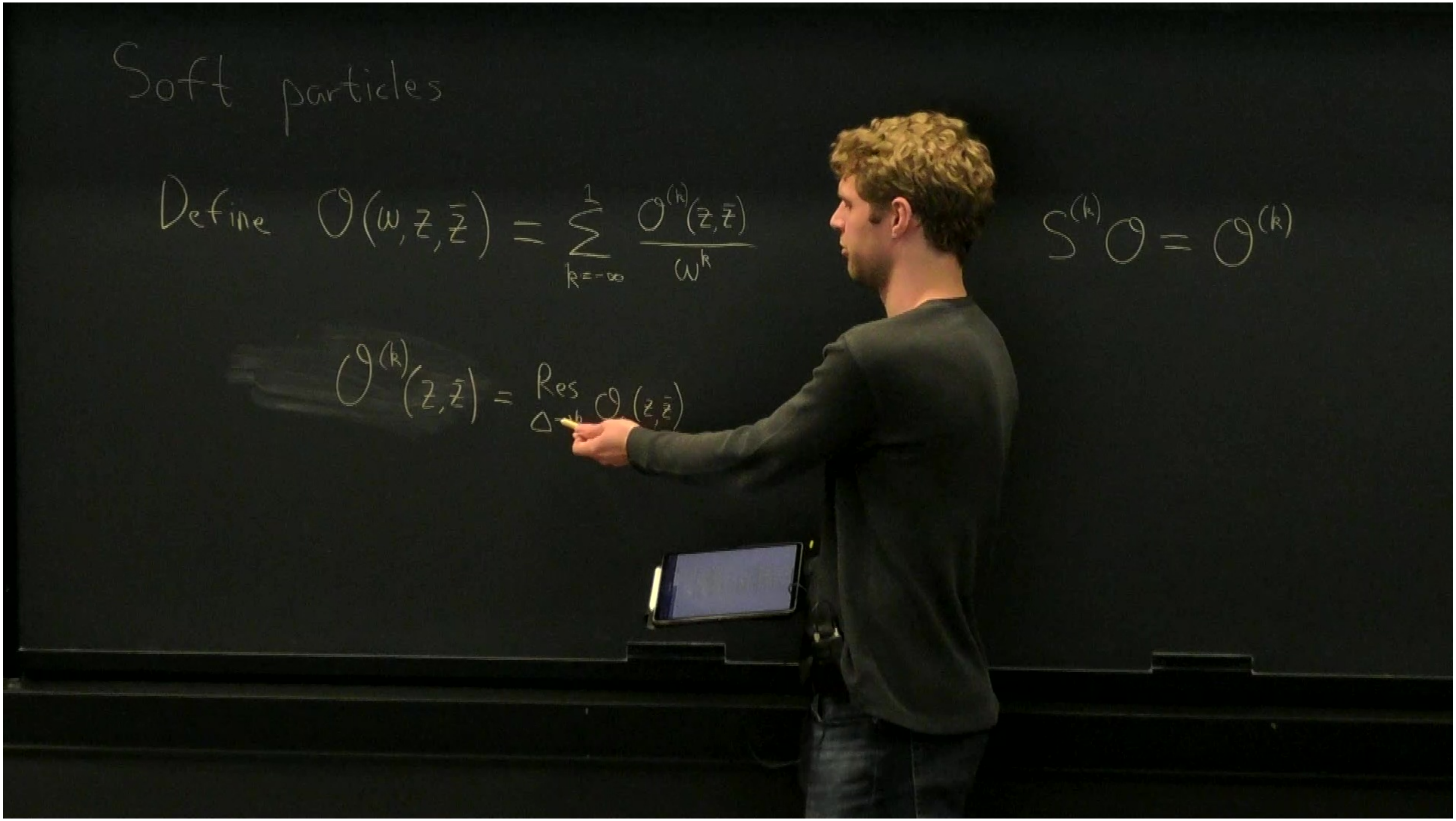
4. $\nexists$ 4-pt momentum space
angle bracket

$$\text{PE}: \ \mathcal{O}^{+,a}_{\Delta_1}(z_1,\bar z_1)\,\mathcal{O}^{+,b}_{\Delta_2}(z_2,\bar z_2) \sim \frac{-if^{ab}{}_c}{z_{12}} \sum_{m=0}^{\infty} B(\Delta_1-1+m, \Delta_2-1)\,\frac{\bar z_{12}^{\,m}}{m!}\,\partial^m_{\bar z_2}\,\mathcal{O}^{+,c}_{\Delta_1+\Delta_2-1}(z_2,\bar z_2)$$

Soft particles

Define $\mathcal{O}(\omega, z, \bar z) = \sum_{k=-\infty}^{1} \frac{\mathcal{O}^{(k)}(z,\bar z)}{\omega^k}$ also $S^{(k)}\mathcal{O} = \mathcal{O}^{(k)}$

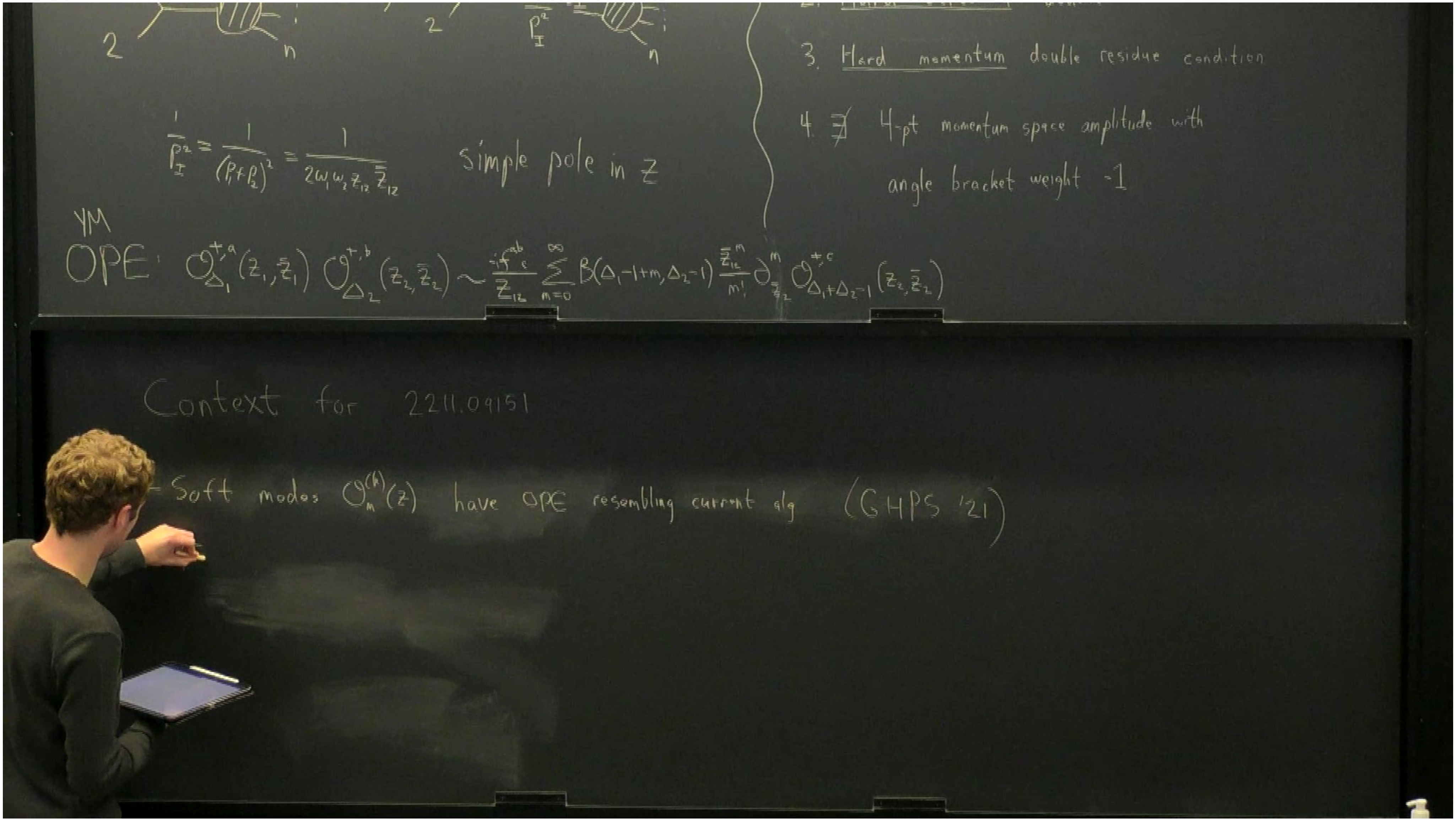

Soft particles
Define $\mathcal{O}(\omega, z, \bar{z}) = \sum_{k=-\infty}^{1} \frac{\mathcal{O}^{(k)}(z,\bar{z})}{\omega^k}$
$S^{(k)}\mathcal{O} = \mathcal{O}^{(k)}$
$\mathcal{O}^{(k)}(z,\bar{z}) = \mathrm{Res}\ \mathcal{O}(z,\bar{z})$

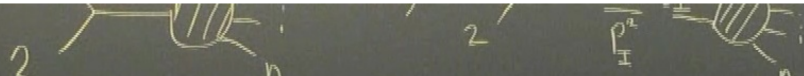
Soft particles

Define $\mathcal{O}(\omega, z, \bar{z}) = \sum_{k=-\infty}^{1} \frac{\mathcal{O}^{(k)}(z,\bar{z})}{\omega^k}$ also $S^{(k)}\mathcal{O} = \mathcal{O}^{(k)}$

$$\mathcal{O}^{(k)}(z,\bar{z}) = \operatorname*{Res}_{\Delta \to k} \mathcal{O}_\Delta(z,\bar{z})$$

$$\mathcal{O}^{(k)}(z,\bar{z}) = \sum_m \frac{\mathcal{O}^{(k)}_m(z)}{\bar{z}^{m+\bar{h}}}$$


3. Hard momentum double residue condition
4. ∄ 4-pt momentum space amplitude with angle bracket weight -1
$\frac{1}{p_I^2} = \frac{1}{(p_1+p_2)^2} = \frac{1}{2\omega_1\omega_2 z_{12}\bar{z}_{12}}$ simple pole in z
YM
OPE: $O^{+,a}_{\Delta_1}(z_1,\bar{z}_1)\, O^{+,b}_{\Delta_2}(z_2,\bar{z}_2) \sim \frac{-if^{ab}{}_c}{z_{12}} \sum_{m=0}^{\infty} B(\Delta_1-1+m,\Delta_2-1)\frac{\bar{z}_{12}^m}{m!}\partial_{\bar{z}_2}^m O^{+,c}_{\Delta_1+\Delta_2-1}(z_2,\bar{z}_2)$
Context for 2211.09151
- Soft modes $O^{(k)}_m(z)$ have OPE resembling current alg (GHPS '21)



$$\frac{1}{p_I^2} = \frac{1}{(p_1+p_2)^2} = \frac{1}{2\omega_1\omega_2 z_{12}\bar{z}_{12}}$$ simple pole in z

3. Hard momentum double residue condition

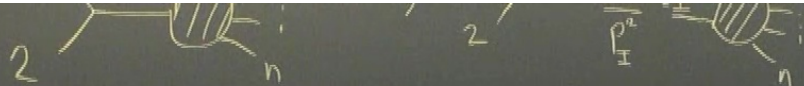
4. $\nexists$ 4-pt momentum space amplitude with angle bracket weight -1

YM

OPE: $$\mathcal{O}^{+,a}_{\Delta_1}(z_1,\bar{z}_1)\,\mathcal{O}^{+,b}_{\Delta_2}(z_2,\bar{z}_2) \sim \frac{-if^{ab}{}_{c}}{z_{12}}\sum_{m=0}^{\infty} B(\Delta_1-1+m,\Delta_2-1)\frac{\bar{z}_{12}^{m}}{m!}\partial_{\bar{z}_2}^{m}\,\mathcal{O}^{+,c}_{\Delta_1+\Delta_2-1}(z_2,\bar{z}_2)$$

Context for 2211.09151

- Soft modes $\mathcal{O}^{(k)}_m(z)$ have OPE resembling current alg. (GHPS '21)
- For pos-helicity gravitons, it's a $w_{1+\infty}$-wedge current algebra



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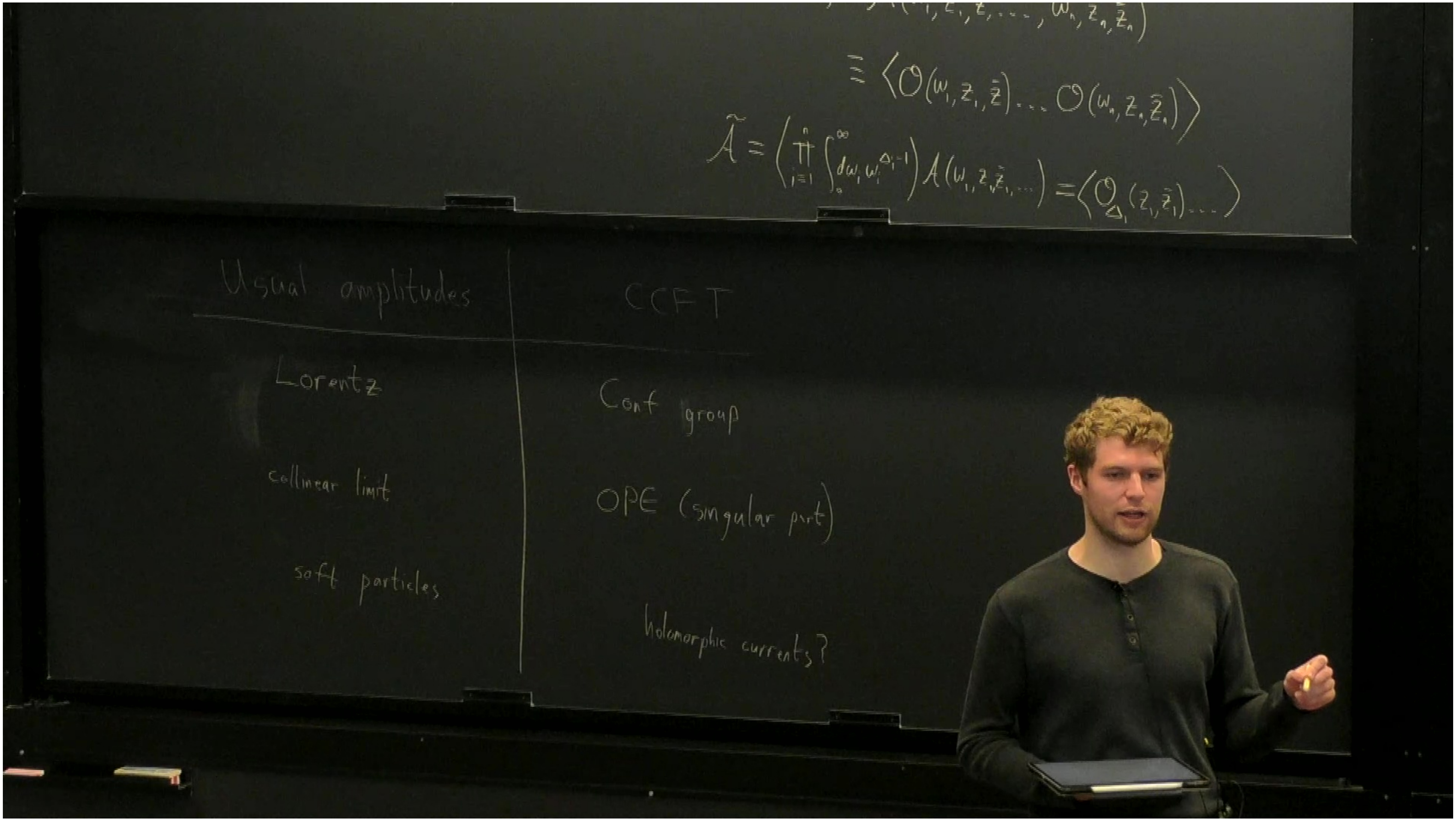
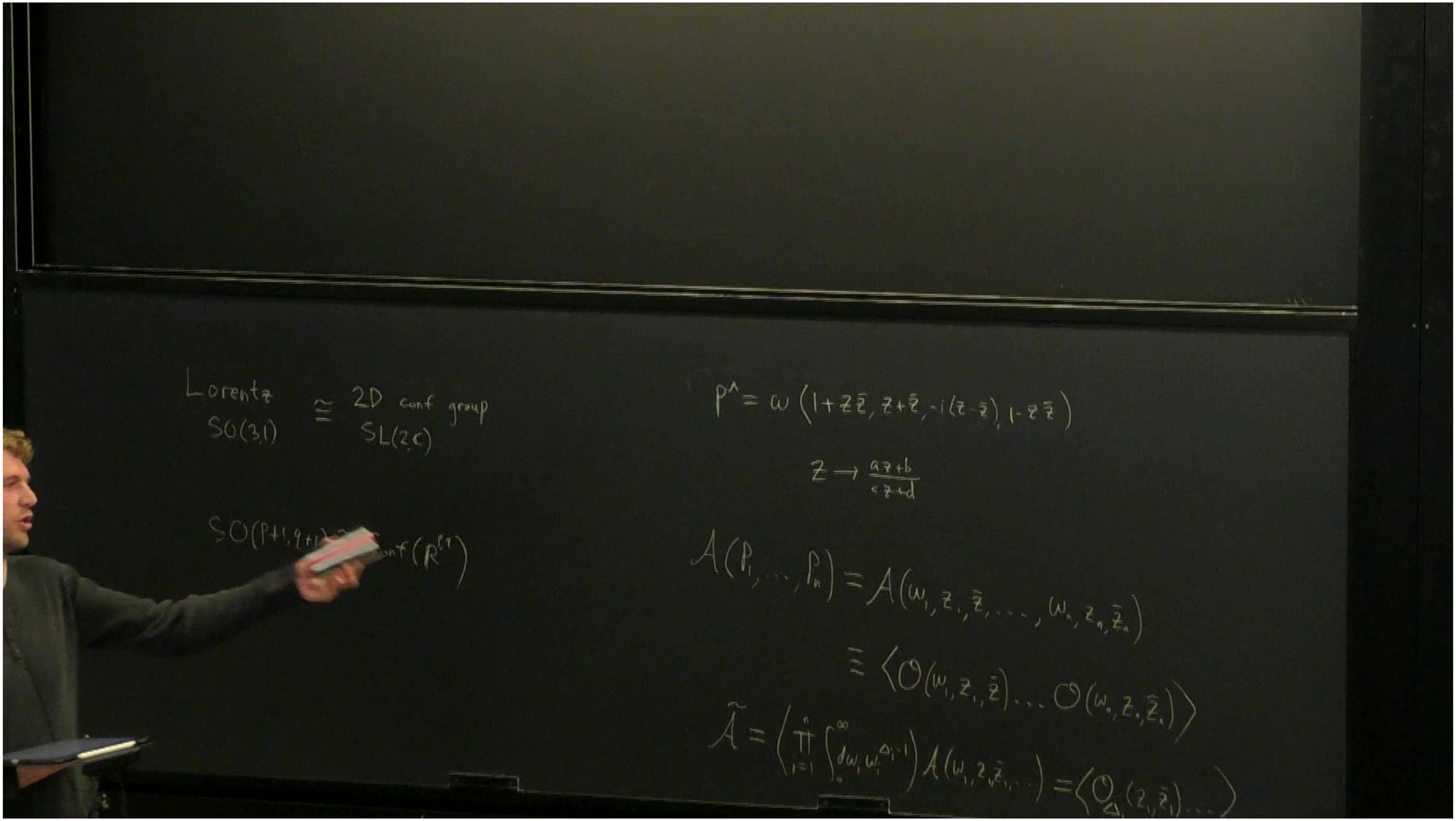
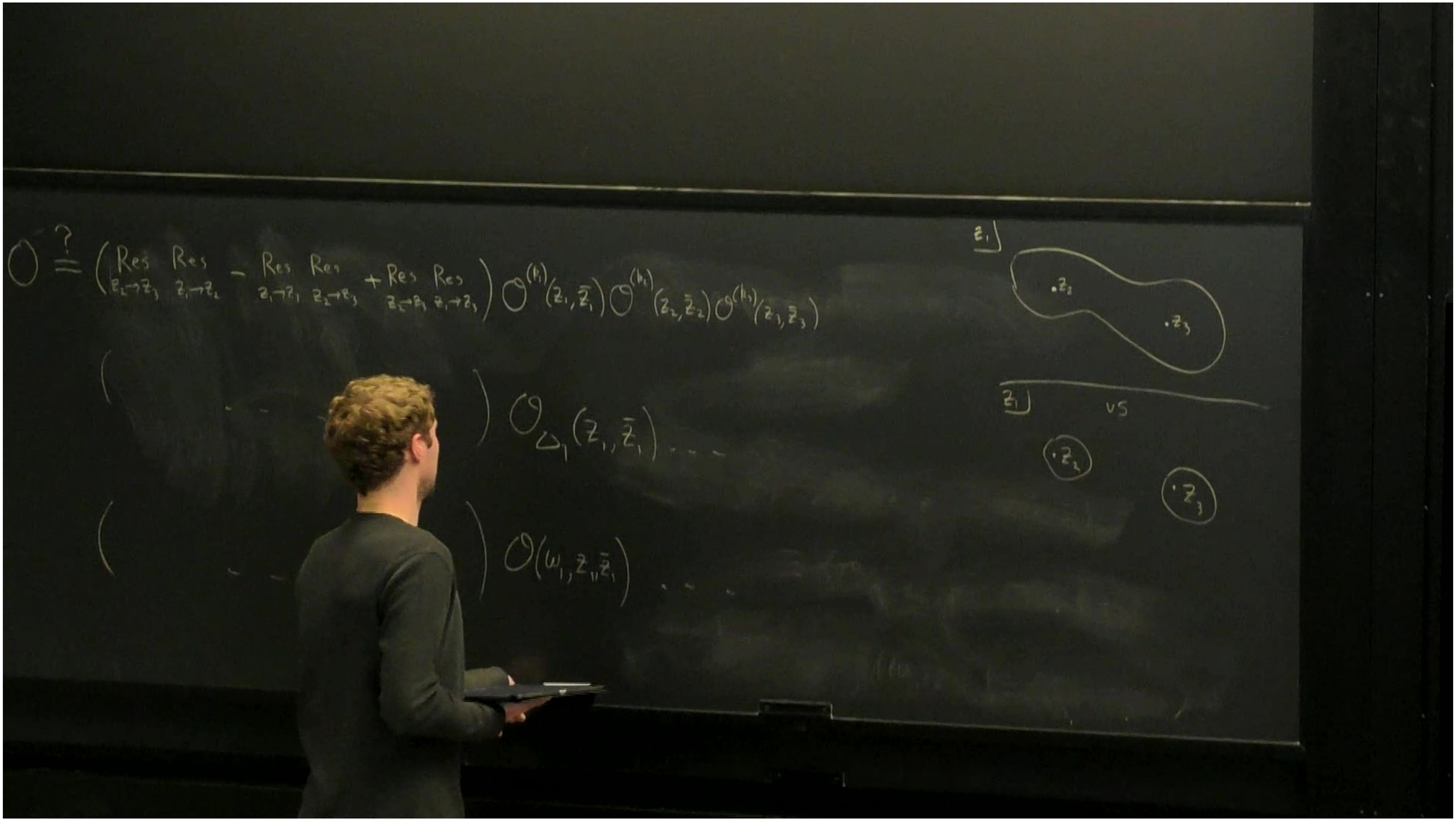
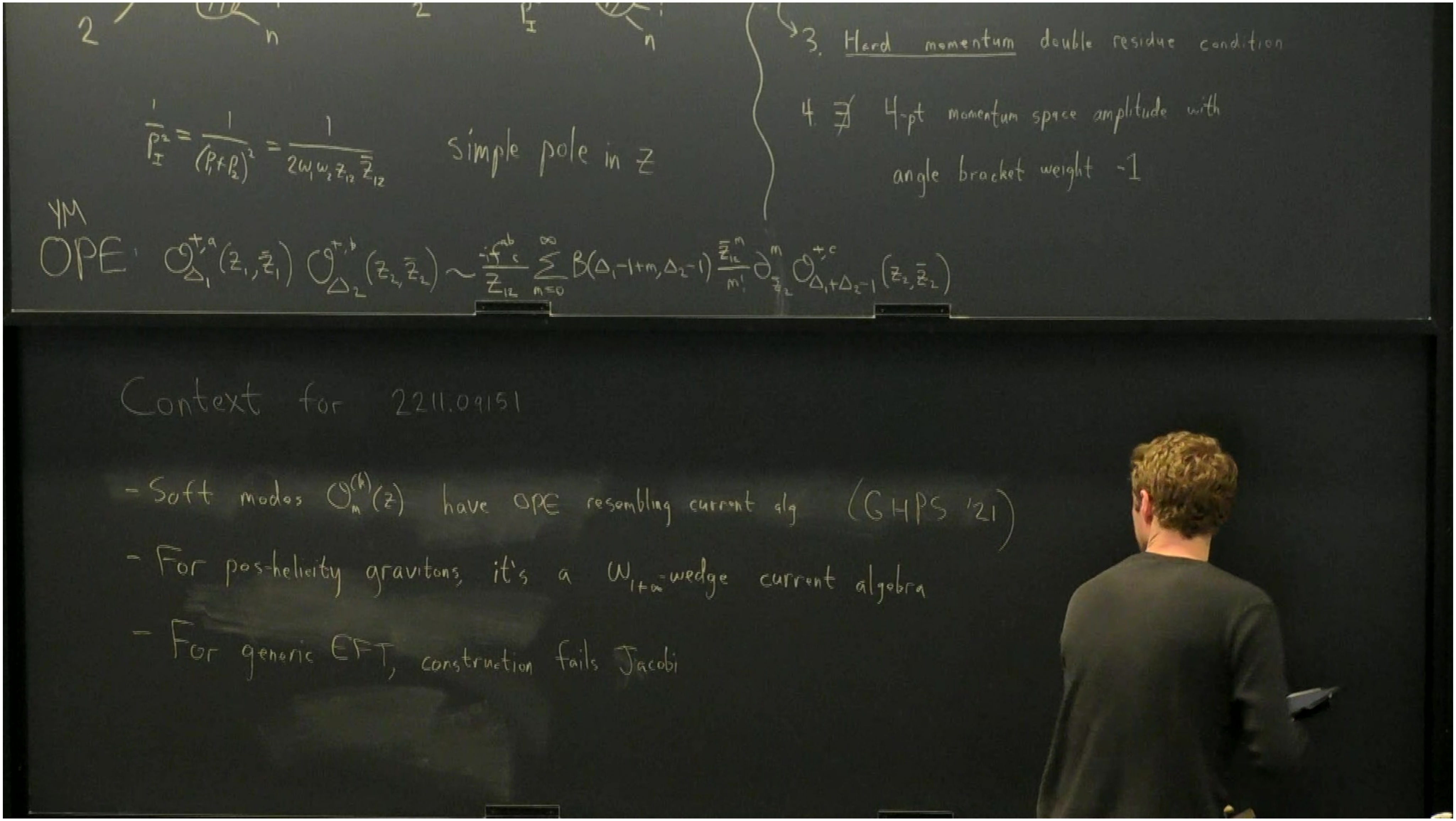
- For generic EFT, construction fails Jacobi


$\equiv \langle \mathcal{O}(\omega_1, z_1, \bar{z}_1) \ldots \mathcal{O}(\omega_n, z_n, \bar{z}_n) \rangle$
$\tilde{A} = \left(\prod_{i=1}^{n} \int_0^\infty d\omega_i\, \omega_i^{\Delta_i - 1} \right) A(\omega_1, z_1, \bar{z}_1, \ldots) = \langle \mathcal{O}_{\Delta_1}(z_1, \bar{z}_1) \ldots \rangle$
Usual amplitudes
CCFT
Lorentz
Conf group
collinear limit
OPE (singular part)
soft particles
holomorphic currents?


Lorentz SO(3,1) ≅ 2D conf group SL(2,C)
SO(p+1,q+1) ≅ conf(R^{p,q})
p^μ = ω(1+zz̄, z+z̄, -i(z-z̄), 1-zz̄)
z → (az+b)/(cz+d)
A(p_1, ..., p_n) = A(ω_1, z_1, z̄_1, ..., ω_n, z_n, z̄_n)
≡ ⟨O(ω_1, z_1, z̄_1) ... O(ω_n, z_n, z̄_n)⟩


O ?= (Res Res − Res Res + Res Res) O(z1, z̄1) O(z2, z̄2) O(z3, z̄3)
vs



3. Hard momentum double residue condition
4. ∄ 4-pt momentum space amplitude with angle bracket weight -1
$\frac{1}{P_I^2} = \frac{1}{(P_1+P_2)^2} = \frac{1}{2\omega_1\omega_2 z_{12}\bar{z}_{12}}$ simple pole in z
YM OPE: $O^{+,a}_{\Delta_1}(z_1,\bar{z}_1)\, O^{+,b}_{\Delta_2}(z_2,\bar{z}_2) \sim \frac{-if^{ab}{}_c}{z_{12}} \sum_{m=0}^{\infty} B(\Delta_1-1+m,\Delta_2-1)\frac{\bar{z}_{12}^m}{m!}\partial^m_{\bar{z}_2} O^{+,c}_{\Delta_1+\Delta_2-1}(z_2,\bar{z}_2)$
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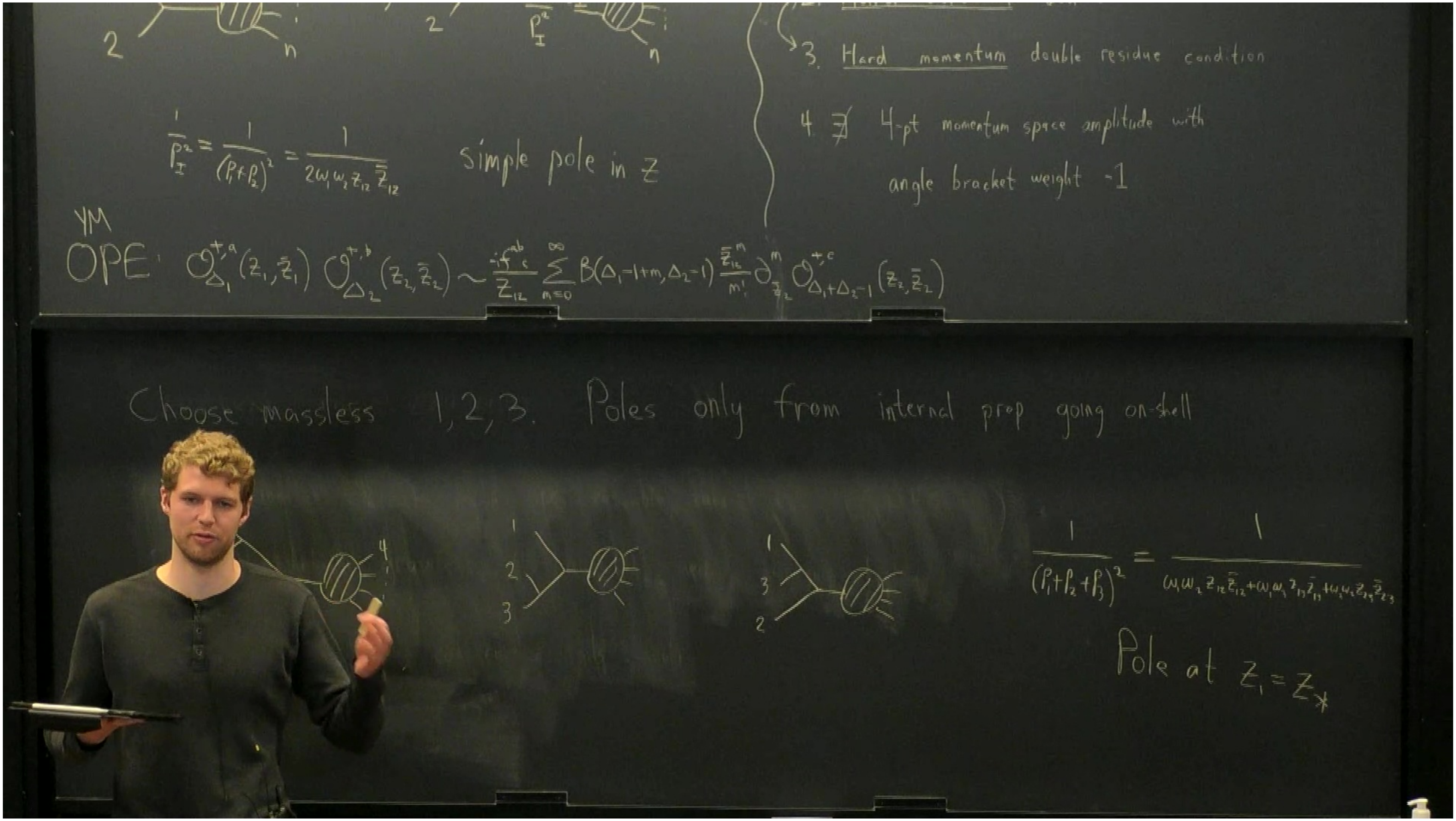
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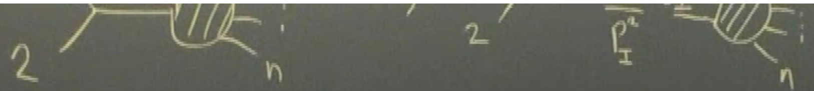
Choose massless 1,2,3. Poles only from internal prop going on-shell
$\frac{1}{(P_1+P_2+P_3)^2} = \frac{1}{\omega_1\omega_2 z_{12}\bar{z}_{12} + \omega_1\omega_3 z_{13}\bar{z}_{13} + \omega_2\omega_3 z_{23}\bar{z}_{23}}$
Pole at $z_1 = z_*$



3. Hard momentum double residue condition

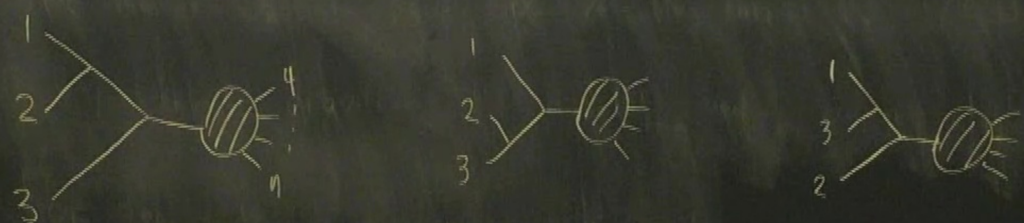
4. $\nexists$ 4-pt momentum space amplitude with angle bracket weight -1

$$\frac{1}{p_I^2} = \frac{1}{(p_1+p_2)^2} = \frac{1}{2\omega_1\omega_2 z_{12}\bar{z}_{12}} \qquad \text{simple pole in } z$$

YM

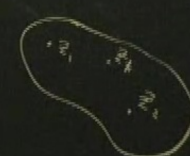
OPE: $$\mathcal{O}^{+,a}_{\Delta_1}(z_1,\bar{z}_1)\,\mathcal{O}^{+,b}_{\Delta_2}(z_2,\bar{z}_2) \sim \frac{-if^{ab}{}_c}{z_{12}} \sum_{m=0}^{\infty} B(\Delta_1-1+m,\Delta_2-1)\frac{\bar{z}_{12}^m}{m!}\partial^m_{\bar{z}_2}\mathcal{O}^{+,c}_{\Delta_1+\Delta_2-1}(z_2,\bar{z}_2)$$

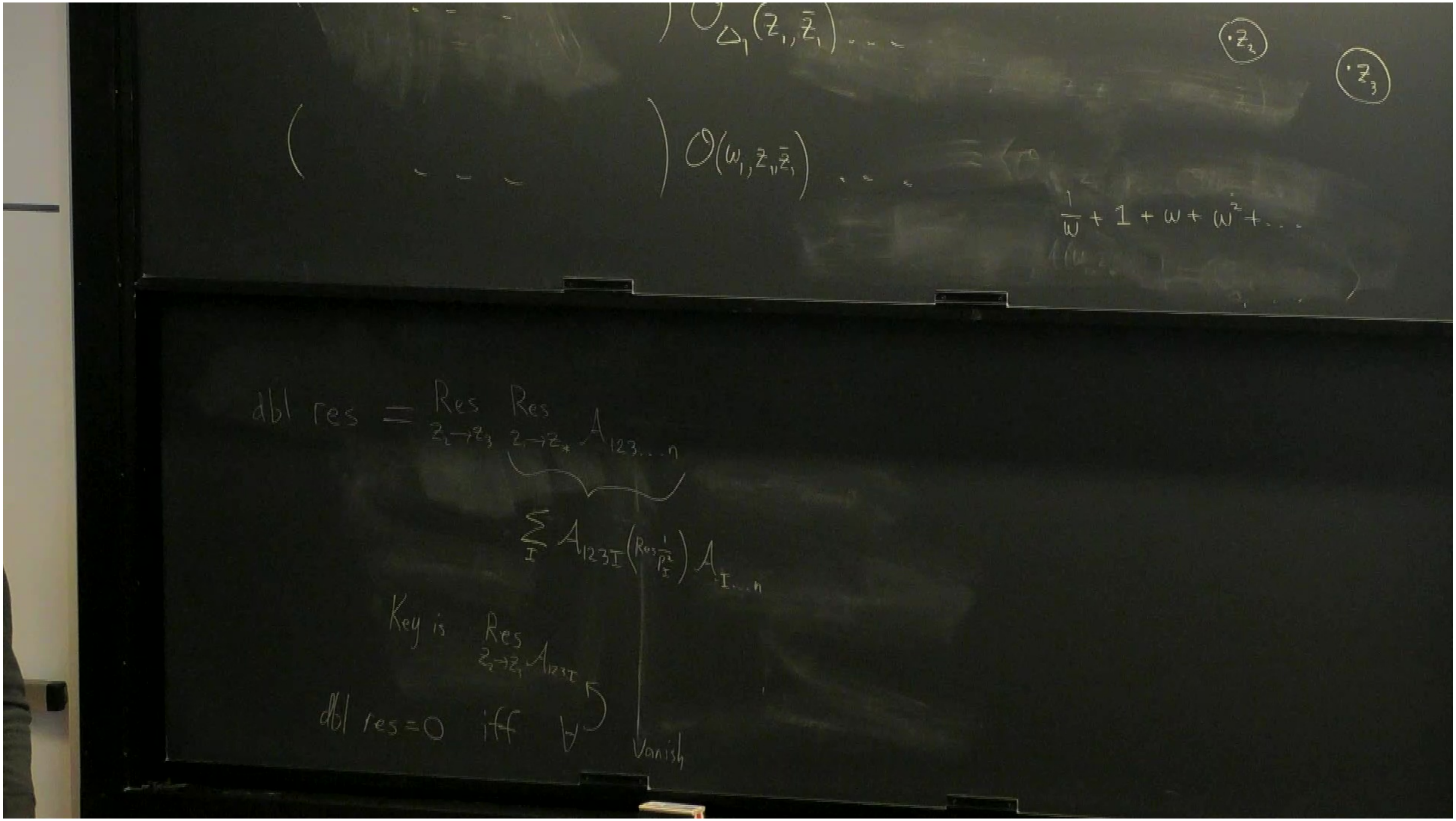
Choose massless 1, 2, 3. Poles only from internal prop going on-shell



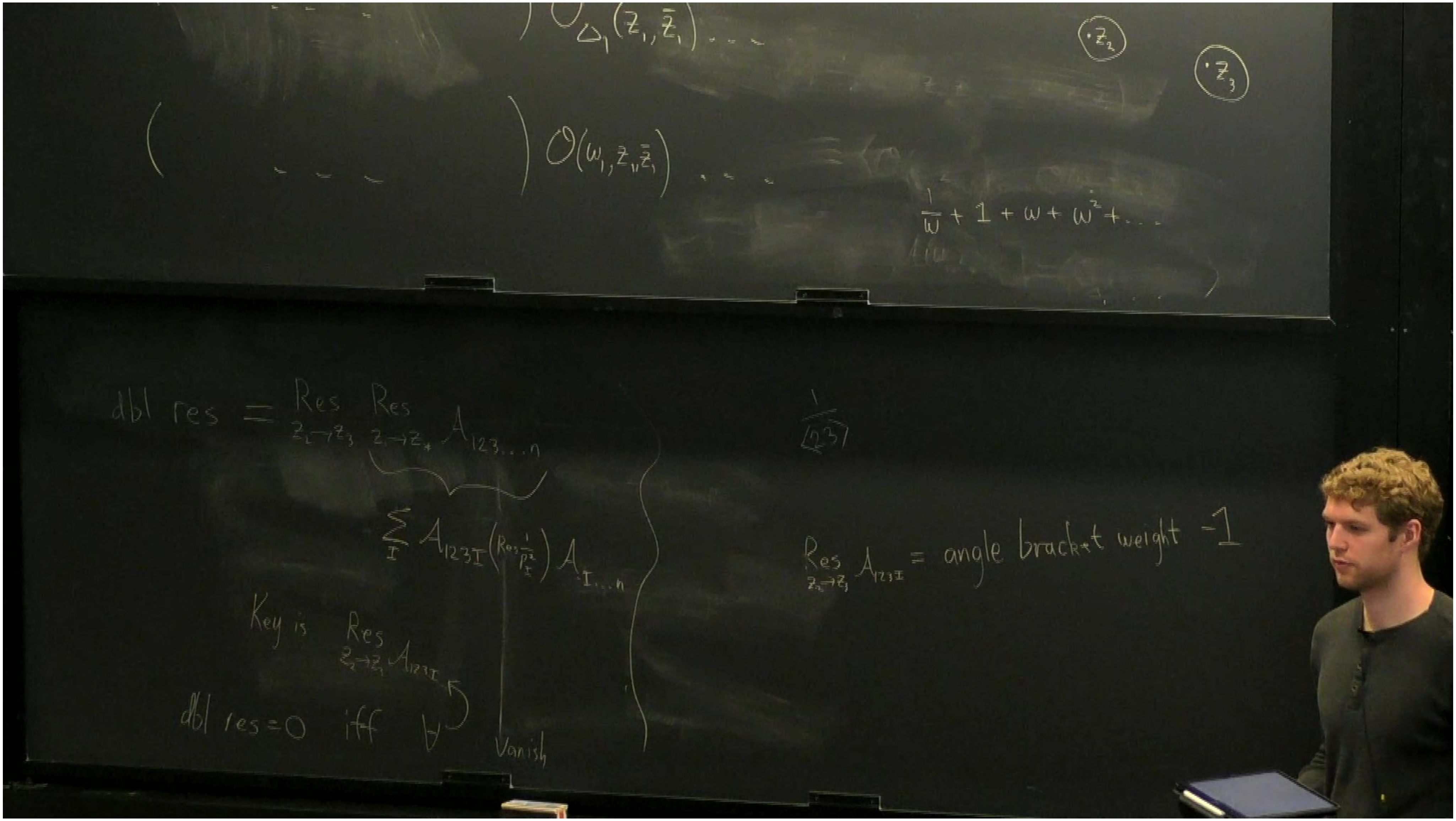
$$\frac{1}{(p_1+p_2+p_3)^2} = \frac{1}{\omega_1\omega_2 z_{12}\bar{z}_{12} + \omega_1\omega_3 z_{13}\bar{z}_{13} + \omega_2\omega_3 z_{23}\bar{z}_{23}}$$

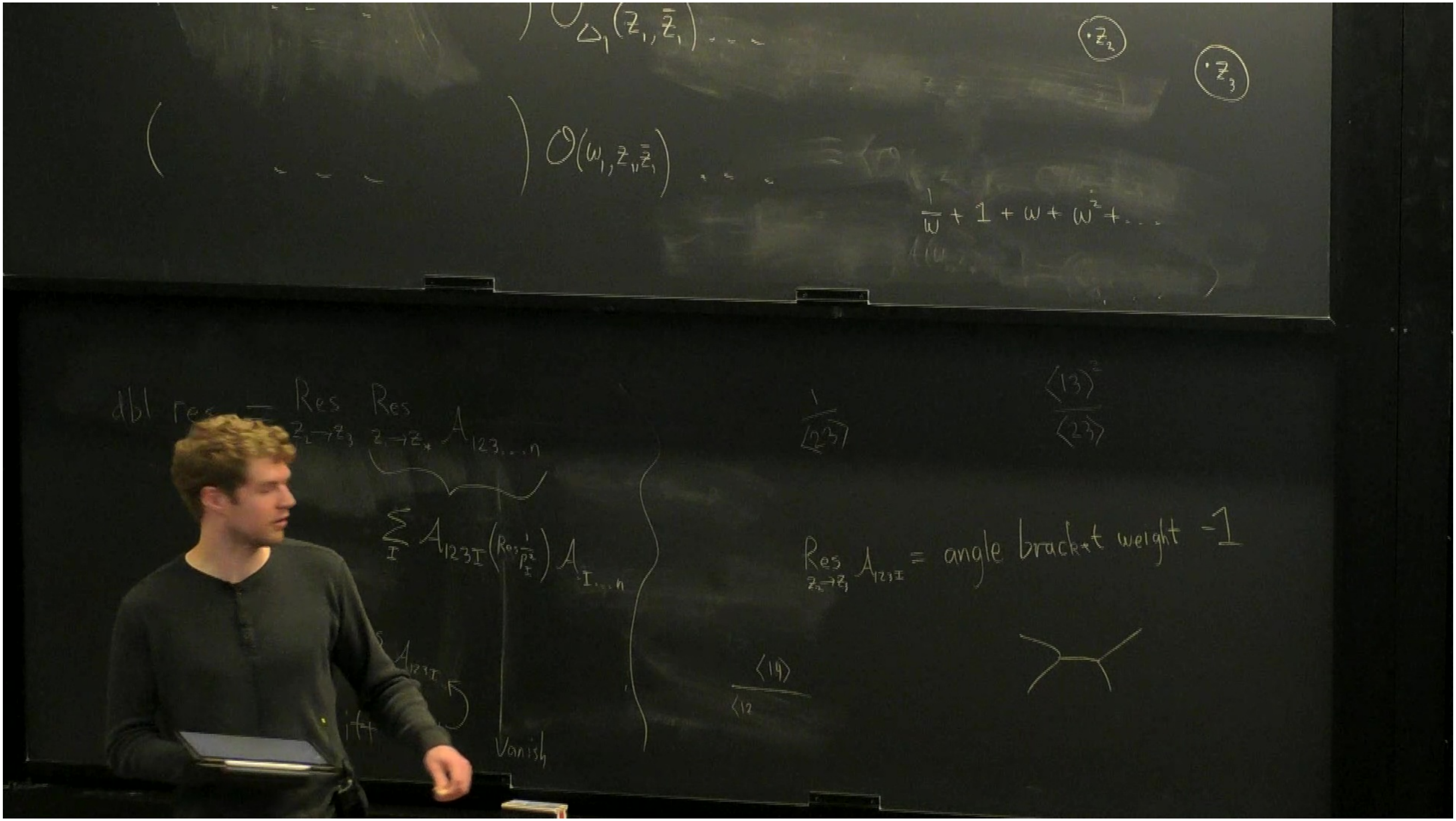
Pole at $z_1 = z_*$

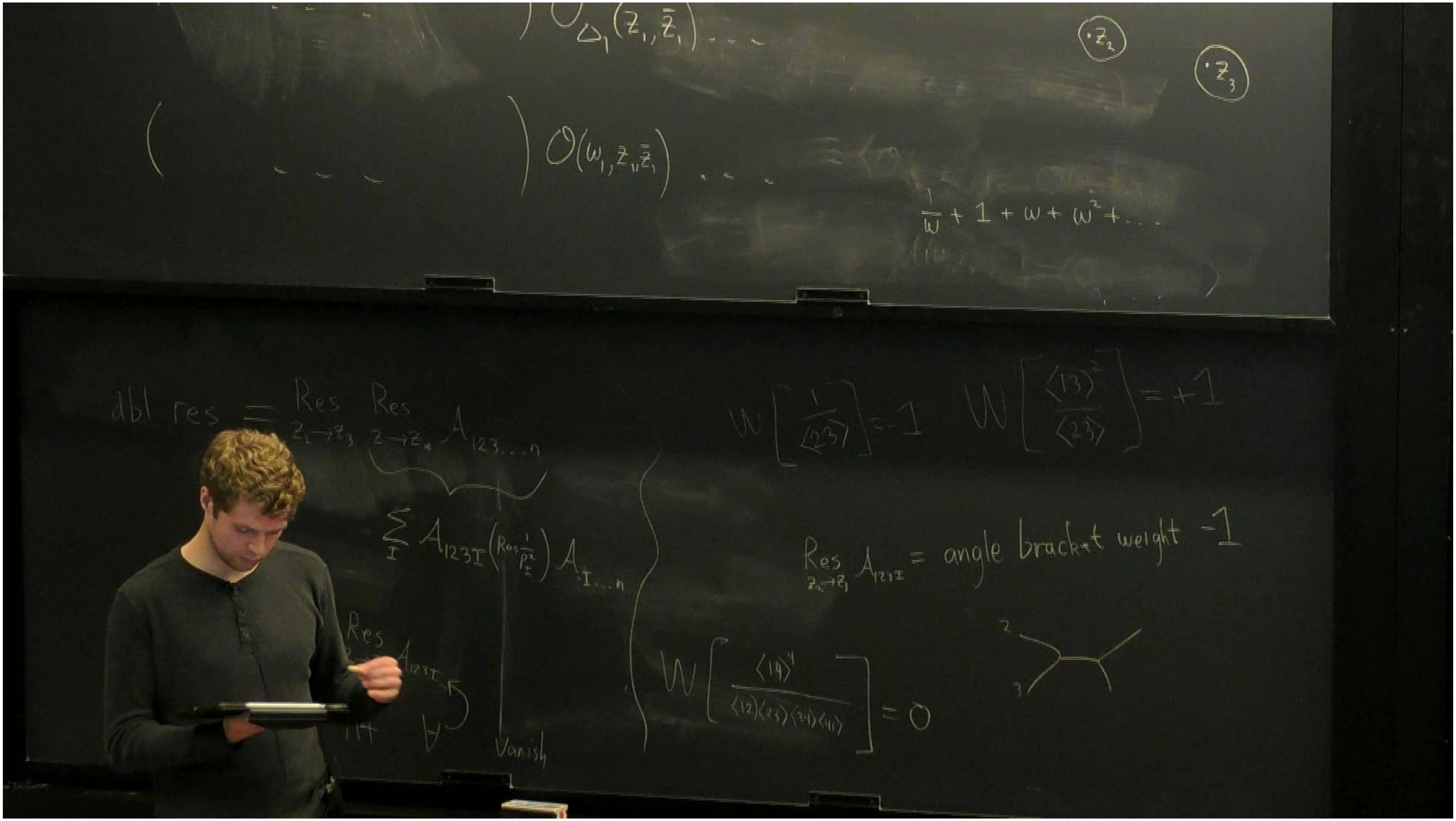




$O_{\Delta_1}(z_1, \bar{z}_1) \ldots$
z_2
z_3
$O(w_1, z_1, \bar{z}_1) \ldots$
$\frac{1}{w} + 1 + w + w^2 + \ldots$
dbl res $= \underset{z_2 \to z_3}{\mathrm{Res}} \; \underset{z_1 \to z_*}{\mathrm{Res}} \; A_{123\ldots n}$
$\sum_I A_{123I} \left(\mathrm{Res} \frac{1}{P_I^2}\right) A_{I\ldots n}$
Key is $\underset{z_2 \to z_1}{\mathrm{Res}} A_{123I}$
dbl res $= 0$ iff A
vanish


dbl res = Res Res A_{123...n}
Key is Res A_{123I}
dbl res = 0 iff ∀
Vanish
Res A_{123I} = angle bracket weight -1
1/w + 1 + w + w² + ...


dbl res = Res Res A_{123...n}
∑ A_{123I} (Res 1/P_I^2) A_{I...n}
vanish
1/⟨23⟩
⟨13⟩^2 / ⟨23⟩
Res A_{123I} = angle bracket weight -1
⟨14⟩ / ⟨12
1/w + 1 + w + w^2 + ...


dbl res = Res Res A
angle bracket weight -1
Vanish