Title: Quantum Field Theory in Curved Spacetime (PM) - 2023-03-31

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Collection: Quantum Field Theory in Curved Spacetime

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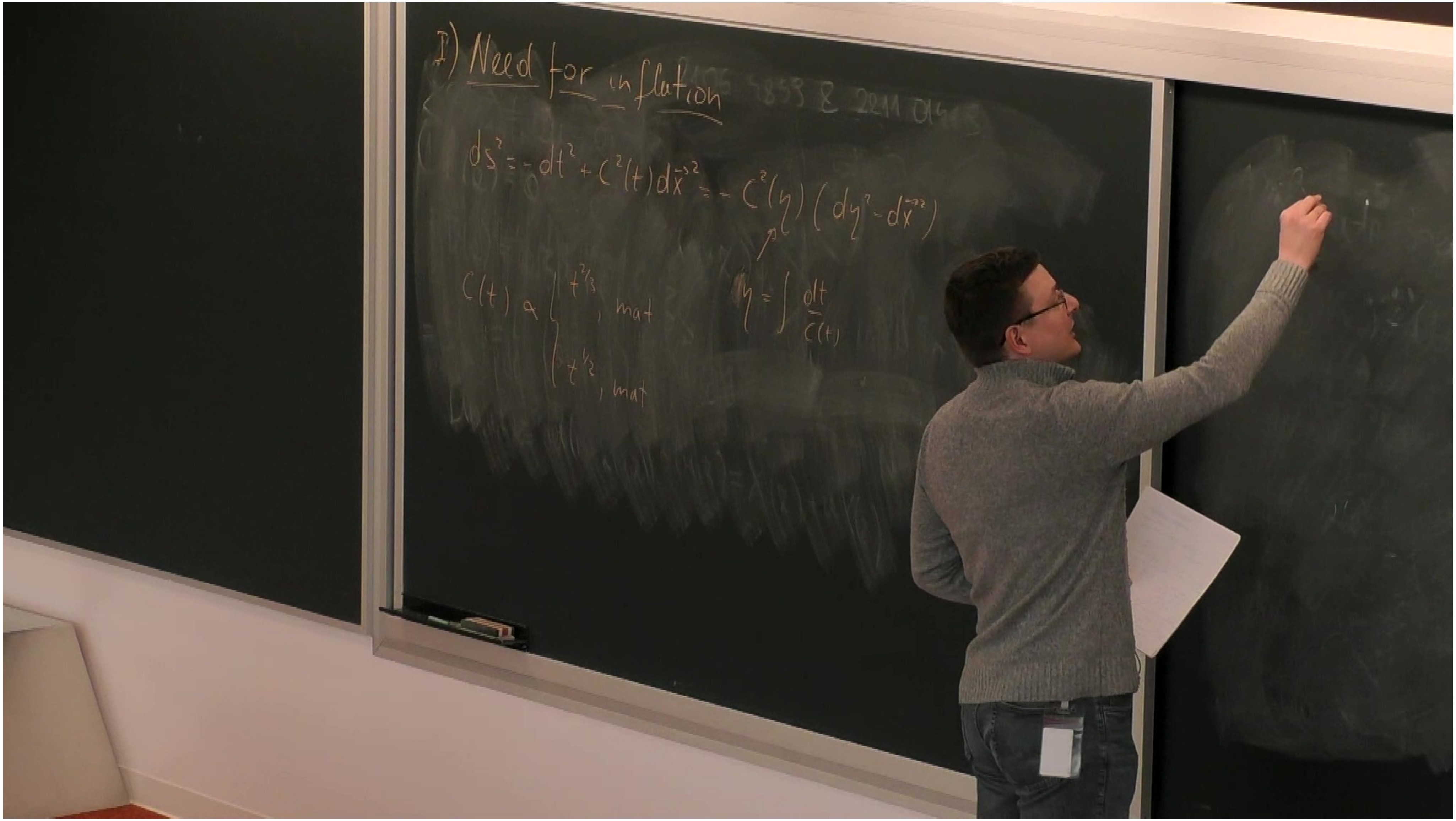
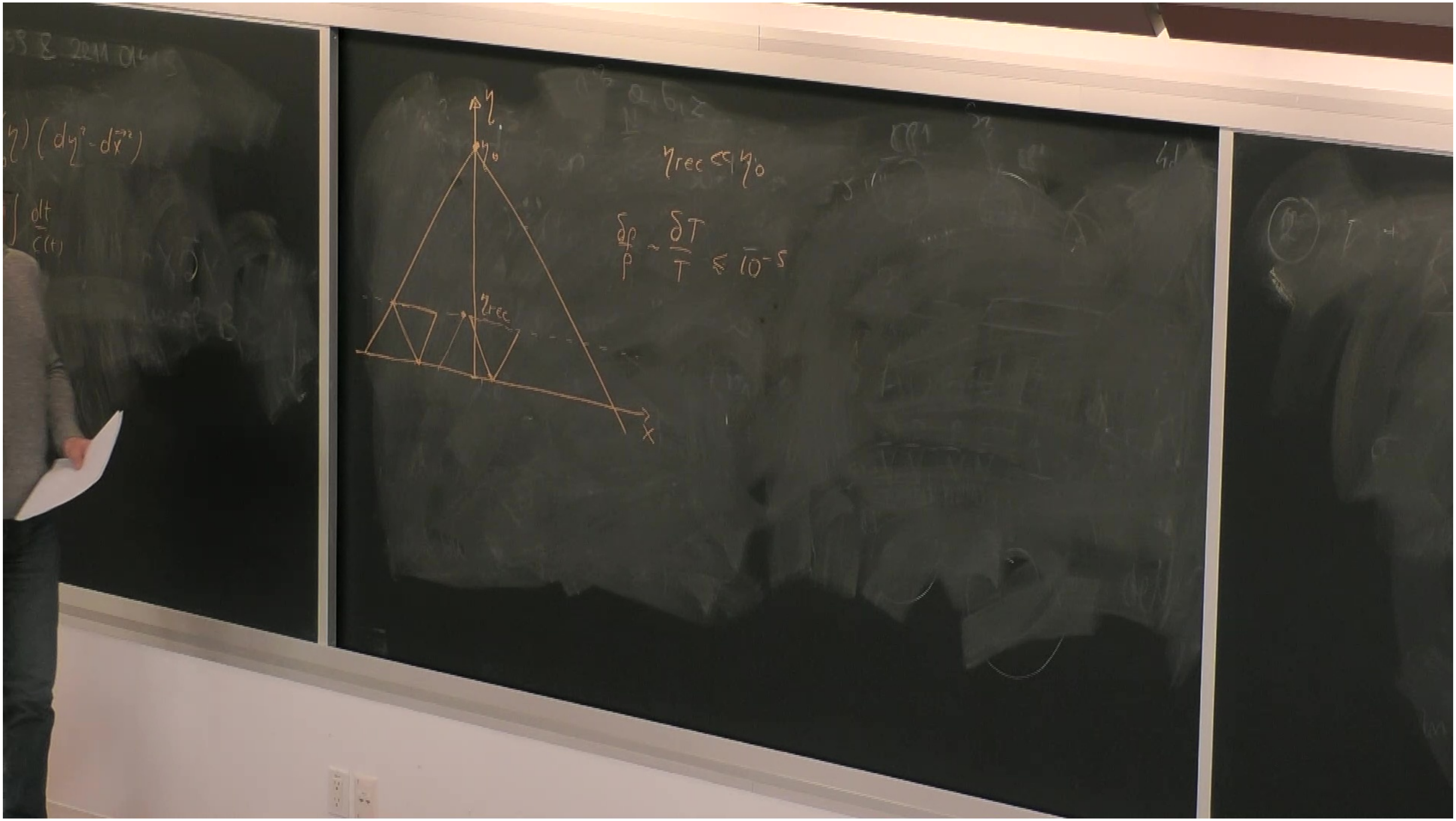
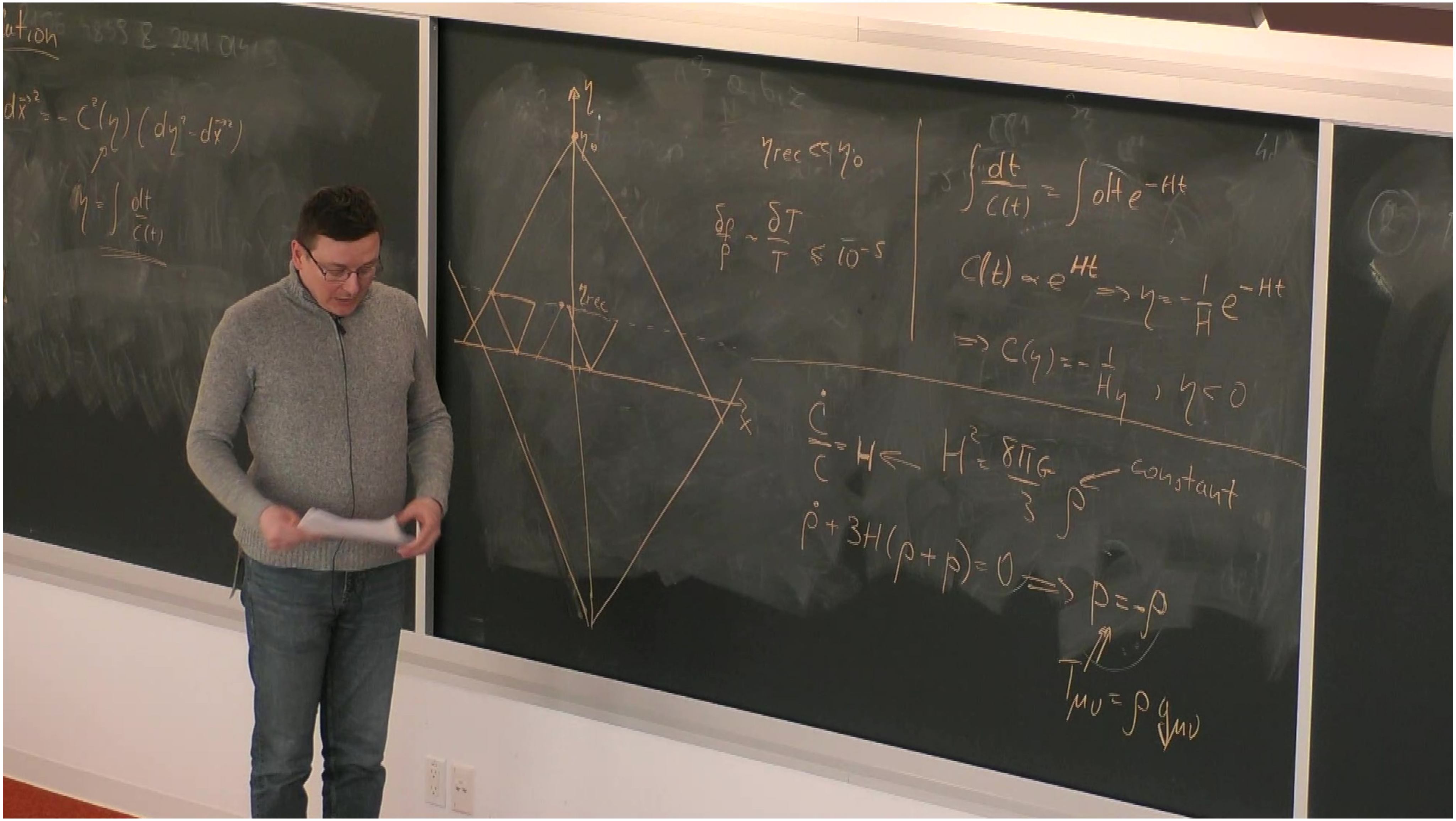
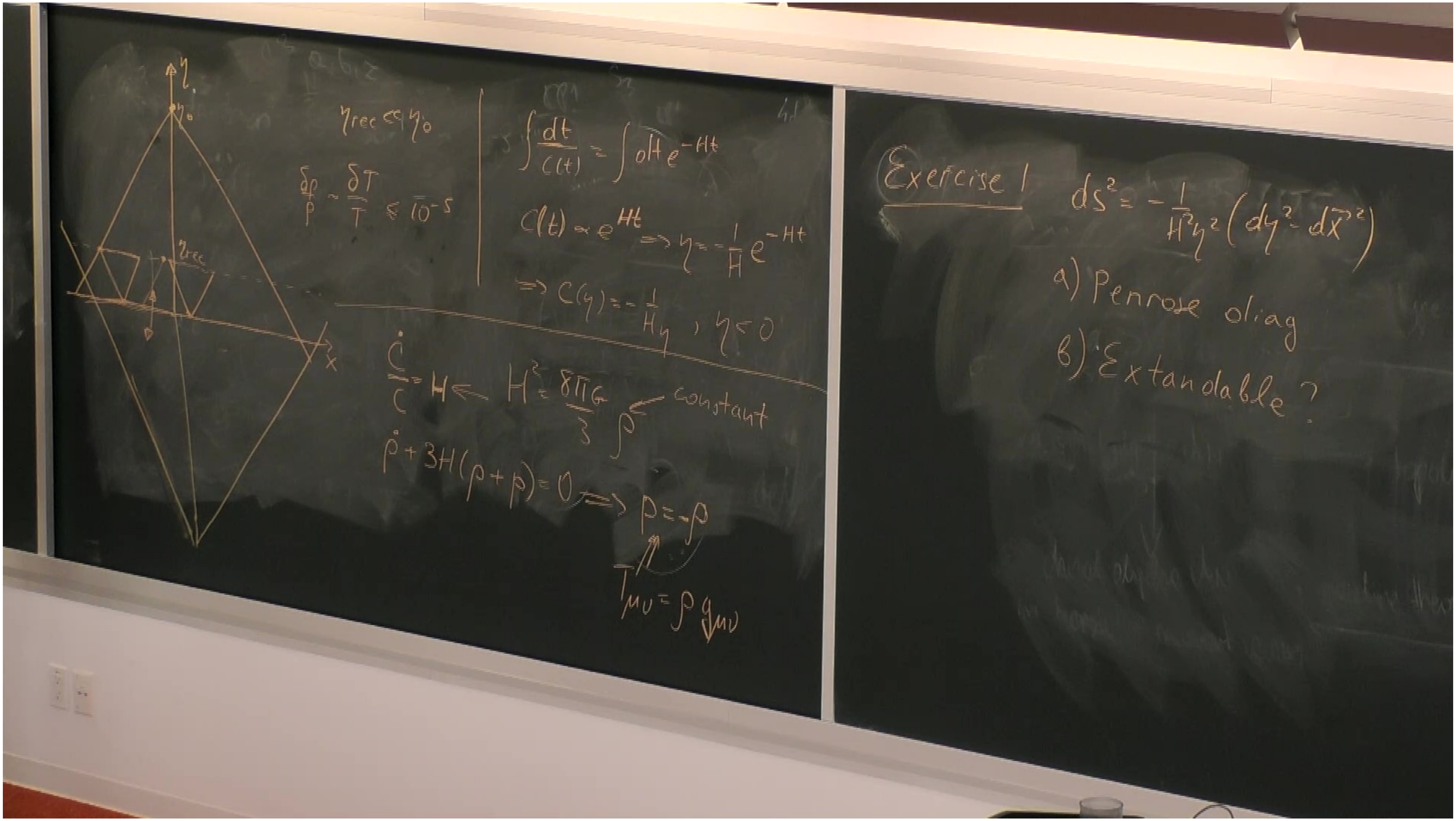
Abstract: https://pitp.zoom.us/j/95867663528?pwd=dmd5Y3FaVHJ0SnJWSlFscEY4cmhYUT09


I) Need for inflation
ds^2 = - dt^2 + C^2(t) dx^2 = - C^2(η) (dη^2 - dx^2)
C(t) ∝ t^2/3, mat
t^1/2, rad
η = ∫ dt/C(t)


η
η₀
η_rec
x
η_rec ≪ η₀
δρ/ρ ~ δT/T ≤ 10⁻⁵
(dη² - dx⃗²)
dt/c(t)


$-C^2(\eta)(d\eta^2 - d\vec{x}^2)$
$\eta = \int \frac{dt}{C(t)}$
η
η_0
η_{rec}
x
$\eta_{rec} \ll \eta_0$
$\frac{\delta\rho}{\rho} \sim \frac{\delta T}{T} \lesssim 10^{-5}$
$\int \frac{dt}{C(t)} = \int dt\, e^{-Ht}$
$C(t) \propto e^{Ht} \Rightarrow \eta = -\frac{1}{H} e^{-Ht}$
$\Rightarrow C(\eta) = -\frac{1}{H\eta}, \ \eta < 0$
$\frac{\dot{C}}{C} = H \Leftarrow H^2 = \frac{8\pi G}{3}\rho \Leftarrow$ constant
$\dot{\rho} + 3H(\rho + p) = 0 \Rightarrow p = -\rho$
$T_{\mu\nu} = \rho\, g_{\mu\nu}$


η
η_0
$\eta_{rec} \ll \eta_0$
$\frac{\delta\rho}{\rho} \sim \frac{\delta T}{T} \leq 10^{-5}$
η_{rec}
x
$\int \frac{dt}{C(t)} = \int dt\, e^{-Ht}$
$C(t) \propto e^{Ht} \Rightarrow \eta = -\frac{1}{H} e^{-Ht}$
$\Rightarrow C(\eta) = -\frac{1}{H\eta}\ , \ \eta < 0$
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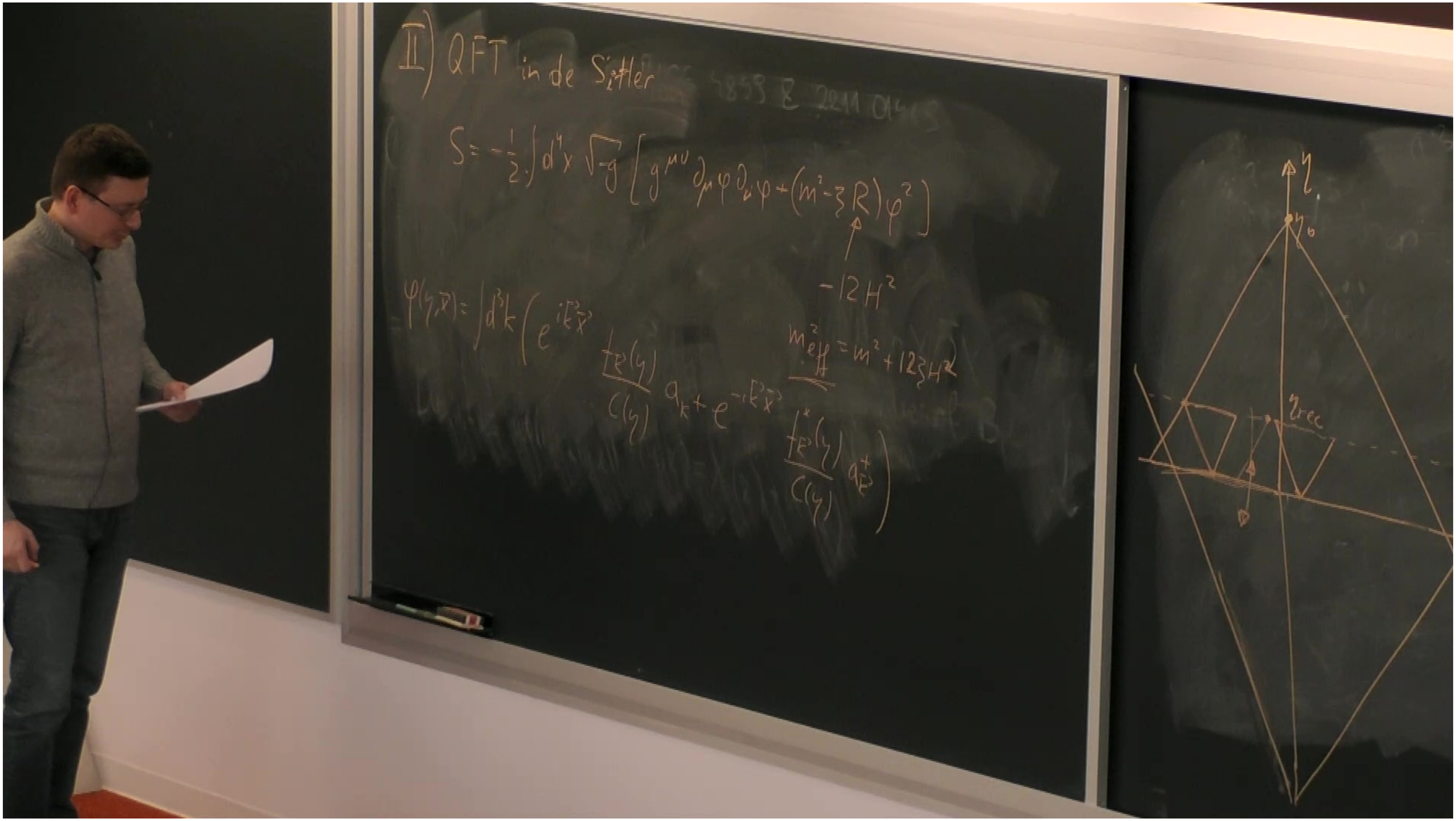
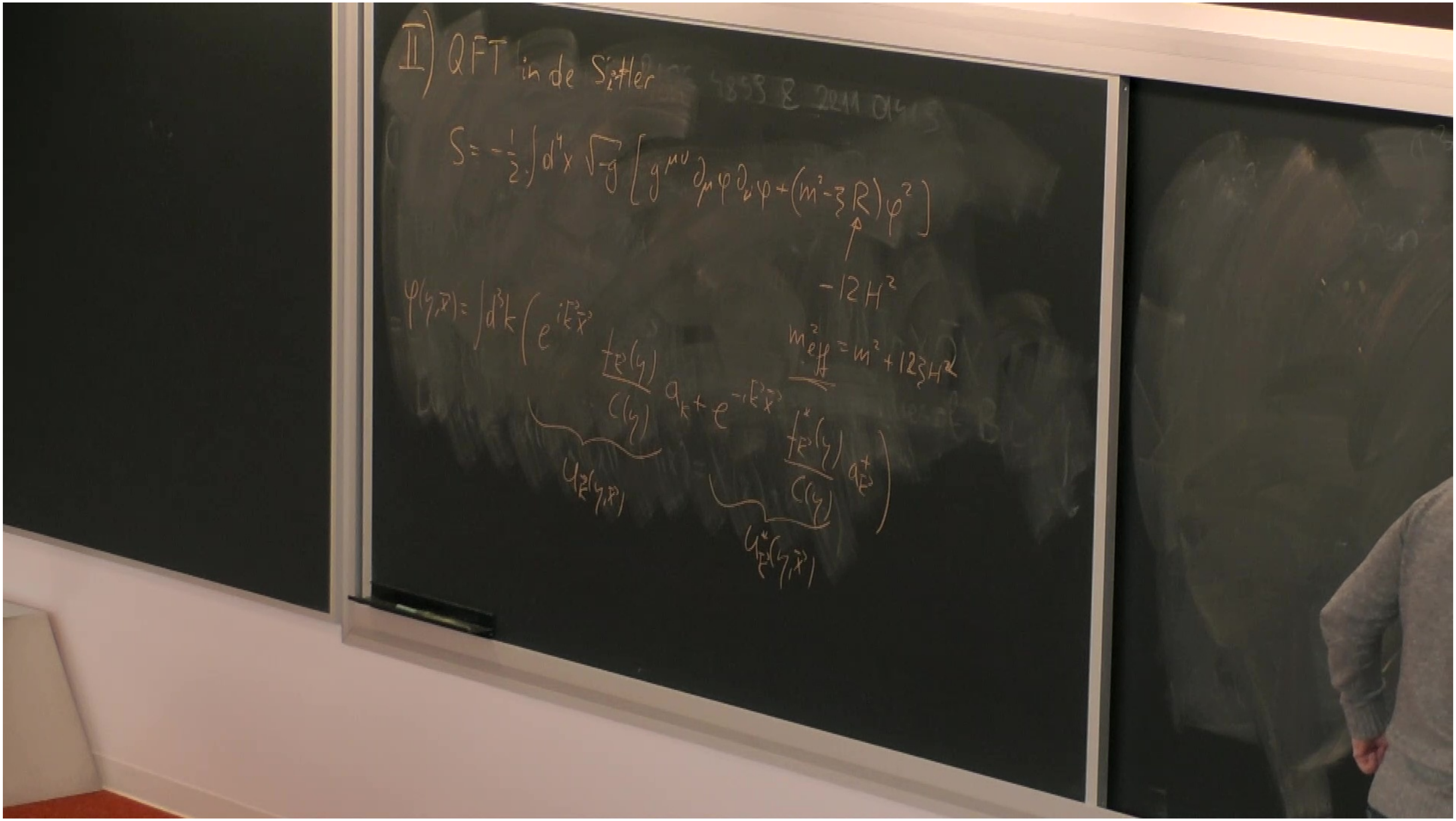
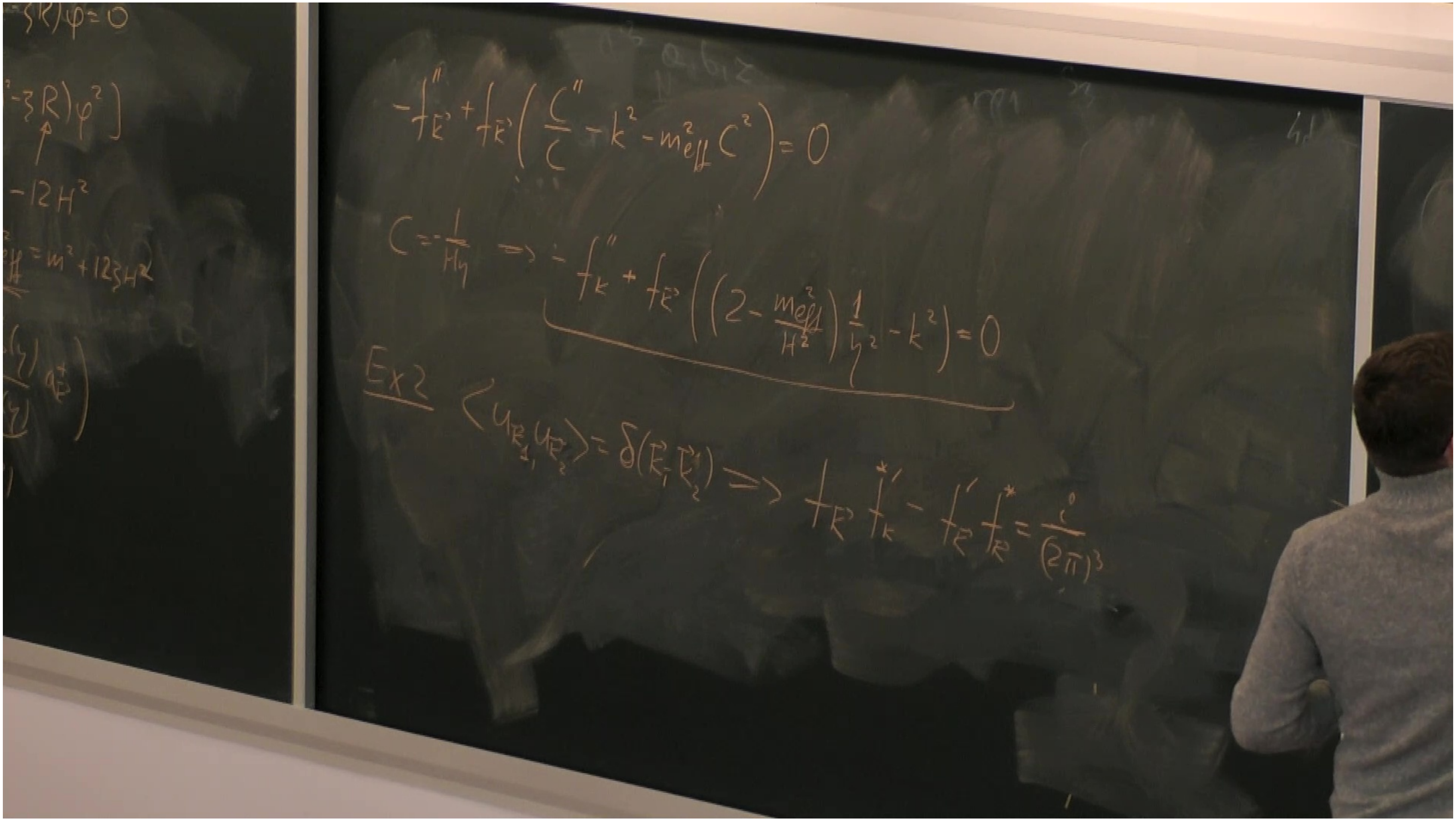
$\dot{\rho} + 3H(\rho + p) = 0 \Rightarrow p = -\rho$
$T_{\mu\nu} = \rho\, g_{\mu\nu}$
Exercise 1
$ds^2 = -\frac{1}{H^2\eta^2}\left(d\eta^2 - d\vec{x}^2\right)$
a) Penrose diag
b) Extandable ?


II) QFT in de Sitter
S = -1/2 ∫d^4x √-g [g^{μν} ∂_μφ ∂_νφ + (m^2 - ξR) φ^2]
-12H^2
φ(η,x) = ∫d^3k (e^{ik·x} f_k(η)/C(η) a_k + e^{-ik·x} f_k^*(η)/C(η) a_k^+)
m_eff^2 = m^2 + 12ξH^2
η
η_0
η_rec


II) QFT in de Sitter
$S = -\frac{1}{2}\int d^4x \sqrt{-g}\left[g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + (m^2 - \xi R)\varphi^2\right]$
$-12H^2$
$m_{eff}^2 = m^2 + 12\xi H^2$
$\varphi(\eta,\vec{x}) = \int d^3k \left(\underbrace{e^{i\vec{k}\vec{x}} \frac{f_{\vec{k}}(\eta)}{C(\eta)}}_{u_{\vec{k}}(\eta,\vec{x})} a_k + \underbrace{e^{-i\vec{k}\vec{x}} \frac{f_{\vec{k}}^*(\eta)}{C(\eta)}}_{u_{\vec{k}}^*(\eta,\vec{x})} a_{\vec{k}}^+ \right)$


$-\xi R)\varphi = 0$
$-\xi R)\varphi^2]$
$-12H^2$
$_{eff}^2 = m^2 + 12\xi H^2$
$-f_{\vec k}'' + f_{\vec k}\left(\frac{C''}{C} - k^2 - m_{eff}^2 C^2\right) = 0$
$C = -\frac{1}{H\eta} \Rightarrow -f_k'' + f_{\vec k}\left(\left(2 - \frac{m_{eff}^2}{H^2}\right)\frac{1}{\eta^2} - k^2\right) = 0$
Ex2
$\langle u_{\vec k_1} u_{\vec k_2}\rangle = \delta(\vec k_1 - \vec k_2) \Rightarrow f_{\vec k} f_k^{*\prime} - f_{\vec k}' f_{\vec k}^* = \frac{i}{(2\pi)^3}$

$\frac{i}{(2\pi)^3}$

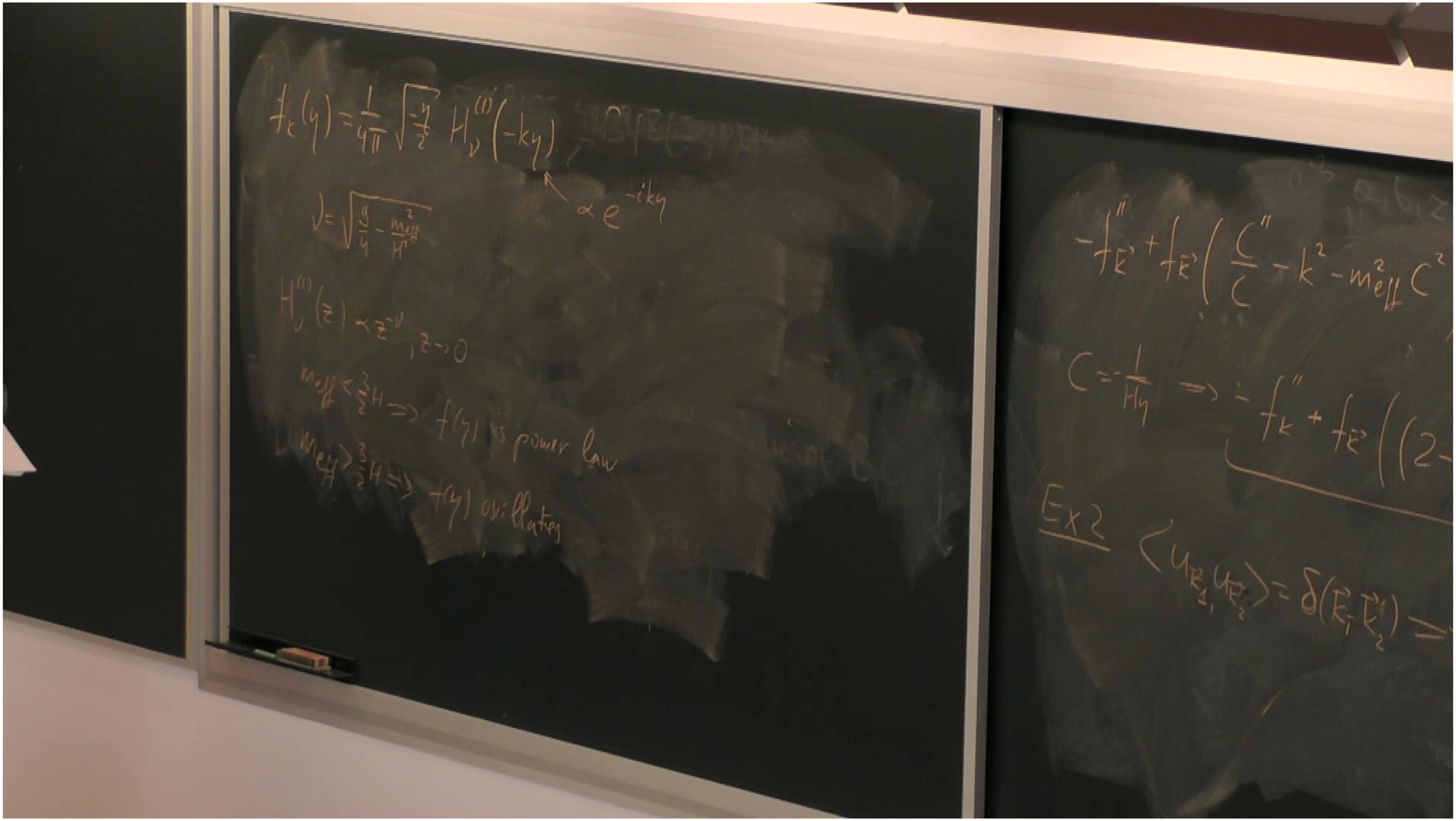
$$m_{eff} = 0 \Longrightarrow f_{\vec{k}}(\eta) = \frac{1}{(2\pi)^{3/2}\sqrt{2k}}\, e^{-ik\eta}\left(1 - \frac{i}{k\eta}\right) \Longleftarrow \text{Bunch-Davies vacuum}$$

$$k|\eta| \gg 1 \Longleftrightarrow \frac{p_{ph}}{H} \gg 1$$

$$k = C(\eta)\, p_{ph} = -\frac{p_{ph}}{H\eta}$$

$$k|\eta| \ll 1$$

Ex 3 Find detetor response for a static detector in dS in BD vacuum


$f_k(\eta) = \frac{1}{4\pi}\sqrt{\frac{-\eta}{2}}\, H_\nu^{(1)}(-k\eta)$
$\propto e^{-ik\eta}$
$\nu = \sqrt{\frac{9}{4} - \frac{m_{eff}^2}{H^2}}$
$H_\nu^{(1)}(z) \propto z^{-\nu}, z \to 0$
$m_{eff} < \frac{3}{2}H \Rightarrow f(\eta)$ is power law
$m_{eff} > \frac{3}{2}H \Rightarrow f(\eta)$ oscillates
$-f_{\vec{k}}'' + f_{\vec{k}}\left(\frac{C''}{C} - k^2 - m_{eff}^2 C^2\right.$
$C = -\frac{1}{H\eta} \Rightarrow -f_k'' + f_{\vec{k}}\left((2-\right.$
Ex 2
$\langle u_{\vec{k}_1}, u_{\vec{k}_2}\rangle = \delta(\vec{k}_1 - \vec{k}_2) \Rightarrow$

$$-f_{\vec{k}}'' + f_{\vec{k}}\left(\frac{C''}{C} - k^2 - m_{eff}^2 C^2\right) = 0$$

$m = 0$

$\xi = \frac{1}{6}$

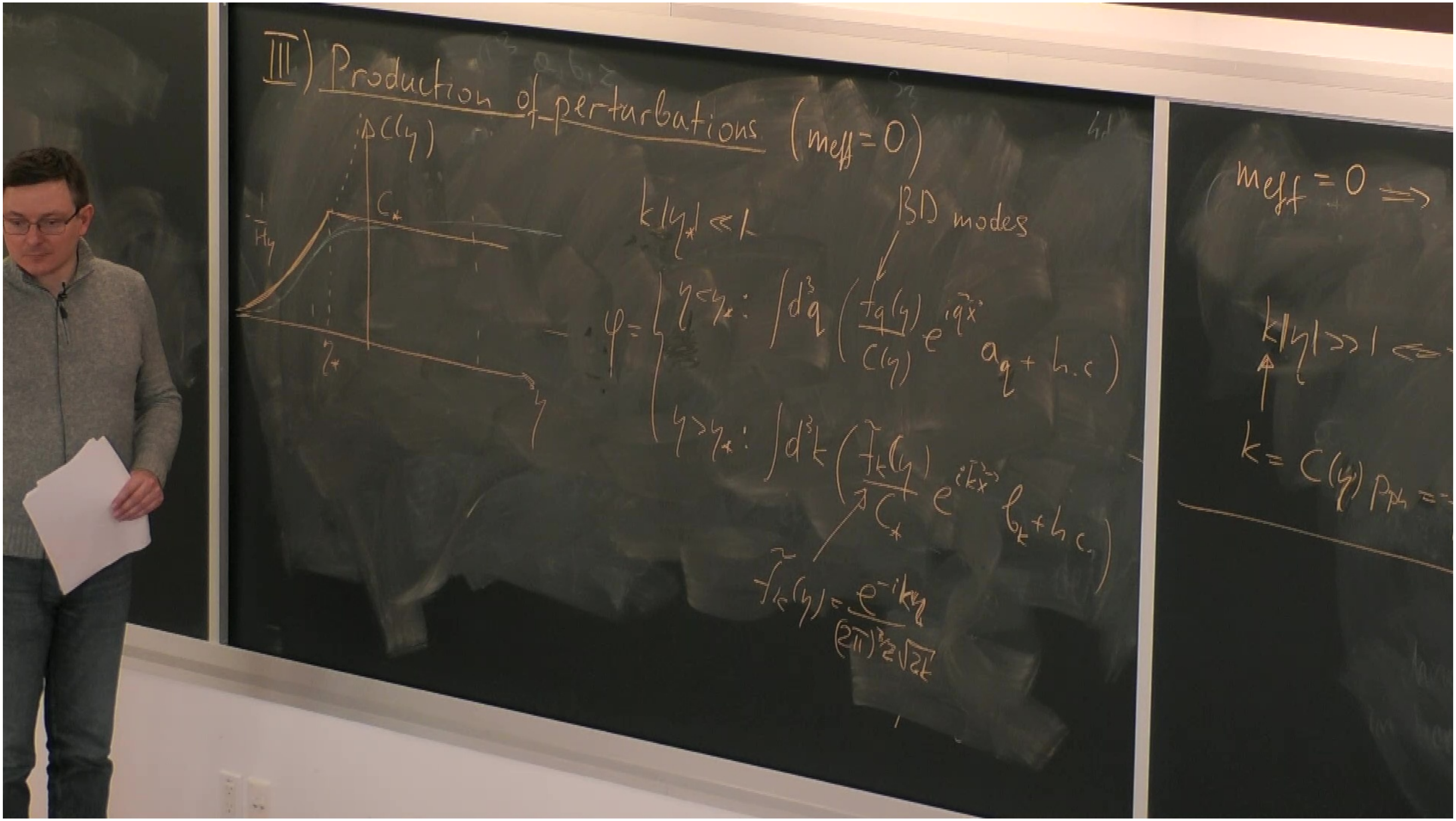
$$C = -\frac{1}{H\eta} \Rightarrow -f_{k}'' + f_{\vec{k}}\left(\left(2 - \cancel{\frac{m_{eff}^2}{H^2}}\right)\frac{1}{\eta^2} - k^2\right) = 0$$

$m_{eff}^2 \to m^2 + \xi\, 12 H^2$

$\eta < 0$

$\eta \to 0^-$

Ex 2 $\langle u_{\vec{k}_1} u_{\vec{k}_2} \rangle = \delta(\vec{k}_1 - \vec{k}_2) \Rightarrow f_{\vec{k}} f_{k}^{*\prime} - f_{\vec{k}}' f_{\vec{k}}^{*} = \frac{i}{(2\pi)^3}$


III) Production of perturbations (meff = 0)
C(η)
C*
η*
η
k|η*| ≪ 1
BD modes
meff = 0 ⟹
k|η| >> 1
k = C(η) pph

...turbations ($m_{eff} = 0$)

$k|\eta_*| \ll 1$ BD modes

$$\varphi = \begin{cases} \eta < \eta_* : \int d^3q \left(\frac{f_q(\eta)}{C(\eta)} e^{i\vec{q}\vec{x}} a_q + h.c. \right) \\ \eta > \eta_* : \int d^3k \left(\frac{\tilde{f}_k(\eta)}{C_*} e^{i\vec{k}\vec{x}} b_k + h.c. \right) \end{cases}$$

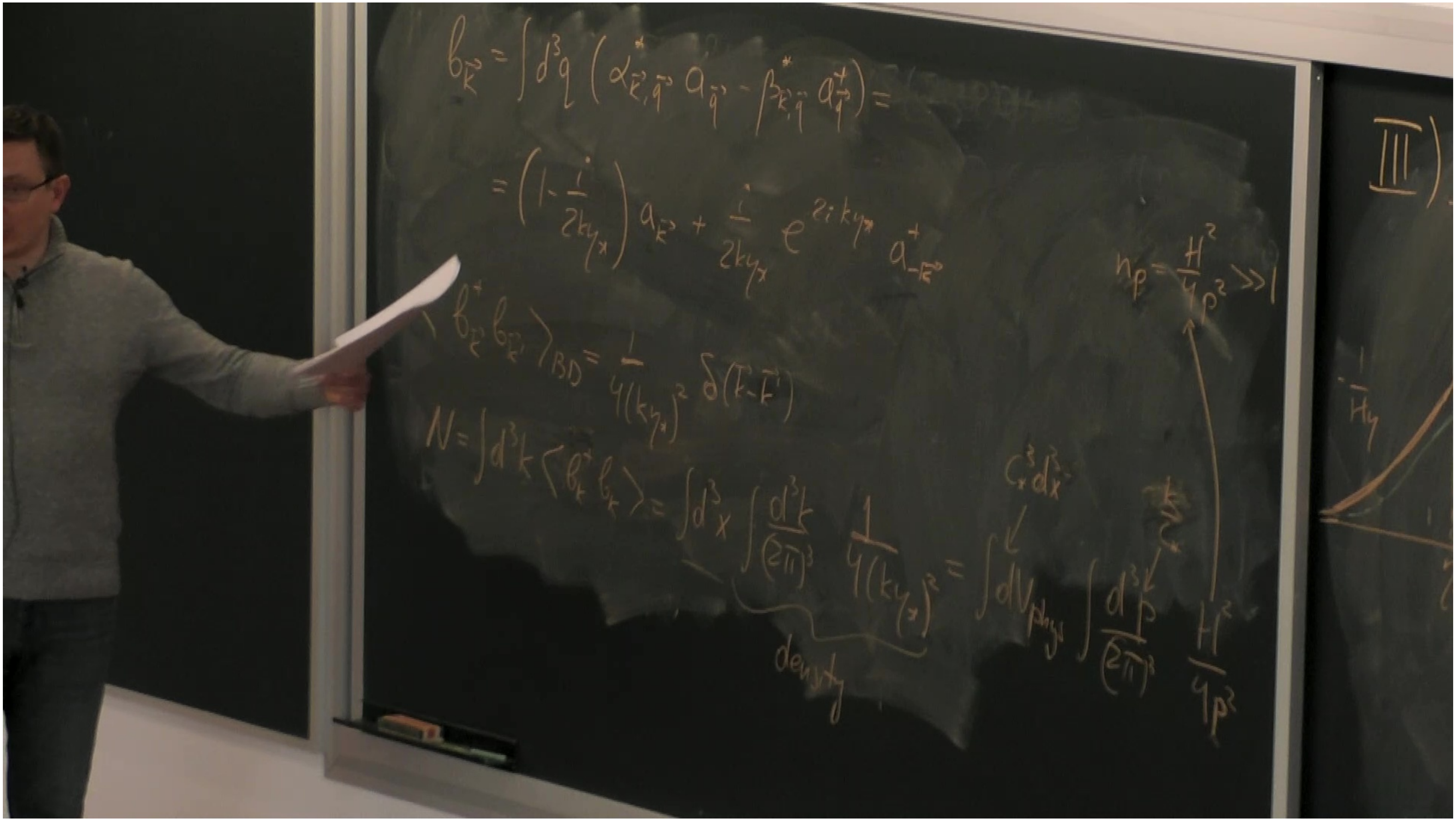
$$\tilde{f}_k(\eta) = \frac{e^{-ik\eta}}{(2\pi)^{3/2}\, 2\sqrt{2k}}$$

Ex 4

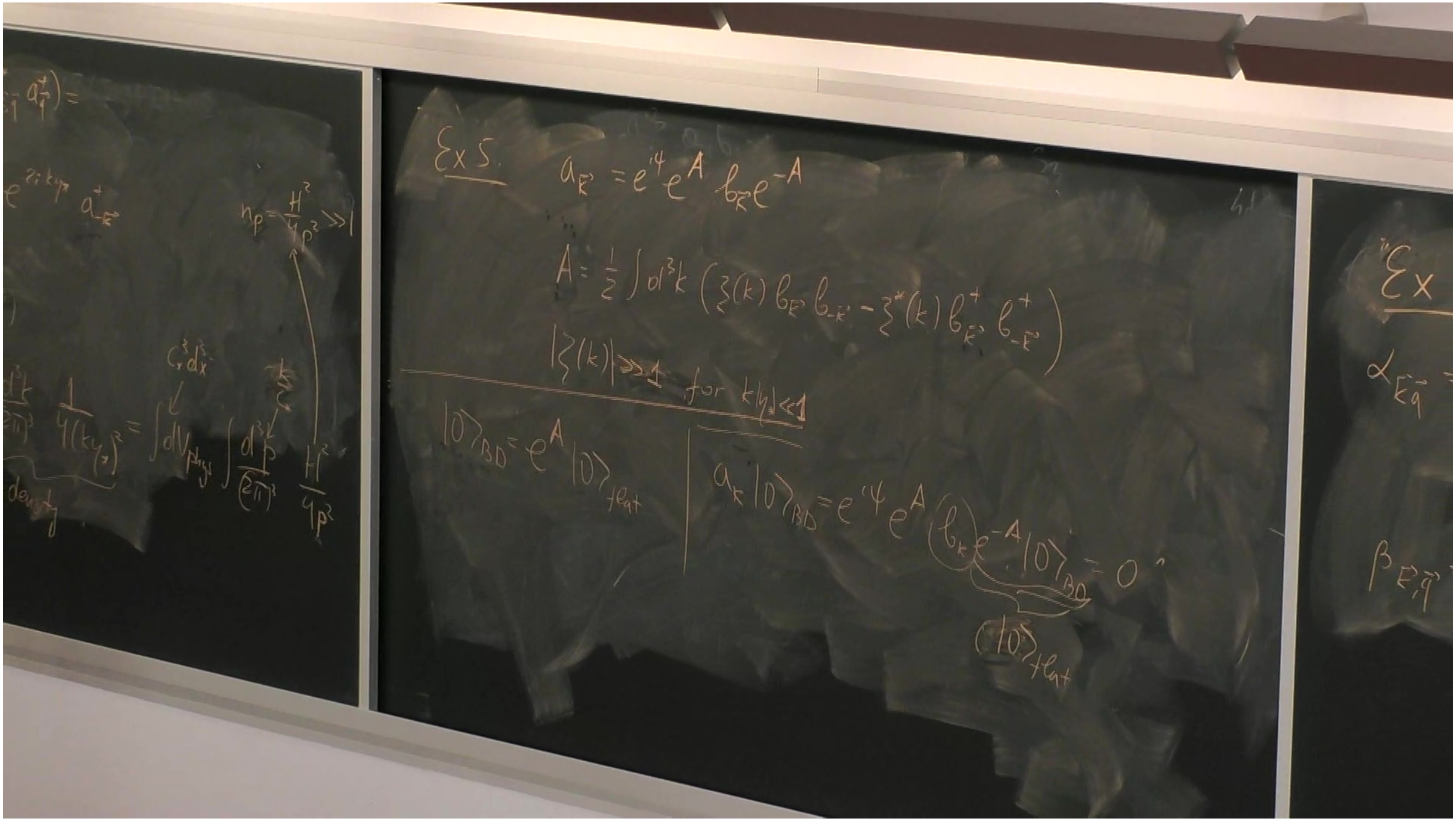
$$\alpha_{\vec{k}\vec{q}} = \langle \tilde{u}_{\vec{k}}, u_{\vec{q}} \rangle = \left(1 + \frac{i}{2k\eta_*} \right) \delta(\vec{k} - \vec{q})$$

flat BD

$$\beta_{\vec{k}\vec{q}} = -\langle \tilde{u}_{\vec{k}}, u^*_{\vec{q}} \rangle = \frac{i}{2k\eta_*} e^{-2ik\eta_*} \delta(\vec{k} + \vec{q})$$



$b_{\vec{k}} = \int d^3q \left(\alpha^*_{\vec{k},\vec{q}}\, a_{\vec{q}} - \beta^*_{\vec{k},\vec{q}}\, a^\dagger_{\vec{q}} \right) =$
$= \left(1 - \frac{i}{2k\eta_*}\right) a_{\vec{k}} + \frac{i}{2k\eta_*} e^{2ik\eta_*} a^\dagger_{-\vec{k}}$
$\langle b^\dagger_{\vec{k}} b_{\vec{k}'} \rangle_{BD} = \frac{1}{4(k\eta_*)^2} \delta(\vec{k}-\vec{k}')$
$N = \int d^3k \langle b^\dagger_k b_k \rangle = \int d^3x \int \frac{d^3k}{(2\pi)^3} \frac{1}{4(k\eta_*)^2} = \int dV_{phys} \int \frac{d^3p}{(2\pi)^3} \frac{H^2}{4p^2}$
density
$n_p = \frac{H^2}{4p^2} \gg 1$
III)


Ex 5. $a_{\vec{k}} = e^{i\varphi} e^{A} b_{\vec{k}} e^{-A}$
$A = \frac{1}{2}\int d^3k \left(\xi(k)\, b_{\vec{k}} b_{-\vec{k}} - \xi^*(k)\, b^+_{\vec{k}} b^+_{-\vec{k}}\right)$
$|\xi(k)| \gg 1$ for $k|\eta_*| \ll 1$
$|0\rangle_{BD} = e^{A} |0\rangle_{flat}$
$a_k |0\rangle_{BD} = e^{i\varphi} e^{A} (b_k e^{-A} |0\rangle_{BD}) = 0$
$|0\rangle_{flat}$
$n_p = \frac{H^2}{4p^2} \gg 1$
density

$$\langle \hat{\varphi}(x)\rangle_{BD} = \dots \langle b\rangle_{BD} + \dots \langle b^+\rangle_{BD} = 0$$

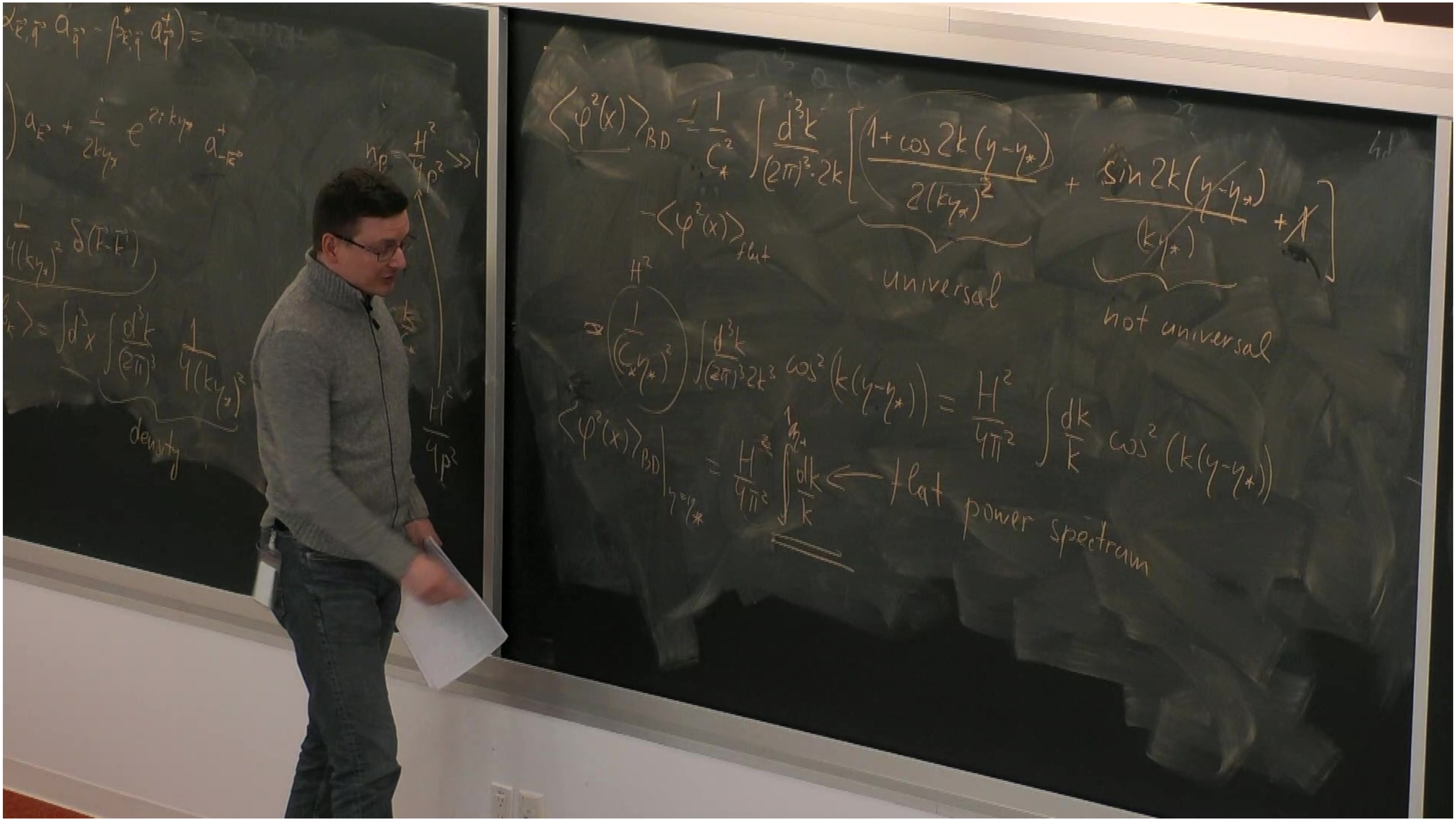
$$\langle \varphi^2(x)\rangle_{BD} \neq 0$$

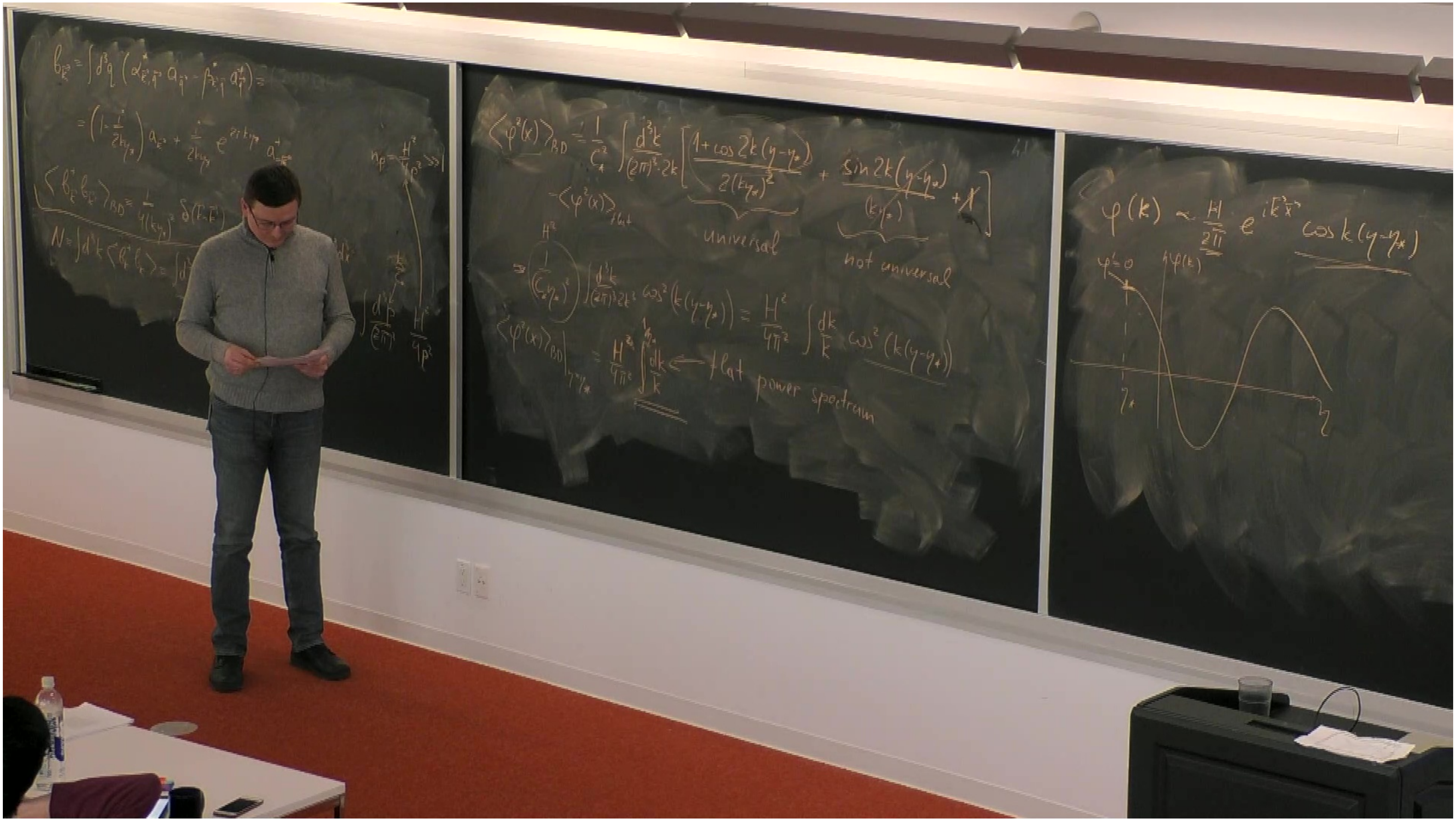
$$\langle b_{\vec{k}}\, b^+_{\vec{k}'}\rangle_{BD} = \left(1 + \frac{1}{4(k\eta_*)^2}\right)\delta(\vec{k}-\vec{k}')$$

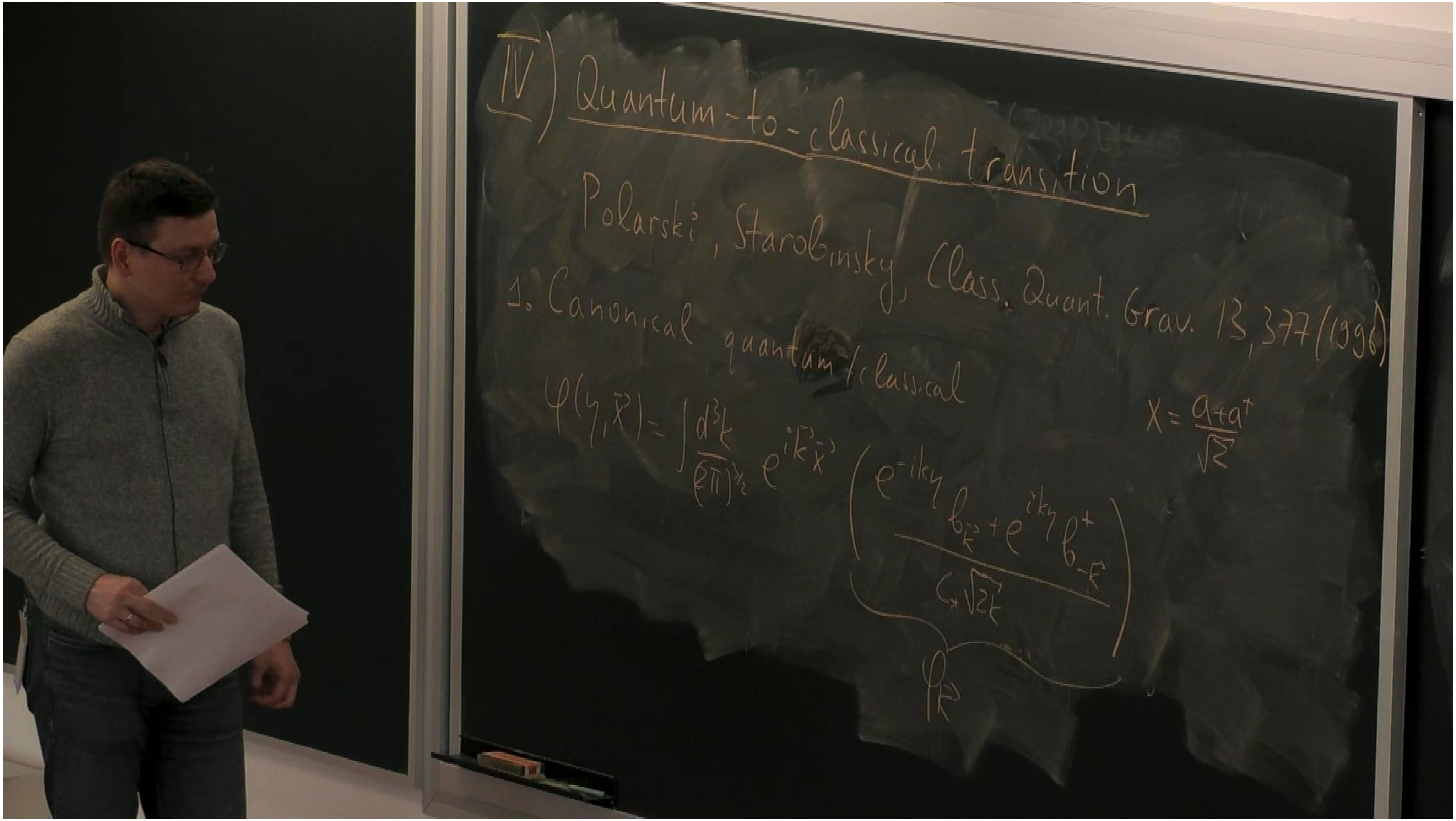
$$\langle b_{\vec{k}}\, b_{\vec{k}'}\rangle_{BD} = \left(\frac{i}{2k\eta_*} + \frac{1}{4(k\eta_*)^2}\right)e^{2ik\eta_*}\,\delta(\vec{k}+\vec{k}')$$

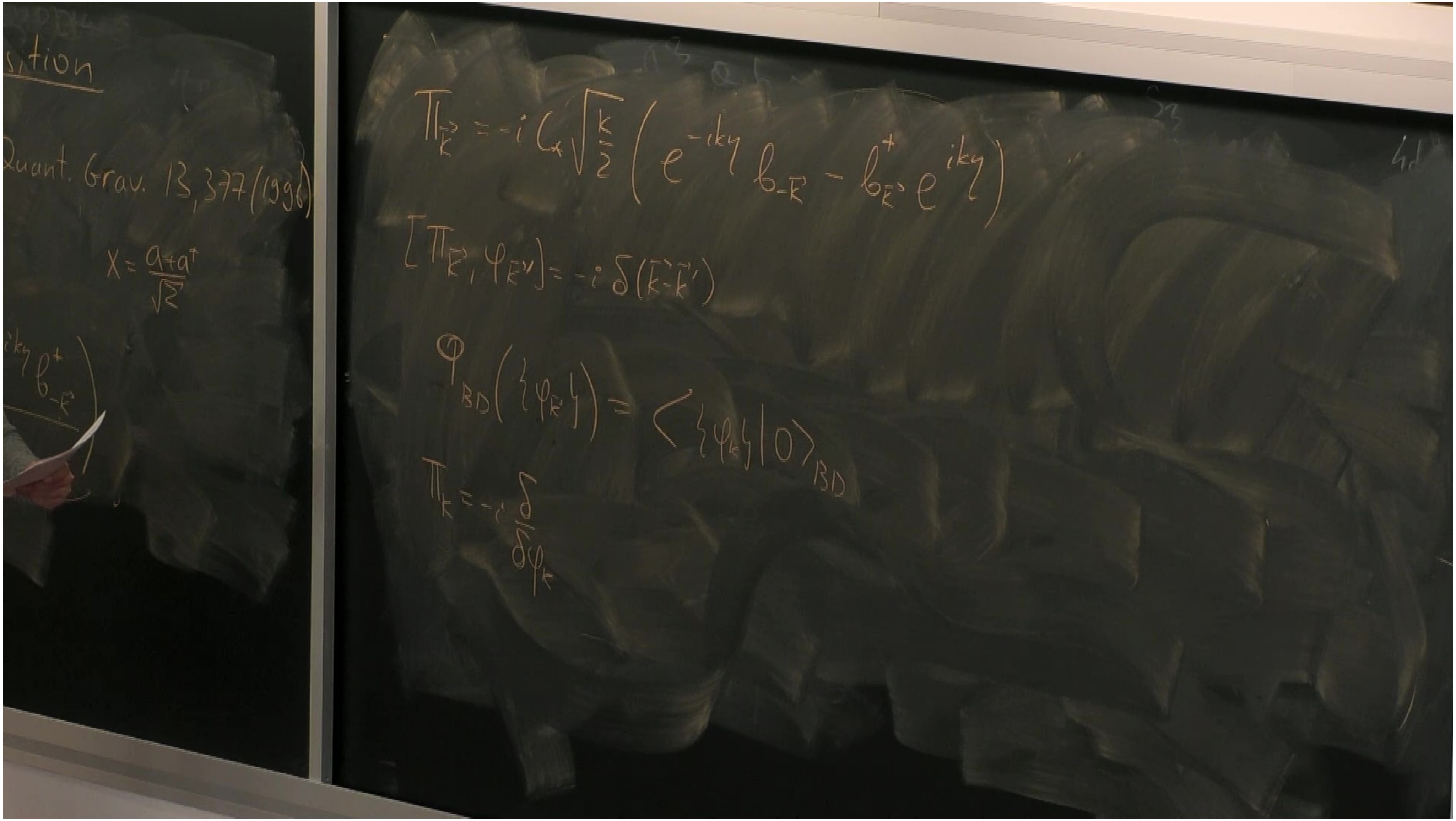
$$\langle b^+_{\vec{k}}\, b^+_{\vec{k}'}\rangle_{BD} = \left(\rule{2em}{0.4pt}\,,\ \rule{2em}{0.4pt}\right)^*$$

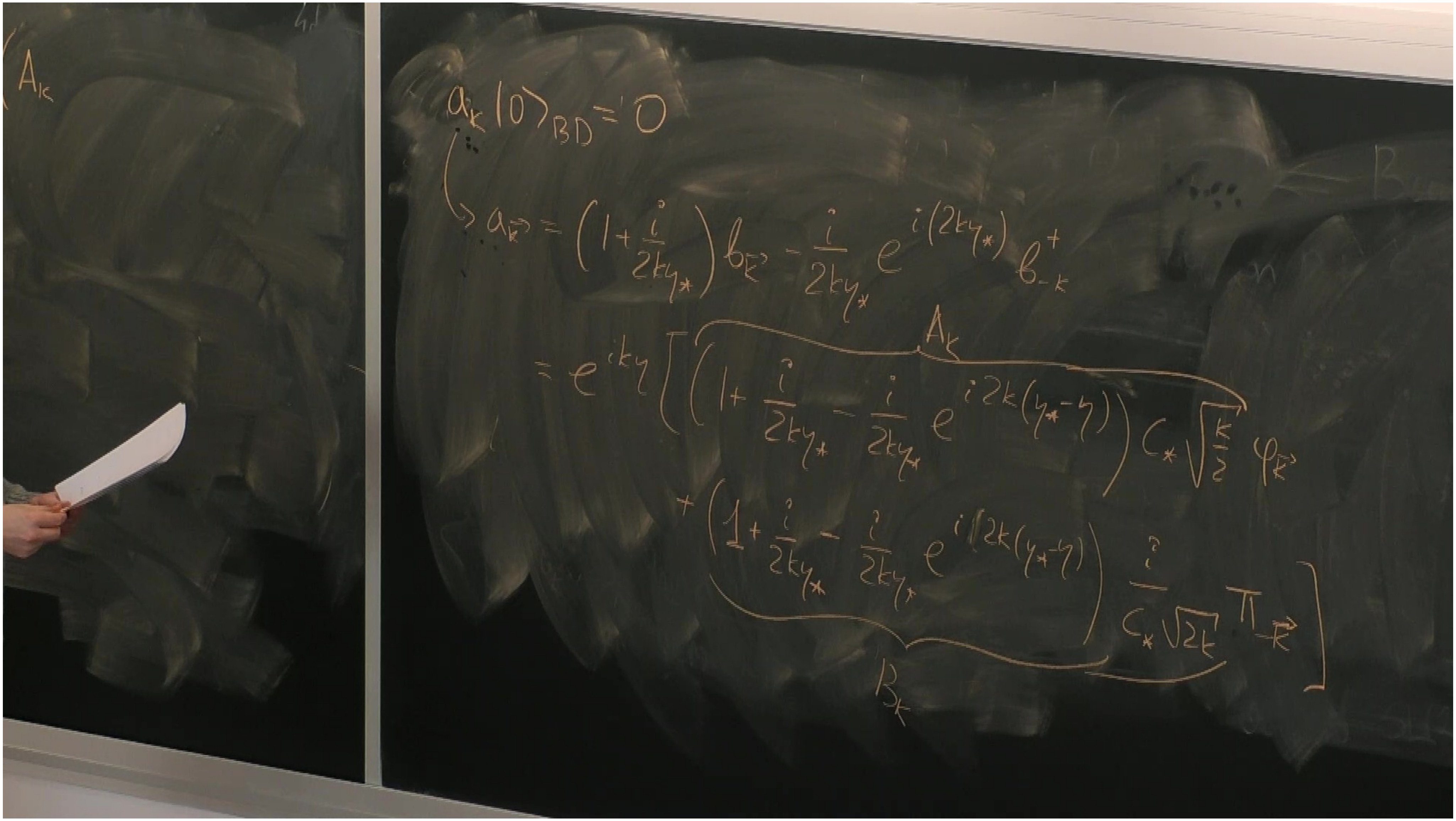
$$\rangle_{BD} = 0$$

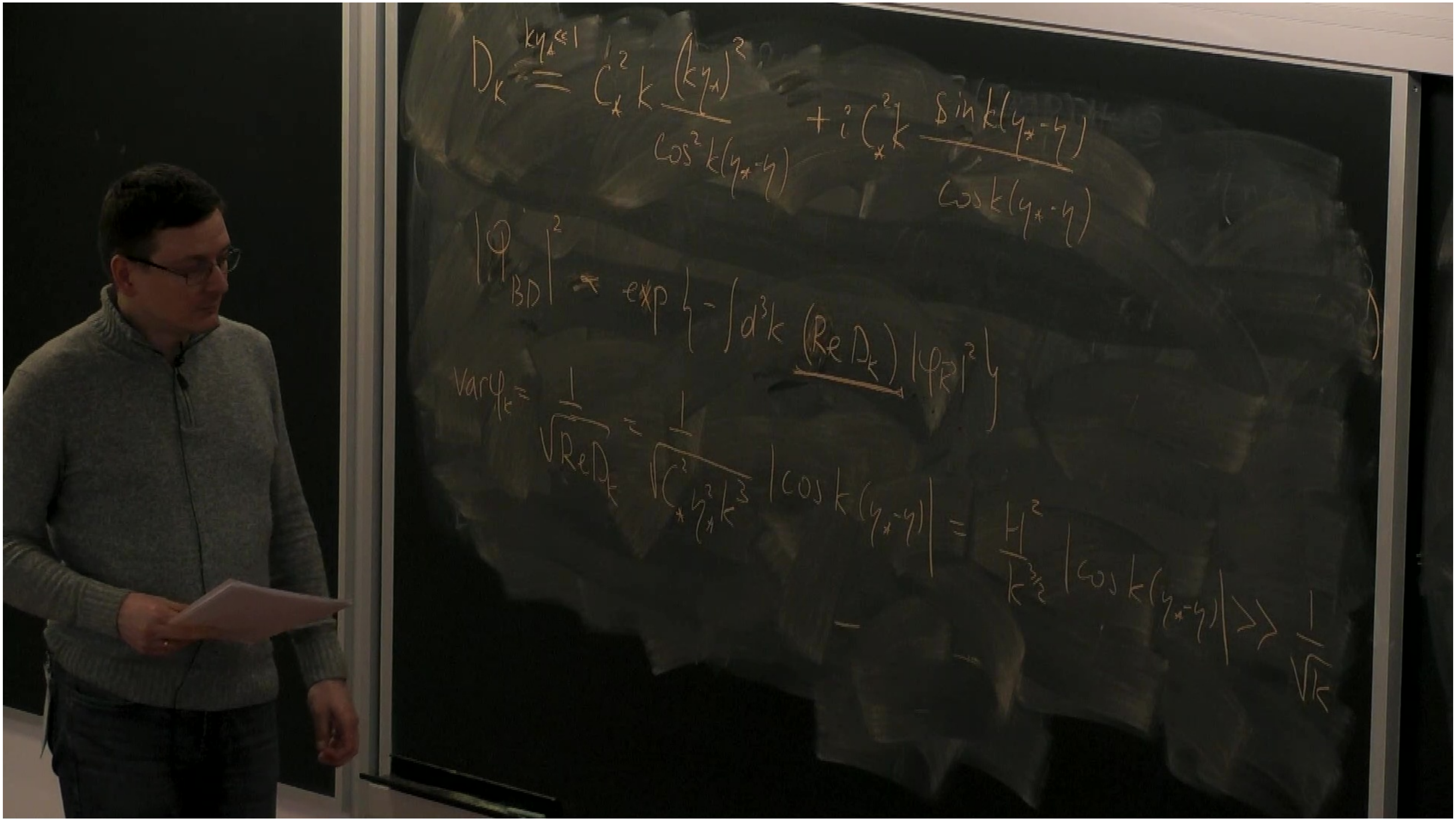

universal
not universal
flat power spectrum
density



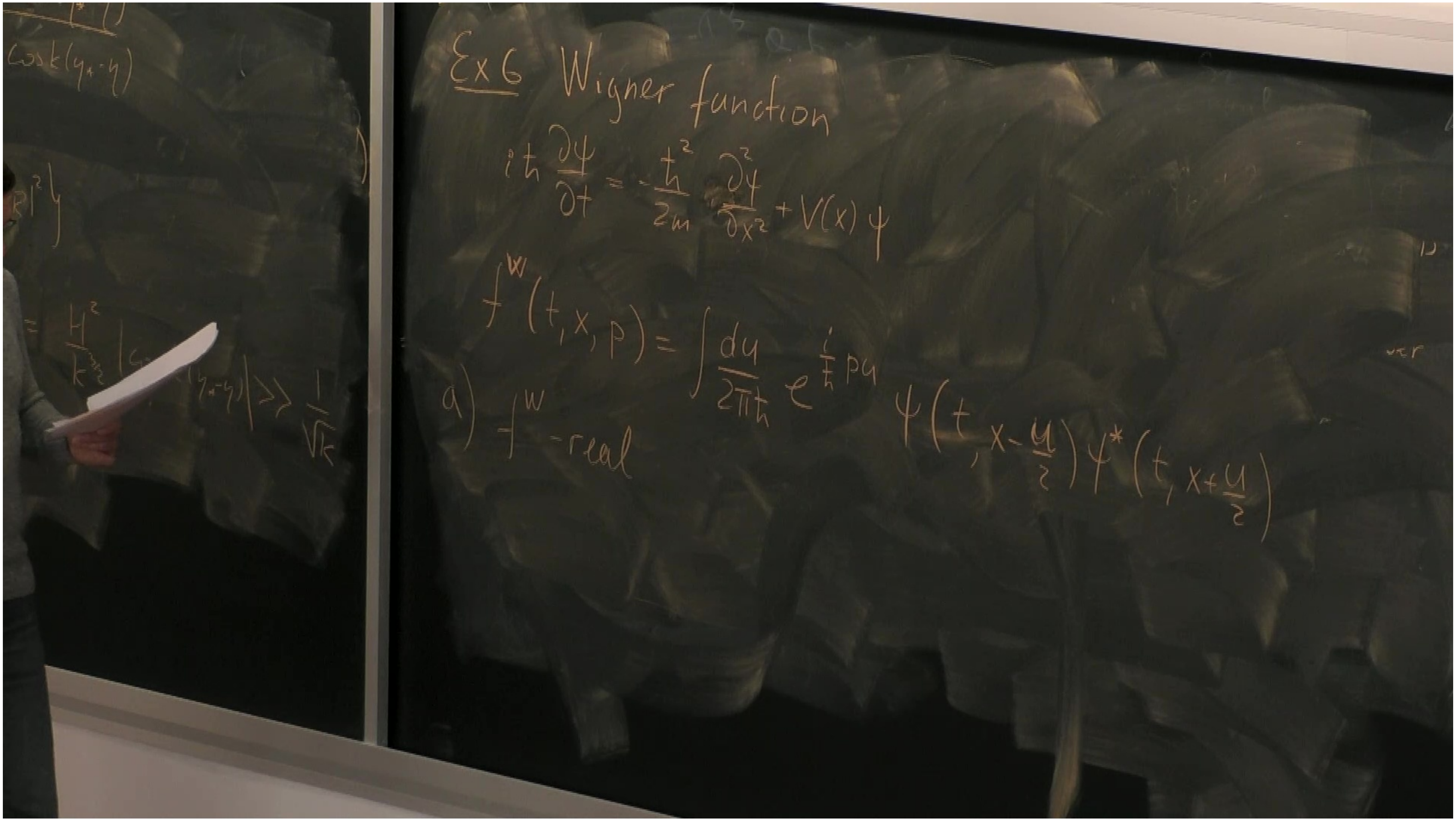

IV) Quantum-to-classical transition
Polarski, Starobinsky, Class. Quant. Grav. 13, 377 (1996)
1. Canonical quantum/classical
$\varphi(\eta, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\vec{x}} \left(\frac{e^{-ik\eta} b_{\vec{k}} + e^{ik\eta} b^{+}_{-\vec{k}}}{c_* \sqrt{2k}} \right)$
$\varphi_{\vec{k}}$
$X = \frac{a + a^{+}}{\sqrt{2}}$


sition
Quant. Grav. 13, 377 (1996)
$x = \frac{a+a^\dagger}{\sqrt{2}}$
$\Pi_{\vec{k}} = -i C_x \sqrt{\frac{k}{2}} \left(e^{-iky} b_{-\vec{k}} - b^\dagger_{\vec{k}} e^{iky} \right)$
$[\Pi_{\vec{k}}, \varphi_{\vec{k}'}] = -i\,\delta(\vec{k}-\vec{k}')$
$\Psi_{BD}(\{\varphi_{\vec{k}}\}) = \langle \{\varphi_k\} | 0 \rangle_{BD}$
$\Pi_k = -i \frac{\delta}{\delta \varphi_k}$


A_k
$a_k |0\rangle_{BD} = 0$
$a_{\vec{k}} = \left(1 + \frac{i}{2ky_*}\right) b_{\vec{k}} - \frac{i}{2ky_*} e^{i(2ky_*)} b^+_{-\vec{k}}$
$= e^{iky}\Big[\underbrace{\left(1 + \frac{i}{2ky_*} - \frac{i}{2ky_*} e^{i2k(y_*-y)}\right) C_* \sqrt{\frac{k}{2}}}_{A_k} \varphi_{\vec{k}}$
$+ \underbrace{\left(1 + \frac{i}{2ky_*} - \frac{i}{2ky_*} e^{i2k(y_*-y)}\right) \frac{i}{C_* \sqrt{2k}}}_{B_k} \pi_{-\vec{k}} \Big]$



$D_k \overset{k\eta_* \ll 1}{=} C_*^2 k \frac{(k\eta_*)^2}{\cos^2 k(\eta_*-\eta)} + i C_*^2 k \frac{\sin k(\eta_*-\eta)}{\cos k(\eta_*-\eta)}$
$|\Psi_{BD}|^2 \approx \exp\{-\int d^3k\, (\mathrm{Re}\, D_k)\, |\varphi_{\vec{k}}|^2\}$
$\mathrm{Var}\,\varphi_k = \frac{1}{\sqrt{\mathrm{Re}\, D_k}} = \frac{1}{\sqrt{C_*^2 \eta_*^2 k^3}} |\cos k(\eta_*-\eta)| = \frac{H^2}{k^{3/2}} |\cos k(\eta_*-\eta)| \gg \frac{1}{\sqrt{k}}$


Ex 6 Wigner function
$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi$
$f^W(t,x,p) = \int \frac{du}{2\pi\hbar} e^{\frac{i}{\hbar}pu} \psi(t, x-\frac{u}{2}) \psi^*(t, x+\frac{u}{2})$
a) f^W - real

$k(y_* - y)$

$$\frac{H^2}{k^{3/2}} |\cos k(y_* - y)| \gg \frac{1}{\sqrt{k}}$$

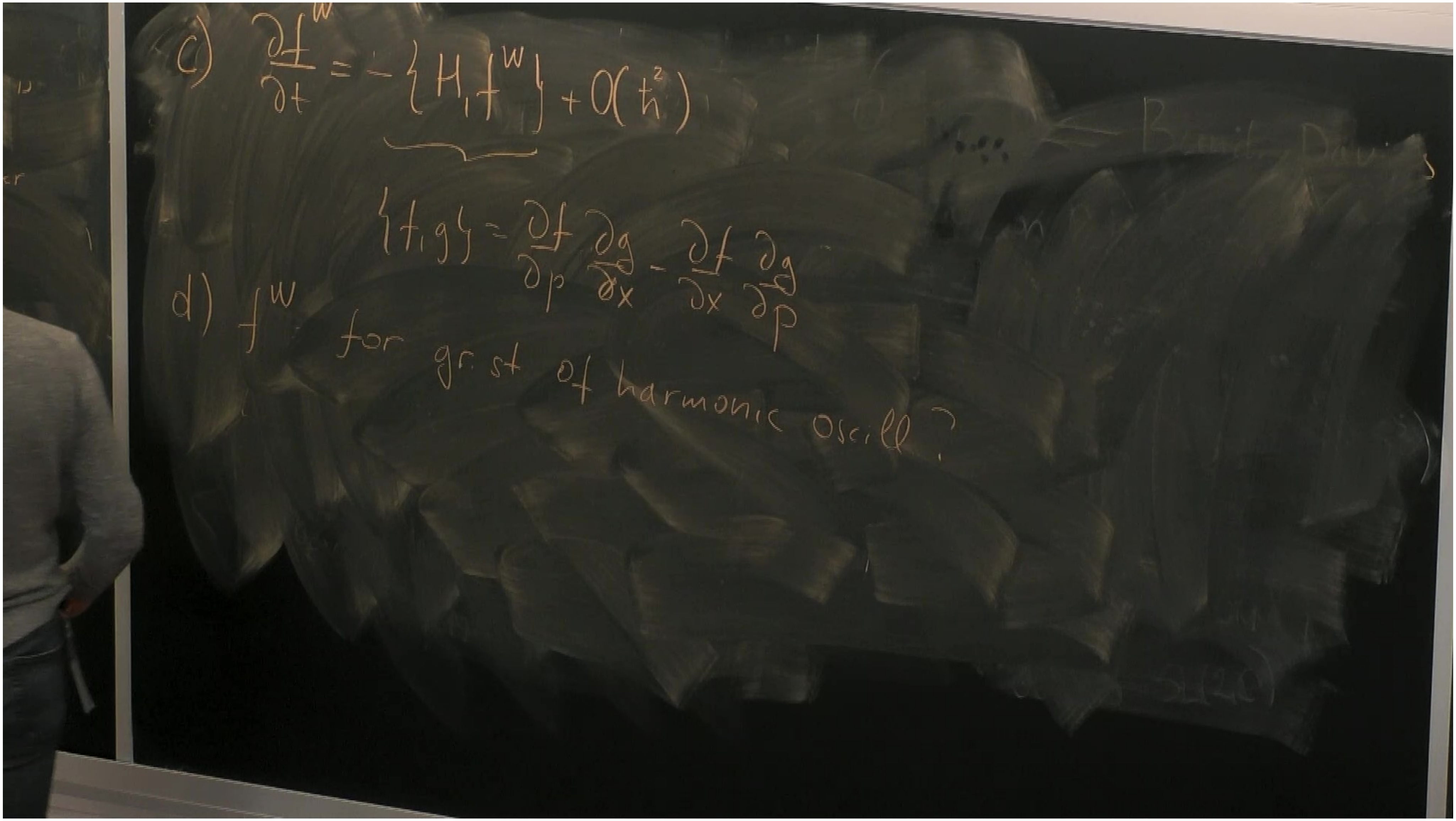
Ex 6 Wigner function

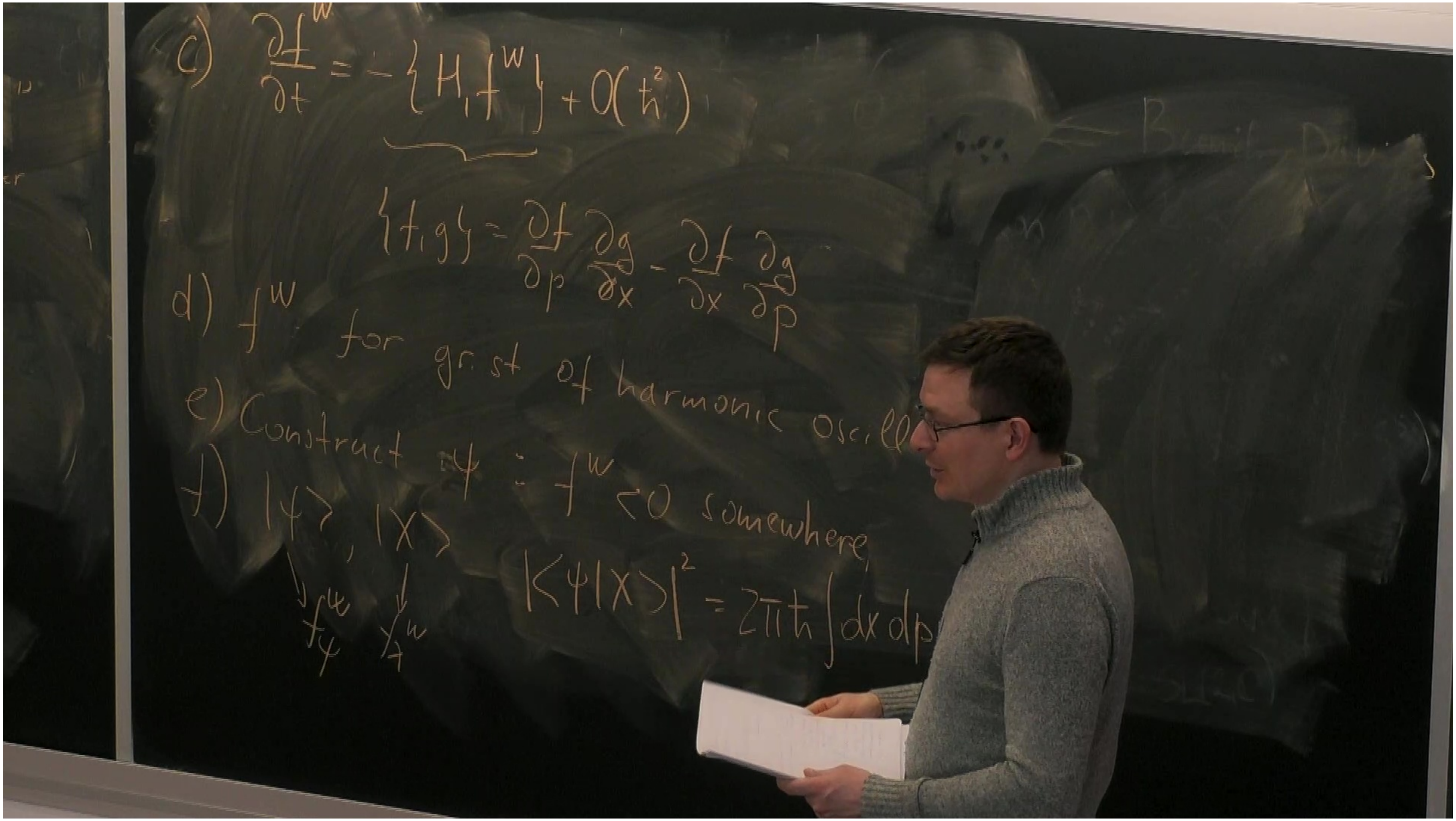
$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\,\psi$$

$$f^W(t,x,p) = \int \frac{du}{2\pi\hbar}\, e^{\frac{i}{\hbar} pu}\, \psi\left(t, x - \frac{u}{2}\right) \psi^*\left(t, x + \frac{u}{2}\right)$$

a) f^W - real

b) $\int dp\, f^W$, $\int dx\, f^W$


c) $\frac{\partial f^W}{\partial t} = -\{H, f^W\} + O(\hbar^2)$
$\{f,g\} = \frac{\partial f}{\partial p}\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial g}{\partial p}$
d) f^W for gr. st of harmonic oscill ?


c) $\frac{\partial f^W}{\partial t} = -\{H, f^W\} + O(\hbar^2)$
$\{f,g\} = \frac{\partial f}{\partial p}\frac{\partial g}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial g}{\partial p}$
d) f^W for gr. st of harmonic oscill
e) Construct ψ : $f^W < 0$ somewhere
f) $|\psi\rangle, |\chi\rangle$
f_ψ^W χ_f^W
$|\langle\psi|\chi\rangle|^2 = 2\pi\hbar \int dx\, dp$