Title: Quantum Field Theory in Curved Spacetime (PM) - 2023-03-24

Speakers: Sergey Sibiryakov

Collection: Quantum Field Theory in Curved Spacetime

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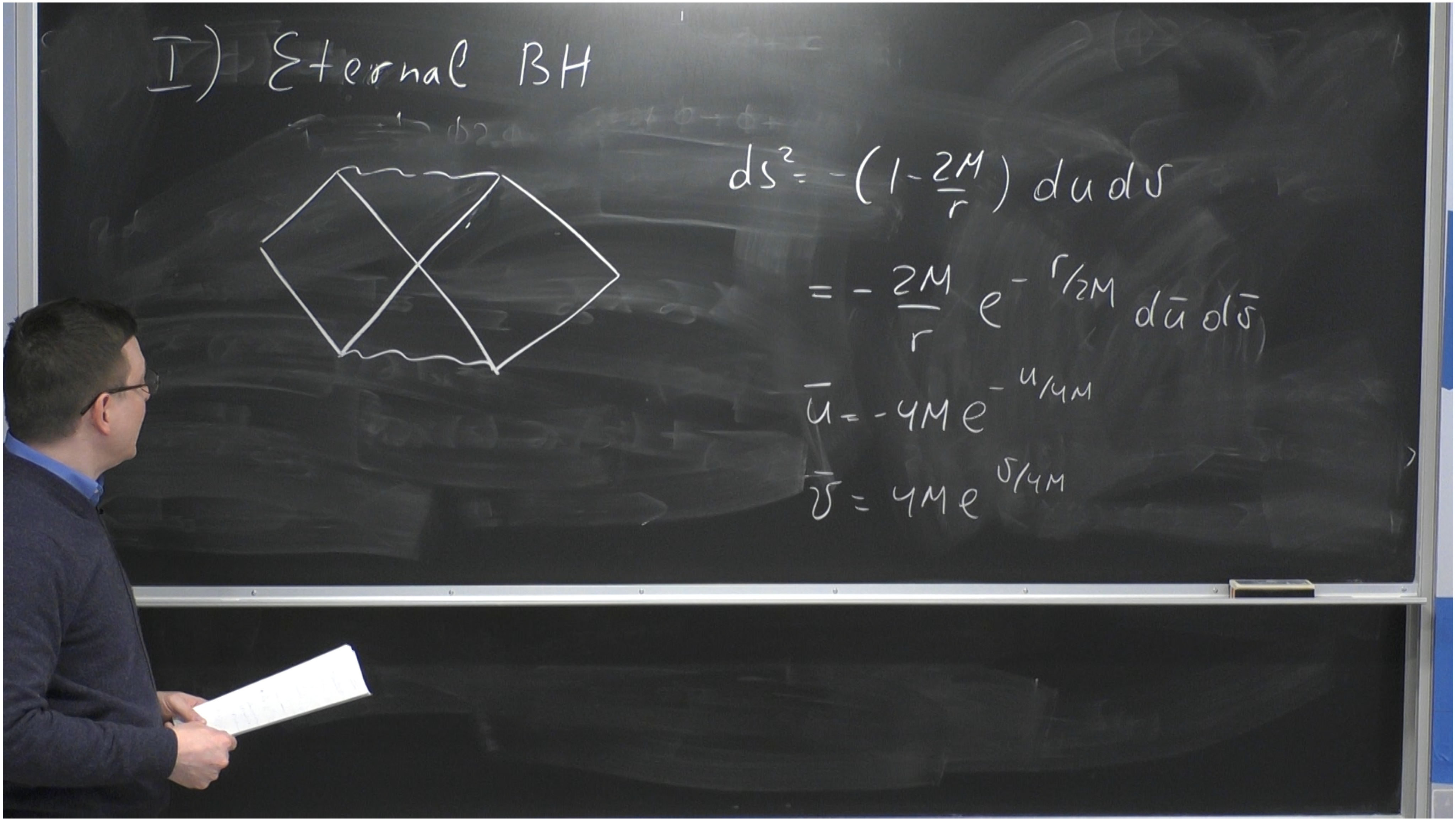
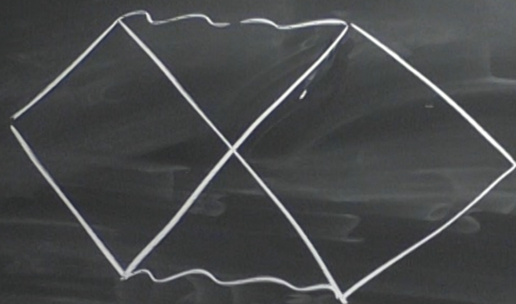
URL: https://pirsa.org/23030097

Abstract: https://pitp.zoom.us/j/95867663528?pwd=dmd5Y3FaVHJ0SnJWSlFscEY4cmhYUT09


I) Eternal BH
$ds^2 = -\left(1-\frac{2M}{r}\right) du\, dv$
$= -\frac{2M}{r} e^{-r/2M} d\bar{u}\, d\bar{v}$
$\bar{u} = -4M e^{-u/4M}$
$\bar{v} = 4M e^{v/4M}$

I) Eternal BH



$$ds^2 = -\left(1-\frac{2M}{r}\right) du\, dv$$

$$= -\frac{2M}{r} e^{-r/2M} d\bar{u}\, d\bar{v}$$

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a) $\varphi^B_{\omega,R} = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega u}$; $\varphi^B_{\omega,L} = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega v}$

$a^B_\omega |0\rangle_B = 0 \leftarrow$ Boulware vacuum

b) $\varphi^{H}_{\omega,R} = \frac{1}{\sqrt{4\pi\omega}}\, e^{-i\omega\bar{u}}$, $\varphi^{H}_{\omega,L} = \frac{1}{\sqrt{4\pi\omega}}\, e^{-i\omega\bar{v}}$

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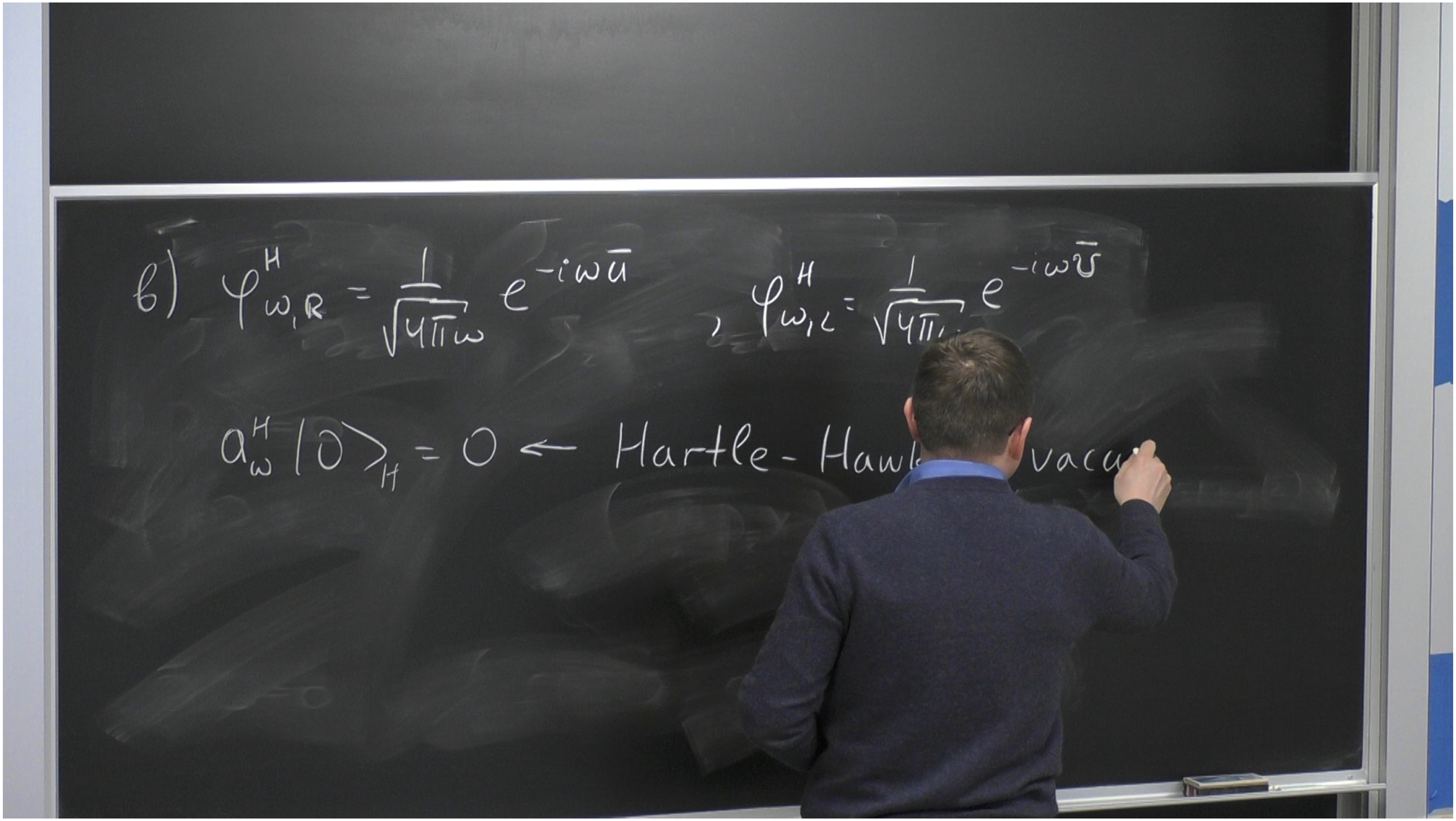
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vacu

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$a^{H}_{\omega} |0\rangle_{H} = 0 \leftarrow$ Hartle-Hawking vacuum

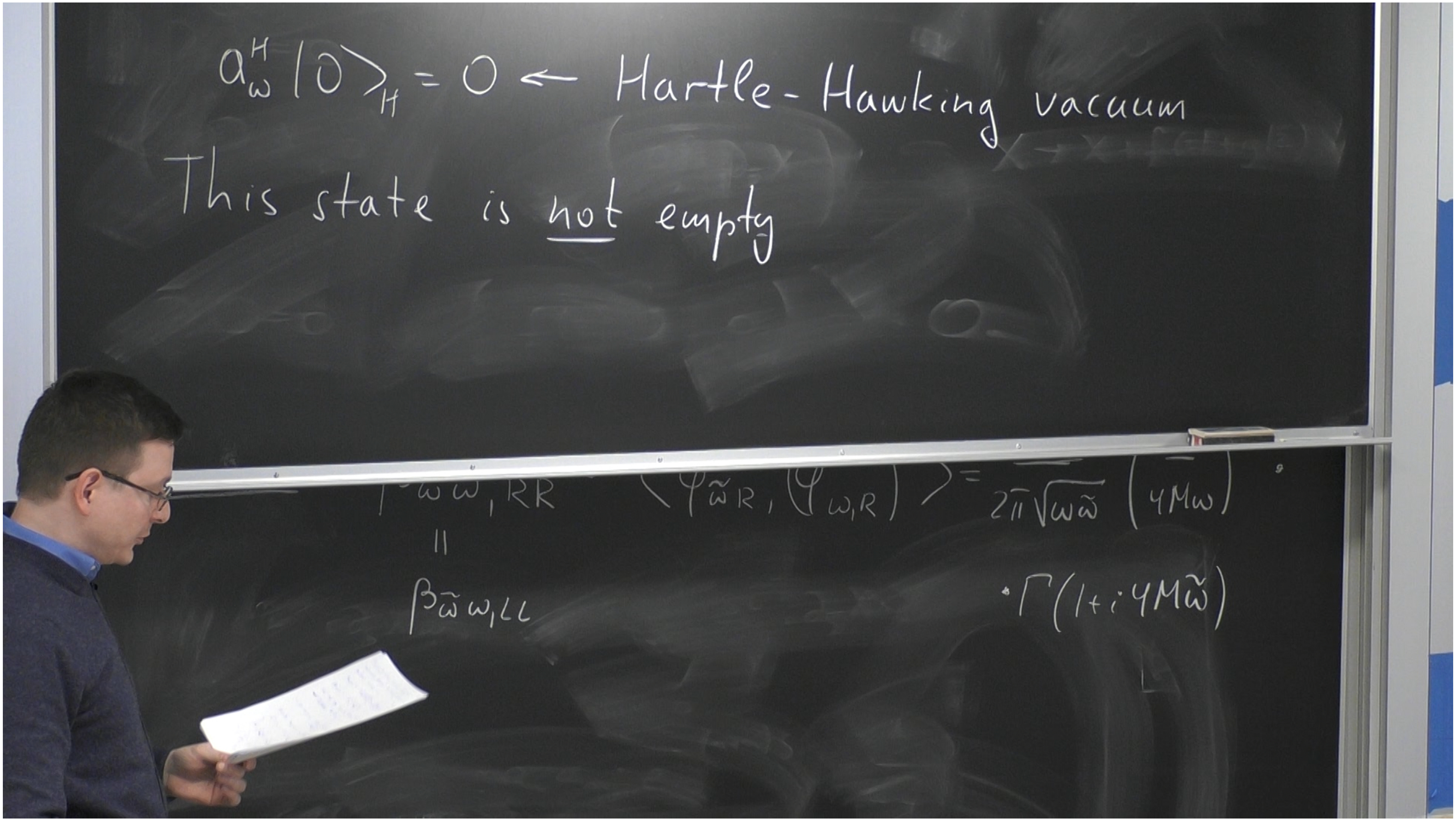
This state is <u>not</u> empty

Ex. 1

$$\beta_{\tilde\omega\omega,RR} = -\left\langle \varphi^{B}_{\tilde\omega R}, \left(\varphi^{H}_{\omega,R}\right)^{*} \right\rangle = \frac{-i}{2\pi\sqrt{\omega\tilde\omega}} \left(\frac{i}{4M\omega}\right)^{i4M\tilde\omega} \cdot \Gamma(1+i\,4M\tilde\omega)$$

$$\parallel$$

$$\beta_{\tilde\omega\omega,LL}$$


a^H_ω |0⟩_H = 0 ← Hartle-Hawking vacuum
This state is not empty
β ω̃ω, LL
·Γ(1+i4Mω̃)

$$G^+(t,r,t',r') = -\frac{1}{4\pi}\ln\left[(\bar{u}-\bar{u}'-i\epsilon)(\bar{v}-\bar{v}'-i\epsilon)\right]$$

$$\|$$

$$\int_0^\infty \frac{d\omega}{4\pi\omega}\left(e^{-i\omega(\bar{u}-\bar{u}')}+e^{-i\omega(\bar{v}-\bar{v}')}\right)$$

$$\partial_{\bar{u}} G^+$$

$$\frac{\quad}{4\pi}\, e \qquad = \qquad 4\pi\left(\bar{u}-\bar{u}'-i\epsilon\right)$$

1. $r = r'$ - fixed

$$\bar{u}-\bar{u}' = -4M\left(e^{-\frac{u}{4M}} - e^{-\frac{u'}{4M}}\right)$$

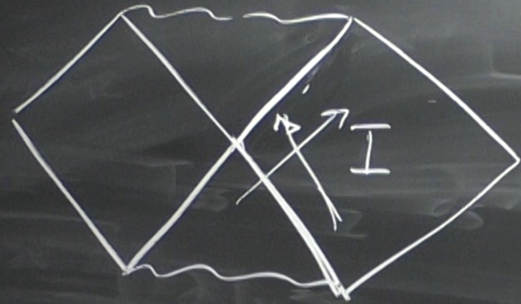
$$=$$

This state is <u>not</u> empty

$\beta_{\tilde{\omega}\omega,LL}$ $\cdot\,\Gamma(1+i\,4M\tilde{\omega})$

$$G^{+}(t,r,t',r) = -\frac{1}{4\pi}\ln\left[\mathrm{ch}\left(\frac{t-t'-i\epsilon}{4M}\right)-1\right]+\tilde{G}(r)$$

$$= -\frac{1}{4\pi}\ln\left[\mathrm{ch}\left(\frac{\tau-\tau'-i\epsilon}{4M\sqrt{1-\frac{2M}{r}}}\right)\right.$$



$$ds^2 = -\left(1-\frac{2M}{r}\right) du\, dv \rightarrow \tau = \sqrt{1-\frac{2M}{r}}\, t$$

$$= -\frac{2M}{r} e^{-r/2M} d\bar{u}\, d\bar{v}$$

$$u = t - r_*$$

$$v = t + r_*$$

$$\bar{u} = -4M e^{-u/4M}$$

$$\bar{v} = 4M e^{v/4M}$$

This state is <u>not</u> empty

$\beta_{\tilde{\omega}\omega,LL}$ $\cdot\Gamma(1+i4M\tilde{\omega})$

$$G^+(t,r,t',r) = -\frac{1}{4\pi}\ln\left[\mathrm{ch}\left(\frac{t-t'-i\epsilon}{4M}\right)-1\right]+\tilde{G}(r)$$

$$\Delta\tau = i8\pi M\sqrt{1-\frac{2M}{r}} \qquad = -\frac{1}{4\pi}\ln\left[\mathrm{ch}\left(\frac{\tau-\tau'-i\epsilon}{4M\sqrt{1-\frac{2M}{r}}}\right)-1\right]+\tilde{G}(r)$$

$$u = t - r_*$$
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a) $\varphi^B_{\omega,R} = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega u}$; $\varphi^B_{\omega,L} = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega v}$

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$\hookrightarrow F_B(\Delta E) = 0$

$$= \int_0^\infty \frac{d\omega}{4\pi\omega}\left(e^{-i\omega(\bar{u}-\bar{u}')} + e^{-i\omega(\bar{v}-\bar{v}')}\right)$$

$$\partial_{\bar{u}} G^+ = (-i)\int_0^\infty \frac{d\omega}{4\pi}\, e^{-i\omega(\bar{u}-\bar{u}')} = -\frac{1}{4\pi(\bar{u}-\bar{u}'-i\epsilon)}$$

$$G_B^+ = -\frac{1}{4\pi}\ln\left[(u-u'-i\epsilon)(v-v'-i\epsilon)\right]$$

b) $\varphi^{H}_{\omega,R} = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega\bar{u}}$, $\varphi^{H}_{\omega,L} = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega\bar{v}}$

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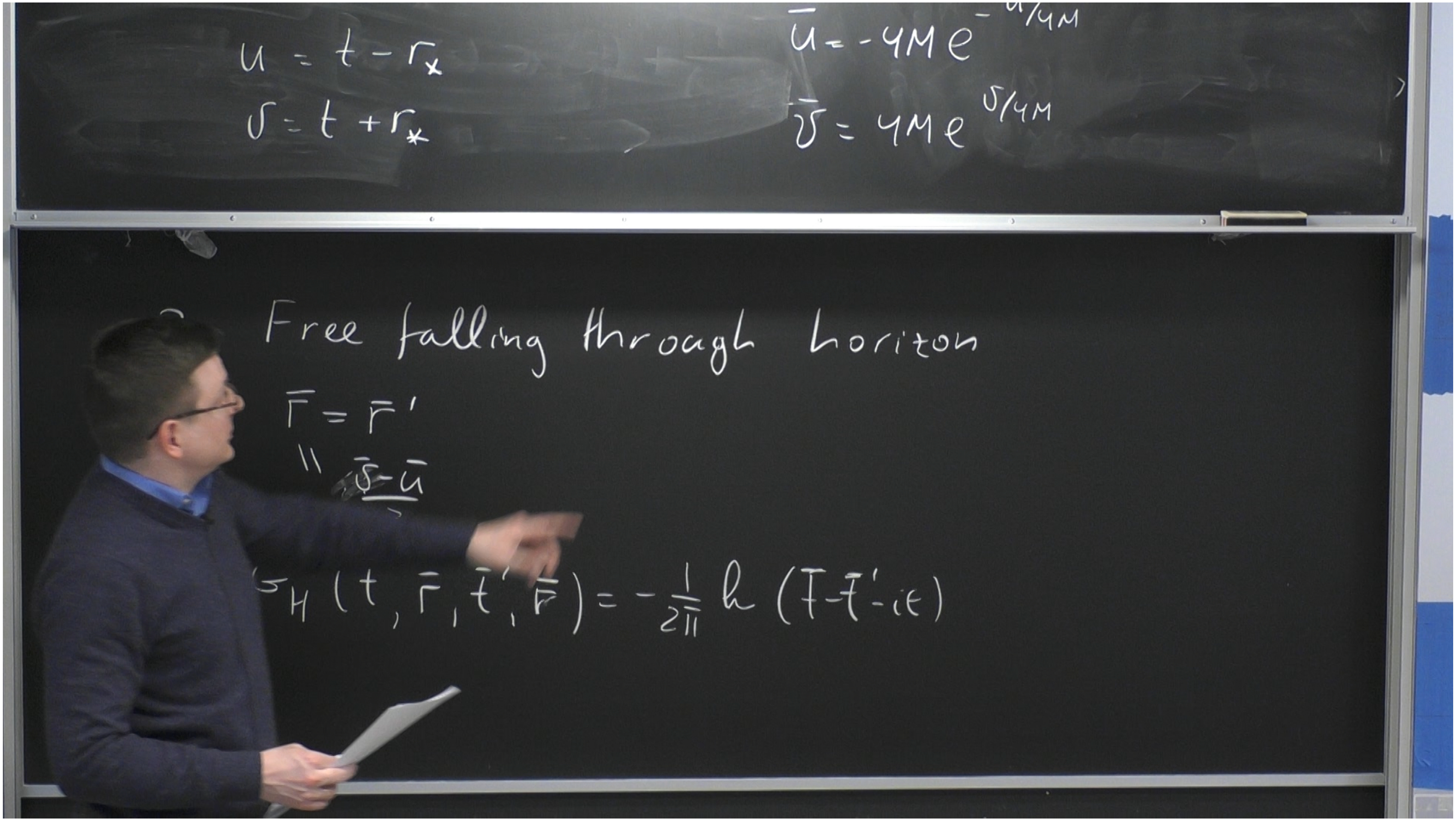
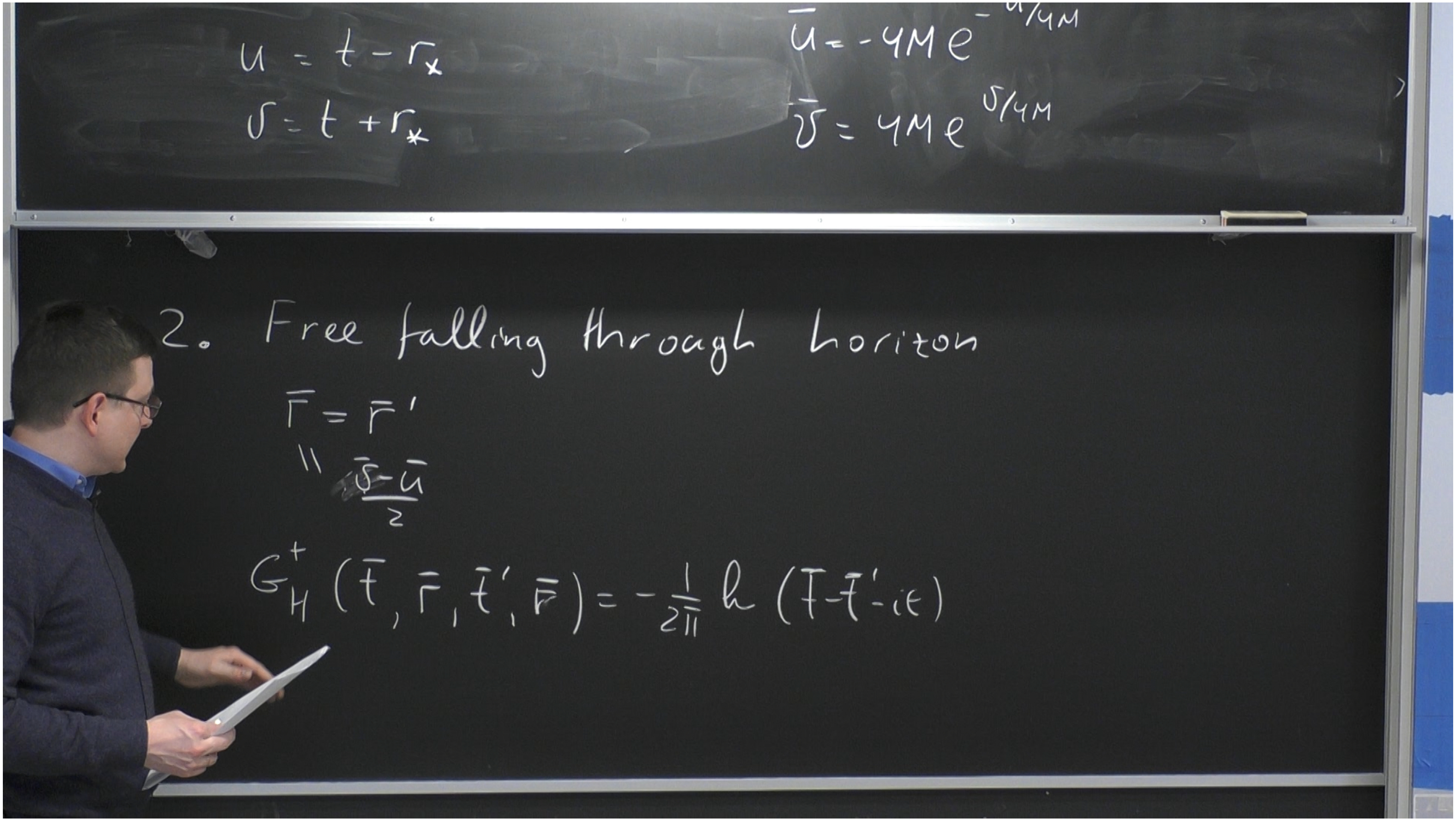

u = t - r*
v = t + r*
ū = -4M e^-u/4M
v̄ = 4M e^v/4M
Free falling through horizon
r̄ = r̄'
v̄ - ū
G_H (t, r̄, t̄', r̄) = - 1/2π ln (T̄ - T̄' - iε)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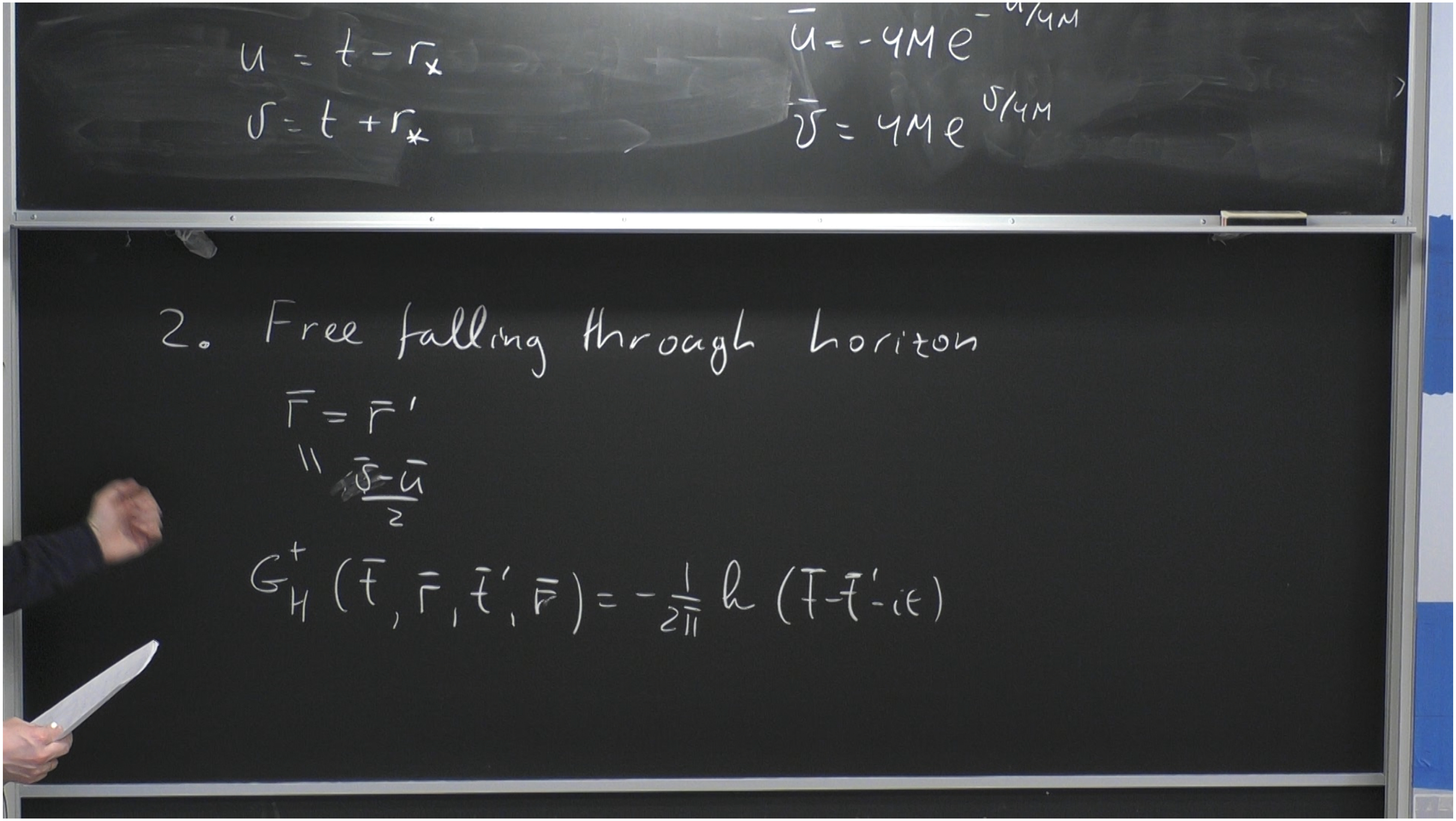
2. Free falling through horiton
$\bar{r} = \bar{r}'$
$\parallel \frac{\bar{v}-\bar{u}}{2}$
$G_H^+(\bar{t}, \bar{r}, \bar{t}', \bar{r}) = -\frac{1}{2\pi} \ln(\bar{t}-\bar{t}'-i\epsilon)$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$$u = t - r_*$$

$$v = t + r_*$$

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2. Free falling through horizon

$$\bar{r} = \bar{r}'$$

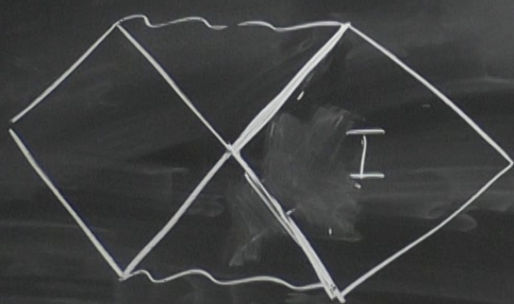
$$\bar{r} = \frac{\bar{v} - \bar{u}}{2}$$

$$G_H^+(\bar{t}, \bar{r}, \bar{t}', \bar{r}) = -\frac{1}{2\pi} \ln(\bar{t} - \bar{t}' - i\epsilon)$$

$_{\beta}(\Delta E) = 0$

I) Eternal BH

$$\varphi^{u}_{\omega,R} = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega\bar{u}} \quad , \quad \varphi^{u}_{\omega,L} = \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega v}$$



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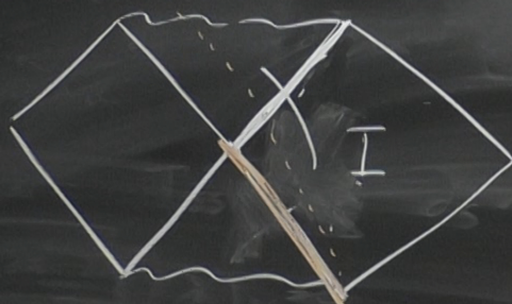

$u = t - r_*$

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$\Gamma_\beta(\Delta E) = 0$

II) HR in a bigger picture

a) GR + horizons + QFT $\Rightarrow$ thermality

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$\overset{?}{\Longleftarrow}$

thermal + horizons + QFT $\Longrightarrow$ GR

T. Jacobson, PRL, 75, 1260 (1995)

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II) H.R in a bigger picture

a) GR + horizons + QFT $\overset{?}{\Longleftarrow}\!\!\Longrightarrow$ thermality

thermal + horizons + QFT $\Longrightarrow$ GR

T. Jacobson, PRL, 75, 1260 (1995)

$$\beta_{\tilde{\omega}\,\omega,\ell\ell} \qquad \cdot\,\Gamma(1+i\,4M\tilde{\omega})$$

b) Trans-Planckian problem

BT , U or HH $\to$ B

$\uparrow$ t , $\omega \propto e^{t/2M}$

$$\beta_{\tilde{\omega}\omega,LL} \qquad \cdot\,\Gamma(1+i4M\tilde{\omega})$$

b) Trans-Planckian problem

BT, U or HH $\rightarrow$ B $\leftarrow$ t, $\omega \propto e^{t/2M}$

Unruh, Schützhold, PRD, 71, 024028 (2005)

$$\parallel$$

$\beta_{\tilde{\omega}\omega,LL}$ $\cdot\Gamma(1+i4M\tilde{\omega})$

b) Trans-Planckian problem

BT, U or HH $\to$ B

$t, \quad \omega \propto e^{t/2M}$

Unruh, Schützhold, PRD, 71, 024028 (2005)

III) Information paradox

Hawking, PRD, 14, 2460 (1976)

Page, PRL, 71, 1291 (1993)

——— " ———, 3743 — " —

Susskind, Thorlasius, Uglum, PRD, 48, 3743 (1993)

III) Information paradox

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Page, PRL, 71, 1291 (1993)

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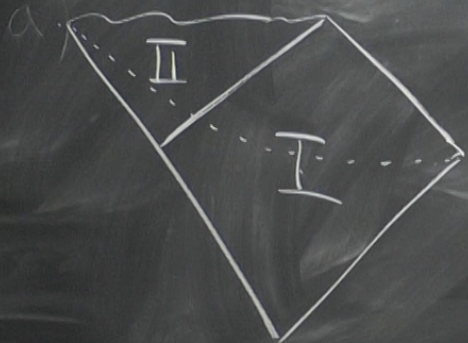
Susskind, Thorlasius, Uglum, PRD, 48, 3743 (1993)

1) First formulation



$$|0\rangle_U \longleftrightarrow \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega \bar{u}} \qquad \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega v}$$

$$|0\rangle_B \longleftrightarrow \frac{1}{\sqrt{4\pi\omega}} e^{-i\omega u} \qquad \text{— " —}$$

$$\varphi_\omega^{B(I)} = \begin{cases} \bar{u} < 0 : & \frac{1}{\sqrt{4\pi\omega}}\, e^{i4M\omega \ln(-\bar{u}/4M)} \\ \bar{u} > 0 : & 0 \end{cases}$$

$$\varphi_\omega^{B(II)} = \begin{cases} \bar{u} < 0 : & 0 \\ \bar{u} > 0 : & \frac{1}{\sqrt{4\pi\omega}}\, e^{-i4M\omega \ln(\bar{u}/4M)} \end{cases}$$

Susskind, Thorlacius, Uglum, PRD, 48, 3743 (1993)

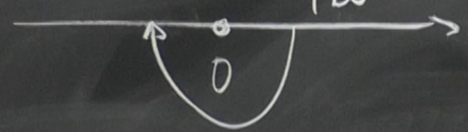
$$\varphi_\omega^{B(I)} \Longrightarrow \tilde{\varphi}_\omega^{U(I)} = \frac{1}{\sqrt{4\pi\omega}} \cdot \frac{1}{\sqrt{1-e^{-8\pi M\omega}}} \times$$

$$\times \begin{cases} e^{i\omega 4M \ln(-\bar{u}/4M)}, & \bar{u}<0 \\ e^{-4\pi M\omega}\, e^{i\omega 4M \ln(\bar{u}/4M)}, & \bar{u}>0 \end{cases}$$

$\left(\bar{u}>0:\ \frac{1}{\sqrt{4\pi\omega}}\, e^{\cdots 4\pi\omega \ln(\bar{u}/4M)}\right)$

$\bar{u}$

$\varphi_\omega^{B(II)}$



$$\tilde{\varphi}_\omega^{u(II)} = \frac{1}{\sqrt{4\pi\omega}\sqrt{1-e^{-8\pi M\omega}}}\begin{cases} e^{-4\pi M\omega}\, e^{-i\omega 4M \ln(-\bar{u}/4M)}, & \bar{u}<0 \\ e^{-i\omega 4M \ln(\bar{u}/4M)}, & \bar{u}>0 \end{cases}$$

$$\tilde{\varphi}_\omega^{u(\mathrm{I},\mathrm{II})} \longrightarrow \tilde{a}_\omega^{u(\mathrm{I},\mathrm{II})} \qquad |\beta_\omega^{\mathrm{I}\,\mathrm{II}}|^2 = \frac{1}{e^{8\pi M\omega}-1}$$

$$\tilde{\varphi}_\omega^{u(\mathrm{I})} = \frac{1}{\sqrt{1-e^{-8\pi M\omega}}}\,\varphi_\omega^{B(\mathrm{I})} + \frac{e^{-4\pi M\omega}}{\sqrt{\quad}}\left(\varphi_\omega^{B(\mathrm{II})}\right)^*$$

$$\tilde{\varphi}_\omega^{u(\mathrm{II})} = \frac{e^{-4\pi M\omega}}{\sqrt{\quad}}\left(\varphi_\omega^{B(\mathrm{I})}\right)^* + \frac{1}{\sqrt{\quad}}\,\varphi_\omega^{B(\mathrm{II})}$$

$$\tilde{a}_\omega^{u(I)} = \frac{1}{\sqrt{}}\, a_\omega^{B(I)} - \frac{e^{-4\pi M\omega}}{\sqrt{}} \left(a_\omega^{B(II)}\right)^\dagger$$

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Ex 3 $\quad K\left(a^B, (a^B)^\dagger\right)$: $\quad \tilde{a}_\omega^{u(I)} = e^{-iK}\, a_\omega^{B(I)}\, e^{iK}$

$$\tilde{a}_\omega^{u(II)} = -e^{-iK}\, a_\omega^{B(II)}\, e^{iK}$$

$\left(\ldots /4M \right),\ \bar{u} > 0$

Ex: $K\left(a^{B}, (a^{B})^{\dagger}\right)$:

$$\tilde{a}^{u(\mathrm{I})}_{\omega} = e^{-iK}\, a^{B(\mathrm{I})}_{\omega}\, e^{iK}$$

$$\tilde{a}^{u(\mathrm{II})}_{\omega} = -e^{-iK}\, a^{B(\mathrm{II})}_{\omega}\, e^{iK}$$

$$0 = \tilde{a}^{u}_{\omega}|0\rangle_{u} = \pm e^{-iK}\, a^{B}_{\omega}\, e^{iK}|0\rangle_{u}$$

$$\hookrightarrow a^{B}_{\omega}\Big(\underbrace{e^{iK}|0\rangle_{u}}_{|0\rangle_{B}}\Big) = 0$$

Susskind, Thorlacius, Uglum, PRD, 48, 3743 (1993)

$$|0\rangle_U = e^{-iK}|0\rangle_B = \sum K_{\omega_1\ldots\omega_k;\omega_1'\ldots\omega_k'} \left(a^{B(I)}_{\omega_1}\right)^\dagger \cdots \left(a^{B(I)}_{\omega_k}\right)^\dagger \cdot \underbrace{\left(a^{B(II)}_{\omega_1'}\right)^\dagger \cdots \left(a^{B(II)}_{\omega_n'}\right)^\dagger}_{\text{not accessible}} |0\rangle_B$$

$$\underline{\rho} = \mathrm{Tr}_{\mathrm{II}}\, |0\rangle_u \langle 0|_u = \sum_{\omega'} K_{\omega_1 \ldots \omega_k ; \omega_1' \ldots \omega_n'} K^*_{\Omega_1 \ldots \Omega_\ell ; \omega_1' \ldots \omega_n'}$$

$$\cdot \, |\omega_1 \ldots \omega_k\rangle_B \langle \Omega_1 \ldots \Omega_\ell|_B$$

Ex 4

$$\rho_I = \frac{1}{Z} \exp\left[-\frac{1}{T_H} \int d\omega \, \omega \left(a_\omega^{B(I)}\right)^\dagger a_\omega^{B(I)} \right]$$

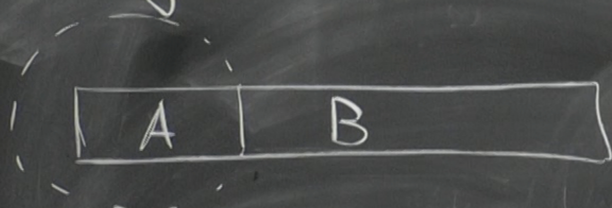
$$\rho = \mathrm{Tr}_{\mathrm{II}}\, |0\rangle_u \langle 0|_u = \sum_{\omega'} K_{\omega_1 \dots \omega_k; \omega_1' \dots \omega_n'} K^*_{\Omega_1 \dots \Omega_\ell; \omega_1' \dots \omega_n'}$$

$$\cdot \, |\omega_1 \dots \omega_k\rangle_B \langle \Omega_1 \dots \Omega_\ell|_B$$

Ex 4

$$\rho_I = \frac{1}{Z} \exp\left[-\frac{1}{T_H} \int d\omega\, \omega \left(a_\omega^{B(I)}\right)^\dagger a_\omega^{B(I)} \right]$$

2) Page's scenario



$N \gg 1$ - spins $|\downarrow\rangle, |\uparrow\rangle$

2^N

$$|\Psi\rangle = \sum U_{AB} |\Psi_A\rangle \otimes |\Psi_B\rangle$$

$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi| = \sum_B U_{AB} U^*_{A'B} |\Psi_A\rangle\langle\Psi_{A'}|$$

$$S_A = -\mathrm{Tr}(\rho_A \ln \rho_A)$$

Ex 5.

1) If $\rho_A \Leftrightarrow$ pure state $\Rightarrow S_A = 0$

2) $S_A \leq N_A \ln 2$

3) If ρ_A is thermal $\Rightarrow S_A = S_{\text{thermodynamic}}$

4)

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Ex 5.

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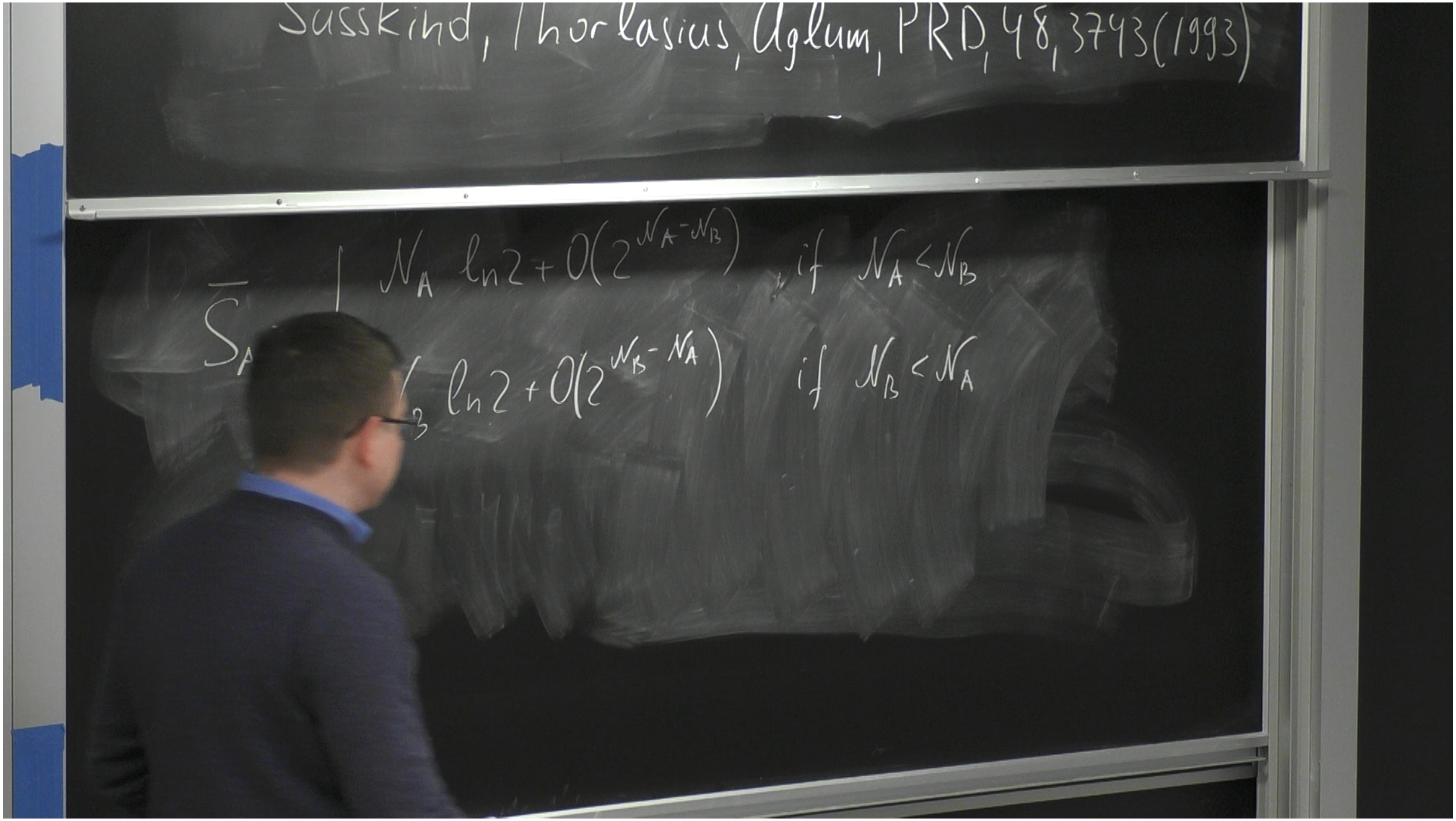
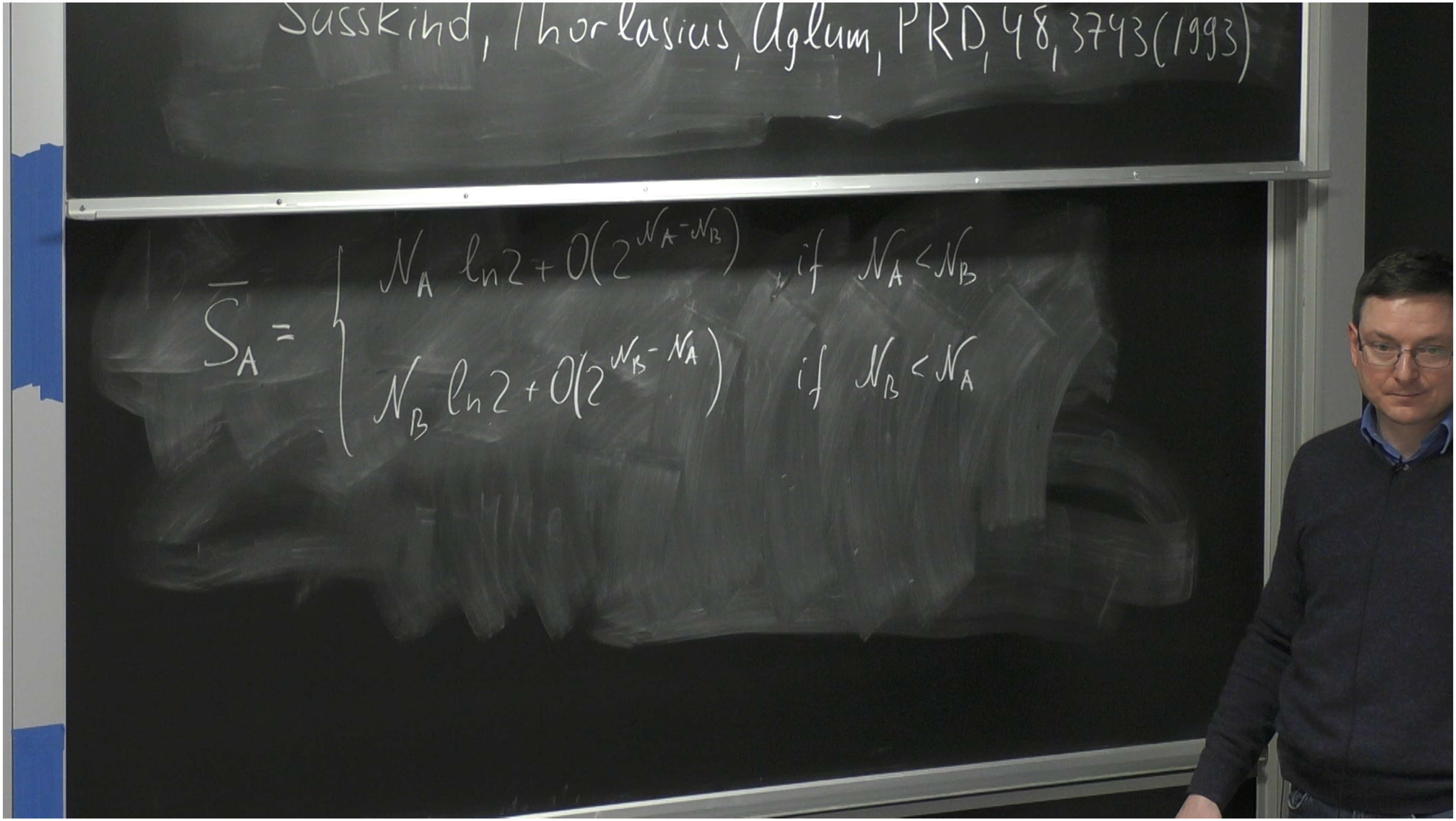
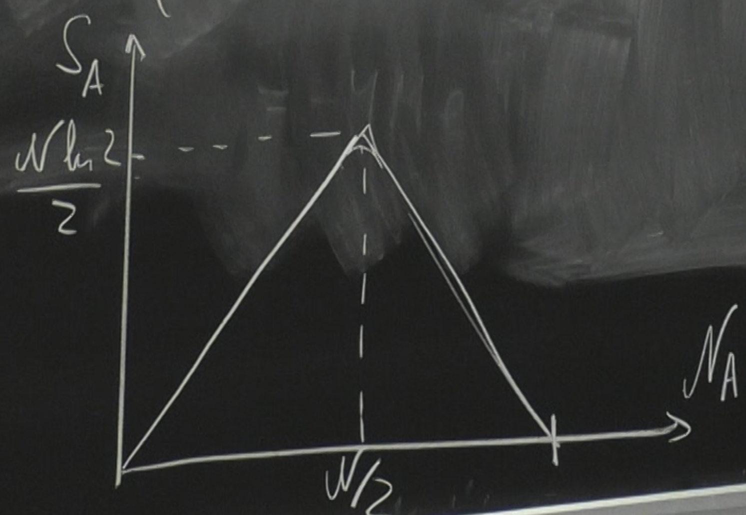

Susskind, Thorlacius, Uglum, PRD, 48, 3793 (1993)
$N_A \ln 2 + O(2^{N_A - N_B})$, if $N_A < N_B$
$\ln 2 + O(2^{N_B - N_A})$ if $N_B < N_A$


Susskind, Thorlasius, Uglum, PRD, 48, 3793 (1993)
$\bar{S}_A = \begin{cases} N_A \ln 2 + O(2^{N_A - N_B}) & \text{if } N_A < N_B \\ N_B \ln 2 + O(2^{N_B - N_A}) & \text{if } N_B < N_A \end{cases}$

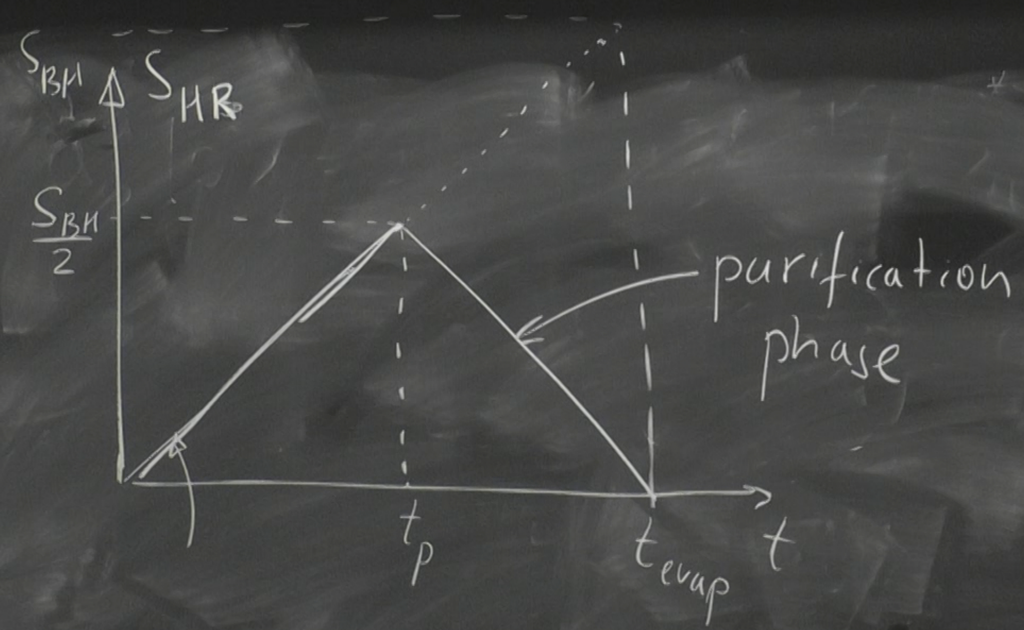
Susskind, Thorlasius, Uglum, PRD, 48, 3793 (1993)

$$\bar{S}_A = \begin{cases} \mathcal{N}_A \ln 2 + O\left(2^{\mathcal{N}_A - \mathcal{N}_B}\right), & \text{if } \mathcal{N}_A < \mathcal{N}_B \\ \mathcal{N}_B \ln 2 + O\left(2^{\mathcal{N}_B - \mathcal{N}_A}\right) & \text{if } \mathcal{N}_B < \mathcal{N}_A \end{cases}$$



3) If ρ_A is thermal $\Rightarrow \tilde{S}_A = S_{\text{thermodynamic}}$

4) $S_A = S_B$



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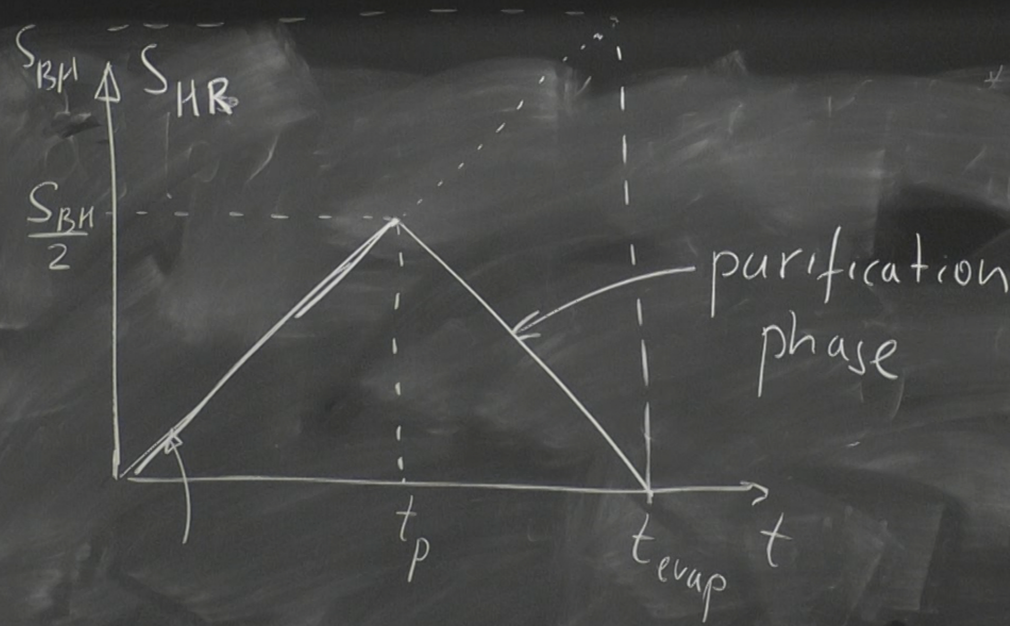

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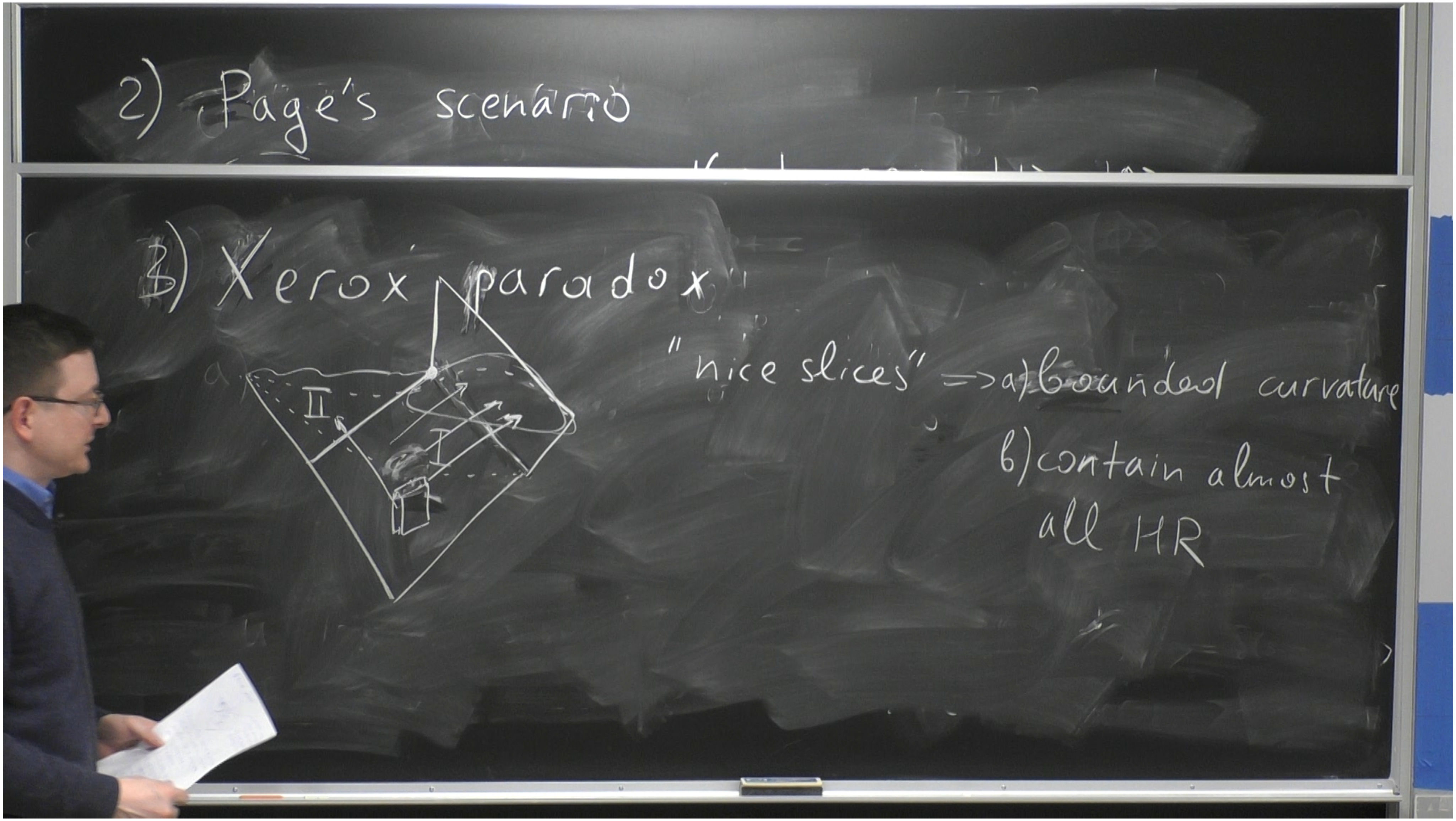
JHEP 09, 002
(2020)


2) Page's scenario
3) Xerox paradox
"nice slices" => a) bounded curvature
b) contain almost
all HR
II
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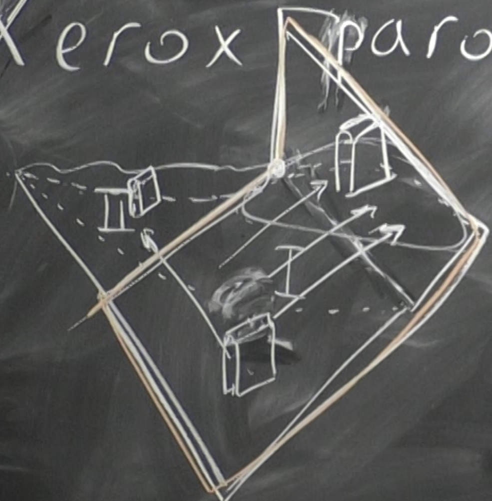

"nice slices" $\Rightarrow$ a) bounded curvature

b) contain almost all HR

$$|\Psi\rangle \to |\Psi\rangle_{out} \otimes |\Psi\rangle_{in}$$