Title: Quantum Field Theory in Curved Spacetime (PM) - 2023-03-17

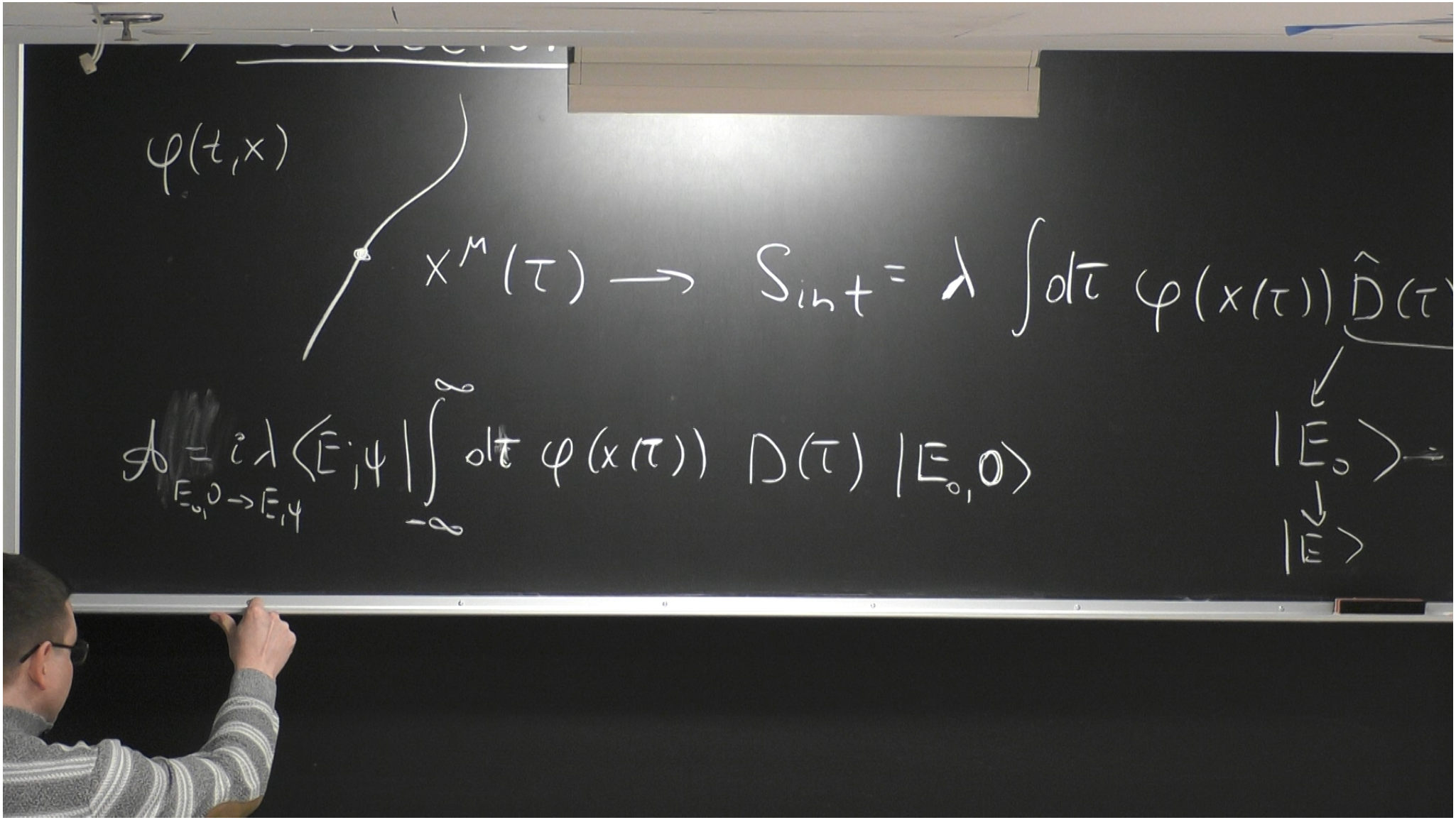
Speakers: Sergey Sibiryakov

Collection: Quantum Field Theory in Curved Spacetime

Date: March 17, 2023 - 2:00 PM

URL: https://pirsa.org/23030096

Abstract: https://pitp.zoom.us/j/95867663528?pwd=dmd5Y3FaVHJ0SnJWSlFscEY4cmhYUT09


$\varphi(t,x)$
$x^M(\tau) \longrightarrow S_{int} = \lambda \int d\tau\, \varphi(x(\tau))\, \hat{D}(\tau)$
$\mathcal{A} = i\lambda \langle E,\psi | \int_{-\infty}^{\infty} d\tau\, \varphi(x(\tau))\, D(\tau) |E_0,0\rangle$
$E_0,0 \to E,\psi$
$|E_0\rangle$
$|E\rangle$

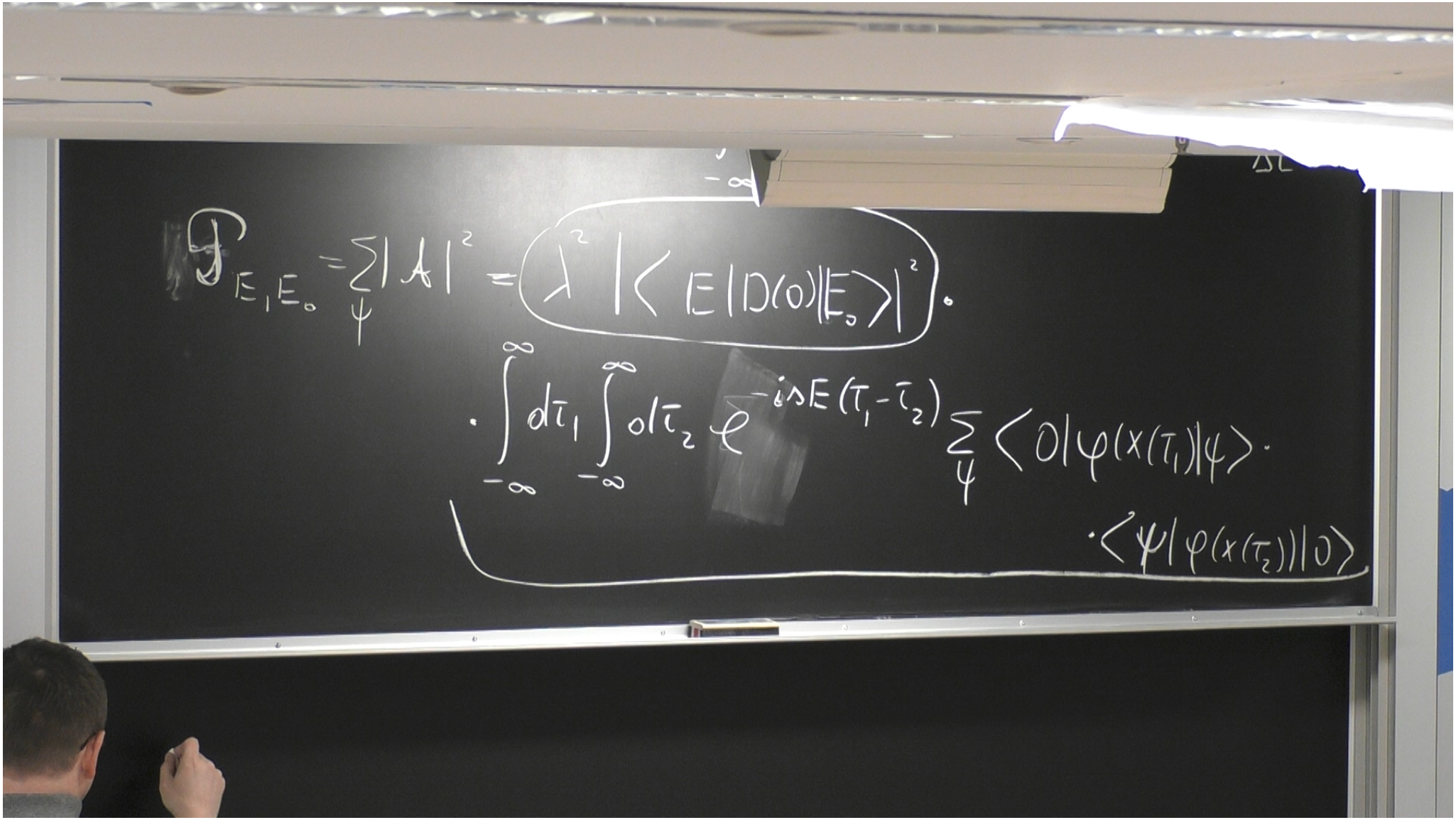
$\mathcal{A} = i\lambda \langle E, \psi | \int_{-\infty} d\tau\, \varphi(x(\tau))\, D(\tau) | E_0, 0 \rangle$

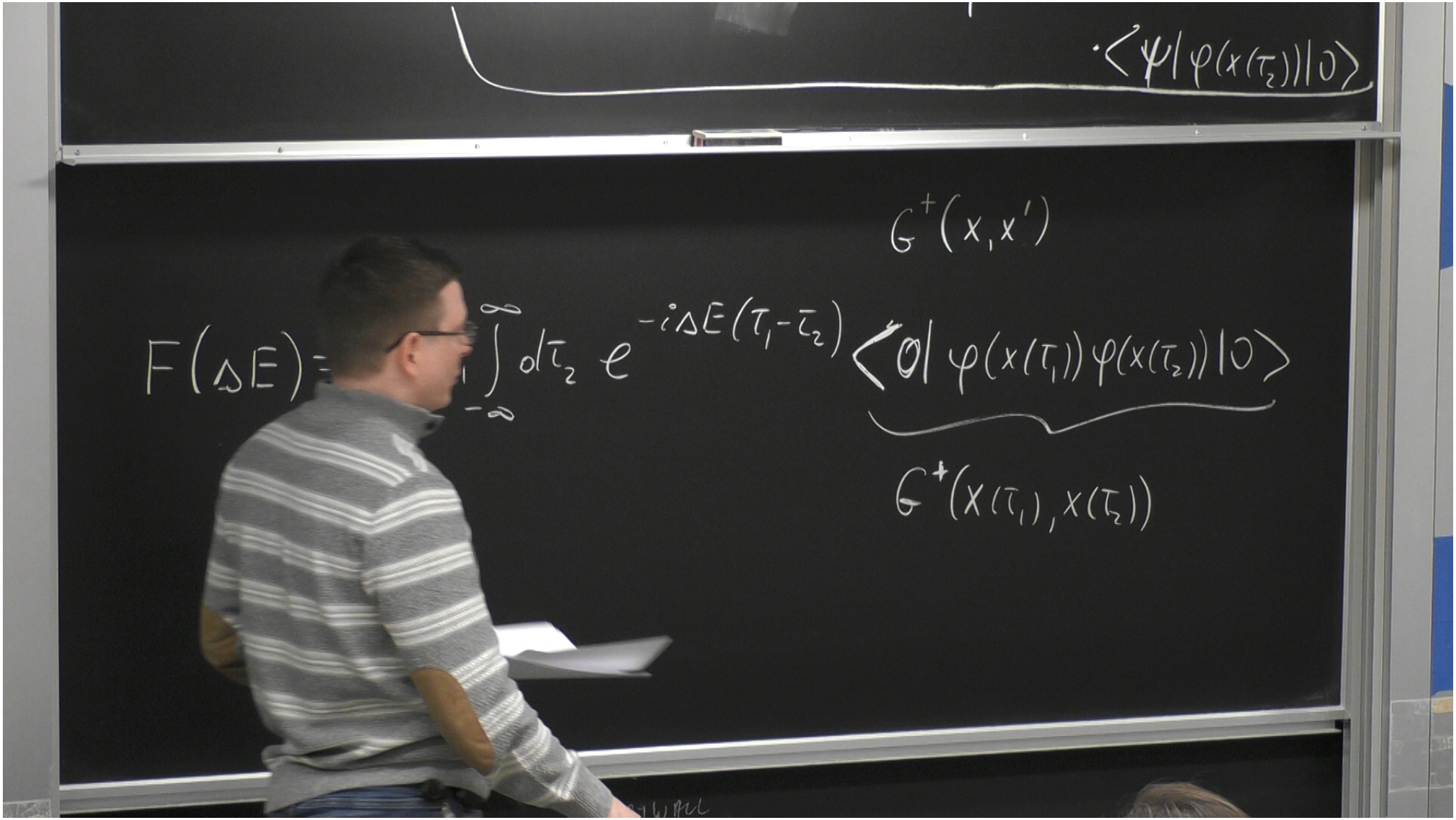
$E_0, 0 \to E, \psi$

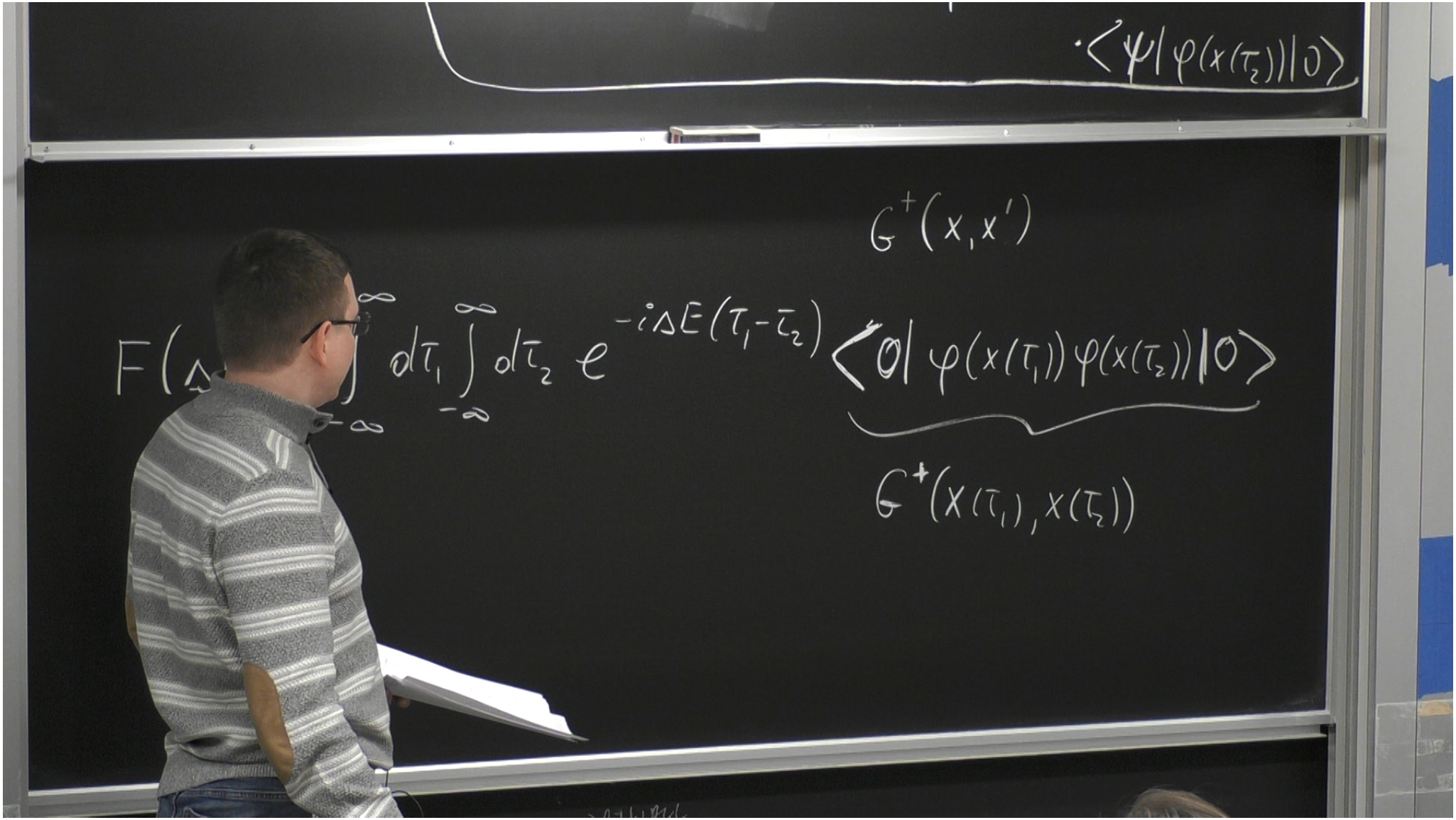
$|E_0\rangle \to |E\rangle$

$$\mathcal{A} = i\lambda \int_{-\infty}^{\infty} \langle E | D(\tau) | E_0 \rangle \langle \psi | \varphi(x(\tau)) | 0 \rangle \, d\tau$$

$$D(\tau) = e^{iH_0\tau} D(0) e^{-iH_0\tau}$$


$\Gamma_{E,E_0} = \sum_\psi |A|^2 = \lambda^2 |\langle E|D(0)|E_0\rangle|^2 \cdot$
$\cdot \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \, e^{-is E(\tau_1-\tau_2)} \sum_\psi \langle 0|\varphi(x(\tau_1))|\psi\rangle \cdot$
$\cdot \langle\psi|\varphi(x(\tau_2))|0\rangle$

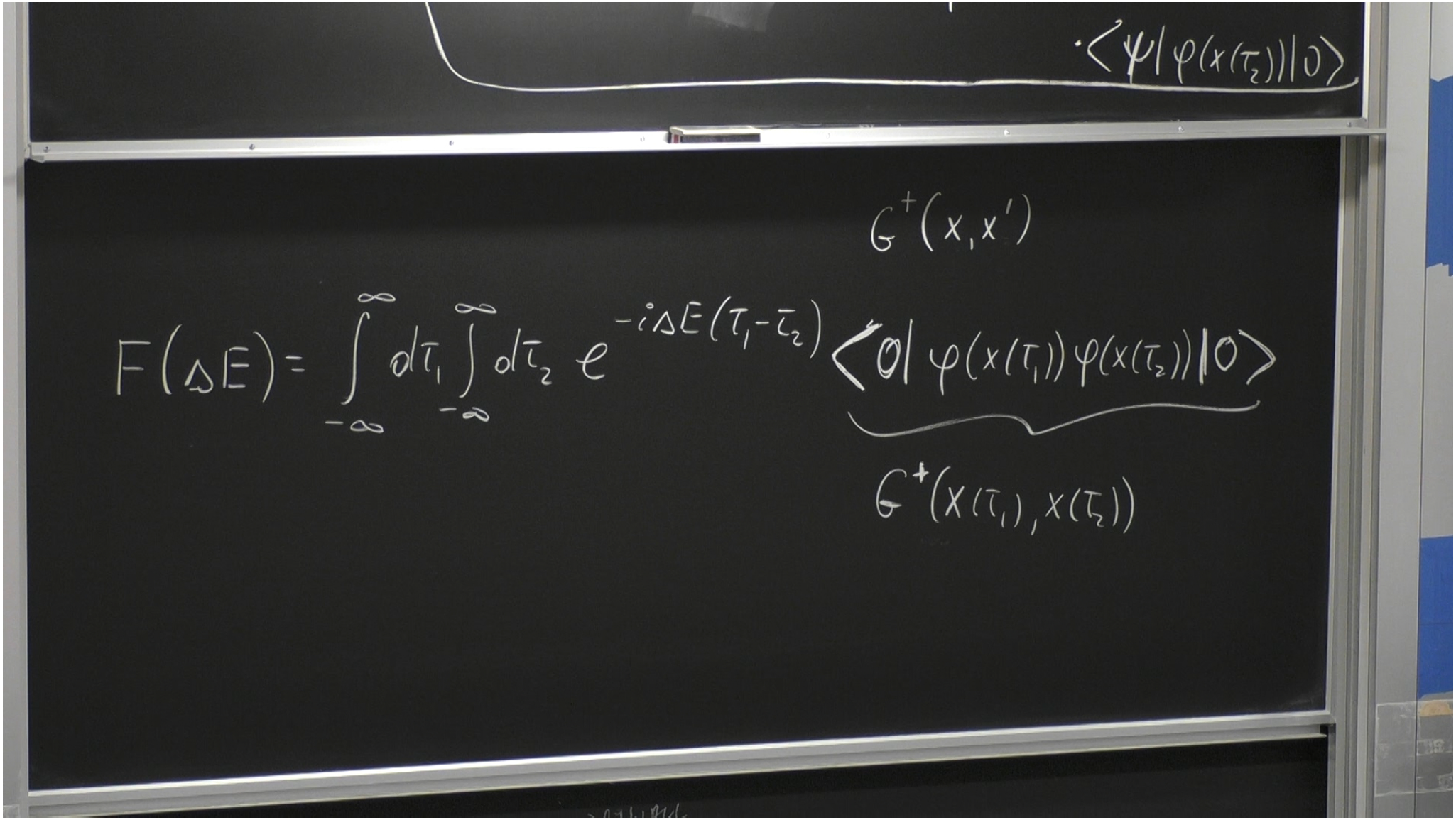

$\cdot \langle \psi | \varphi(x(\tau_2)) | 0 \rangle$
$G^+(x, x')$
$F(\Delta E) = \int_{-\infty}^{\infty} d\tau_2 \, e^{-i\Delta E(\tau_1 - \tau_2)} \langle 0 | \varphi(x(\tau_1)) \varphi(x(\tau_2)) | 0 \rangle$
$G^+(x(\tau_1), x(\tau_2))$

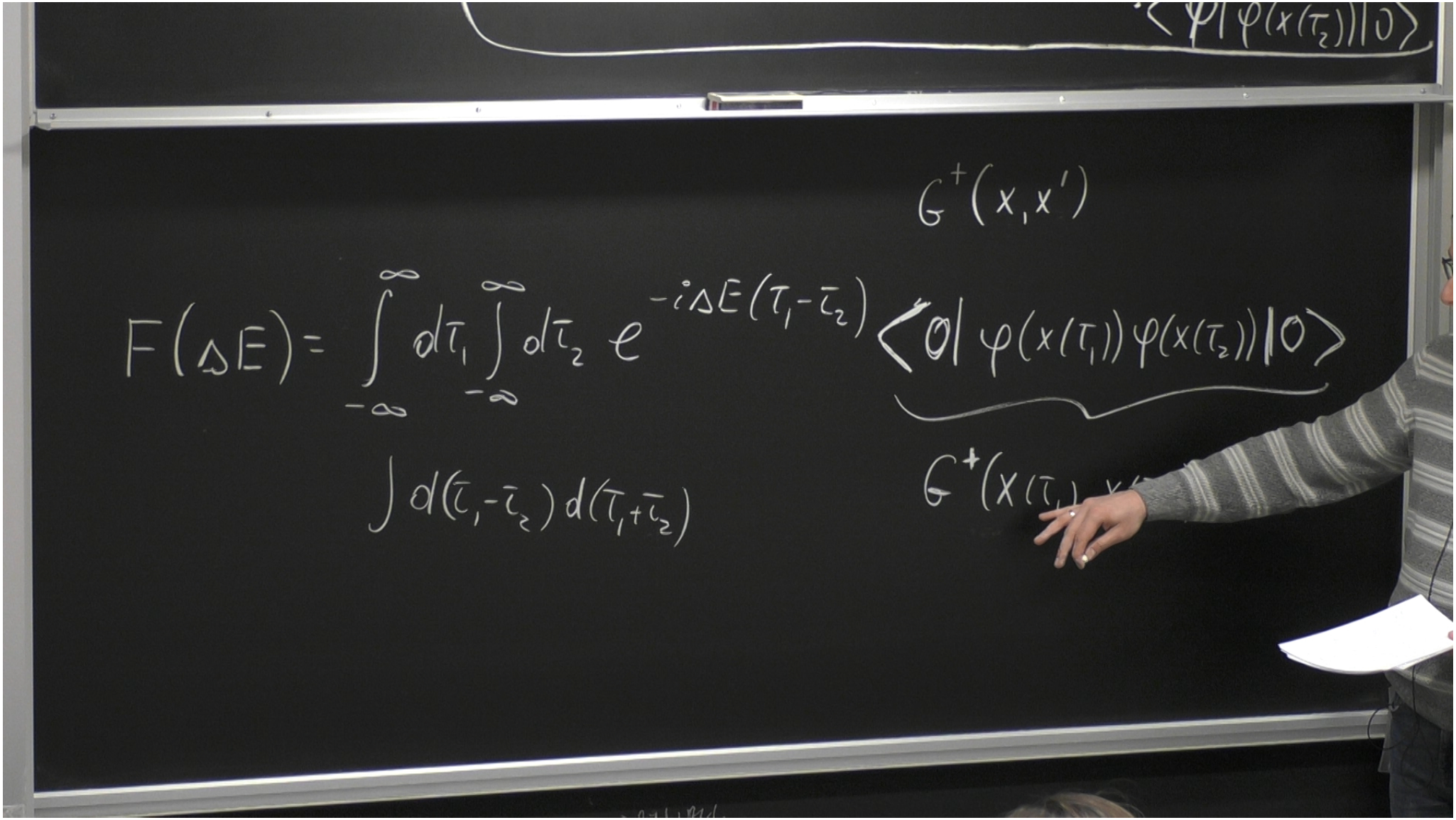

$\cdot \langle \psi | \varphi(x(\tau_2)) | 0 \rangle$
$G^+(x, x')$
$F(\Delta$
$\int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \, e^{-i\Delta E(\tau_1 - \tau_2)} \langle 0 | \varphi(x(\tau_1)) \varphi(x(\tau_2)) | 0 \rangle$
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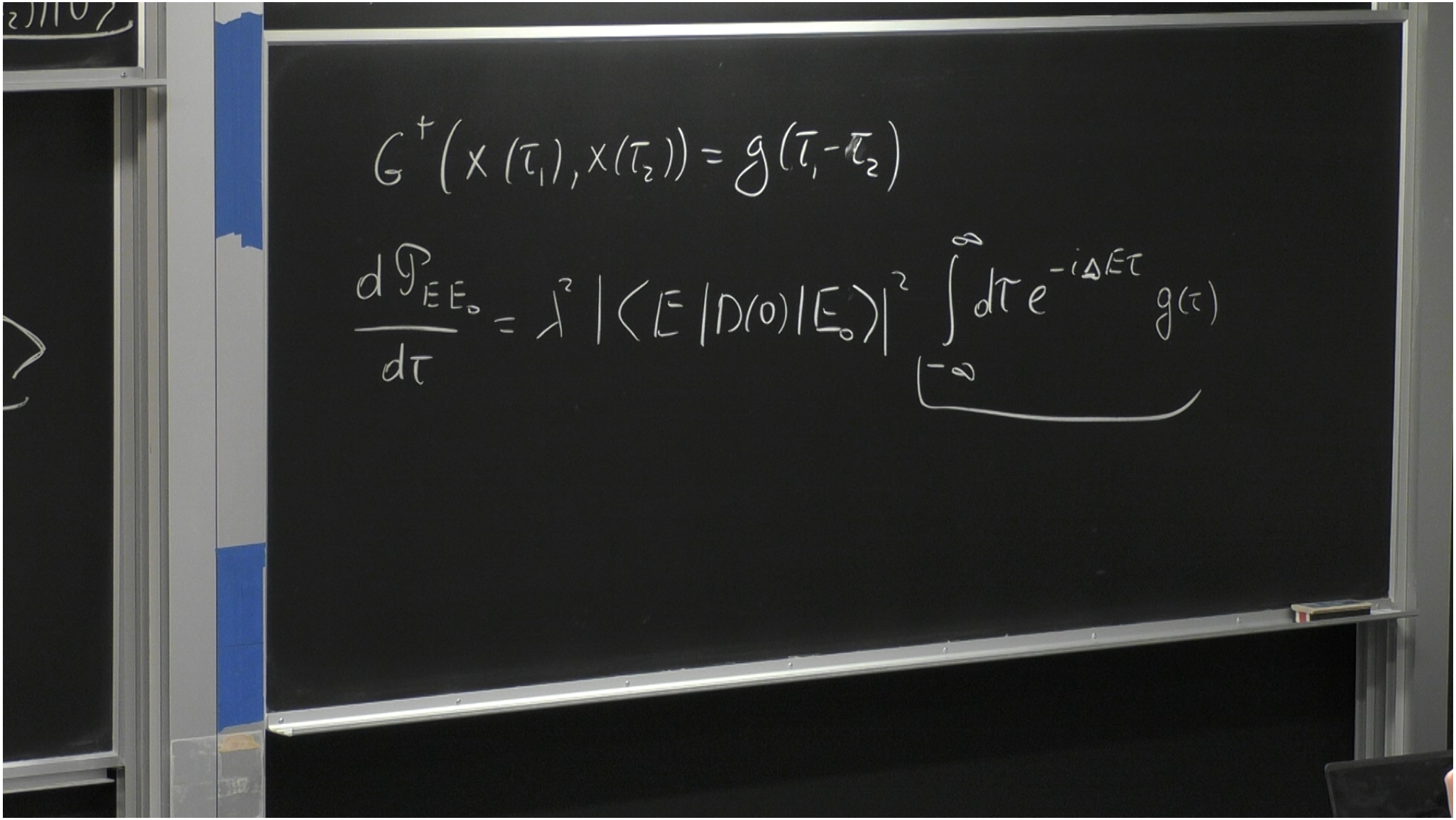
$\cdot \langle \psi | \varphi(x(\tau_2)) | 0 \rangle$

$G^+(x, x')$

$$F(\Delta E) = \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \, e^{-i\Delta E(\tau_1 - \tau_2)} \underbrace{\langle 0 | \varphi(x(\tau_1)) \varphi(x(\tau_2)) | 0 \rangle}_{G^+(x(\tau_1), x(\tau_2))}$$


$\cdot \langle \psi | \varphi(x(\tau_2)) | 0 \rangle$
$G^+(x, x')$
$F(\Delta E) = \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2 \, e^{-i\Delta E(\tau_1 - \tau_2)} \langle 0 | \varphi(x(\tau_1)) \varphi(x(\tau_2)) | 0 \rangle$
$G^+(x(\tau_1), x(\tau_2))$


$\langle \varphi | \varphi(x(\tau_2)) | 0 \rangle$
$G^+(x, x')$
$F(\Delta E) = \int_{-\infty}^{\infty} d\bar{\tau}_1 \int_{-\infty}^{\infty} d\bar{\tau}_2 \, e^{-i\Delta E(\bar{\tau}_1 - \bar{\tau}_2)} \langle 0 | \varphi(x(\bar{\tau}_1)) \varphi(x(\bar{\tau}_2)) | 0 \rangle$
$\int d(\bar{\tau}_1 - \bar{\tau}_2) \, d(\bar{\tau}_1 + \bar{\tau}_2)$
$G^+(x(\bar{\tau}_1)$


$G^{+}(x(\tau_1), x(\tau_2)) = g(\tau_1 - \tau_2)$
$\frac{d\mathcal{P}_{E E_0}}{d\tau} = \lambda^2 |\langle E|D(0)|E_0\rangle|^2 \int_{-\infty}^{\infty} d\tau\, e^{-i\Delta E \tau}\, g(\tau)$

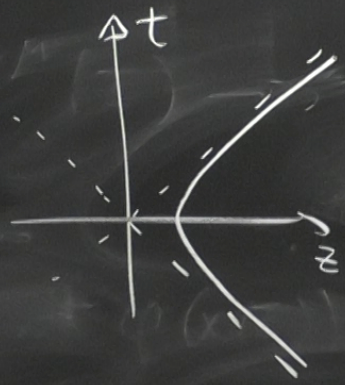
I) Detector response

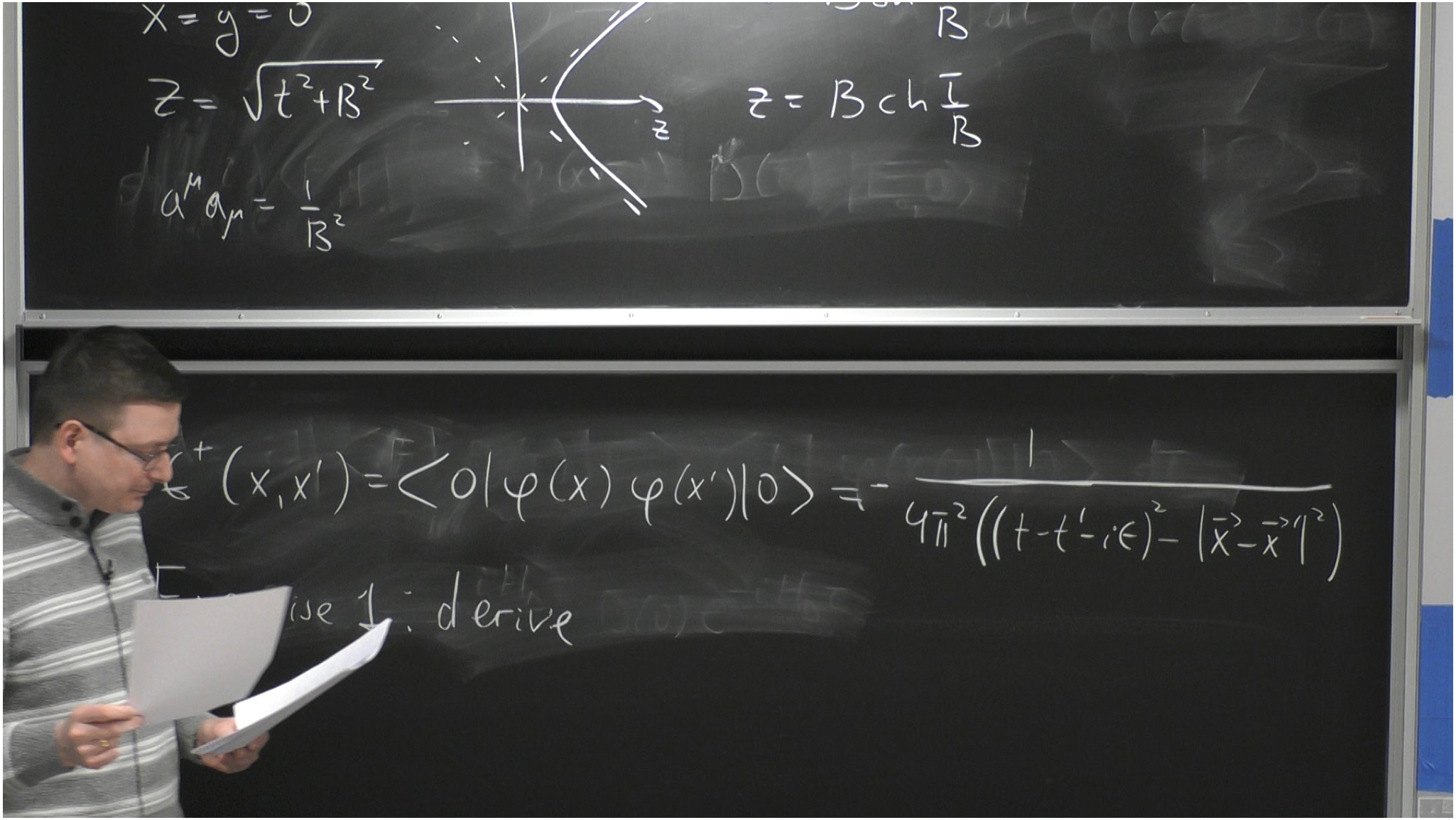
4d Minkowski

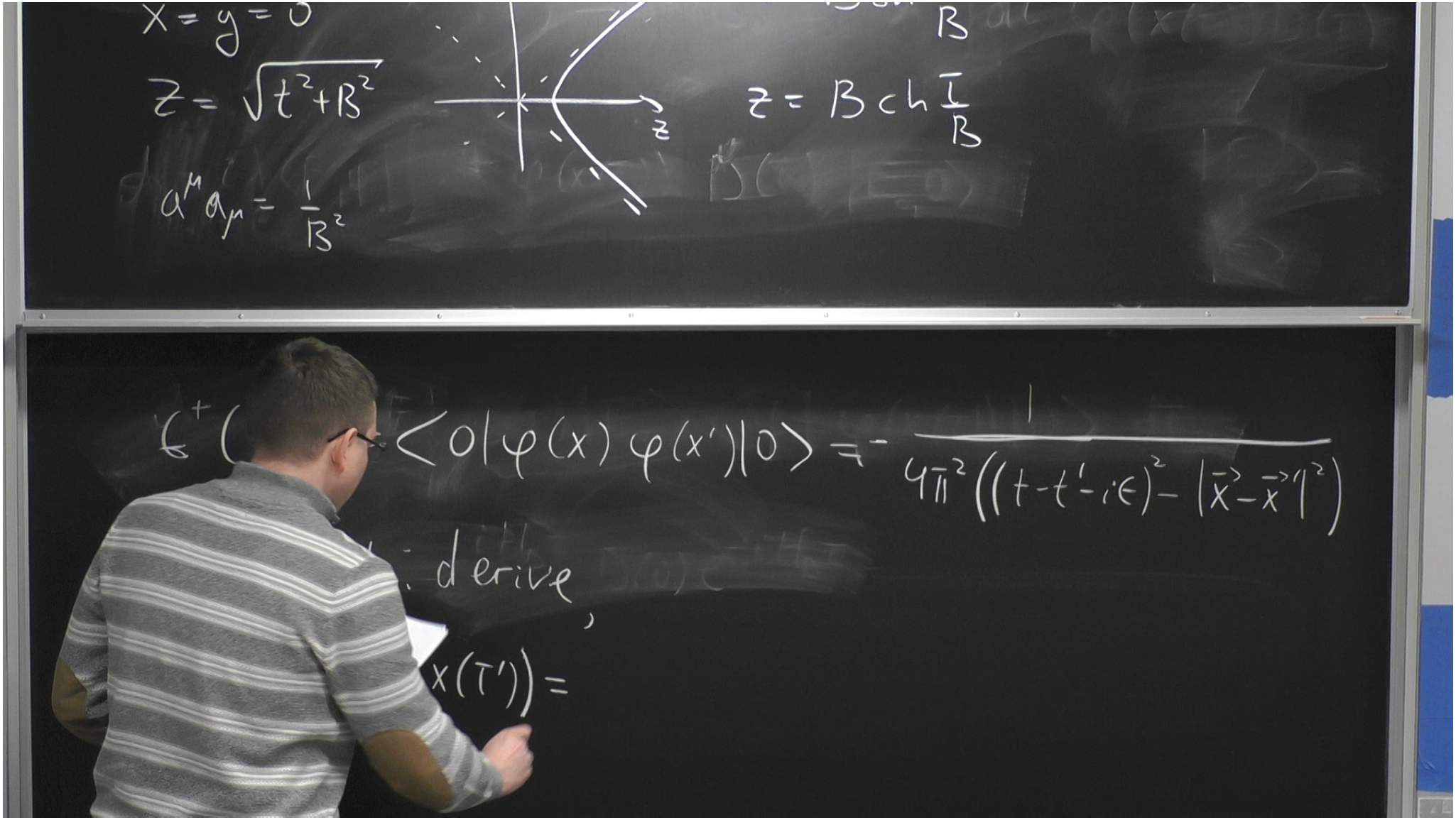
Unruh, PRD 14, 871 (1976)

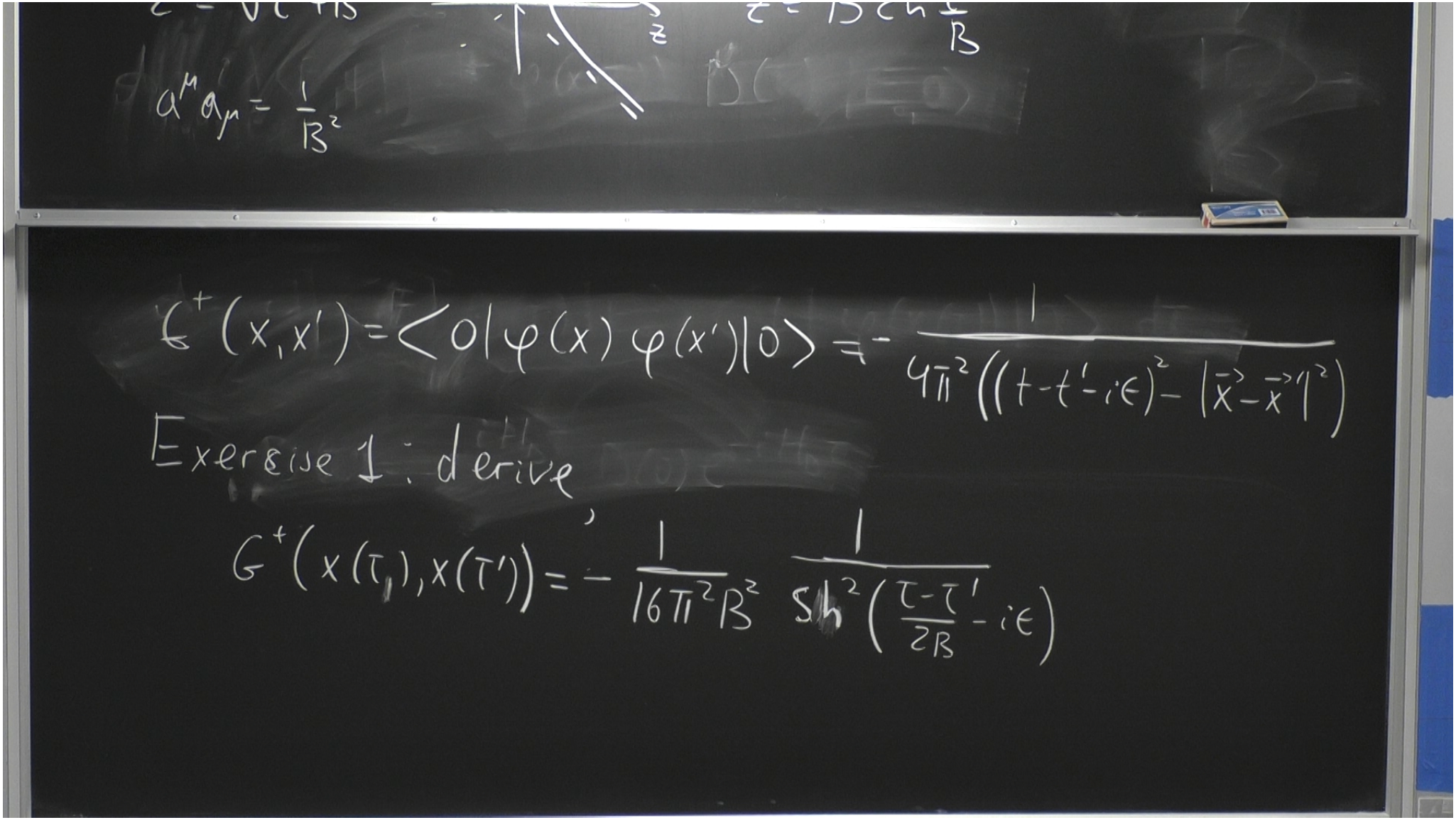
$x = y = 0$

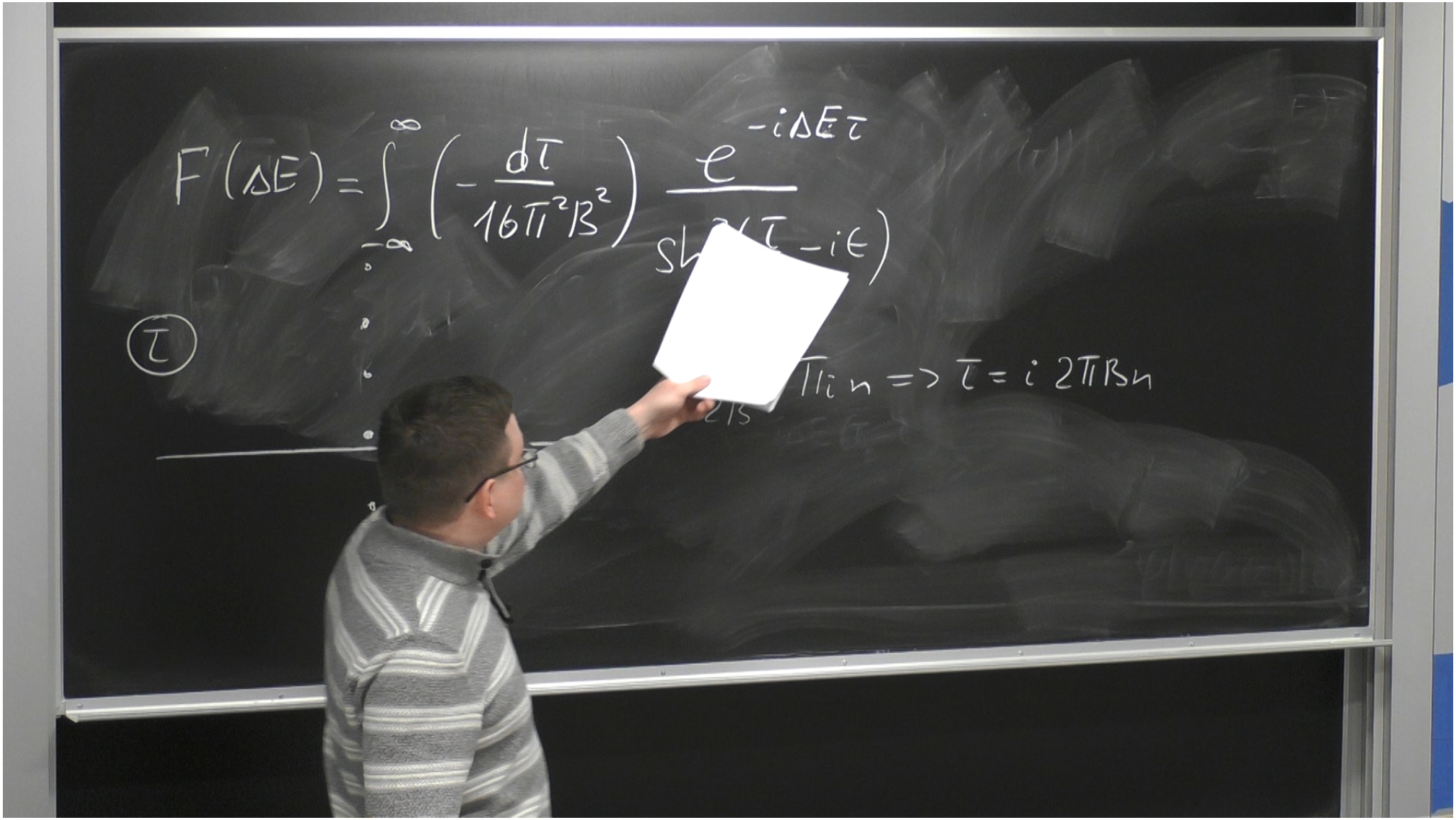
$z = \sqrt{t^2 + B^2}$

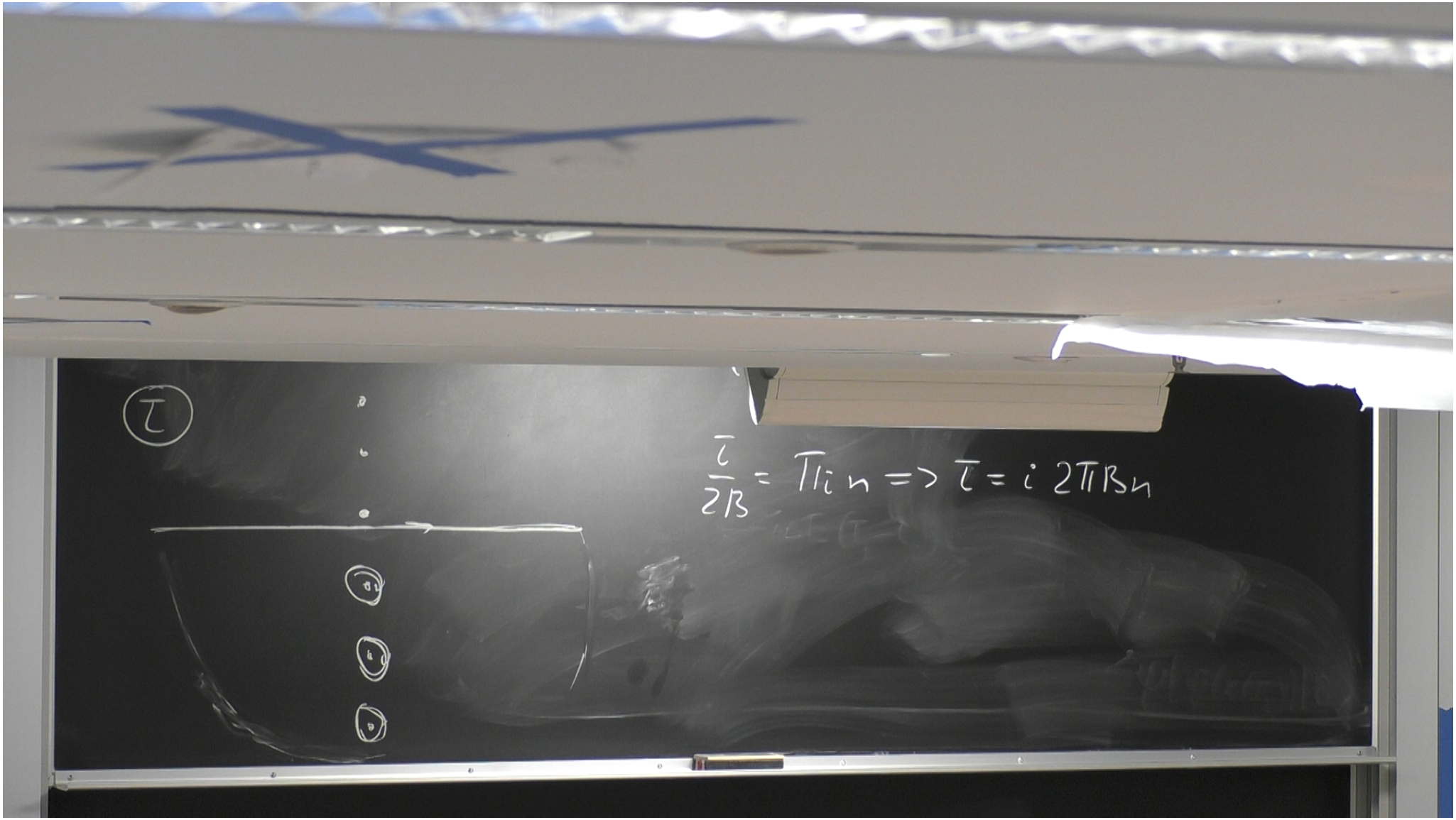


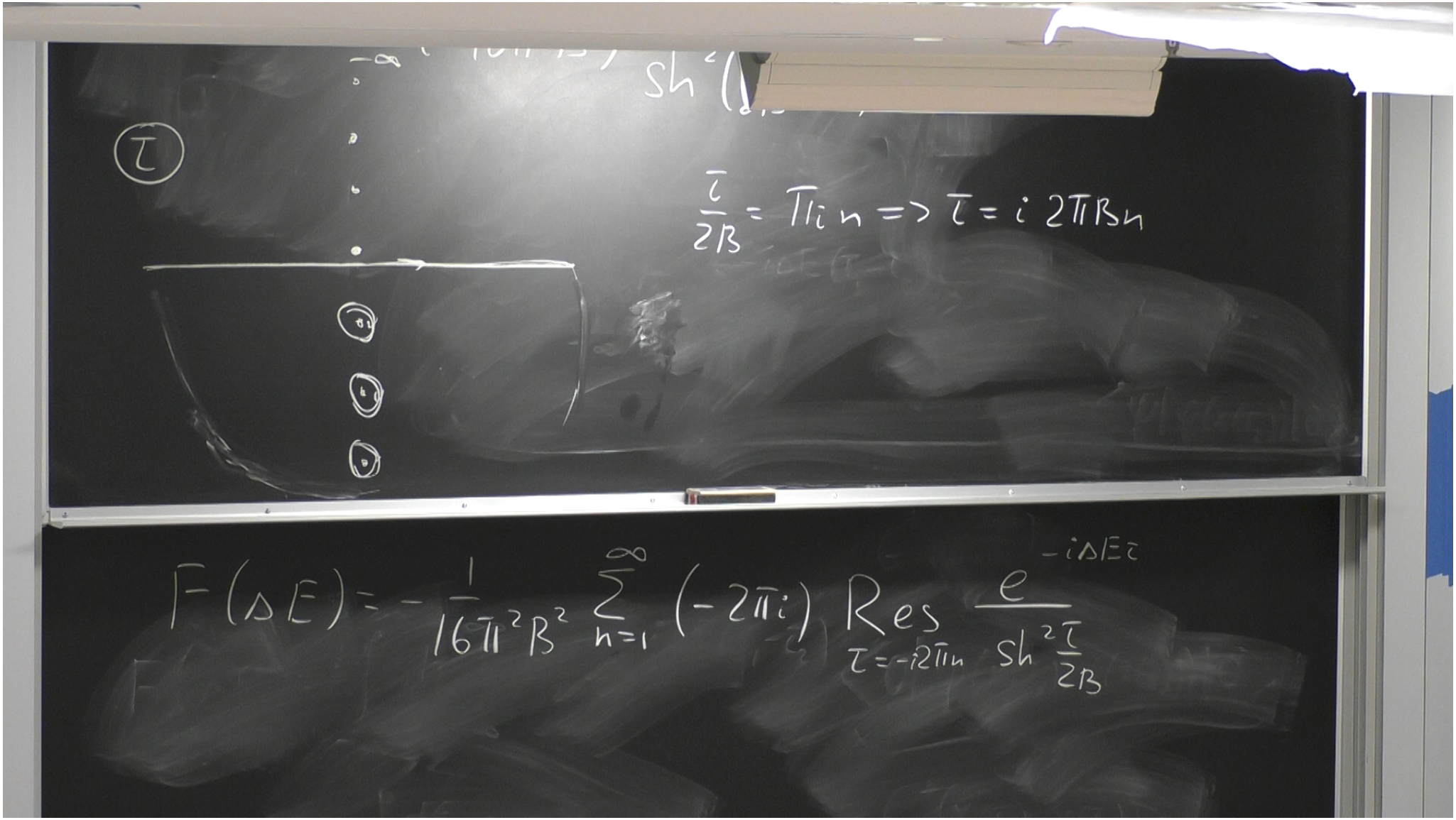

x = y = 0
z = √(t² + B²)
a^μ a_μ = 1/B²
z
B
z = B ch t/B
ε⁺(x, x') = ⟨0|φ(x) φ(x')|0⟩ = − 1 / (4π²((t − t' − iε)² − |x⃗ − x⃗'|²))
ise 1: derive

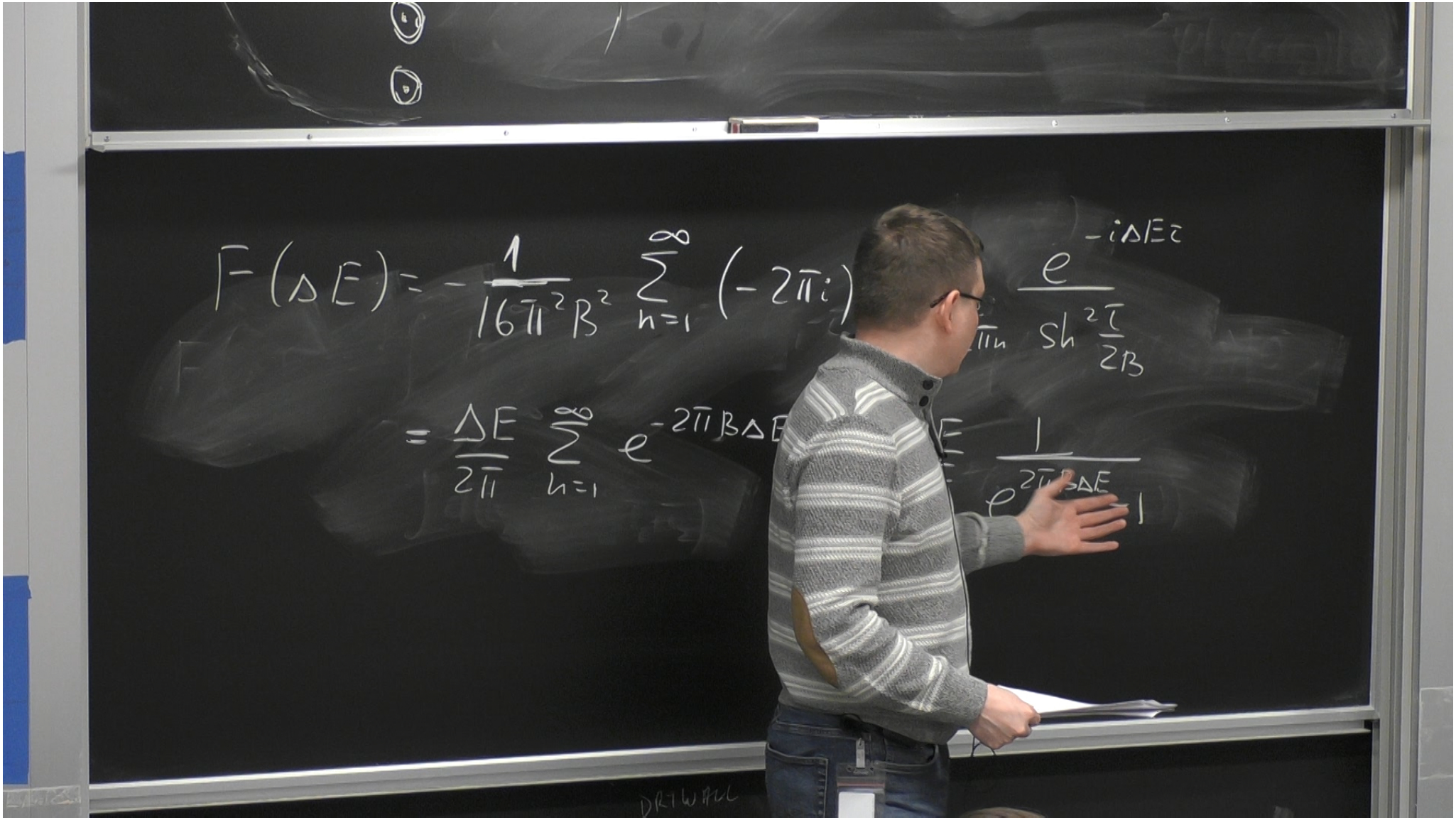

x = y = 0
z = √(t² + B²)
a^μ a_μ = 1/B²
z
B
z = B ch τ/B
G⁺(< 0 | φ(x) φ(x') | 0 > = − 1 / (4π² ((t − t' − iε)² − |x⃗ − x⃗'|²))
derive
x(τ')) =

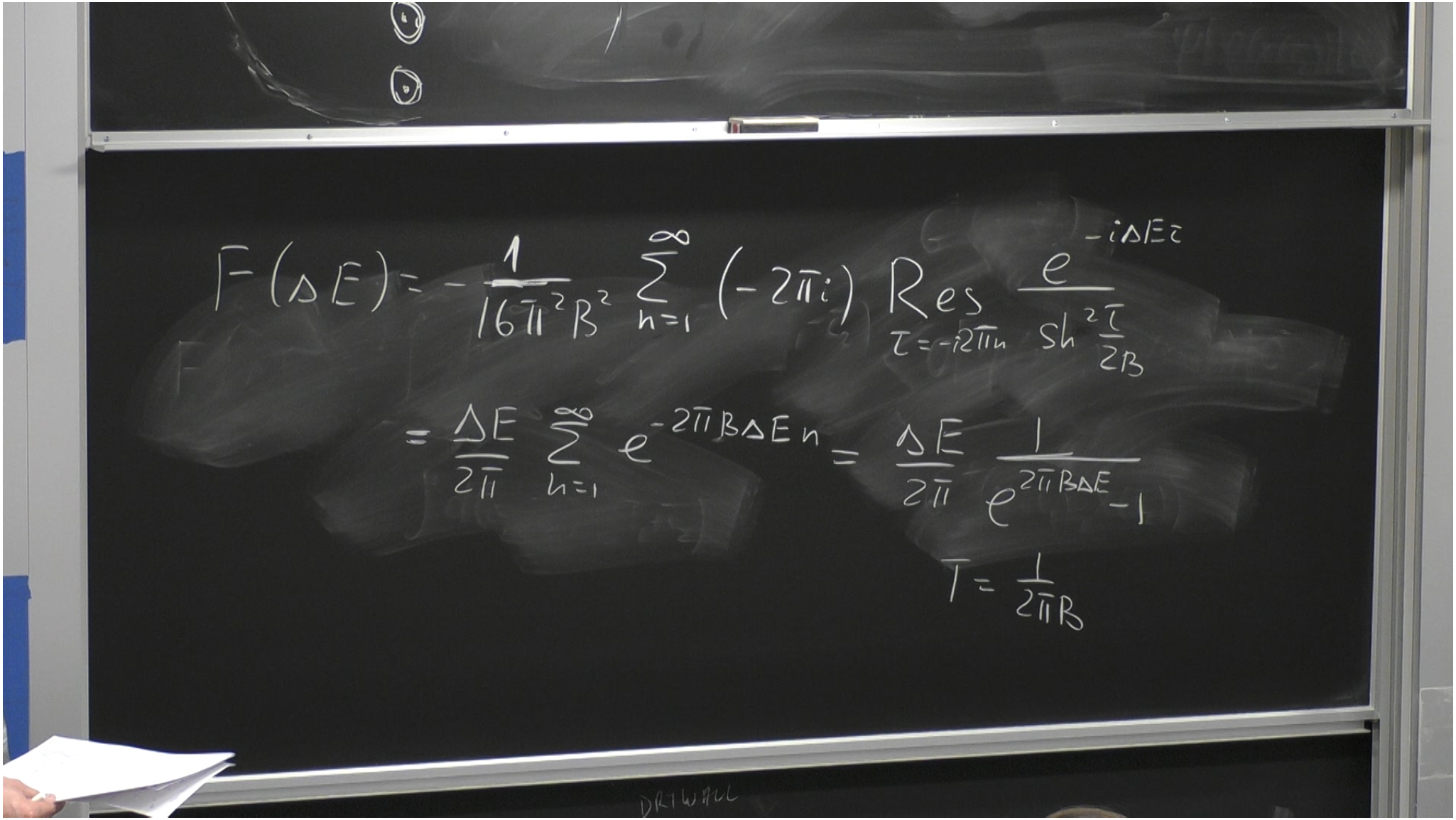

$a^\mu a_\mu = \frac{1}{B^2}$
z
$z = \frac{1}{B}$
$G^+(x,x') = \langle 0|\varphi(x)\,\varphi(x')|0\rangle = -\frac{1}{4\pi^2\left((t-t'-i\epsilon)^2 - |\vec{x}-\vec{x}'|^2\right)}$
Exercise 1: derive
$G^+(x(\tau),x(\tau')) = -\frac{1}{16\pi^2 B^2}\frac{1}{\sinh^2\left(\frac{\tau-\tau'}{2B} - i\epsilon\right)}$


$F(\Delta E) = \int_{-\infty}^{\infty} \left(-\frac{d\tau}{16\pi^2 B^2}\right) \frac{e^{-i\Delta E\tau}}{\mathrm{Sh}^2(\tau - i\epsilon)}$
τ
$\tau = i n \Rightarrow \tau = i\, 2\pi B n$


τ
τ/2B = Tli n => τ = i 2TIBn


τ
$\frac{\tau}{2B} = \pi i n \Rightarrow \tau = i\,2\pi B n$
$F(\Delta E) = -\frac{1}{16\pi^2 B^2} \sum_{n=1}^{\infty} (-2\pi i) \operatorname{Res}_{\tau=-i2\pi n} \frac{e^{-i\Delta E \tau}}{\operatorname{sh}^2 \frac{\tau}{2B}}$


$F(\Delta E) = -\frac{1}{16\pi^2 B^2} \sum_{n=1}^{\infty} (-2\pi i)$
$-i\Delta E\tau$
e
$\pi n \; \mathrm{Sh}^2 \frac{\tau}{2B}$
$= \frac{\Delta E}{2\pi} \sum_{n=1}^{\infty} e^{-2\pi B \Delta E}$
$E \; \frac{1}{e^{2\pi B \Delta E} - 1}$


$F(\Delta E) = -\frac{1}{16\pi^2 B^2} \sum_{n=1}^{\infty} (-2\pi i) \operatorname{Res}_{\tau=-i2\pi n} \frac{e^{-i\Delta E \tau}}{\operatorname{sh}^2 \frac{\tau}{2B}}$
$= \frac{\Delta E}{2\pi} \sum_{n=1}^{\infty} e^{-2\pi B \Delta E n} = \frac{\Delta E}{2\pi} \frac{1}{e^{2\pi B \Delta E} - 1}$
$T = \frac{1}{2\pi B}$

Detector response

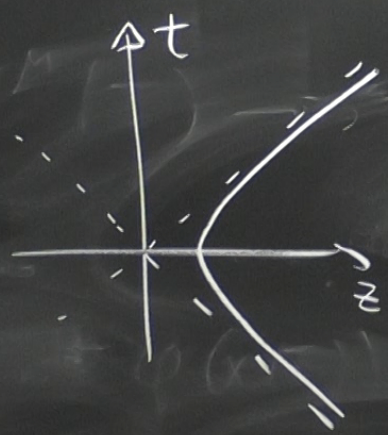
4d Minkowski

, PRD 14, 871 (1976)

$= 0$

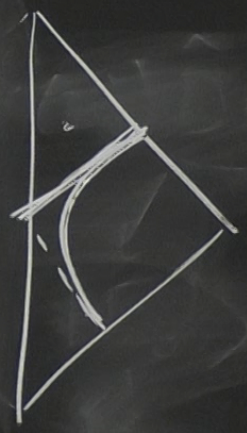
$\sqrt{t^2+B^2}$

$= \frac{1}{B^2}$

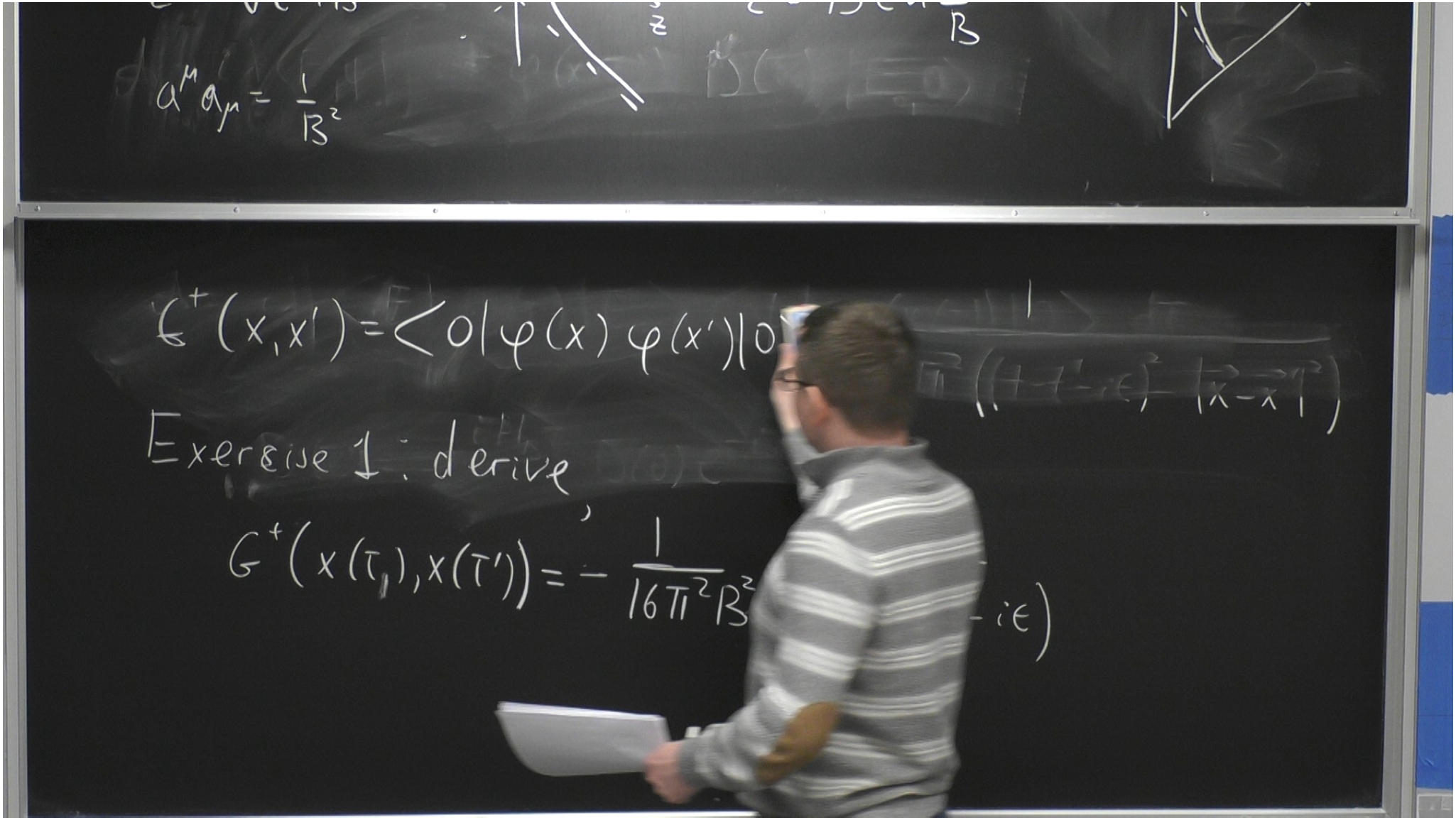


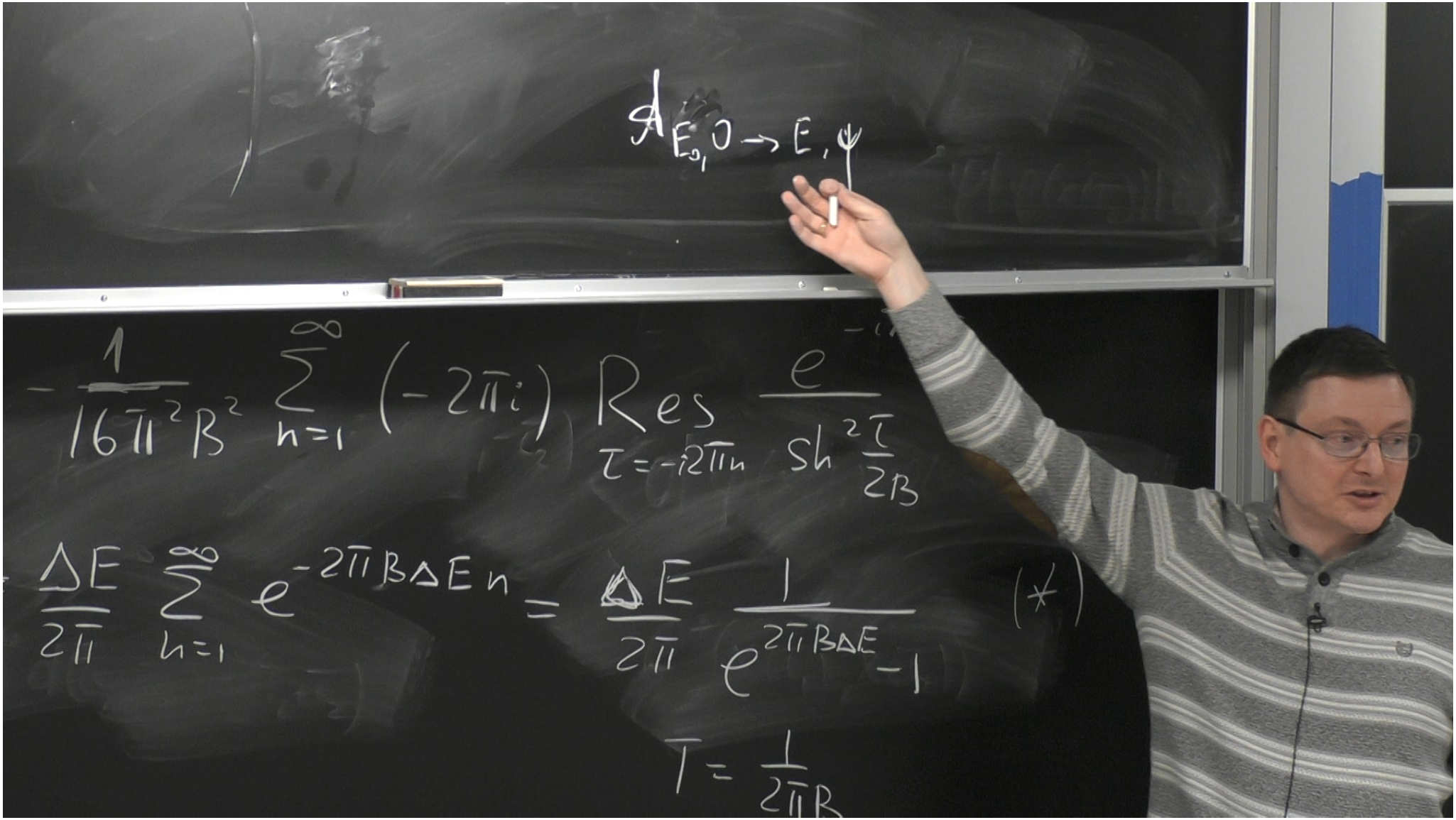
$t = B \operatorname{sh} \frac{\tau}{B}$

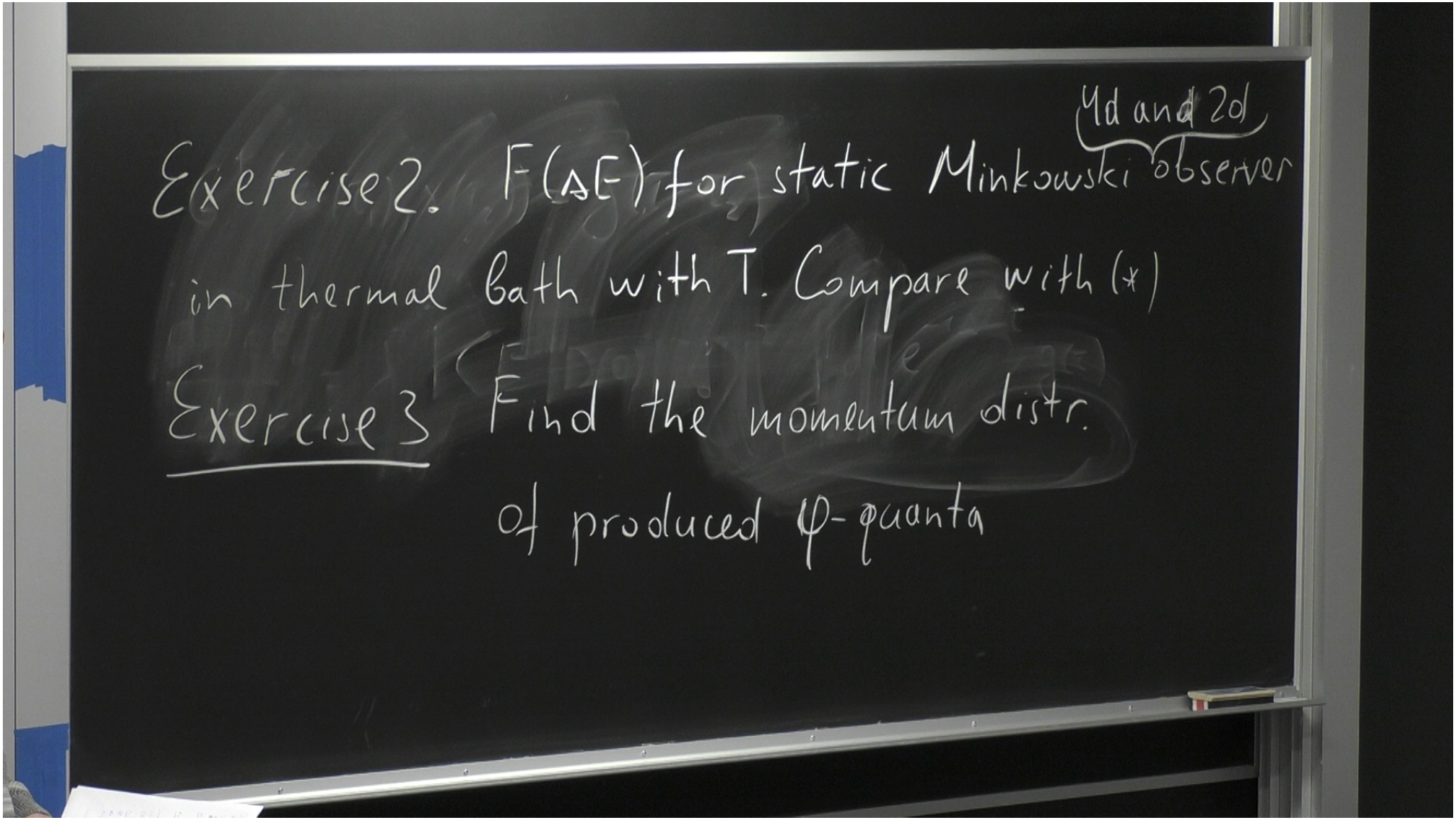
$z = B \operatorname{ch} \frac{\tau}{B}$



$16\pi^2 B \operatorname{sh}\left(\frac{\tau}{2B} - i\epsilon\right)$

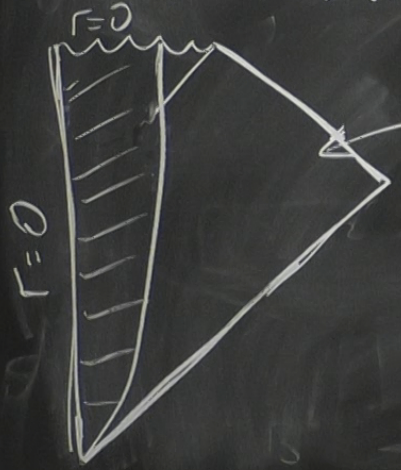

$a^\mu a_\mu = \frac{1}{B^2}$
$G^+(x,x') = \langle 0|\varphi(x)\,\varphi(x')|0$
Exercise 1: derive
$G^+(x(\tau),x(\tau')) = -\frac{1}{16\pi^2 B^2}$
$-i\epsilon)$


$\mathcal{A}_{E_0,0 \to E,\psi}$
$-\frac{1}{16\pi^2 B^2} \sum_{n=1}^{\infty} (-2\pi i) \operatorname{Res}_{\tau=-i2\pi n} \frac{e^{-(\cdot}}{\operatorname{Sh}^2 \frac{\tau}{2B}}$
$\frac{\Delta E}{2\pi} \sum_{n=1}^{\infty} e^{-2\pi i B \Delta E n} = \frac{\Delta E}{2\pi} \frac{1}{e^{2\pi i B \Delta E} - 1}$ (*)
$T = \frac{1}{2\pi B}$


Exercise 2. $F(\Delta E)$ for static Minkowski observer (4d and 2d)
in thermal bath with T. Compare with (*)
Exercise 3 Find the momentum distr.
of produced φ-quanta

II) Particle prod. in grav. collapse

Hawking, Commun. Math. Phys. 43, 199 (1975)

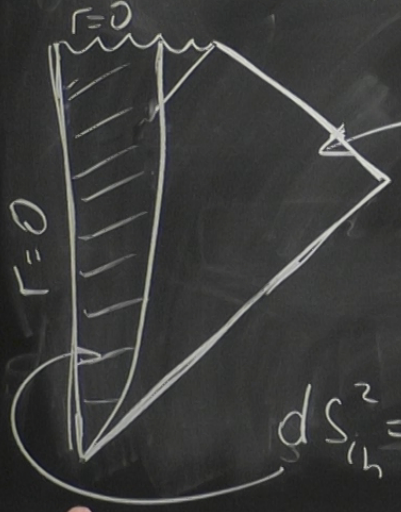


$$ds^2_{out} = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2M}{r}, \quad f(r) = f'_h(r - r_h) + \dots$$

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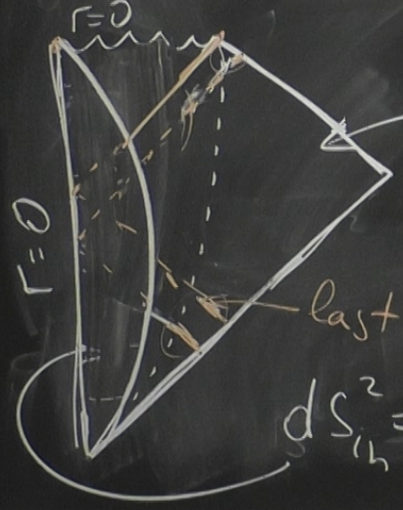
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$$f(r) = 1 - \frac{2M}{r}, \quad f(r) = f_h'(r - r_h) + \dots$$

$$ds^2_{in} = -d\tilde{t}^2 + dr^2 + r^2 d\Omega^2$$

II) Particle prod. in grav. collapse

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$$ds^2_{out} = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

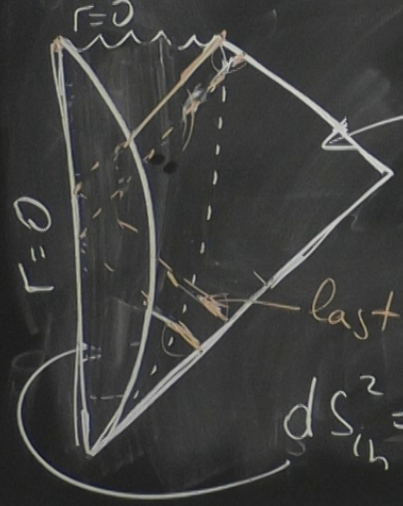
$$f(r) = 1 - \frac{2M}{r}, \quad f(r) = f'_h(r - r_h) + \dots$$

$$ds^2_{in} = -dt^2 + dr^2 + r^2 d\Omega^2$$

in 2d , $\varkappa > 0$

II) Particle prod. in grav. collapse

Hawking, Commun. Math. Phys. 43, 199 (1975)

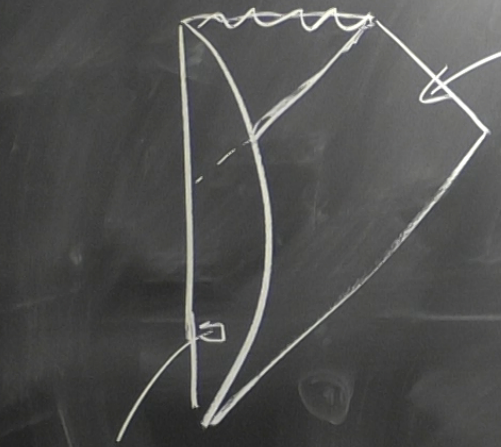


$$dS^2_{out} = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{2M}{r}, \quad f(r) = f'_h(r - r_h) + \dots$$

$$dS^2_{in} = -dt^2 + dr^2 + r^2 d\Omega^2$$

2d model



$ds^2_{out} = -f(r)\, du\, dv$

$u, v = t \mp r_*$

$\int \frac{dr}{f(r)} = r + 2M \ln\left(\frac{r}{2M} - 1\right)$

$ds^2_{in} = -d\tilde{u}\, d\tilde{v}$

$\tilde{u}, \tilde{v} = \tilde{t} \mp r$

$ds^2_{in} = -d\tilde{u}\, d\tilde{v}$

$\tilde{u}, \tilde{v} = \tilde{t} \mp r$

a) $d\tilde{u} = dt - dr = d\tilde{t}\,(1 - \dot{r}_s)$, $\dot{r}_s \equiv \frac{dr_s}{d\tilde{t}}$

$d\tilde{v} = d\tilde{t}\,(1 + \dot{r}_s)$

$du = dt - \frac{dr}{f(r)} = d\tilde{t}\left(\frac{dt}{d\tilde{t}} - \frac{\dot{r}_s}{f(r_s)}\right)$

$dv = d\tilde{t}\left(\frac{dt}{d\tilde{t}} + \frac{\dot{r}_s}{f(r_s)}\right)$

$ds^2_{in} = -d\tilde{u}\, d\tilde{v}$

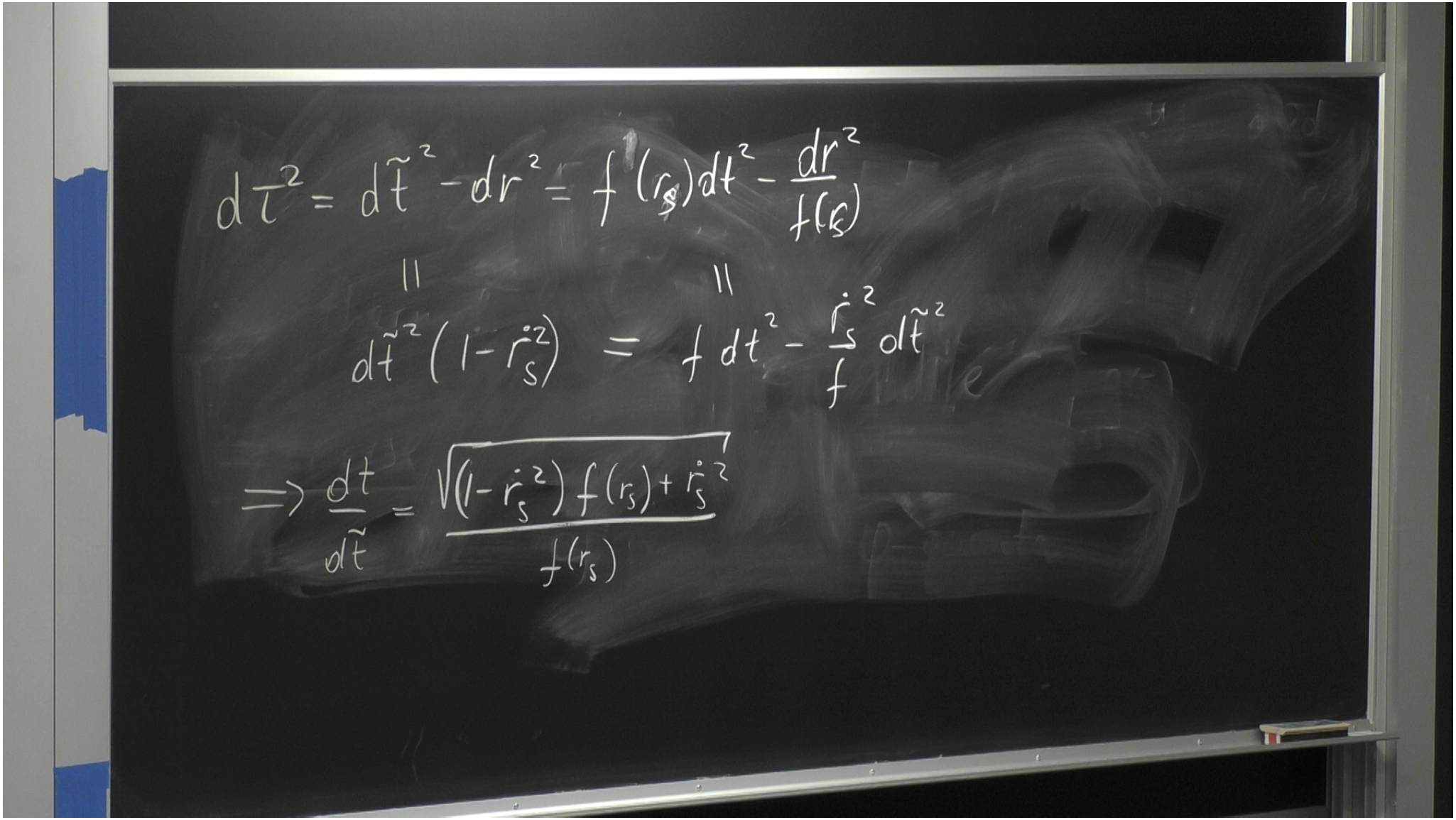
$\tilde{u}, \tilde{v} = \tilde{t} \mp r$

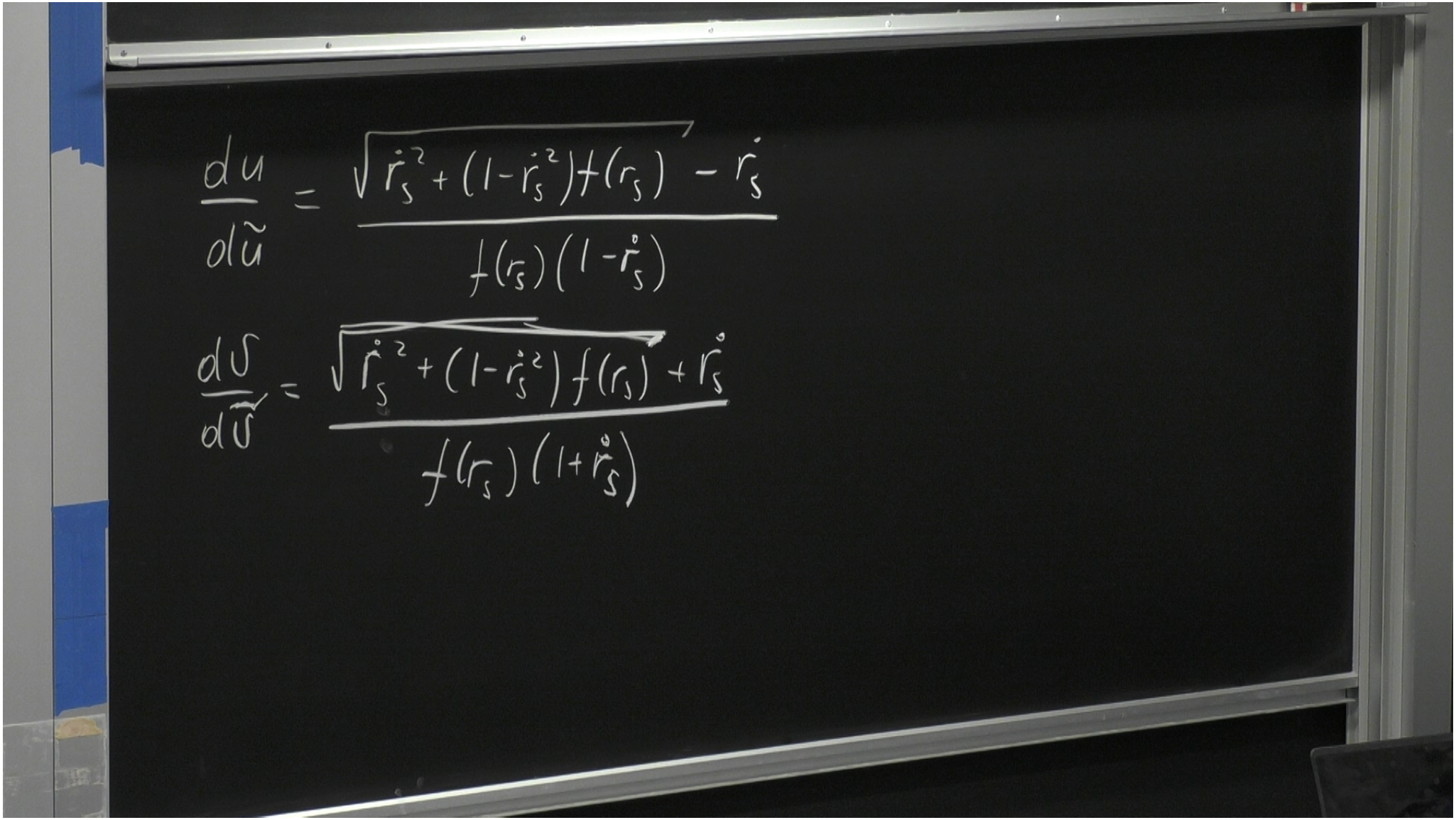
a) $d\tilde{u} = dt - dr = d\tilde{t}\left(1 - \dot{r}_s\right), \quad \dot{r}_s \equiv \frac{dr_s}{d\tilde{t}}$

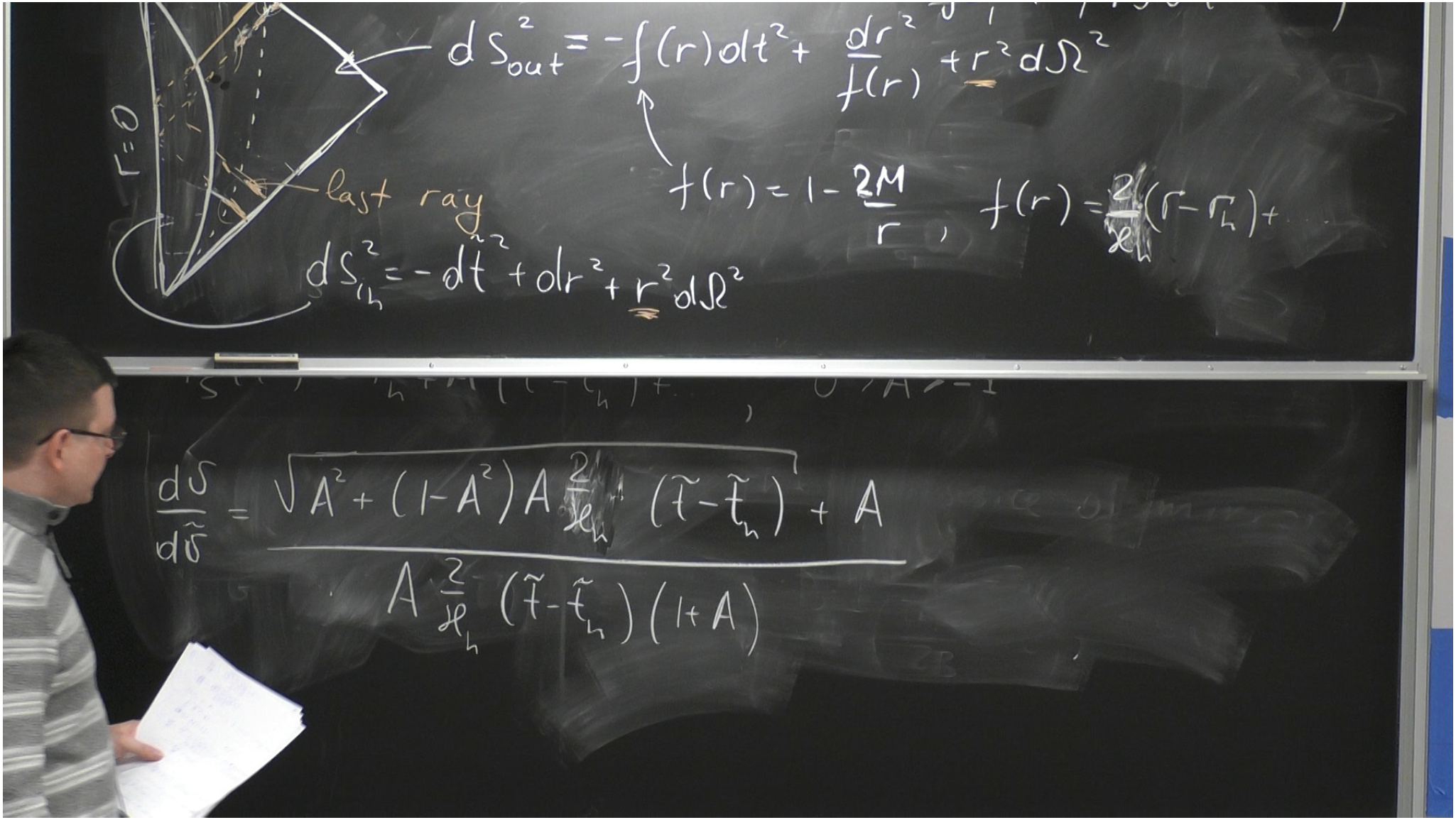
$d\tilde{v} = d\tilde{t}\left(1 + \dot{r}_s\right)$

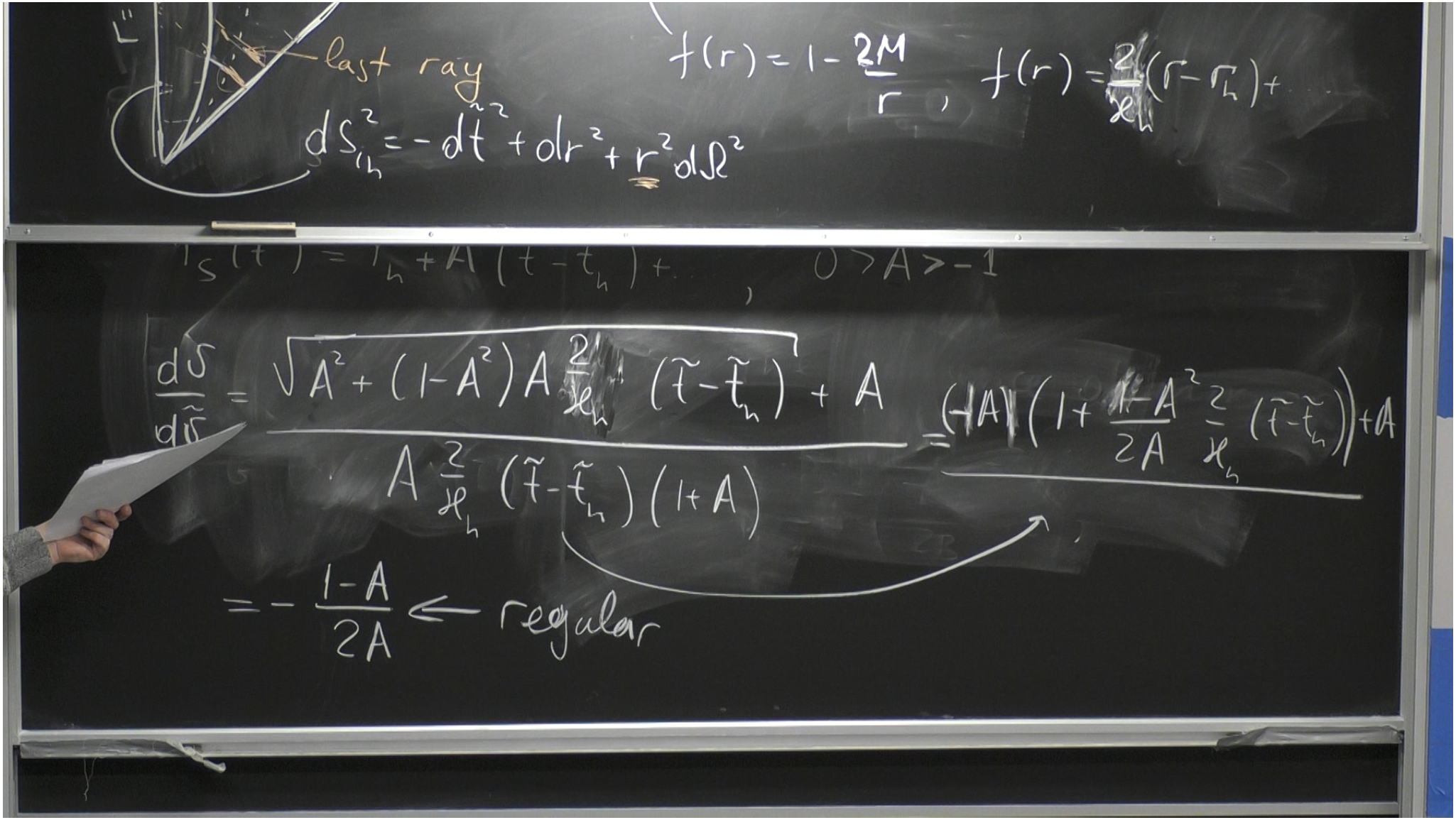
$du = dt - \frac{dr}{f(r)} = d\tilde{t}\left(\frac{dt}{d\tilde{t}} - \frac{\dot{r}_s}{f(r_s)}\right)$

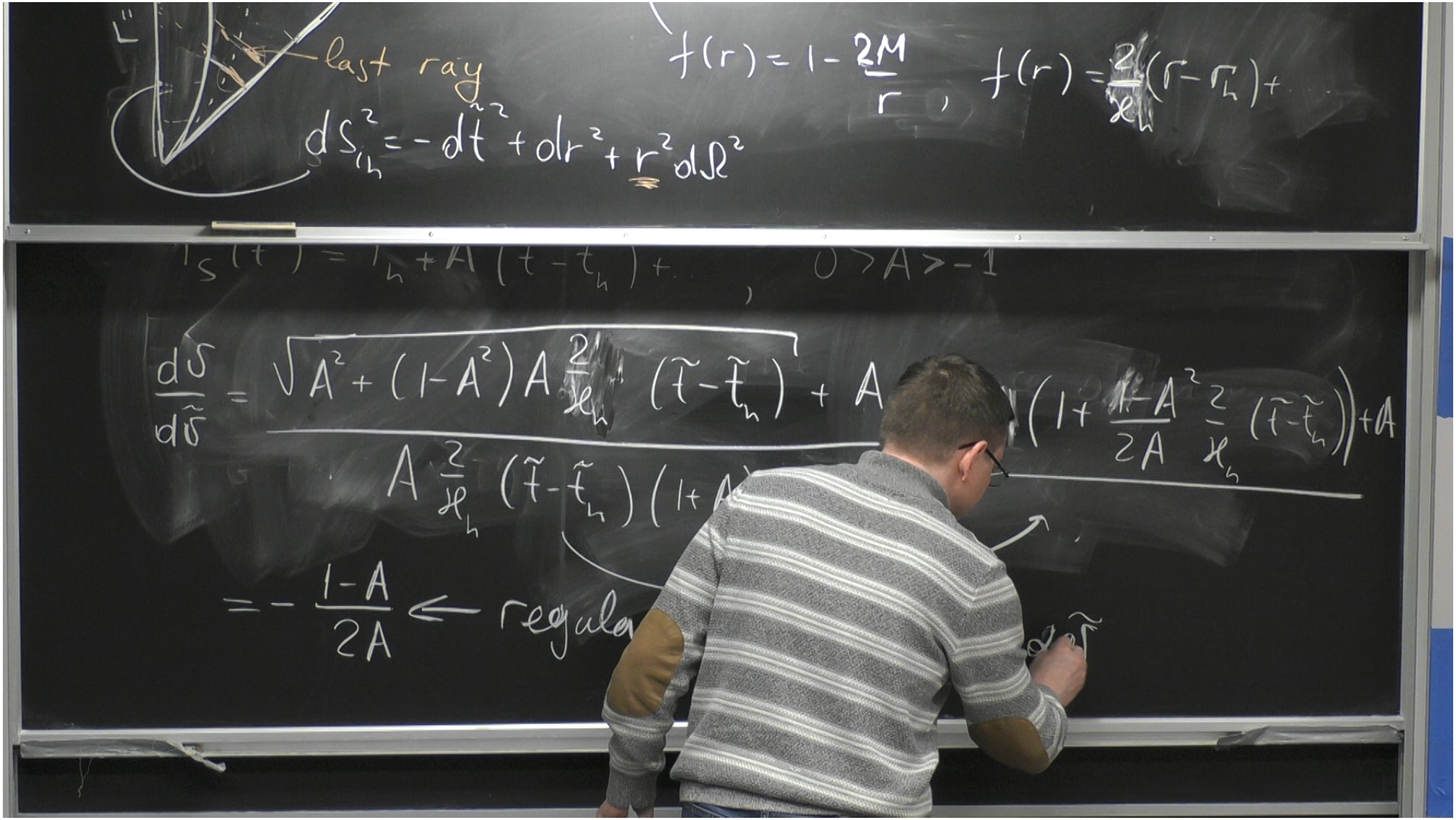
$dv = d\tilde{t}\left(\frac{dt}{d\tilde{t}} + \frac{\dot{r}_s}{f(r_s)}\right)$

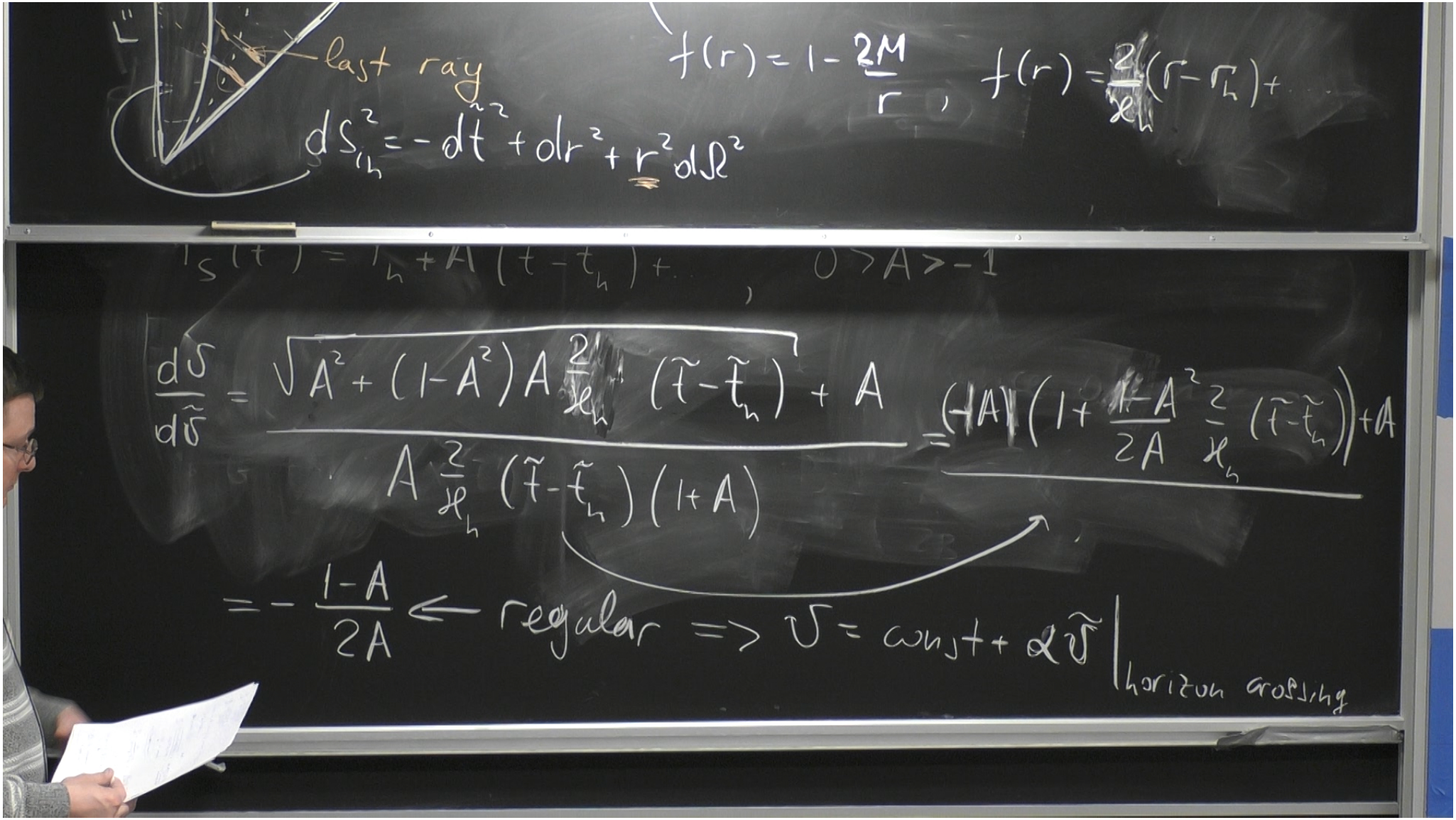

$d\tau^2 = d\tilde{t}^2 - dr^2 = f(r_s)dt^2 - \frac{dr^2}{f(r_s)}$
$d\tilde{t}^2(1-\dot{r}_s^2) = f\,dt^2 - \frac{\dot{r}_s^2}{f}\,d\tilde{t}^2$
$\Rightarrow \frac{dt}{d\tilde{t}} = \frac{\sqrt{(1-\dot{r}_s^2)f(r_s)+\dot{r}_s^2}}{f(r_s)}$

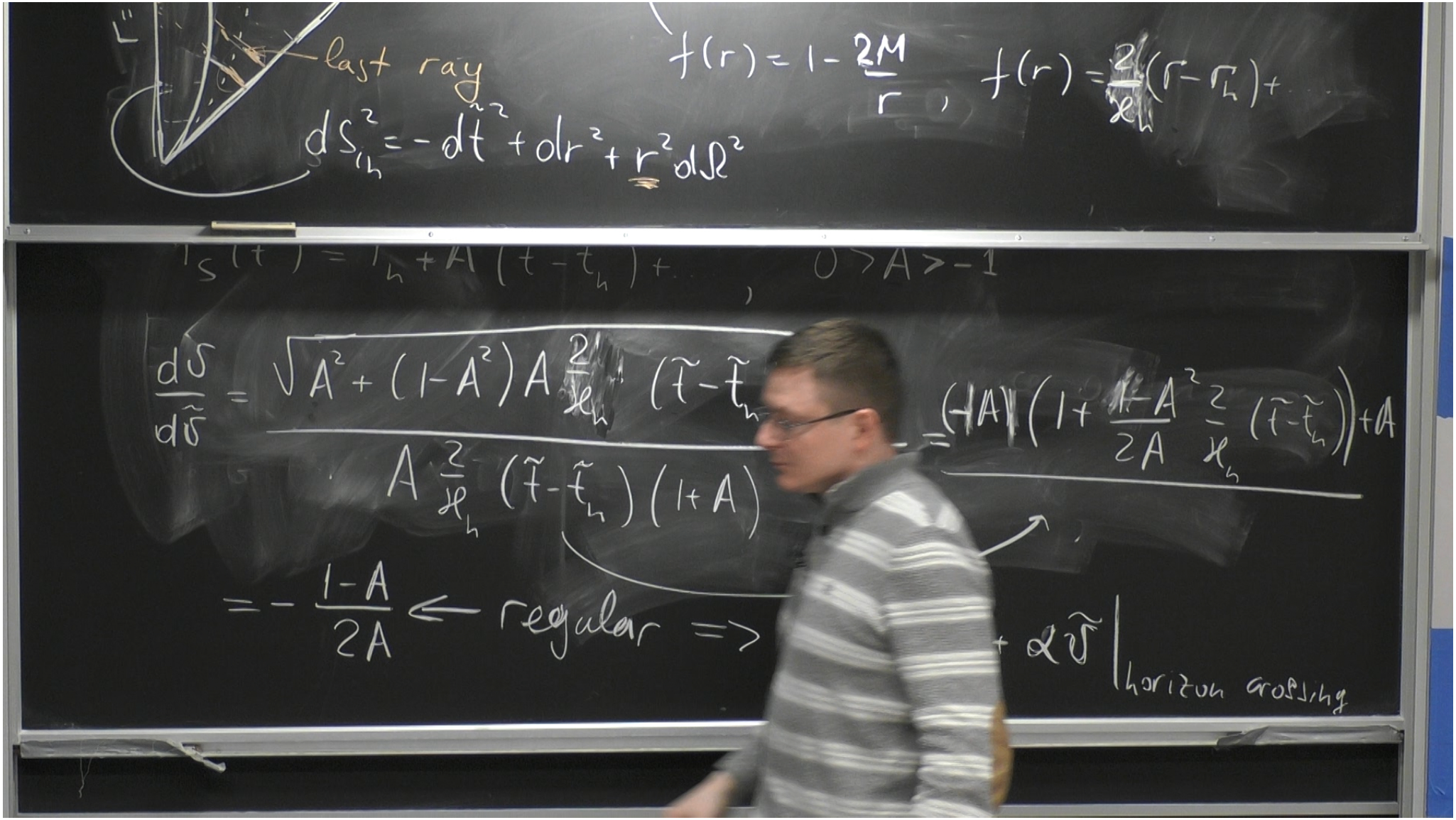

$\frac{du}{d\tilde{u}} = \frac{\sqrt{\dot{r}_s^2 + (1-\dot{r}_s^2) f(r_s)} - \dot{r}_s}{f(r_s)(1-\dot{r}_s)}$
$\frac{dv}{d\tilde{v}} = \frac{\sqrt{\dot{r}_s^2 + (1-\dot{r}_s^2) f(r_s)} + \dot{r}_s}{f(r_s)(1+\dot{r}_s)}$

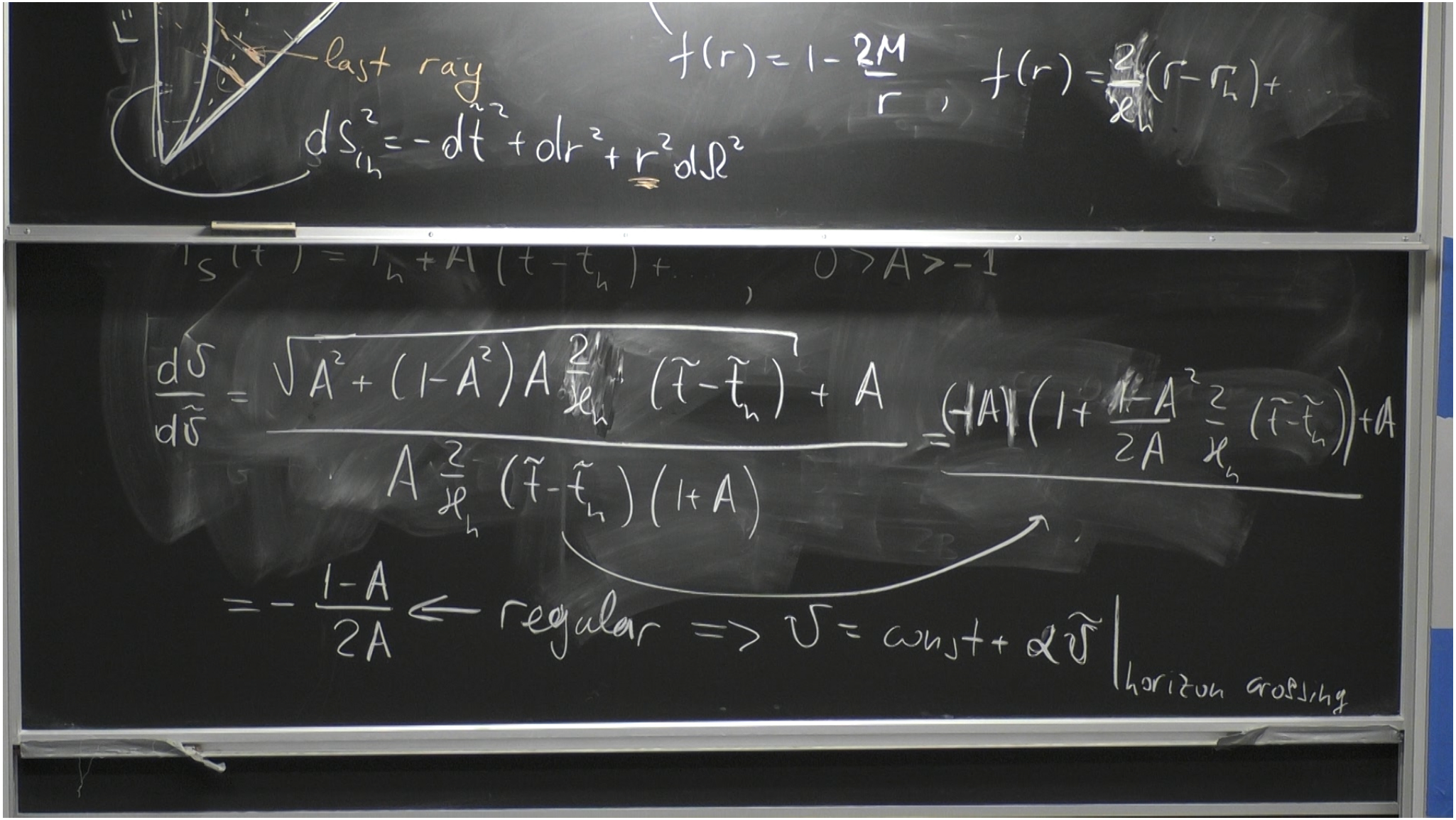

$ds^2_{out} = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$
$f(r) = 1 - \frac{2M}{r}$
$f(r) = \frac{2}{\mathcal{R}_h}(r - r_h) + \dots$
$r = 0$
last ray
$ds^2_{in} = -d\tilde{t}^2 + d\tilde{r}^2 + r^2 d\Omega^2$
$\frac{d\tilde{v}}{d\tilde{r}} = \frac{\sqrt{A^2 + (1-A^2)A\frac{2}{\mathcal{R}_h}(\tilde{t} - \tilde{t}_h)} + A}{A\frac{2}{\mathcal{R}_h}(\tilde{t} - \tilde{t}_h)(1+A)}$

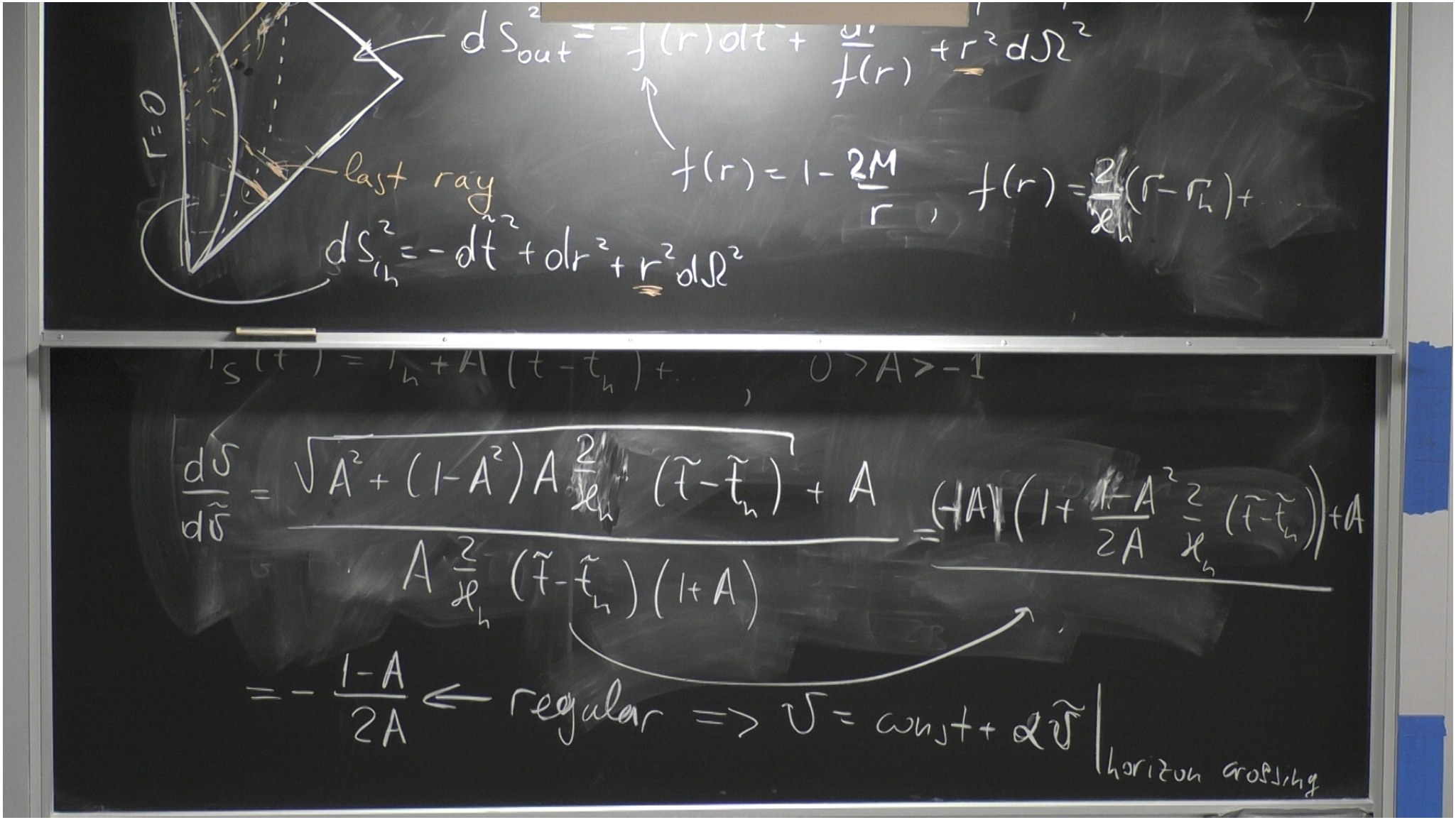

last ray
$f(r) = 1 - \frac{2M}{r}$, $f(r) = \frac{2}{\mathcal{R}_h}(r - r_h) + \dots$
$ds^2_{in} = -dt^2 + dr^2 + r^2 d\Omega^2$
$r_s(t) = r_h + A(t - t_h) + \dots$, $0 > A > -1$
$\frac{dv}{d\tilde{v}} = \frac{\sqrt{A^2 + (1-A^2) A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)} + A}{A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)(1+A)} = \frac{(1+A)\left(1 + \frac{1-A^2}{2A} \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)\right) + A}{\dots}$
$= -\frac{1-A}{2A}$ ← regular

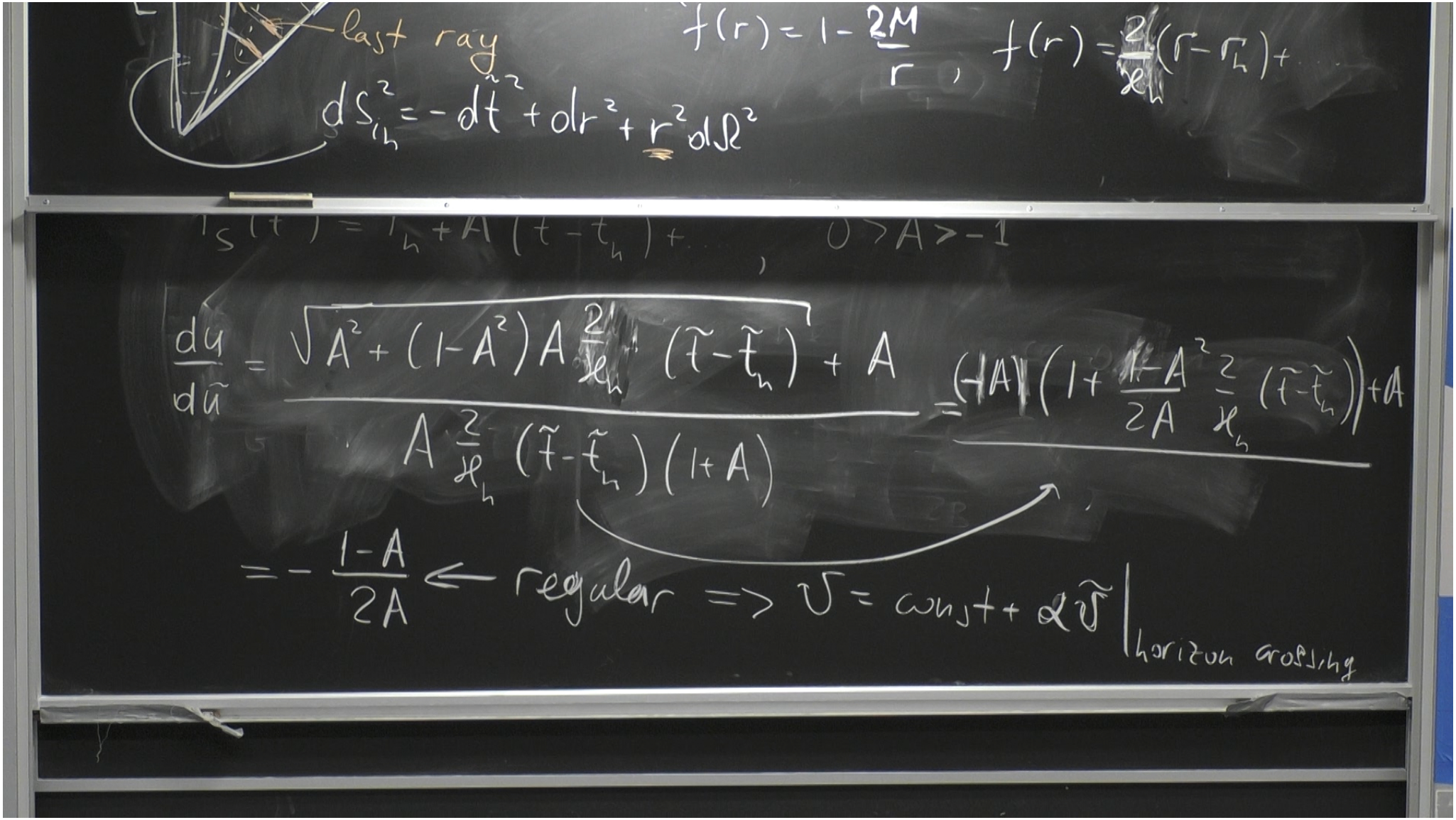

last ray
$f(r) = 1 - \frac{2M}{r}$, $f(r) = \frac{2}{\kappa_h}(r - r_h) +$
$ds^2_{in} = -d\hat{t}^2 + dr^2 + r^2 d\Omega^2$
$0 > A > -1$
$\frac{d\sigma}{d\tilde{v}} = \frac{\sqrt{A^2 + (1-A^2) A \frac{2}{\kappa_h} (\tilde{t} - \tilde{t}_h)} + A}{A \frac{2}{\kappa_h} (\tilde{t} - \tilde{t}_h)(1+A)}$
$\left(1 + \frac{1-A^2}{2A} \frac{2}{\kappa_h} (\tilde{t} - \tilde{t}_h)\right) + A$
$= -\frac{1-A}{2A}$ ← regula

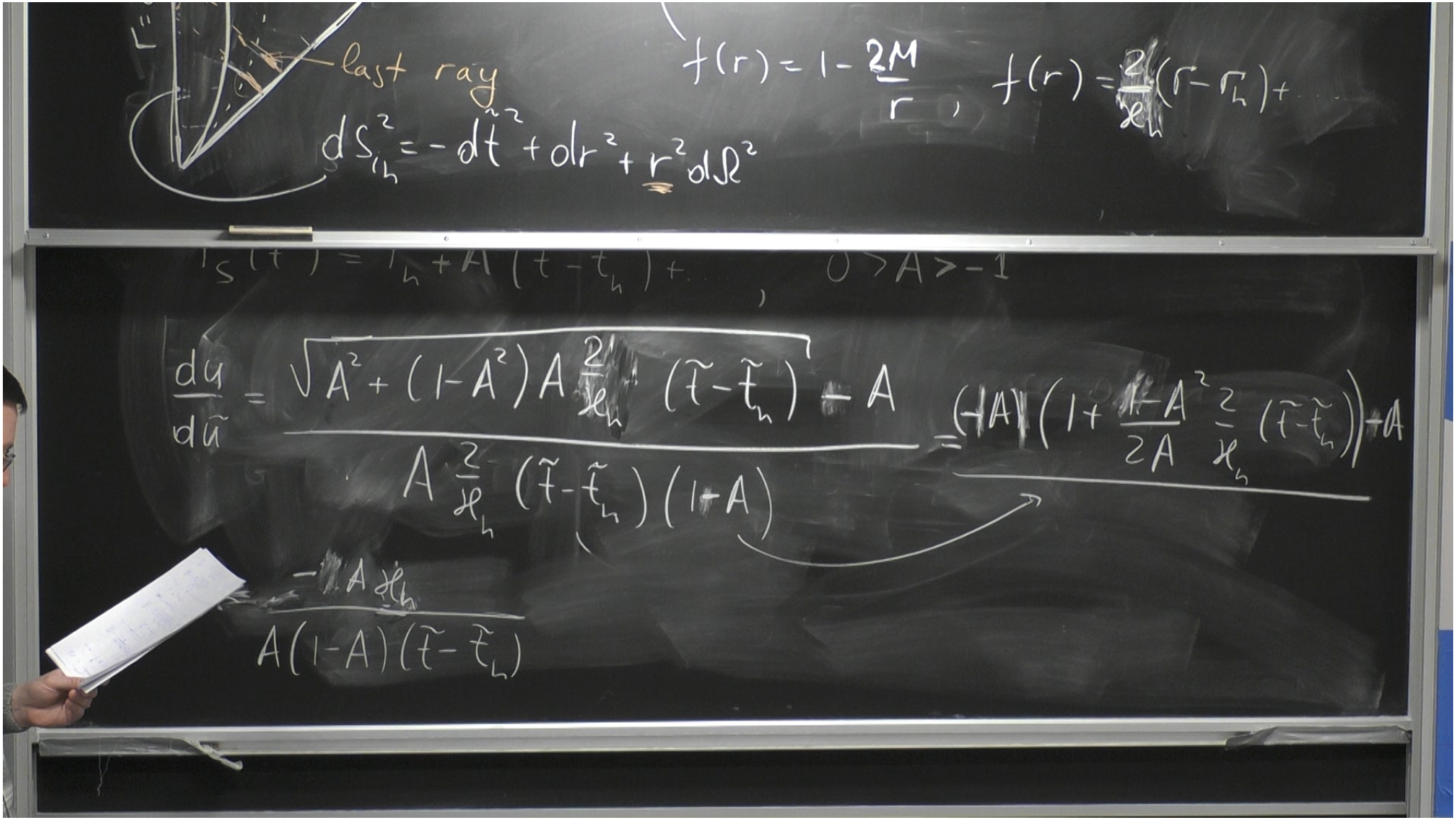

last ray
$f(r) = 1 - \frac{2M}{r}$, $f(r) = \frac{2}{\mathcal{H}_h}(r - r_h) + \dots$
$ds^2_{in} = -dt^2 + dr^2 + r^2 d\Omega^2$
$r_s(t) = r_h + A(t - t_h) + \dots$, $0 > A > -1$
$\frac{dv}{d\tilde{v}} = \frac{\sqrt{A^2 + (1-A^2) A \frac{2}{\mathcal{H}_h} (\tilde{t} - \tilde{t}_h)} + A}{A \frac{2}{\mathcal{H}_h} (\tilde{t} - \tilde{t}_h)(1+A)} = \frac{(+A)\left(1 + \frac{1-A^2}{2A} \frac{2}{\mathcal{H}_h} (\tilde{t} - \tilde{t}_h)\right) + A}{}$
$= -\frac{1-A}{2A}$ ← regular ⇒ $v = \text{const} + \alpha \tilde{v}$ | horizon crossing

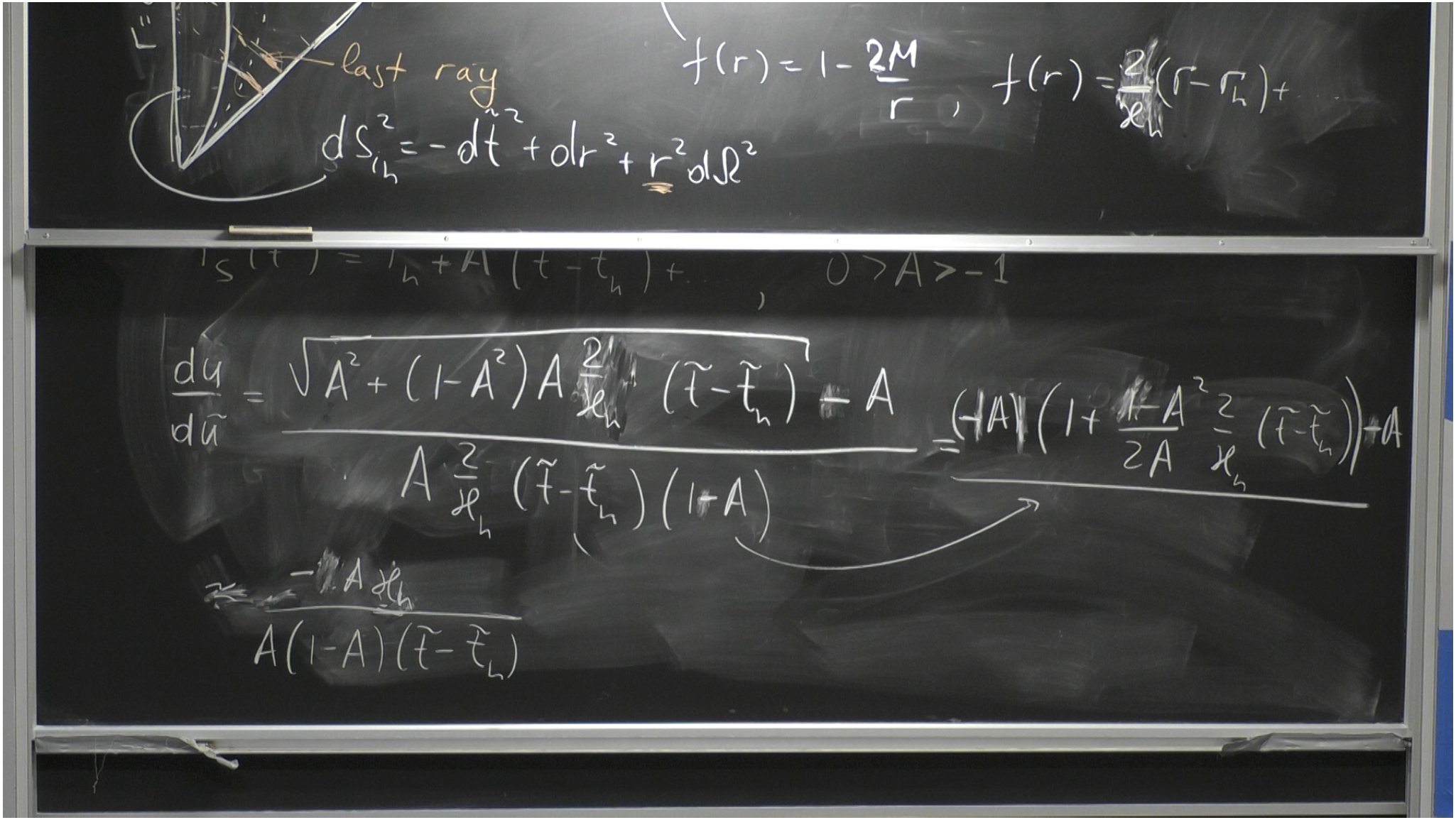

last ray
$f(r) = 1 - \frac{2M}{r}$, $f(r) = \frac{2}{\mathcal{R}_h}(r - r_h) +$
$ds^2_{in} = -dt^2 + dr^2 + r^2 d\Omega^2$
$0 > A > -1$
$\frac{d\tilde{v}}{d\tilde{v}} = \frac{\sqrt{A^2 + (1-A^2)A\frac{2}{\mathcal{R}_h}(\tilde{t} - \tilde{t}_h)}}{A\frac{2}{\mathcal{R}_h}(\tilde{t} - \tilde{t}_h)(1+A)}$
$= \frac{(1+A)\left(1 + \frac{1-A^2}{2A}\frac{2}{\mathcal{R}_h}(\tilde{t} - \tilde{t}_h)\right) + A}{}$
$= -\frac{1-A}{2A}$ ← regular =>
$+ \alpha\tilde{v}\Big|_{\text{horizon crossing}}$


last ray
$f(r) = 1 - \frac{2M}{r}$, $f(r) = \frac{2}{\mathcal{R}_h}(r - r_h) + \dots$
$ds^2_{in} = -dt^2 + dr^2 + r^2 d\Omega^2$
$r_s(t) = r_h + A(t - t_h) + \dots$, $0 > A > -1$
$\frac{dv}{d\tilde{v}} = \frac{\sqrt{A^2 + (1-A^2) A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)} + A}{A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)(1+A)} = \frac{(1+A)\left(1 + \frac{1-A^2}{2A} \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)\right) + A}{}$
$= -\frac{1-A}{2A}$ ← regular $\Rightarrow v = const + \alpha \tilde{v}$ | horizon crossing


$ds^2_{out} = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$
$f(r) = 1 - \frac{2M}{r}$, $f(r) = \frac{2}{\mathcal{H}_h}(r - r_h) + \dots$
$r = 0$
last ray
$ds^2_{in} = -d\tilde{t}^2 + dr^2 + r^2 d\Omega^2$
$t_s(\tilde{t}) = t_h + A(\tilde{t} - \tilde{t}_h) + \dots$, $0 > A > -1$
$\frac{dv}{d\tilde{v}} = \frac{\sqrt{A^2 + (1-A^2) A \frac{2}{\mathcal{H}_h}(\tilde{t} - \tilde{t}_h)} + A}{A \frac{2}{\mathcal{H}_h}(\tilde{t} - \tilde{t}_h)(1+A)} = \frac{(1+A)\left(1 + \frac{1-A^2}{2A} \frac{2}{\mathcal{H}_h}(\tilde{t} - \tilde{t}_h)\right) + A}{}$
$= -\frac{1-A}{2A}$ ← regular $\Rightarrow v = \omega_{in} t + \alpha \tilde{v}$ | horizon crossing


last ray
$f(r) = 1 - \frac{2M}{r}$, $f(r) = \frac{2}{\varkappa_h}(r - r_h) + \dots$
$ds^2_{in} = -dt^2 + dr^2 + r^2 d\Omega^2$
$r_s(t) = r_h + A(t - t_h) + \dots$, $0 > A > -1$
$\frac{dv}{d\tilde{u}} = \frac{\sqrt{A^2 + (1-A^2) A \frac{2}{\varkappa_h} (\tilde{t} - \tilde{t}_h)} + A}{A \frac{2}{\varkappa_h} (\tilde{t} - \tilde{t}_h)(1+A)} = \frac{(+A)\left(1 + \frac{1-A^2}{2A} \frac{2}{\varkappa_h} (\tilde{t} - \tilde{t}_h)\right) + A}{}$
$= -\frac{1-A}{2A}$ ← regular $\Rightarrow v = \text{const} + \alpha \tilde{v}\Big|_{\text{horizon crossing}}$

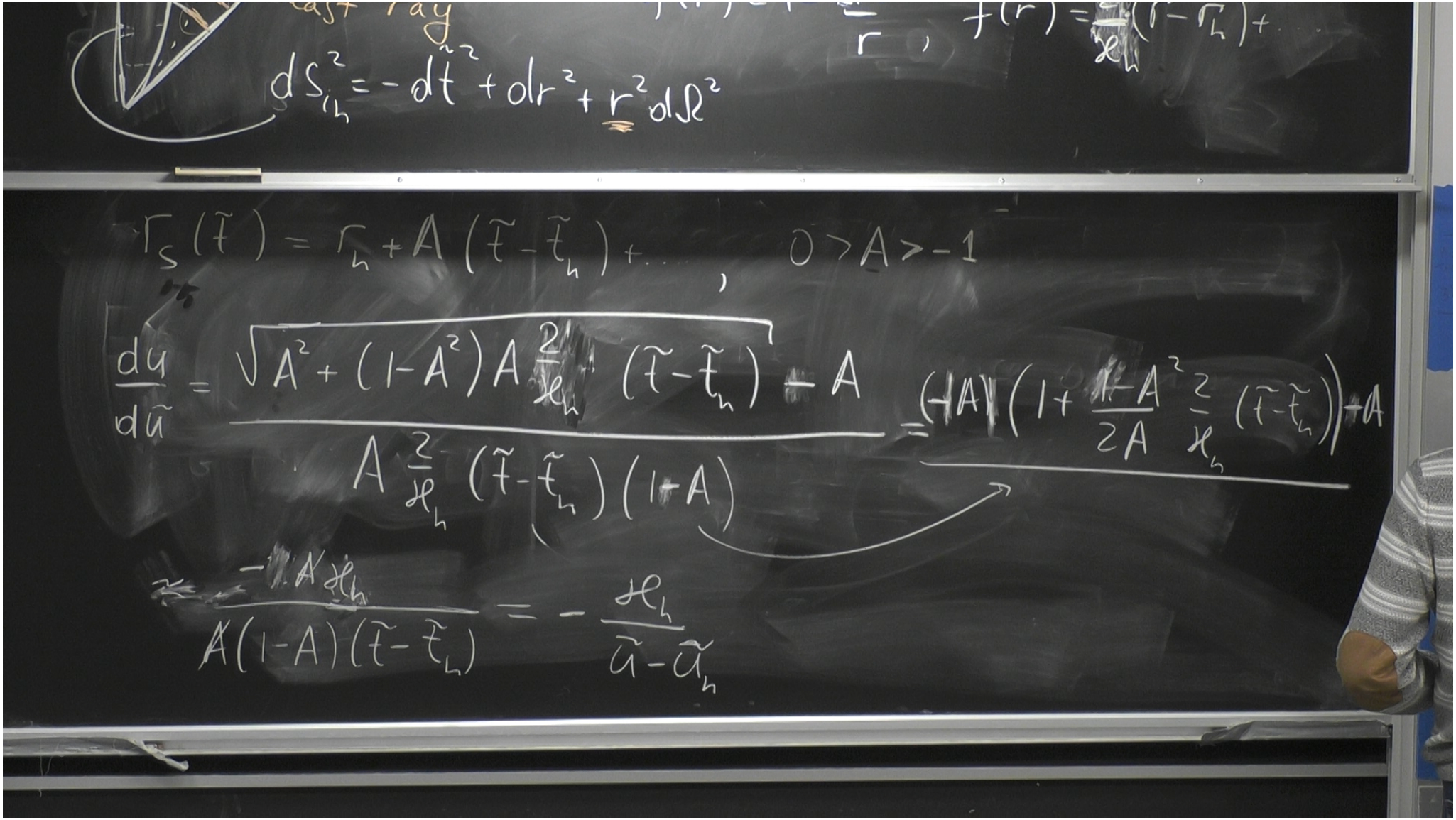

last ray
$f(r) = 1 - \frac{2M}{r}$, $f(r) = \frac{2}{\mathcal{R}_h}(r - r_h) + \dots$
$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2$
$t_s(\tilde{t}) = t_h + A(\tilde{t} - \tilde{t}_h) + \dots$, $0 > A > -1$
$\frac{du}{d\tilde{u}} = \frac{\sqrt{A^2 + (1-A^2) A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)} - A}{A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)(1+A)} = \frac{(1+A)\left(1 + \frac{1-A^2}{2A} \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)\right) + A}{}$
$\frac{-1 \cdot A \mathcal{R}_h}{A(1-A)(\tilde{t} - \tilde{t}_h)}$

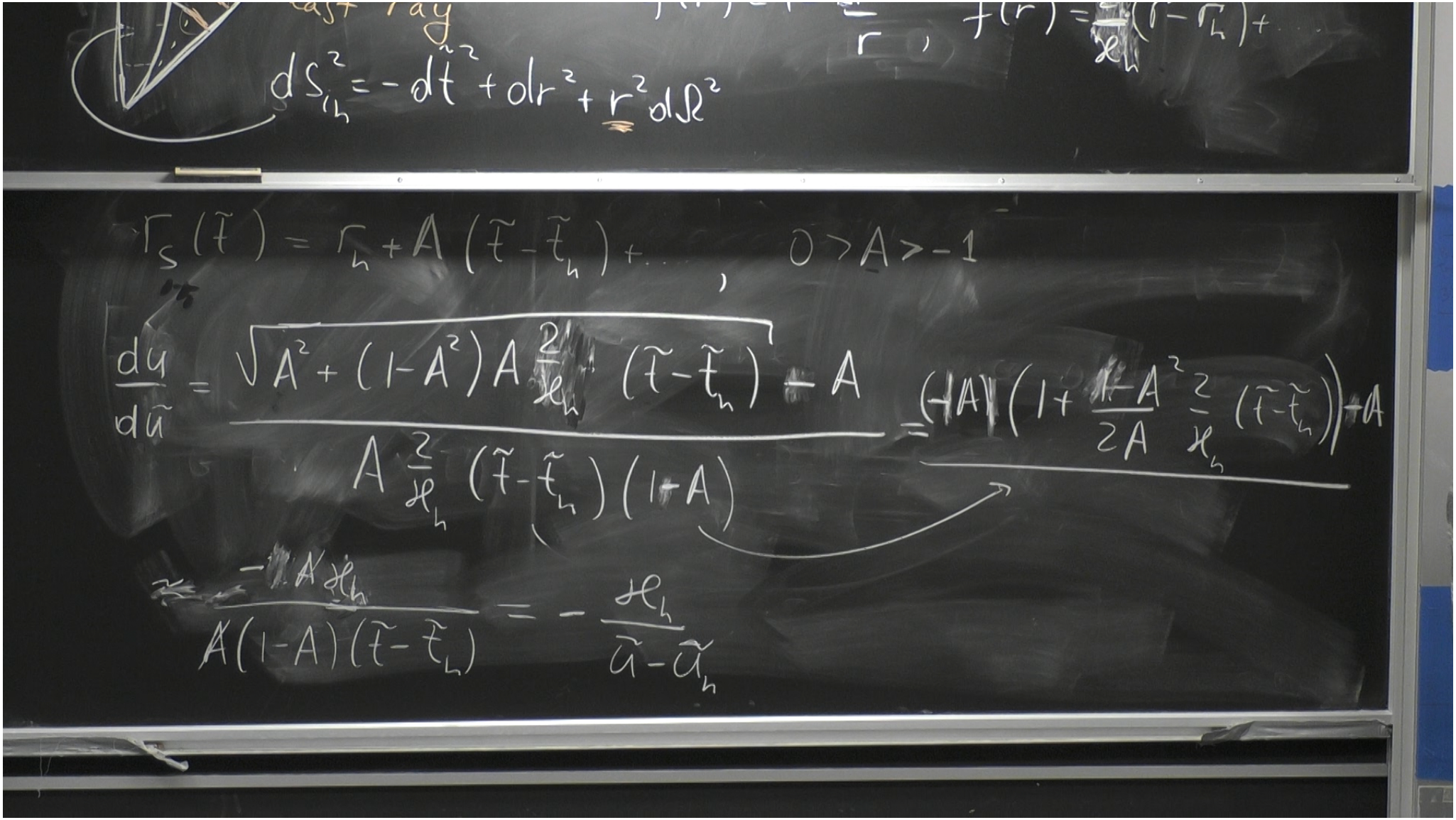

last ray
$f(r) = 1 - \frac{2M}{r}$, $f(r) = \frac{2}{\mathcal{R}_h}(r - r_h) +$
$ds^2_{in} = -dt^2 + dr^2 + r^2 d\Omega^2$
$0 > A > -1$
$\frac{du}{d\tilde{u}} = \frac{\sqrt{A^2 + (1-A^2) A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)} - A}{A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)(1-A)} = \frac{(-A)\left(1 + \frac{1-A^2}{2A} \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)\right) + A}{}$
$\approx \frac{-A \mathcal{R}_h}{A(1-A)(\tilde{t} - \tilde{t}_h)}$

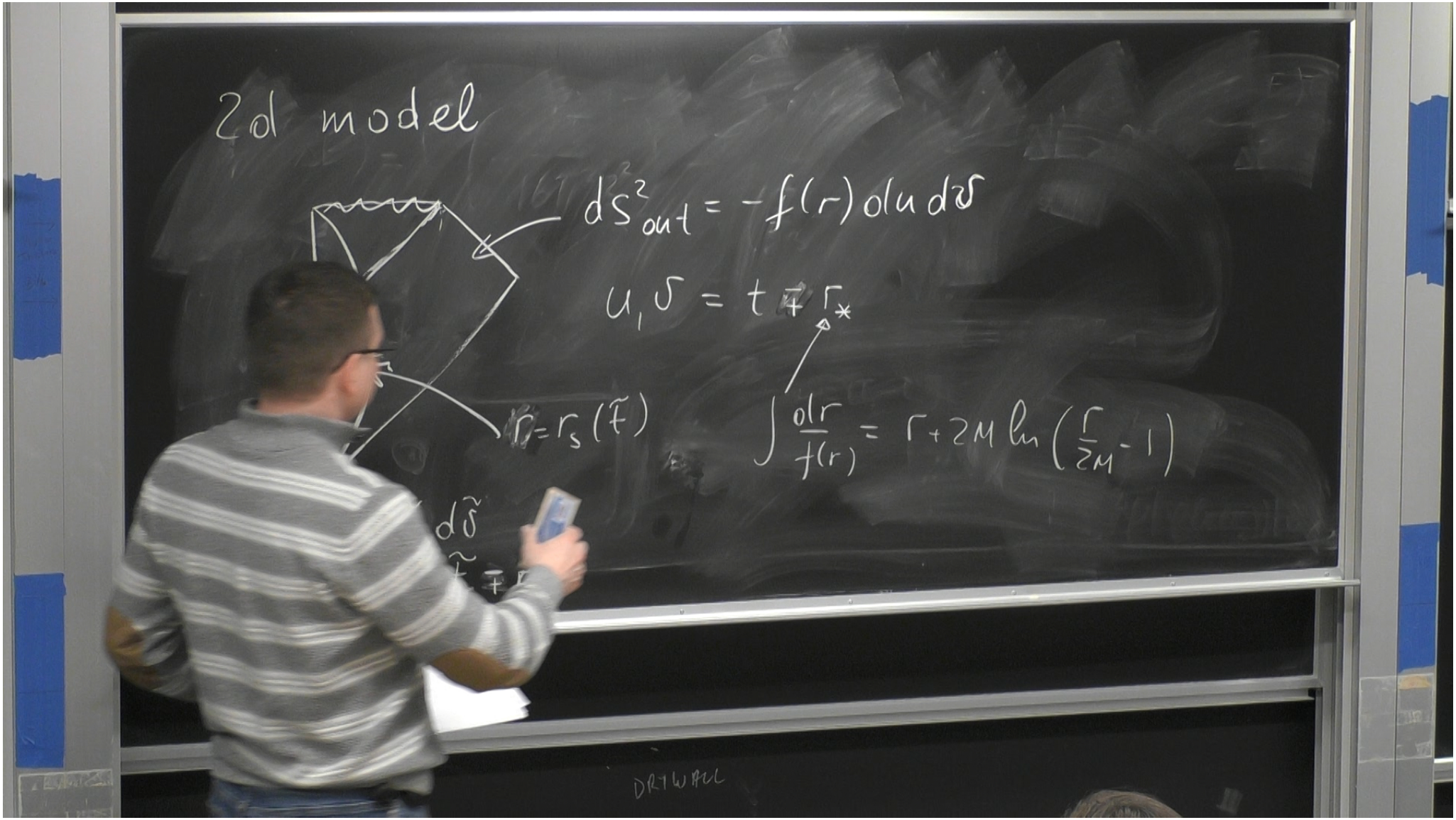
$ds_{in} = -du\,d\sigma$

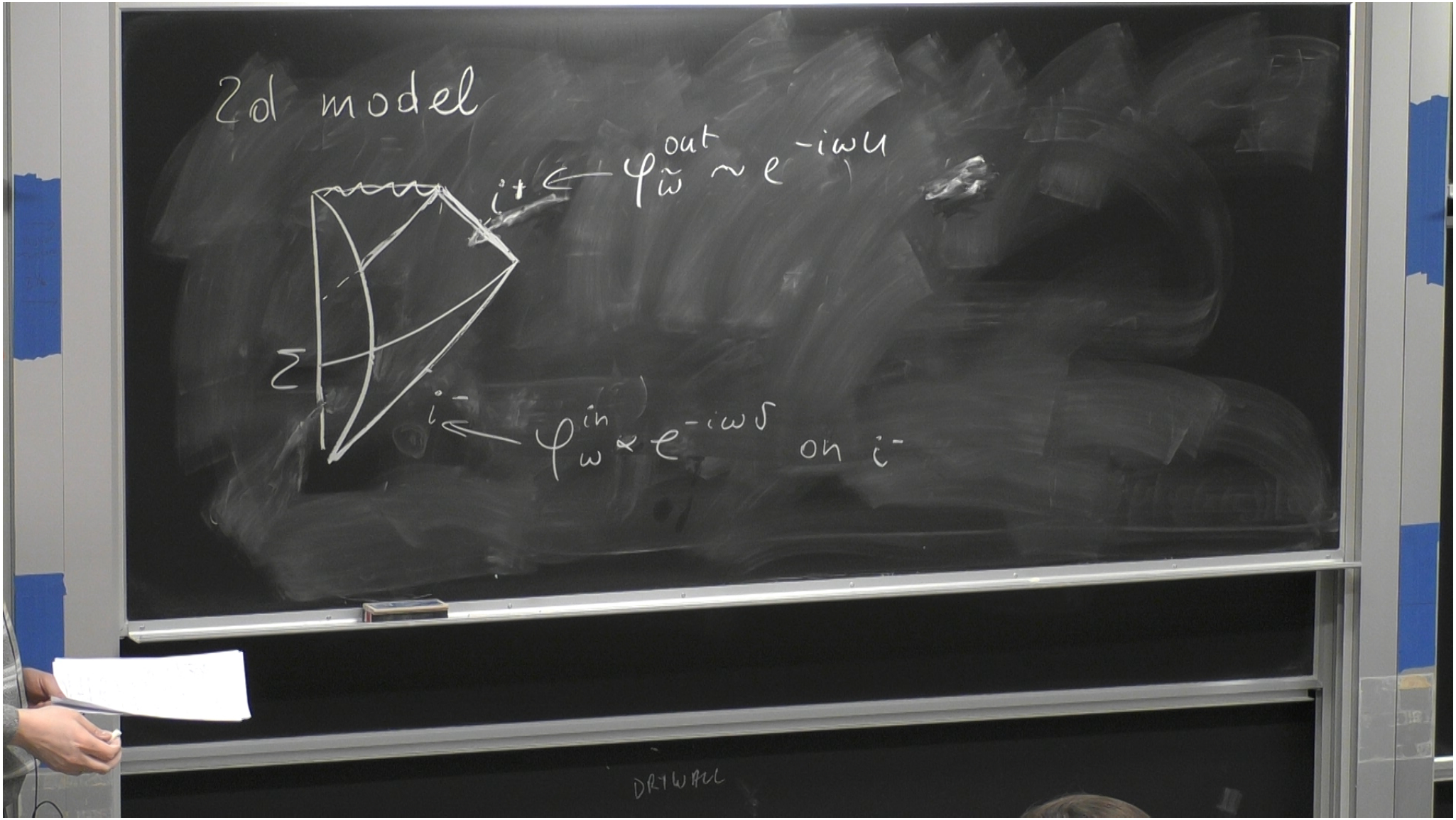
$\tilde{u}, \tilde{v} = \tilde{t} \mp r$

$$\tilde{u} = \tilde{t} - r = \tilde{t} - r_n - A(\tilde{t} - \tilde{t}_n) = \tilde{u}_n + (1-A)(\tilde{t} - \tilde{t}_n)$$


$ds^2_{in} = -dt^2 + dr^2 + r^2 d\Omega^2$
$r_s(\tilde{t}) = r_h + A(\tilde{t} - \tilde{t}_h) + \dots$, $0 > A > -1$
$\frac{dy}{d\tilde{u}} = \frac{\sqrt{A^2 + (1-A^2) A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)} - A}{A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)(1+A)}$
$= \frac{(+A)\left(1 + \frac{1-A^2}{2A} \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)\right) + A}{}$
$= \frac{-A \mathcal{R}_h}{A(1-A)(\tilde{t} - \tilde{t}_h)} = -\frac{\mathcal{R}_h}{\tilde{u} - \tilde{u}_h}$


$ds^2_{in} = -dt^2 + dr^2 + r^2 d\Omega^2$
$r_s(\tilde{t}) = r_h + A(\tilde{t} - \tilde{t}_h) + \dots$, $0 > A > -1$
$\frac{dy}{d\tilde{u}} = \frac{\sqrt{A^2 + (1-A^2) A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)} - A}{A \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)(1+A)} = \frac{(+A)\left(1 + \frac{1-A^2}{2A} \frac{2}{\mathcal{R}_h} (\tilde{t} - \tilde{t}_h)\right) + A}{}$
$= \frac{-A \mathcal{R}_h}{A(1-A)(\tilde{t} - \tilde{t}_h)} = -\frac{\mathcal{R}_h}{\tilde{u} - \tilde{u}_h}$


2d model
$ds^2_{out} = -f(r)\,du\,dv$
$u, v = t \mp r_*$
$r = r_s(\tilde{t})$
$\int \frac{dr}{f(r)} = r + 2M \ln\left(\frac{r}{2M} - 1\right)$
$d\tilde{v}$
DRYWALL


2d model
i^+
Σ
i^-
$\varphi^{out}_{\tilde{\omega}} \sim e^{-i\omega u}$
$\varphi^{in}_{\omega} \propto e^{-i\omega v}$ on i^-
DRYWALL

$i^- \leftarrow \varphi^{in}_\omega \propto e^{-i\omega v}$ on i^-

b) on $\bar{\Sigma}$:

$$\varphi^{in}_\omega = \frac{1}{\sqrt{4\pi\omega}}\left(e^{-i\omega v} - e^{-i\omega u}\right)$$

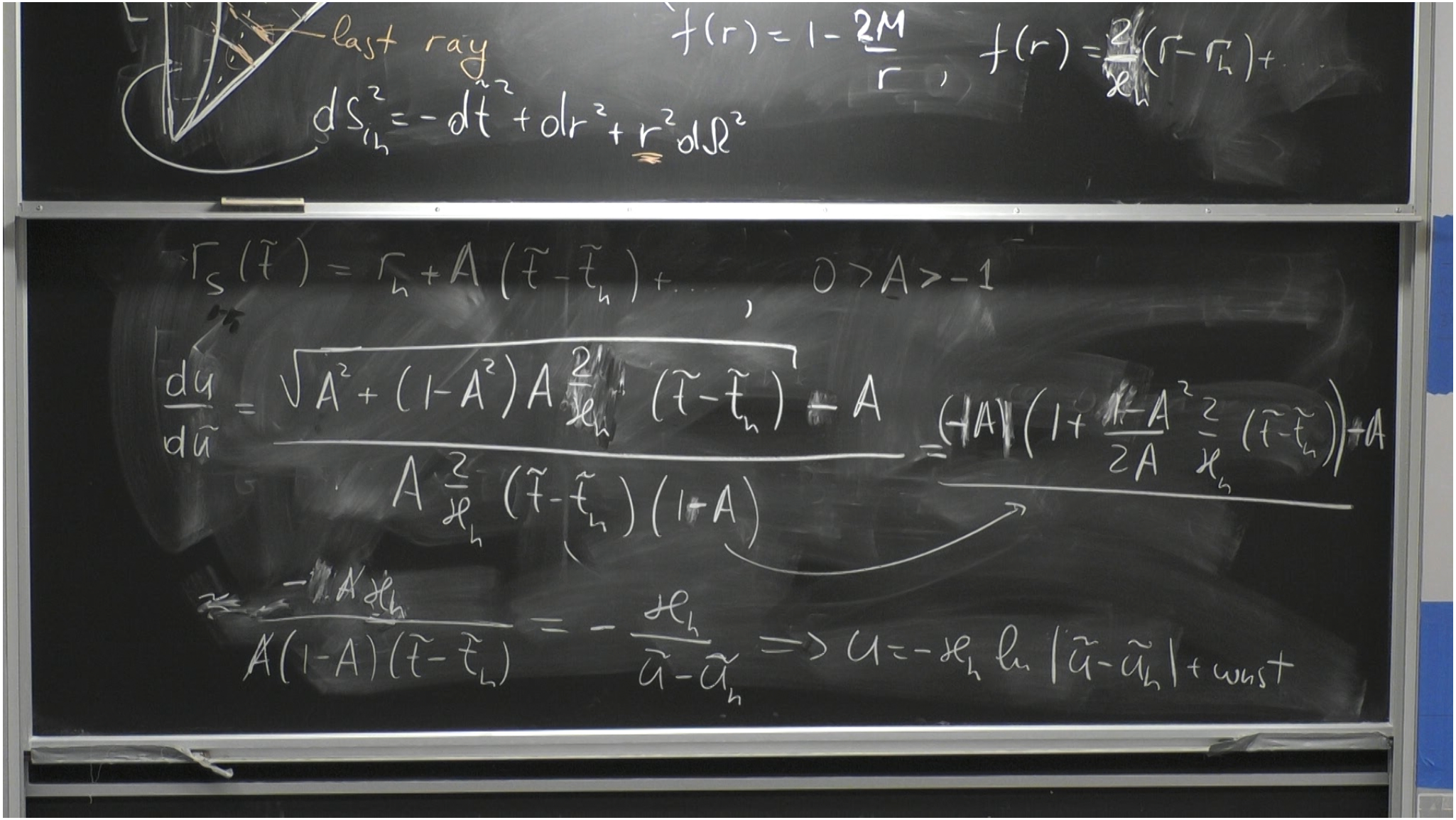
$$\varphi^{out}_{\tilde{\omega}} = \frac{1}{\sqrt{4\pi\omega}}\left(e^{-i\tilde{\omega} u} - e^{-i\tilde{\omega} u_0(v)}\right)$$

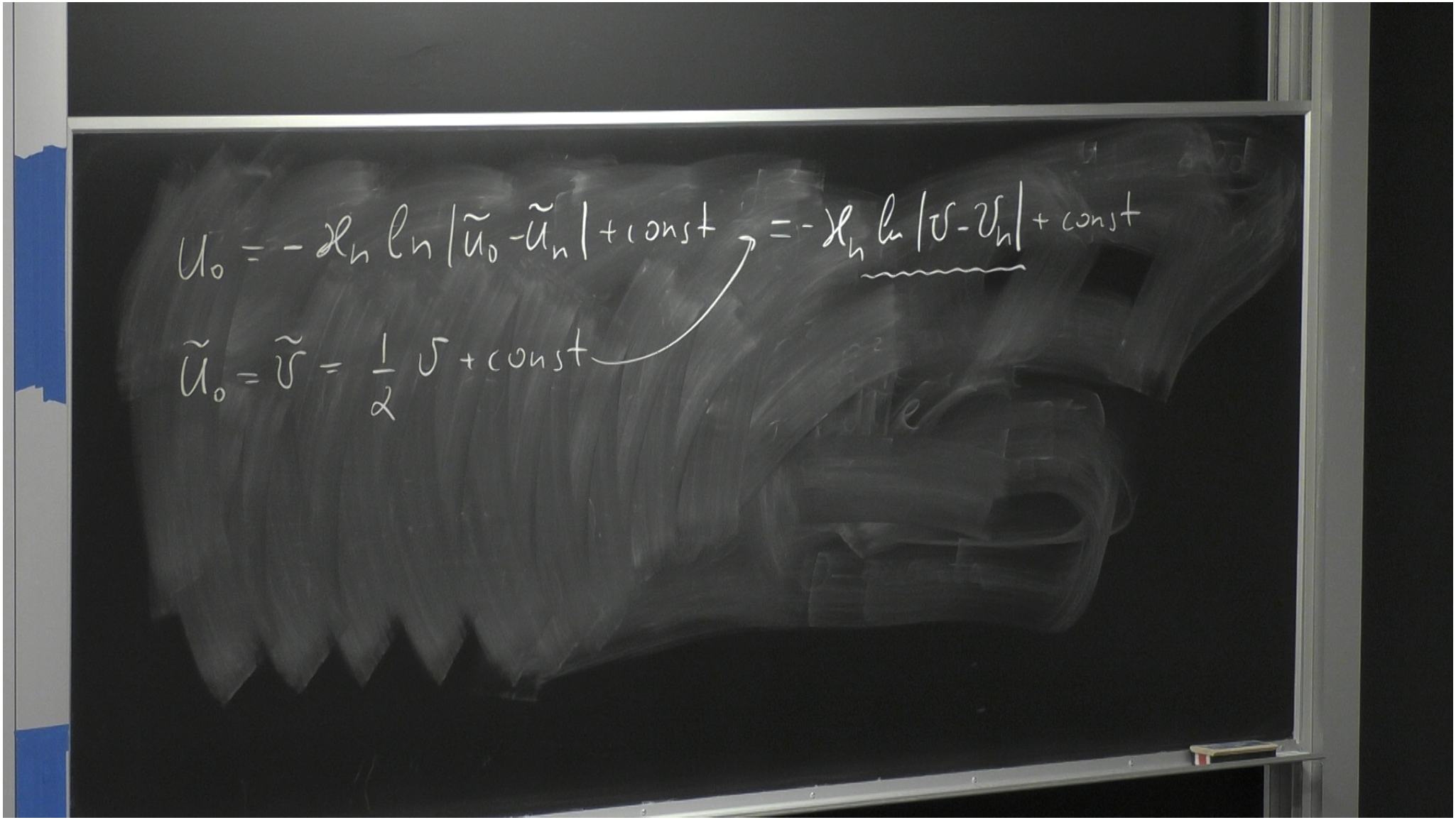
$i^- \leftarrow \varphi^{in}_\omega \propto e^{-i\omega v}$ on i^-

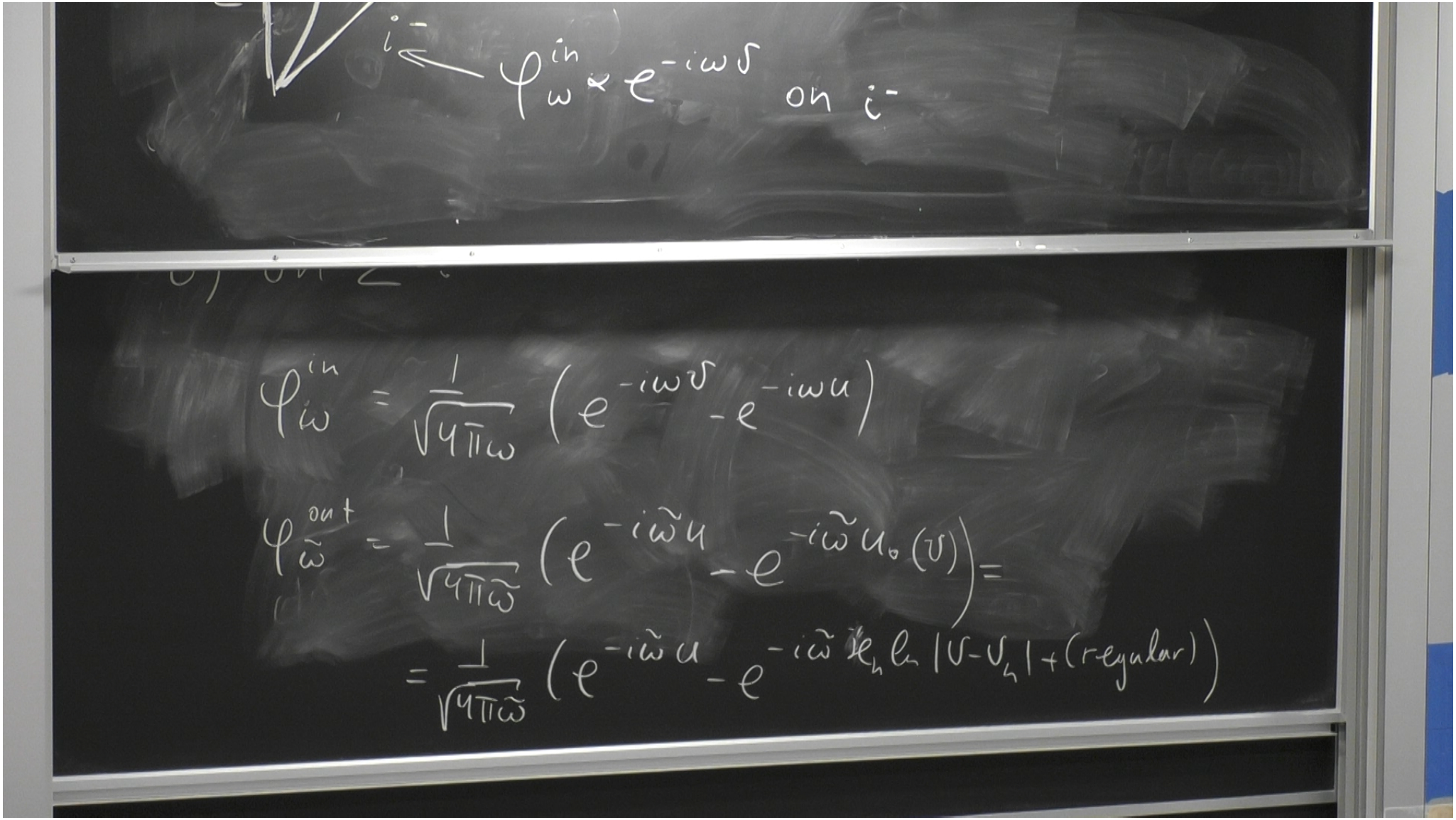
b) on $\bar{\Sigma}$:

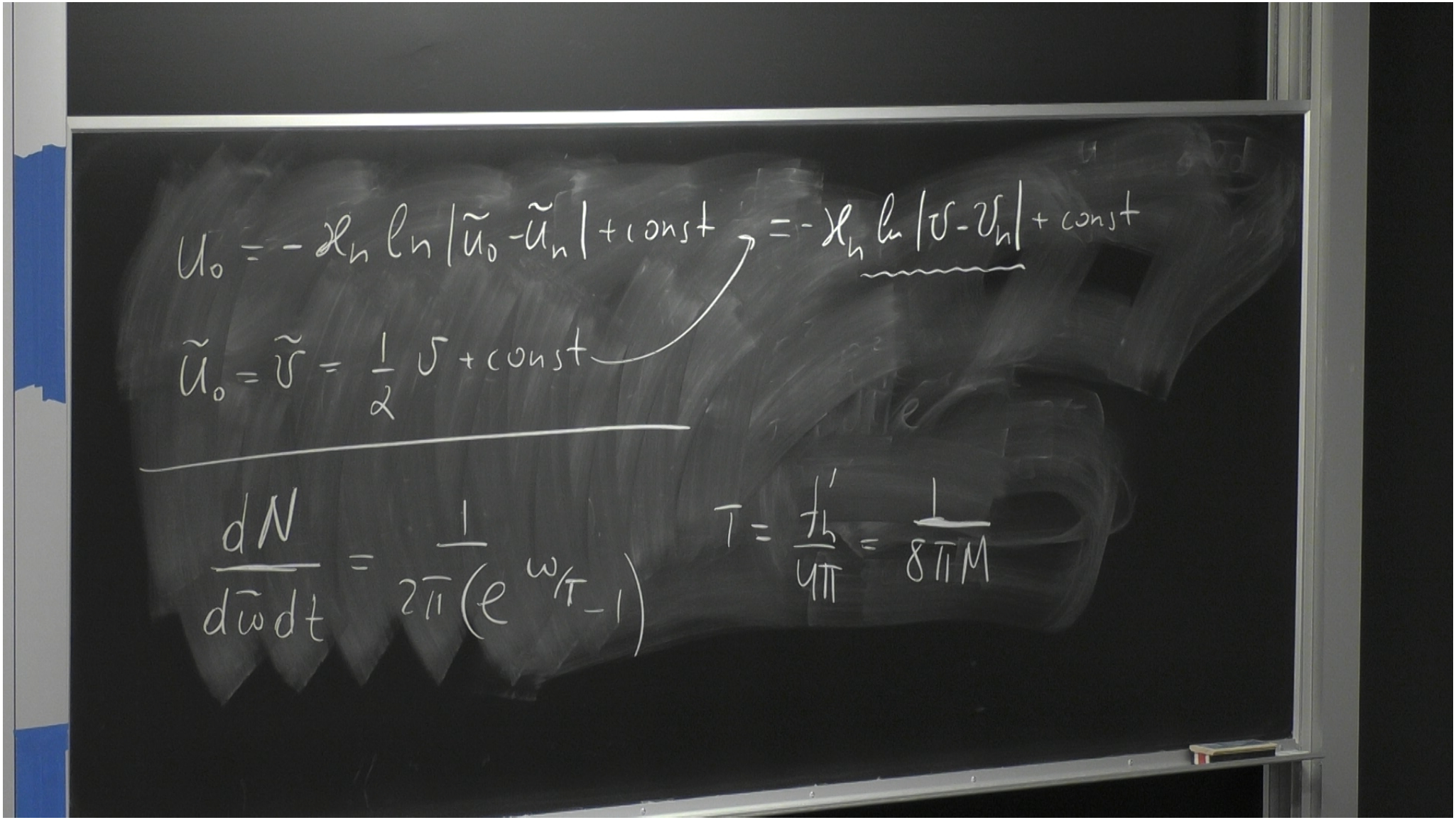
$$\varphi^{in}_\omega = \frac{1}{\sqrt{4\pi\omega}}\left(e^{-i\omega v} - e^{-i\omega u}\right)$$

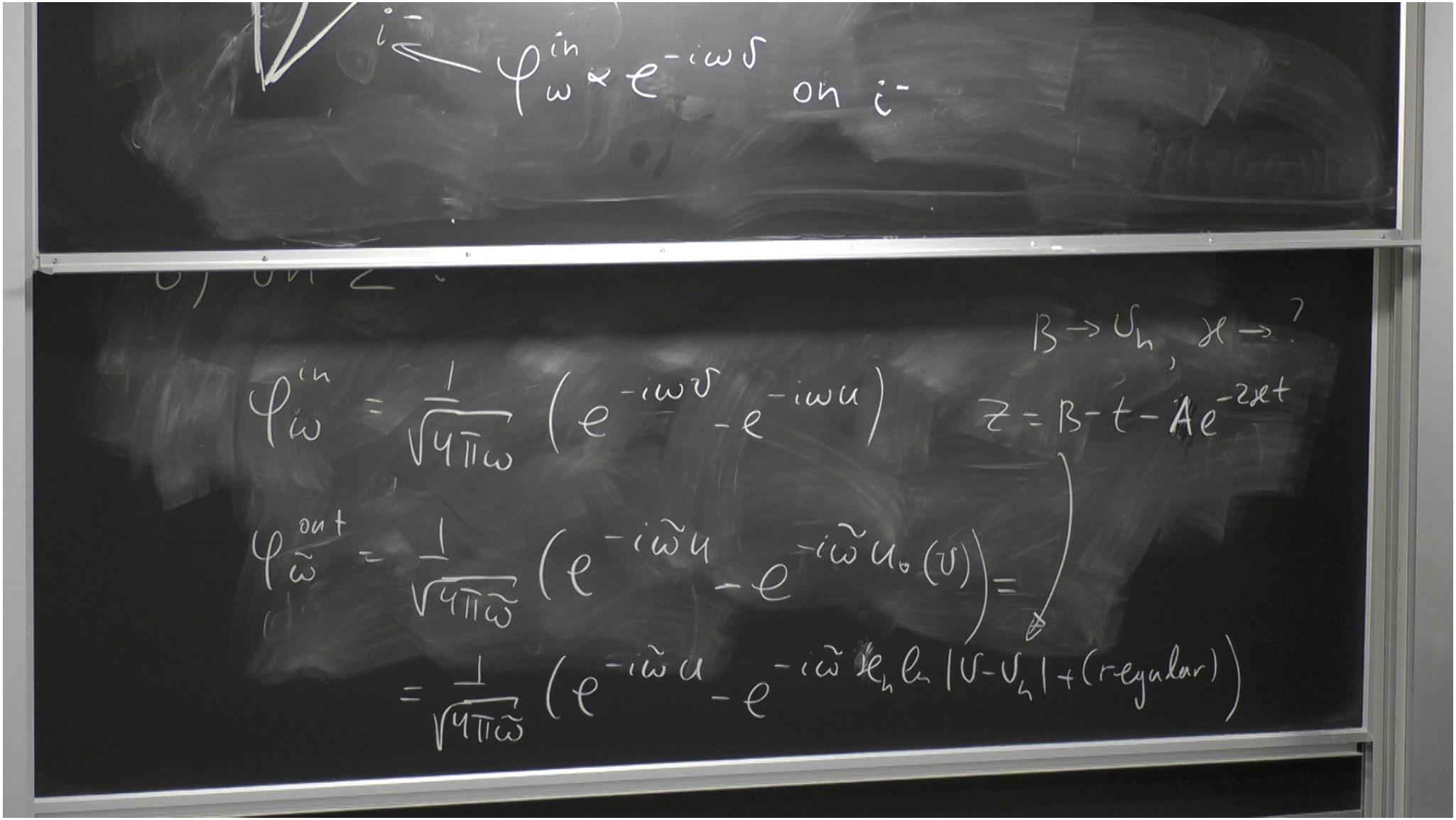
$$\varphi^{out}_{\tilde{\omega}} = \frac{1}{\sqrt{4\pi\omega}}\left(e^{-i\tilde{\omega} u} - e^{-i\tilde{\omega} u_0(v)}\right)$$

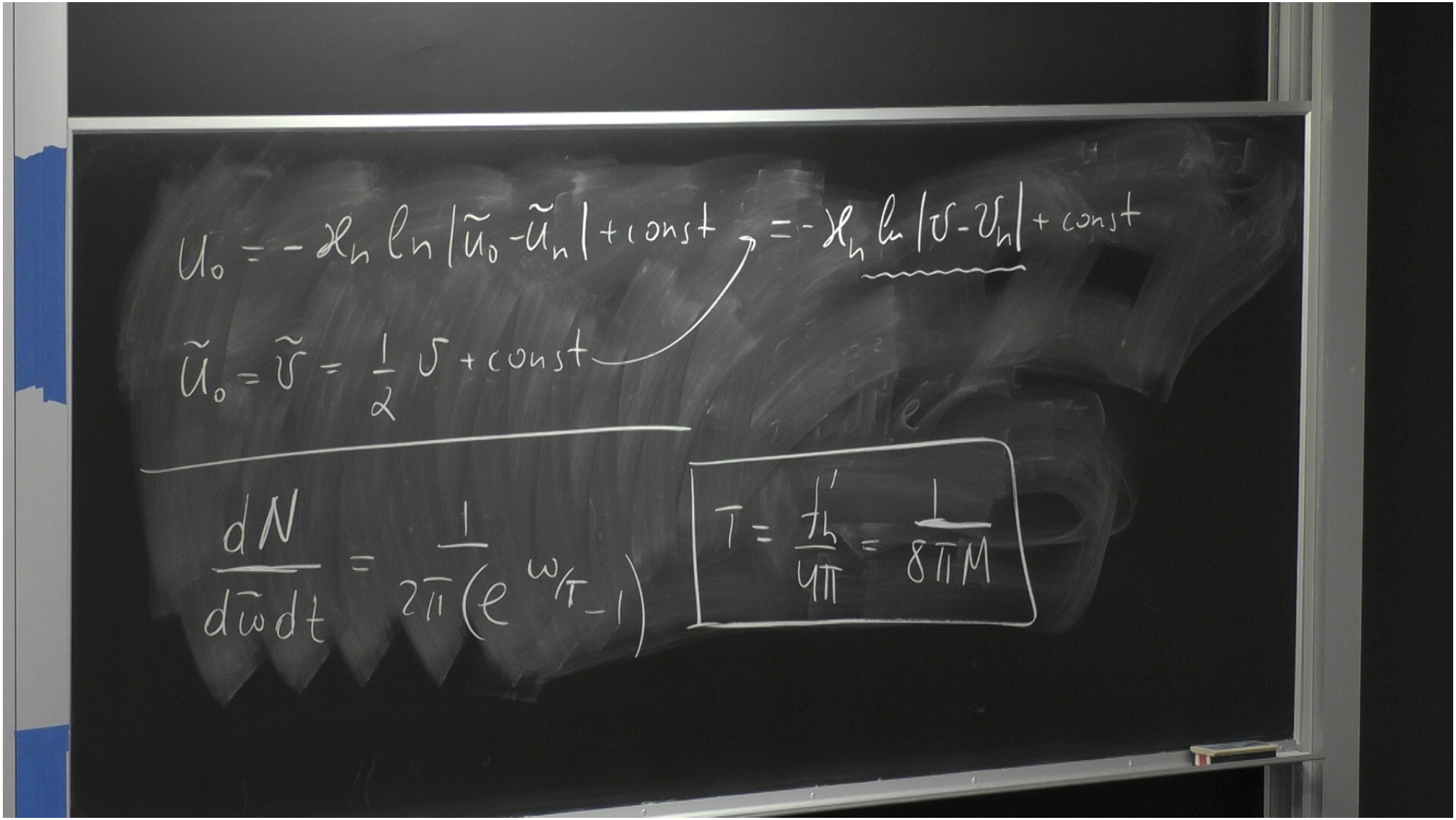

last ray
$f(r) = 1 - \frac{2M}{r}$, $f(r) = \frac{2}{\mathcal{H}_h}(r - r_h) + \dots$
$ds^2_{in} = -d\tilde{t}^2 + dr^2 + r^2 d\Omega^2$
$r_s(\tilde{t}) = r_h + A(\tilde{t} - \tilde{t}_h) + \dots$, $0 > A > -1$
$\frac{du}{d\tilde{u}} = \frac{\sqrt{A^2 + (1-A^2) A \frac{2}{\mathcal{H}_h}(\tilde{t} - \tilde{t}_h)} - A}{A \frac{2}{\mathcal{H}_h}(\tilde{t} - \tilde{t}_h)(1-A)} = \frac{(1-A)\left(1 + \frac{1-A^2}{2A} \frac{2}{\mathcal{H}_h}(\tilde{t} - \tilde{t}_h)\right) + A}{}$
$= \frac{-A \mathcal{H}_h}{A(1-A)(\tilde{t} - \tilde{t}_h)} = -\frac{\mathcal{H}_h}{\tilde{u} - \tilde{u}_h} \Rightarrow u = -\mathcal{H}_h \ln|\tilde{u} - \tilde{u}_h| + \text{const}$

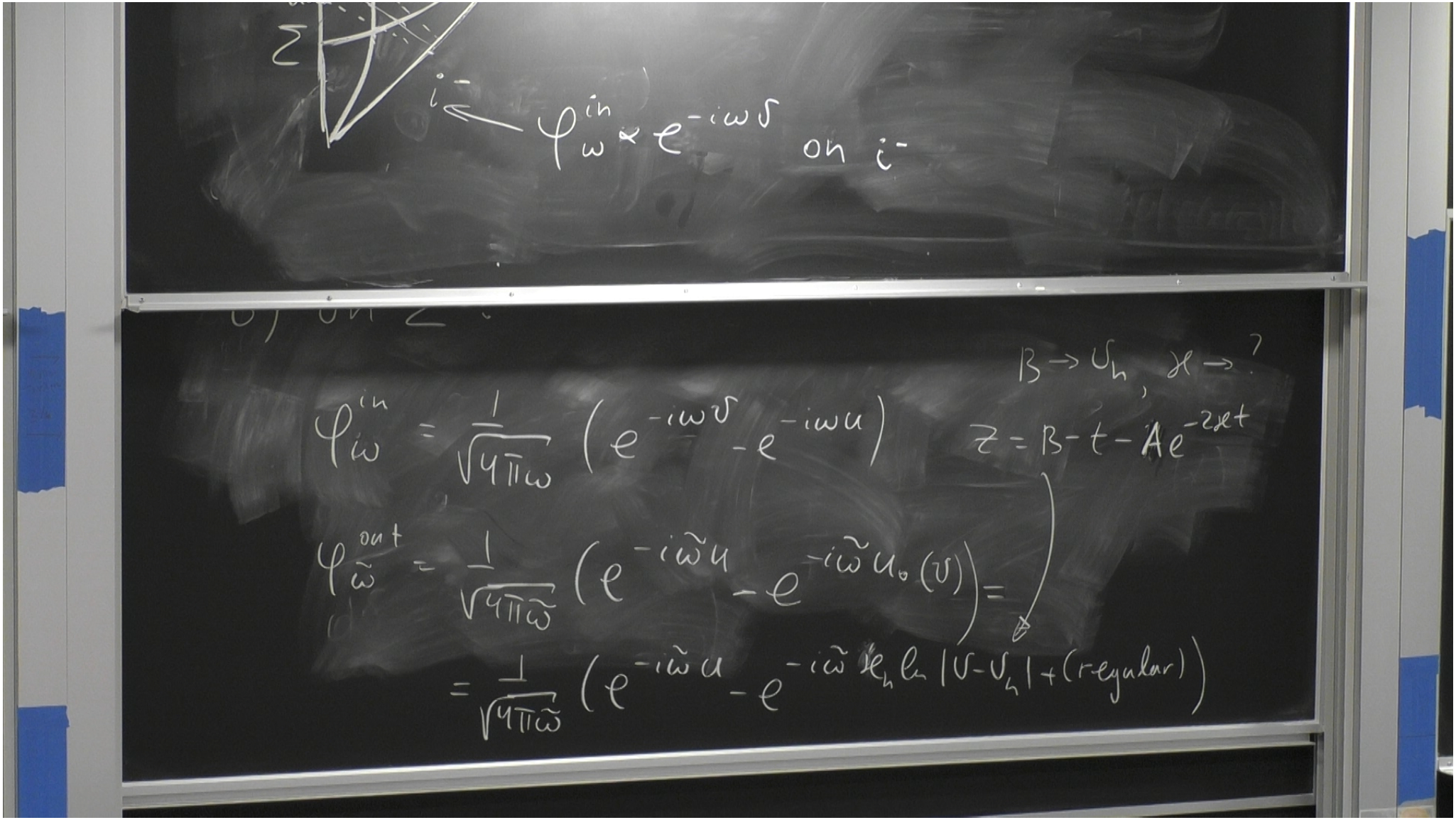

$u_0 = -\alpha_n \ln|\tilde{u}_0 - \tilde{u}_n| + \text{const} = -\alpha_n \ln|v - v_n| + \text{const}$
$\tilde{u}_0 = \tilde{v} = \frac{1}{\alpha} v + \text{const}$

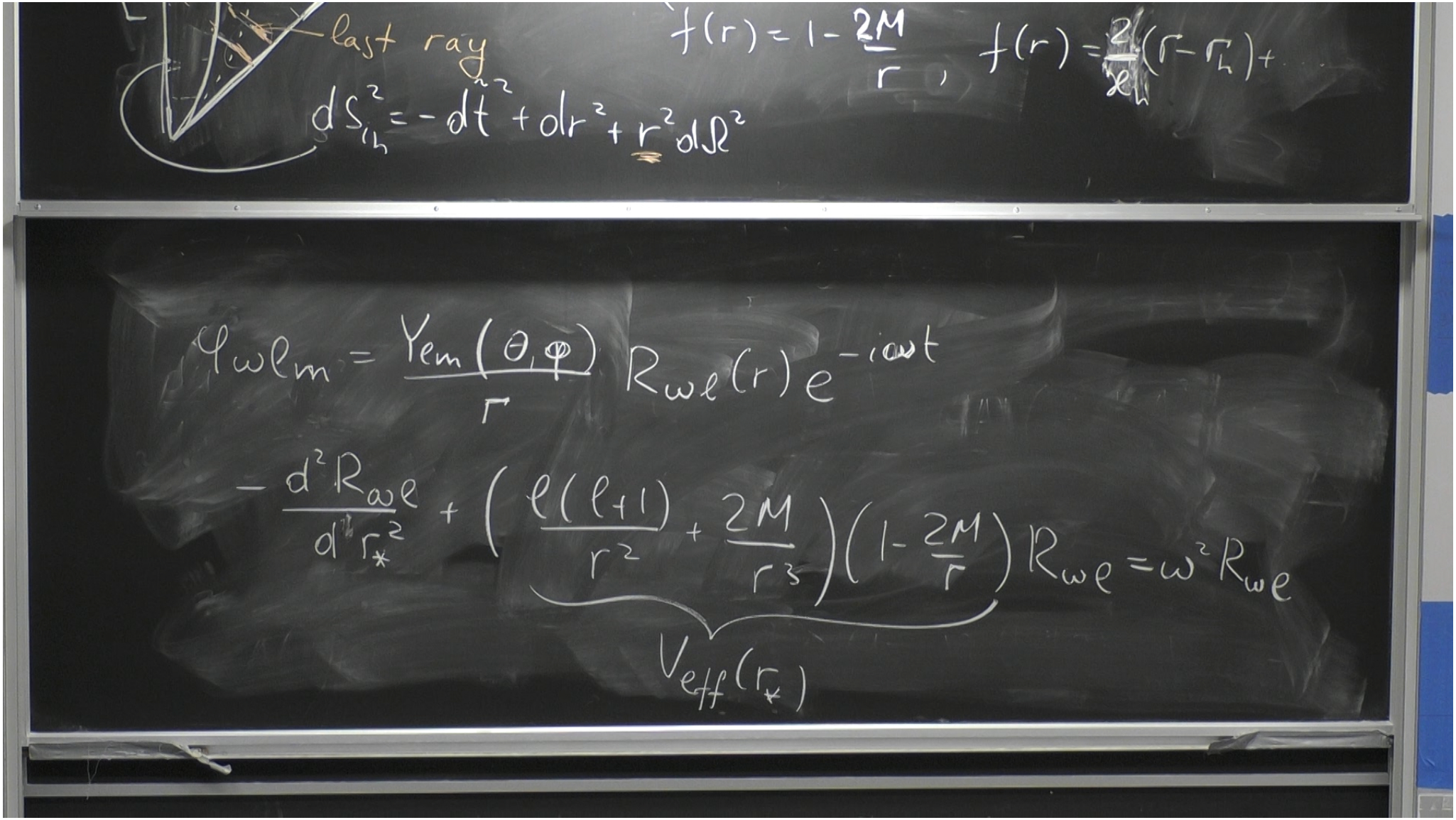

$i^- \leftarrow \varphi^{in}_\omega \propto e^{-i\omega v}$ on i^-
$\varphi^{in}_\omega = \frac{1}{\sqrt{4\pi\omega}}\left(e^{-i\omega v} - e^{-i\omega u}\right)$
$\varphi^{out}_{\tilde\omega} = \frac{1}{\sqrt{4\pi\tilde\omega}}\left(e^{-i\tilde\omega u} - e^{-i\tilde\omega u_0(v)}\right) =$
$= \frac{1}{\sqrt{4\pi\tilde\omega}}\left(e^{-i\tilde\omega u} - e^{-i\tilde\omega \kappa_h \ln|v-v_h|} + (\text{regular})\right)$

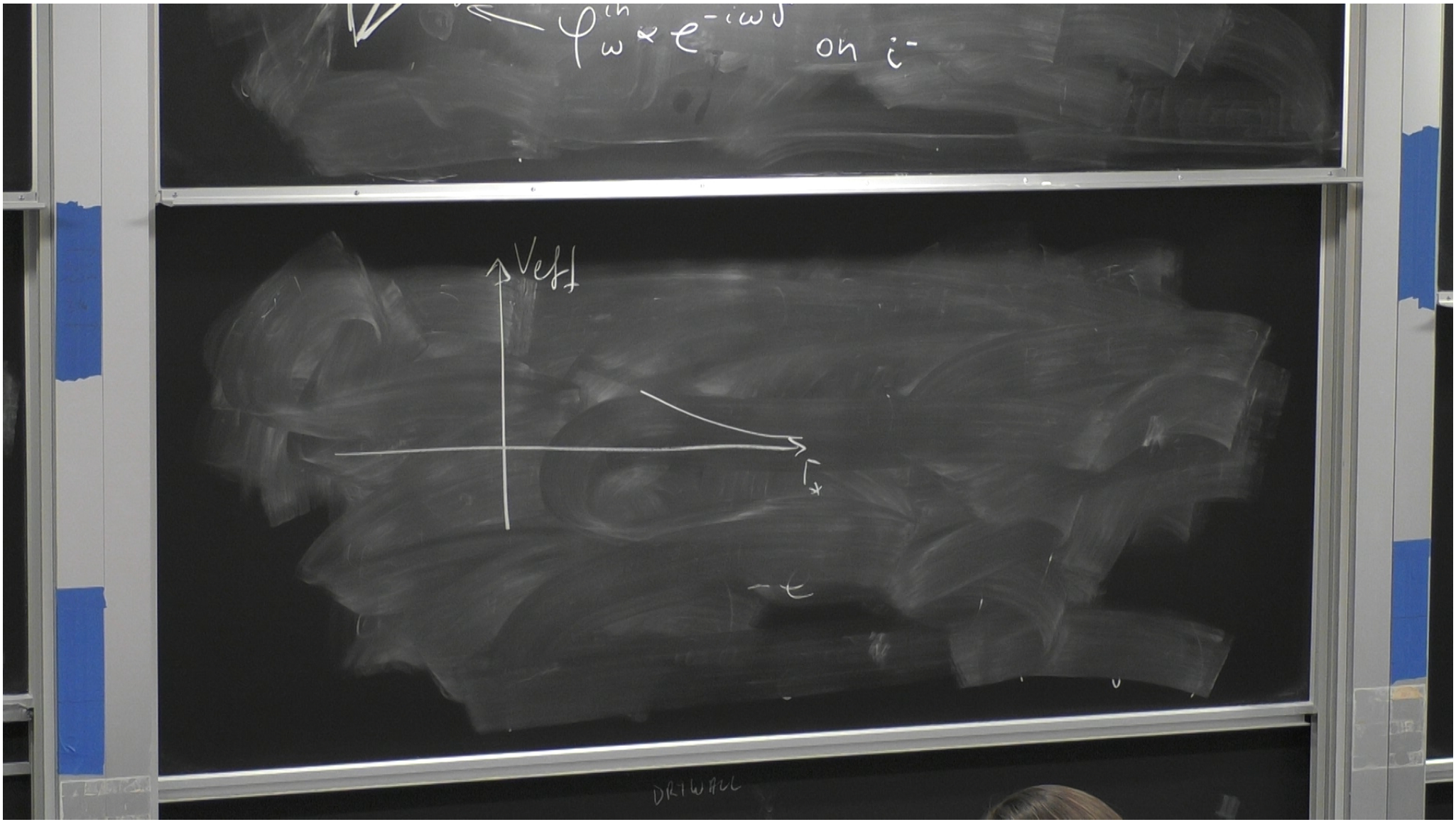

$u_0 = -\varkappa_n \ln|\tilde{u}_0 - \tilde{u}_n| + \text{const} = -\varkappa_n \ln|v - v_n| + \text{const}$
$\tilde{u}_0 = \tilde{v} = \frac{1}{\alpha} v + \text{const}$
$\frac{dN}{d\omega dt} = \frac{1}{2\pi\left(e^{\omega/T} - 1\right)}$
$T = \frac{t_h'}{4\pi} = \frac{1}{8\pi M}$

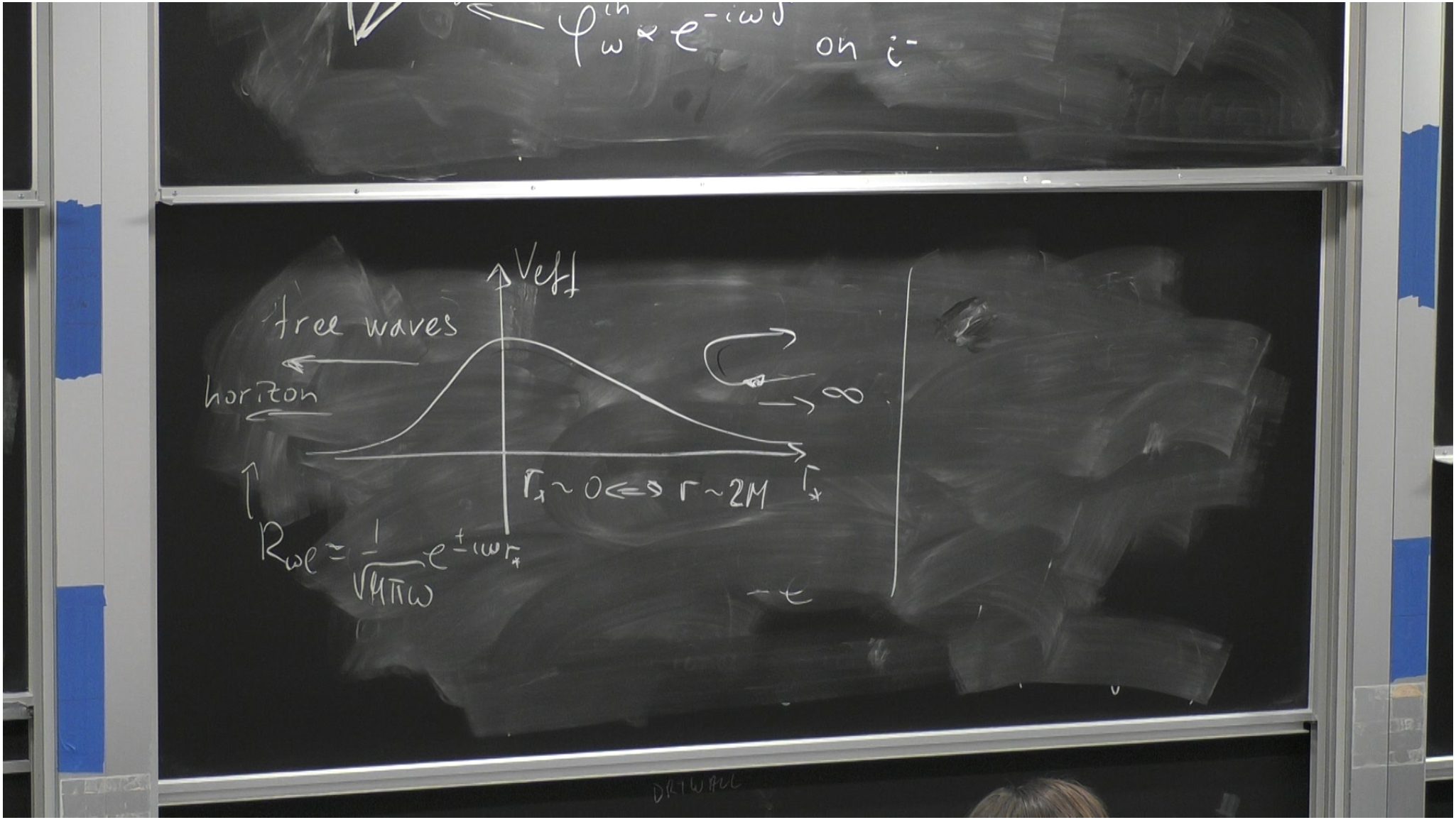

i^-
$\varphi^{in}_\omega \propto e^{-i\omega v}$ on i^-
$\mathcal{B} \to v_h$, $\mathcal{H} \to ?$
$\varphi^{in}_\omega = \frac{1}{\sqrt{4\pi\omega}}\left(e^{-i\omega v} - e^{-i\omega u}\right)$
$z = \mathcal{B} - t - Ae^{-2\varkappa t}$
$\varphi^{out}_{\tilde\omega} = \frac{1}{\sqrt{4\pi\tilde\omega}}\left(e^{-i\tilde\omega u} - e^{-i\tilde\omega u_0(v)}\right) =$
$= \frac{1}{\sqrt{4\pi\tilde\omega}}\left(e^{-i\tilde\omega u} - e^{-i\tilde\omega \varkappa_h h_h \ln|v - v_h|} + (\text{regular})\right)$

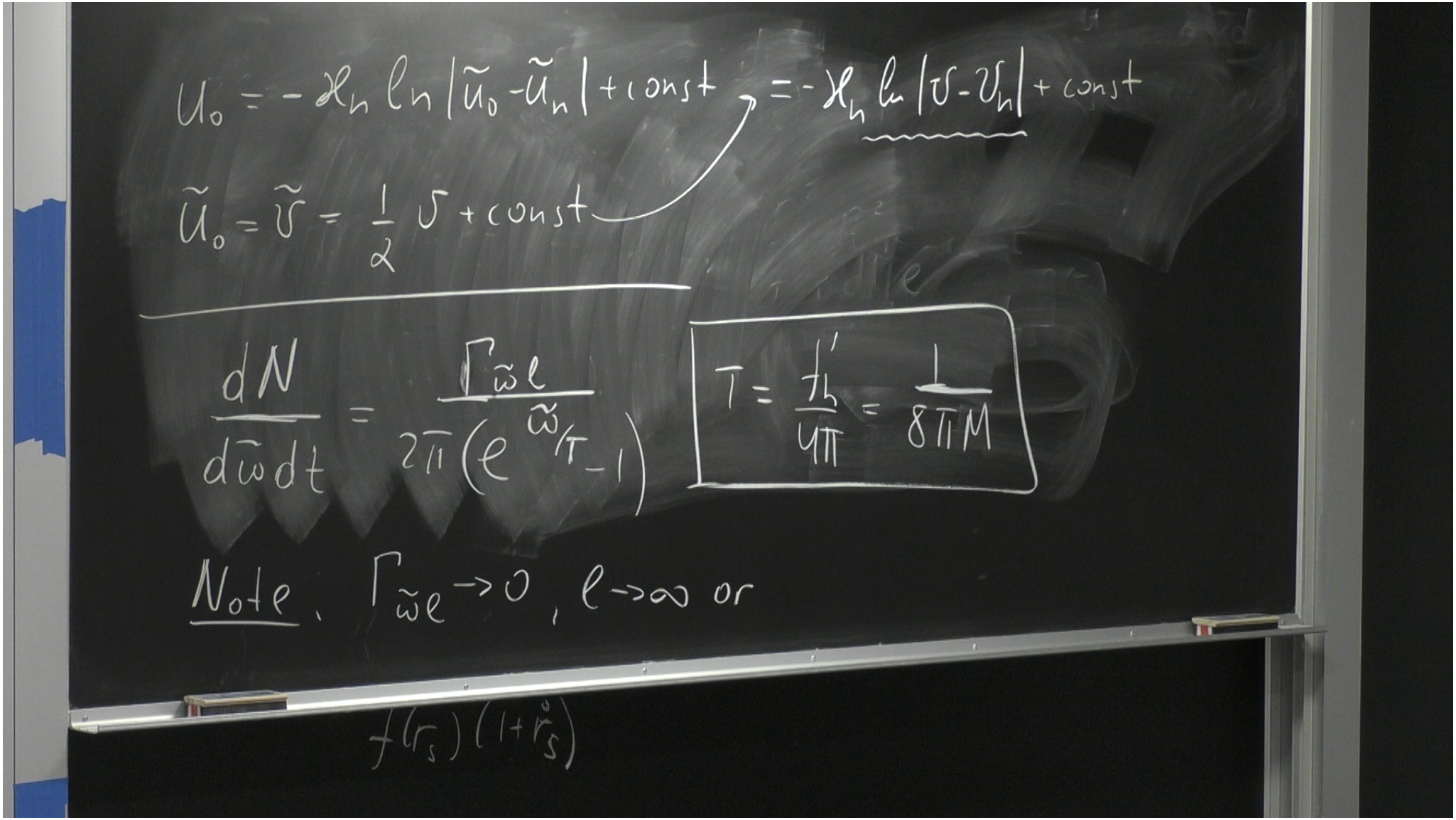

$U_0 = -\varkappa_n \ln|\tilde{u}_0 - \tilde{u}_n| + \text{const} = -\varkappa_n \ln|v - v_n| + \text{const}$
$\tilde{u}_0 = \tilde{v} = \frac{1}{\alpha} v + \text{const}$
$\frac{dN}{d\tilde{\omega}dt} = \frac{1}{2\pi\left(e^{\omega/T} - 1\right)}$
$T = \frac{t_h'}{4\pi} = \frac{1}{8\pi M}$


Σ
i^-
$\varphi^{in}_\omega \propto e^{-i\omega v}$ on i^-
$\varphi^{in}_\omega = \frac{1}{\sqrt{4\pi\omega}}\left(e^{-i\omega v} - e^{-i\omega u}\right)$
$B \to v_h$, $\mathcal{H} \to$?
$z = B - t - Ae^{-2\kappa t}$
$\varphi^{out}_{\tilde\omega} = \frac{1}{\sqrt{4\pi\tilde\omega}}\left(e^{-i\tilde\omega u} - e^{-i\tilde\omega u_0(v)}\right) =$
$= \frac{1}{\sqrt{4\pi\tilde\omega}}\left(e^{-i\tilde\omega u} - e^{-i\tilde\omega \kappa_h \ln|v - v_h|} + (\text{regular})\right)$


last ray
$f(r) = 1 - \frac{2M}{r}$, $f(r) = \frac{2}{\kappa_h}(r - r_h) + \ldots$
$ds^2_{sh} = -dt^2 + dr^2 + r^2 d\Omega^2$
$\varphi_{\omega \ell m} = \frac{Y_{\ell m}(\theta,\phi)}{r} R_{\omega \ell}(r) e^{-i\omega t}$
$-\frac{d^2 R_{\omega \ell}}{dr_*^2} + \left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3} \right)\left(1 - \frac{2M}{r}\right) R_{\omega \ell} = \omega^2 R_{\omega \ell}$
$V_{eff}(r_*)$


$\varphi^{in}_{\omega} \propto e^{-i\omega v}$ on i^-
V_{eff}
r_*
$-t$
DRYWALL


$\varphi^{in}_{\omega} \propto e^{-i\omega v}$ on i^-
V_{eff}
free waves
horizon
∞
$r_* \sim 0 \Leftrightarrow r \sim 2M$
r_*
$R_{\omega\ell} \simeq \frac{1}{\sqrt{4\pi\omega}} e^{\pm i\omega r_*}$
$-t$


$U_0 = -\varkappa_n \ln|\tilde{u}_0 - \tilde{u}_n| + \text{const} = -\varkappa_h \ln|v - v_h| + \text{const}$
$\tilde{u}_0 = \tilde{v} = \frac{1}{\alpha} v + \text{const}$
$\frac{dN}{d\tilde{\omega}dt} = \frac{\Gamma_{\tilde{\omega}\ell}}{2\pi\left(e^{\tilde{\omega}/T} - 1\right)}$
$T = \frac{t_h'}{4\pi} = \frac{1}{8\pi M}$
Note. $\Gamma_{\tilde{\omega}\ell} \to 0$, $\ell \to \infty$ or
$f(r_s)\left(1 + \dot{r}_s^2\right)$

$\varphi^{in}_{\omega} \propto e^{-i\omega v}$ on i^-

