Title: Quantum Field Theory in Curved Spacetime (PM) - 2023-03-10

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Collection: Quantum Field Theory in Curved Spacetime

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URL: https://pirsa.org/23030095

Abstract: https://pitp.zoom.us/j/95867663528?pwd=dmd5Y3FaVHJ0SnJWSlFscEY4cmhYUT09

I) Recap

$$(\Box - m^2 + \xi R)\varphi = 0$$

$$\varphi(x) = \sum_i \left(u_i(x)\, a_i + u_i^*(x)\, a_i^+\right) = \sum_j \left(\tilde{u}_j \tilde{a}_j + \tilde{u}_j^* \tilde{a}_j^+\right)$$

$$\langle u_i, u_j^* \rangle = 0$$

$$\langle u_i, u_j \rangle = \delta_{ij}$$

$$\langle u_i^*, u_j^* \rangle = -\delta_{ij}$$

$$\tilde{a}_j = \sum_i \left(\alpha_{ji} a_i - \beta^*_{ji} a_i^\dagger \right)$$

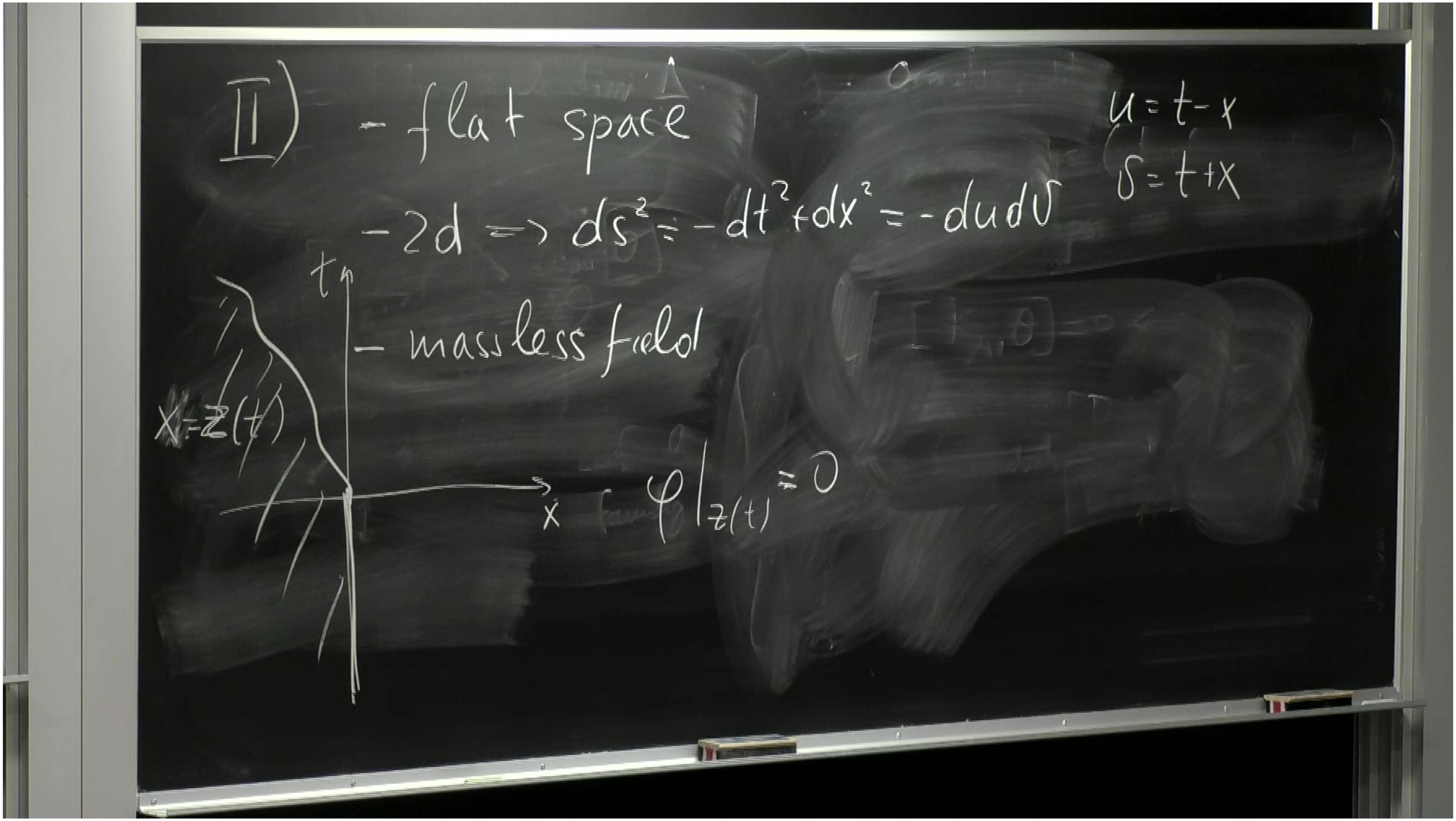
$$\alpha_{ji} = \langle \tilde{u}_j , u_i \rangle$$

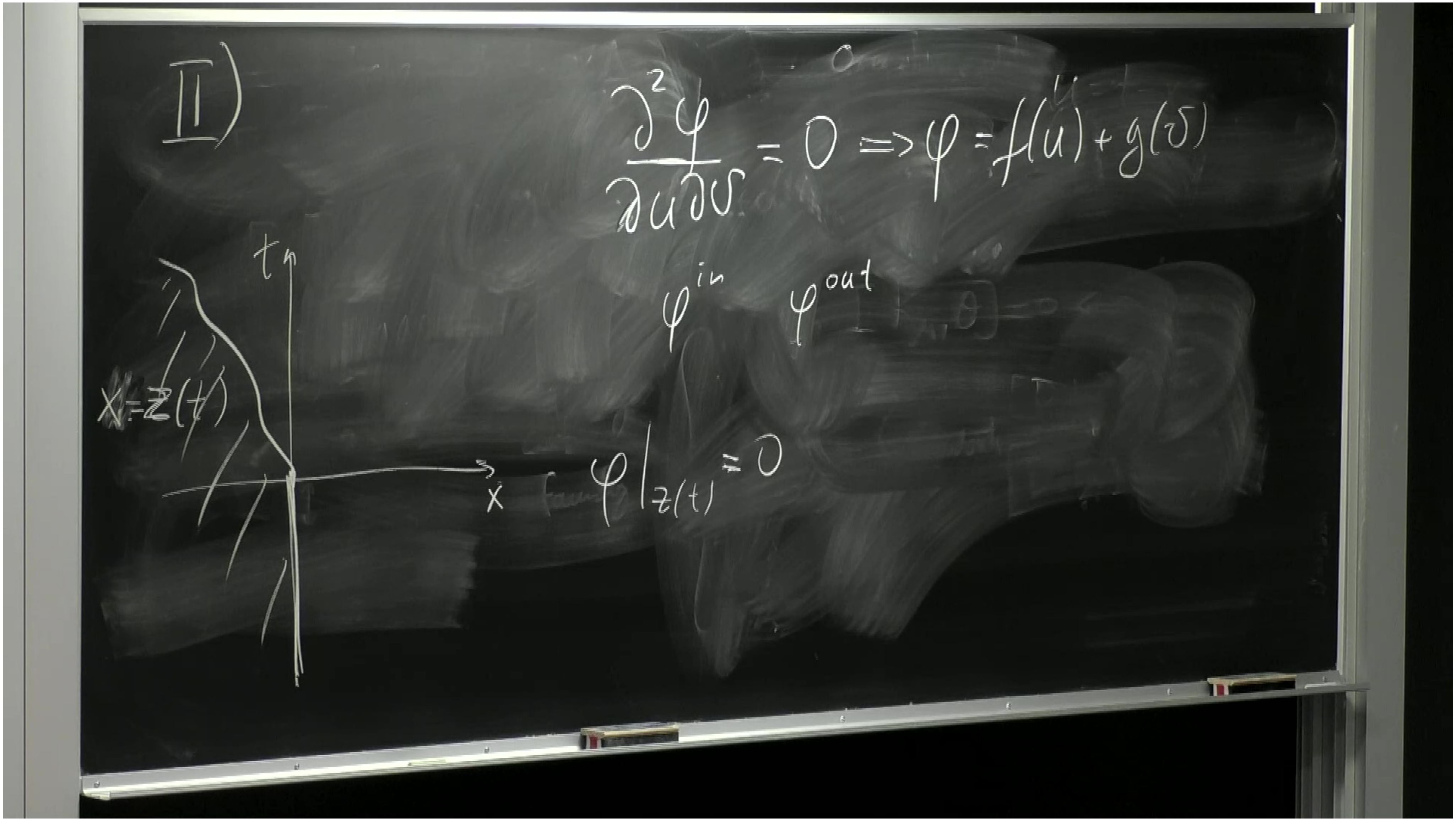
$$\beta_{ji} = -\langle \tilde{u}_j , u_i^* \rangle$$

$$= \langle \tilde{u}_j^* , u_i \rangle^*$$

$$\langle \varphi_1 , \varphi_2 \rangle = -i \int_\Sigma d\sigma \sqrt{g_\Sigma}\, n^\mu \left(\varphi_1 \partial_\mu \varphi_2^* - \partial_\mu \varphi_1 \cdot \varphi_2^* \right)$$

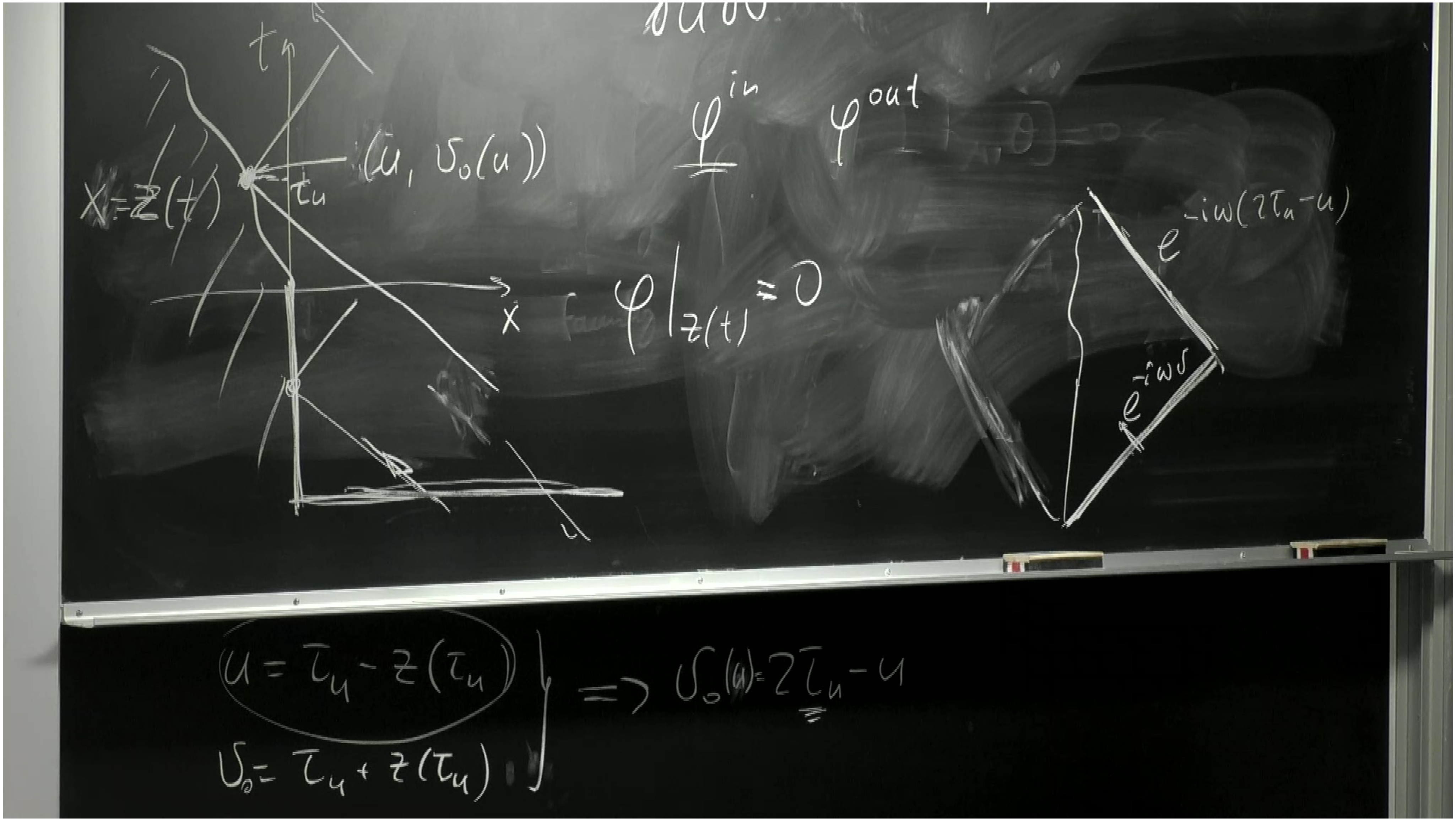
$$\langle 0 | \tilde{a}_j^\dagger a_j | 0 \rangle = \sum_i | \beta_{ij} |^2$$


II) - flat space
- 2d ⇒ $ds^2 = -dt^2 + dx^2 = -du\,dv$
$u = t - x$
$v = t + x$
- massless field
$x = z(t)$
t
x
$\varphi|_{z(t)} = 0$


II)
$\frac{\partial^2 \varphi}{\partial u \partial v} = 0 \Rightarrow \varphi = f(u) + g(v)$
φ^{in} φ^{out}
$x = z(t)$
t
x
$\varphi|_{z(t)} = 0$

$$\varphi_\omega^{in} = \frac{1}{\sqrt{4\pi\omega}}\left(e^{-i\omega v} - e^{-i\omega(2\tau_u - u)}\right) \quad t < 0$$

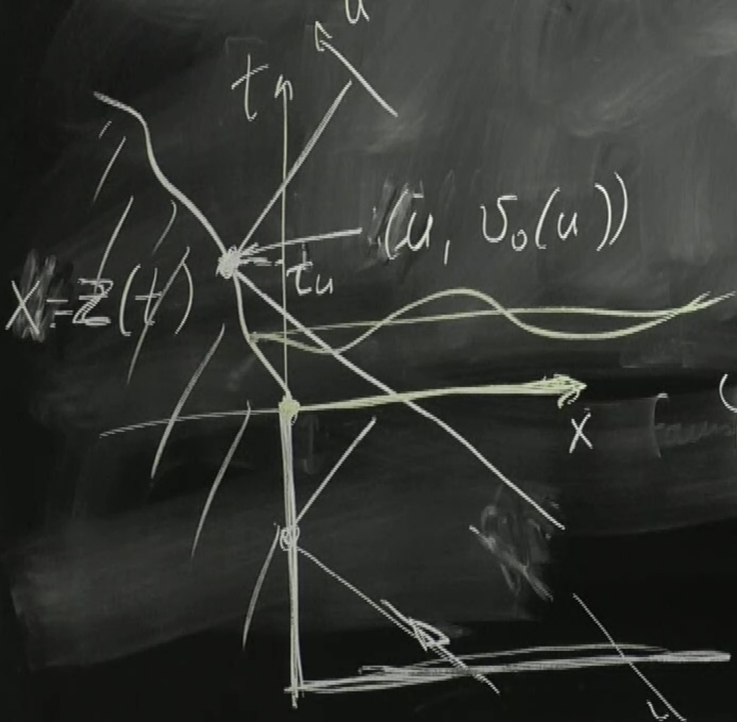
$$\left.\begin{aligned} u &= \tau_u - z(\tau_u) \\ v_0 &= \tau_u + z(\tau_u) \end{aligned}\right\} \Rightarrow v_0(u) = 2\tau_u - u$$



t
$(u, v_0(u))$
$x = z(t)$
τ_u
x
u
φ^{in}
φ^{out}
$\varphi|_{z(t)} = 0$
$e^{-i\omega(2\tau_u - u)}$
$e^{-i\omega v}$
$u = \tau_u - z(\tau_u)$
$v_0 = \tau_u + z(\tau_u)$
$\Rightarrow v_0(u) = 2\tau_u - u$

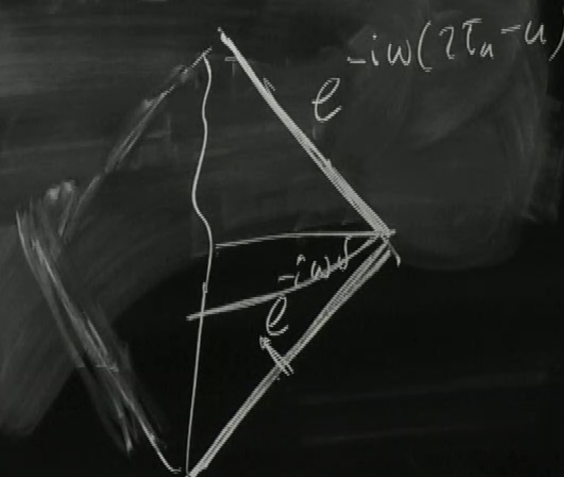
II)

$$\frac{\partial^2 \varphi}{\partial u \partial v} = 0 \Rightarrow \varphi = f(u) + g(v)$$



φ^{in} φ^{out}

$\varphi|_{z(t)} = 0$



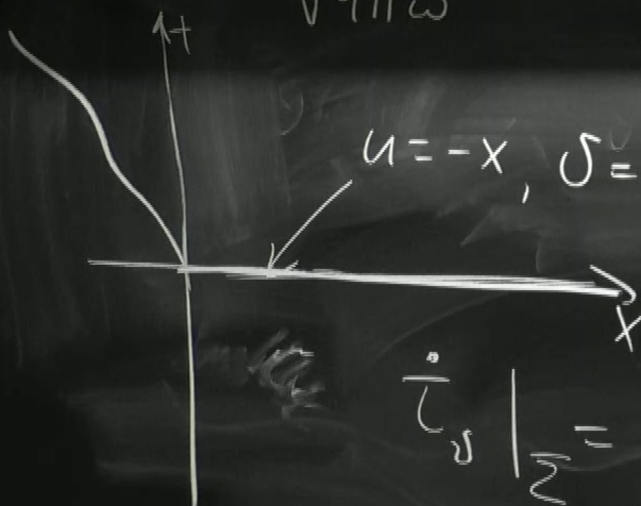
$$\varphi_\omega^{out} = \frac{1}{\sqrt{4\pi\omega}} \left(e^{-i\omega u} - e^{-i\omega(2\tau_v - v)} \right)$$

$$\tau_v + z(\tau_v) = v$$

$$\beta_{\bar{\omega}\omega} = -\left\langle \varphi^{out}_{\tilde{\omega}}, \left(\varphi^{in}_{\omega}\right)^* \right\rangle =$$

$$= i \int_0^\infty dx \left(\varphi^{out}_{\tilde{\omega}} \dot{\varphi}^{in}_{\omega} - \dot{\varphi}^{out}_{\tilde{\omega}} \varphi^{in}_{\omega} \right) \Big|_{t=0}$$

$$\langle u_i^*, u_j^* \rangle = -\delta_{ij}$$



$u = -x, \; v = x$

$$\dot{\tau}_v \big|_{\Sigma} = \tau_x'$$

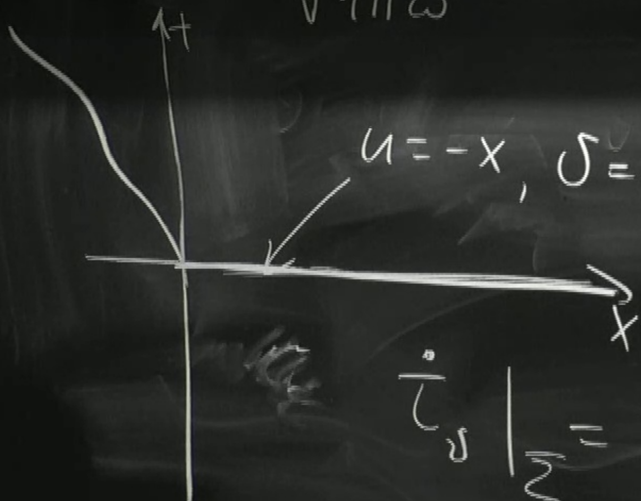
$$\beta_{\tilde{\omega}\omega} = -\int_0^\infty \frac{dx}{2\pi} \sqrt{\frac{\omega}{\tilde{\omega}}} \left(e^{i(\tilde{\omega}+\omega)x} + e^{i(\tilde{\omega}-\omega)x - 2i\tilde{\omega}\tau_x} \right)$$

$$\tau_x + z(\tau_x) = x$$

$$\langle u_i, u_j \rangle = \delta_{ij}$$

$$\langle u_i^*, u_j^* \rangle = -\delta_{ij}$$

$$\frac{dN}{d\tilde{\omega}} = \int_0^\infty |\beta_{\tilde{\omega}\omega}|^2 \, d\omega$$



$$\beta_{\tilde{\omega}\omega} = -\int_0^\infty \frac{dx}{2\pi} \sqrt{\frac{\omega}{\tilde{\omega}}} \left(e^{i(\tilde{\omega}+\omega)x} + e^{i(\tilde{\omega}-\omega)x - 2i\tilde{\omega}\tau_x} \right)$$

$$\tau_x + z(\tau_x) = x$$

$$\dot{\tau}_v \big|_z = \tau_x'$$

$$\tau_v - \alpha \tau_v = v \Rightarrow \tau_v = \frac{v}{1-\alpha} \Rightarrow \tau_x = \frac{x}{1-\alpha}$$

$$\beta_{\tilde{\omega}\omega} = -\int_0^\infty \frac{dx}{2\pi} \sqrt{\frac{\omega}{\tilde{\omega}}} \left(e^{i(\tilde{\omega}+\omega)x - \epsilon x} + e^{-i\left(\frac{1+\alpha}{1-\alpha}\tilde{\omega}+\omega\right)x - \epsilon x} \right)$$

$$= \frac{\sqrt{\omega\tilde{\omega}}}{2\pi} \left(\frac{2\alpha}{1-\alpha}\right) \frac{1}{(\tilde{\omega}+\omega)\left(\frac{1+\alpha}{1-\alpha}\tilde{\omega}+\omega\right)}$$

$$\int_0^\infty dx\, e^{iAx} = \lim_{\epsilon \to 0} \int_0^\infty dx\, e^{iAx - \epsilon x} = \lim_{\epsilon \to 0} \left[\frac{1}{-iA - \epsilon} \right] = \frac{i}{A}$$

$$u_i + u_i^*(x)\, a_i^+) = \sum_j \left(\tilde{u}_j \tilde{a}_j + \tilde{u}_j^* \tilde{a}_j^+ \right)$$

$$\frac{dN}{d\tilde{\omega}} = \int_0^\infty |\beta_{\tilde{\omega}\omega}|^2 \, d\omega$$

$$\frac{dN}{d\tilde{\omega}} = \frac{1}{\pi^2 \tilde{\omega}} \left(\frac{1}{4\alpha} \ln \frac{1+\alpha}{1-\alpha} - \frac{1}{2} \right)$$

$$\langle \varphi^{in}_{\omega} \varphi^{in}_{\omega'} \rangle = \delta(\omega - \omega')$$

$$\sum_i \to \int d\omega$$

$$N_j = {}_{in}\langle 0 | \tilde{a}^{\dagger}_j \tilde{a}_j | 0 \rangle_{in} = \sum_i |\beta_{ji}|^2$$

out

$$N_j \to dN_{\tilde{\omega}} \qquad j \to d\tilde{\omega}$$

$$\frac{dN}{d\tilde{\omega}} = \int_0^{\infty} |\beta_{\tilde{\omega}\omega}|^2 \, d\omega$$

$$\frac{dN}{d\tilde\omega} = \frac{1}{\pi^2 \tilde\omega}\left(\frac{1}{4\alpha}\ln\frac{1+\alpha}{1-\alpha} - \frac{1}{2}\right)$$

$$\langle \varphi^{in}_{\omega} \varphi^{in}_{\omega'} \rangle = \delta(\omega - \omega')$$

$$\sum_i \to \int d\omega$$

$$N_j = {}_{in}\langle 0 | \tilde a^{+}_j \tilde a_j | 0 \rangle_{in} = \sum_i |\beta_{ji}|^2$$

out

$$N_j \to dN_{\tilde\omega} \qquad j \to d\tilde\omega$$

$$\int dN = \int \frac{d\tilde\omega}{\tilde\omega}$$

$$\frac{dN}{d\tilde\omega} = \int_0^\infty |\beta_{\tilde\omega \omega}|^2 d\omega$$

$$\tau_{\upsilon}\Big|_{\Sigma} = \tau'_x$$

$$\tau_x + z(\tau_x) = x$$

$N_j = {}_{in}\langle 0 | a_j^\dagger a_j | 0 \rangle_{in} = \sum_i |\beta_{\tilde{\omega} i}|^2$

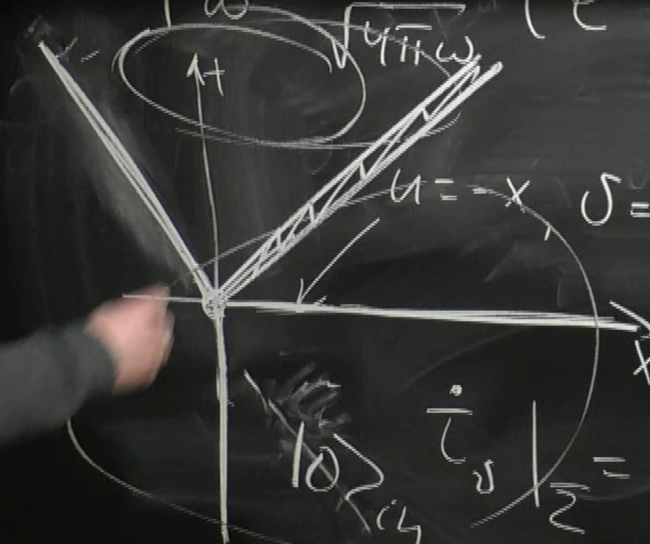
out

$N_j \to dN_{\tilde{\omega}} \qquad j \to d\tilde{\omega}$

$\int dN = \int \frac{d\tilde{\omega}}{\tilde{\omega}}$

$$\frac{dN}{d\tilde{\omega}} = \int_0^\infty |\beta_{\tilde{\omega}\omega}|^2 \, d\omega$$

$$\varphi_\omega^{in} = \frac{1}{\sqrt{4\pi\omega}}\left(e^{-i\omega v} - e^{-i\omega u}\right)$$



$u = -x \quad v = x$

$$\beta_{\tilde{\omega}\omega} = -\int_0^\infty \frac{dx}{2\pi}\sqrt{\frac{\omega}{\tilde{\omega}}}\left(e^{i(\tilde{\omega}+\omega)x} + e^{i(\tilde{\omega}-\omega)x - 2i\tilde{\omega}\tau_x}\right)$$

$\tau_x + z(\tau_x) = x$

$|0\rangle_{in} \quad \dot{\tau}_v\big|_\Sigma = \tau'_x$

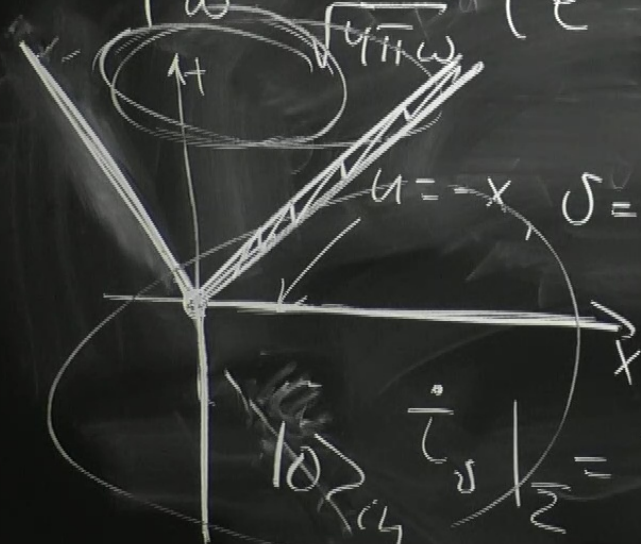
$N_j \to dN_{\tilde\omega}$ $\quad j \to d\tilde\omega$

out

$$\int dN = \int \frac{d\tilde\omega}{\tilde\omega}$$

$$\frac{dN}{d\tilde\omega} = \int_0^\infty |\beta_{\tilde\omega\omega}|^2 \, d\omega$$

$$\varphi_\omega^{in} = \frac{1}{\sqrt{4\pi\omega}}\left(e^{-i\omega v} - e^{-i\omega u}\right)$$



$u = -x \quad v = x$

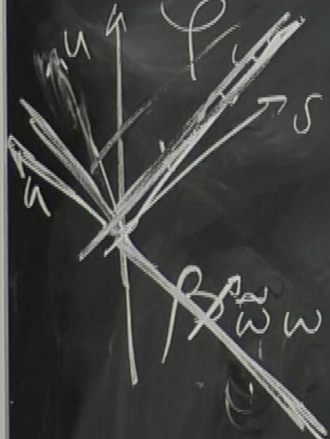
$$\beta_{\tilde\omega\omega} = -\int_0^\infty \frac{dx}{2\pi}\sqrt{\frac{\omega}{\tilde\omega}}\left(e^{i(\tilde\omega+\omega)x} + e^{i(\tilde\omega-\omega)x - 2i\tilde\omega\tau_x}\right)$$

$\tau_x + z(\tau_x) = x$

$$\dot\tau_v \big|_{\Sigma} = \tau_x'$$

$$\tau_v - \alpha \tau_v = v \Rightarrow \underbrace{\tau_v = \frac{v}{1-\alpha}} \Rightarrow \tau_x = \frac{x}{1-\alpha}, \quad v>0$$

$$\beta_{\tilde\omega\omega} = -\int^{\infty} \frac{dx}{\ldots}\sqrt{\frac{\tilde\omega}{\ldots}}\left(e^{i(\tilde\omega+\omega)x-\epsilon x} - e^{-i\left(\frac{1+\alpha}{1-\alpha}\tilde\omega+\omega\right)x-\epsilon x}\right)$$

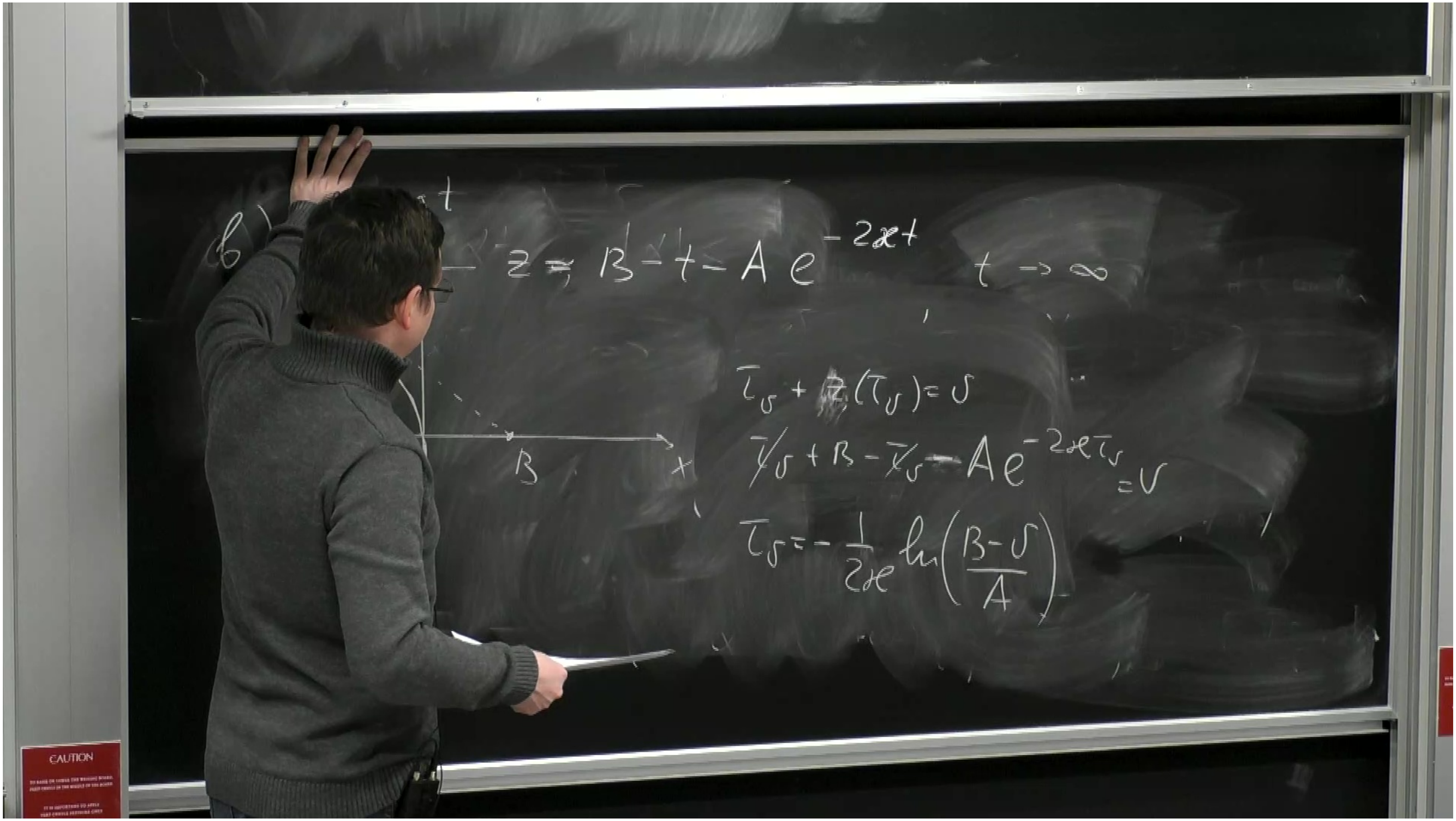


$$\varphi^{out}_{\omega} = \frac{1}{\sqrt{4\pi\omega}}\left(e^{-i\omega u} - e^{-i\omega(2\tau_v - v)}\right) \qquad \frac{2}{1-\alpha}-1=$$

$$e^{-i\omega u} - e^{-i\omega\left(\frac{1+\alpha}{1-\alpha}v\right)}\Big|_{v=0} \qquad v + z(\tau_v) = v$$

$$\beta_{\tilde\omega\omega} = -\left\langle \varphi^{out}_{\tilde\omega}, \left(\varphi^{in}_{\omega}\right)^{*}\right\rangle = \qquad e^{-i\omega v}$$

$$= i\int_0^{\infty} dx\left(\varphi^{out}_{\tilde\omega}\,\dot\varphi^{in}_{\omega} - \dot\varphi^{out}_{\tilde\omega}\,\varphi^{in}_{\omega}\right)\Big|_{t=0}$$


$z = B - t - A e^{-2\varkappa t}$
$t \to \infty$
$\tau_v + z(\tau_v) = v$
$\cancel{\tau_v} + B - \cancel{\tau_v} - A e^{-2\varkappa \tau_v} = v$
$\tau_v = -\frac{1}{2\varkappa} \ln\left(\frac{B-v}{A}\right)$
B
x
CAUTION

$$\beta_{\tilde{\omega}\omega} = -\frac{1}{2\pi}\sqrt{\frac{\omega}{\tilde{\omega}}}\left(\int_0^{\infty} e^{i(\omega+\tilde{\omega})x}\,dx + \int_0^{B} dx\, e^{i(\tilde{\omega}-\omega)x + \frac{i\tilde{\omega}}{2\varkappa}\ln\left(\frac{B-x}{A}\right)}\right)$$

$$\approx -\frac{1}{2\pi}\sqrt{\frac{\omega}{\tilde{\omega}}}\int_0^{\infty} dy\, e^{i(\tilde{\omega}-\omega)(B-y)}\left(\frac{y}{A}\right)^{\frac{i\tilde{\omega}}{\varkappa}} =$$

$$y = B-x$$

$$\Gamma(z) = \int_0^{\infty} dy\, y^{z-1} e^{-y}$$

$$= -\frac{1}{2\pi}\sqrt{\frac{\omega}{\tilde{\omega}}}\, e^{i(\tilde{\omega}-\omega)B}\, \frac{\left(iA(\tilde{\omega}-\omega)\right)^{-\frac{i\tilde{\omega}}{\varkappa}}}{i(\tilde{\omega}-\omega)}\,\Gamma\left(1+\frac{i\tilde{\omega}}{\varkappa}\right)$$

$$|\beta_{\tilde\omega\omega}|^2 = \frac{1}{(2\pi)^2}\,\frac{\omega}{\tilde\omega}\,\frac{e^{-\frac{\pi\tilde\omega}{2\varkappa}}}{(\omega-\tilde\omega)^2}\,\Gamma\left(1+\frac{i\tilde\omega}{\varkappa}\right)\Gamma\left(1-\frac{i\tilde\omega}{\varkappa}\right)$$

$$\Gamma(z+1) = z\,\Gamma(z)$$

$$\Gamma(z)\,\Gamma(1-z) = \frac{\pi}{\sin\pi z}$$

$$\omega > \tilde\omega$$

$$(-i)^{-\frac{i\tilde\omega}{\varkappa}} = \left(e^{-i\frac{\pi}{2}}\right)^{-\frac{i\tilde\omega}{\varkappa}} = e^{-\frac{\pi\tilde\omega}{2\varkappa}}$$

$$\frac{1}{2\pi\varkappa}\,\frac{\omega}{(\omega-\tilde\omega)^2}\,\frac{1}{e^{\frac{2\pi\tilde\omega}{\varkappa}}-1} \Rightarrow \frac{dN}{d\tilde\omega} = \underbrace{\frac{1}{2\pi\varkappa}\,\frac{1}{e^{\frac{2\pi\tilde\omega}{\varkappa}}-1}}\int\frac{d\omega}{\omega}$$

$$T = \frac{\varkappa}{2\pi}$$

$$\bar\tau_v + z(\bar\tau_v) = v$$

$$\bar\tau_v + B - \bar\tau_v - A e^{-2\kappa\bar\tau_v} = v$$

$$\bar\tau_v = -\frac{1}{2\kappa}\ln\left(\frac{B-v}{A}\right), \quad v < B$$

$$u_f = \tau_f - z(\tau_f) = \tau_f - \left(B - \tau_f - A e^{-2\kappa\tau_f}\right) \approx 2\tau_f$$

$$x_f = v_0 = \tau_f + z(\tau_f) = B - A e^{-2\kappa\tau_f} = B - A e^{-\kappa u_f}$$

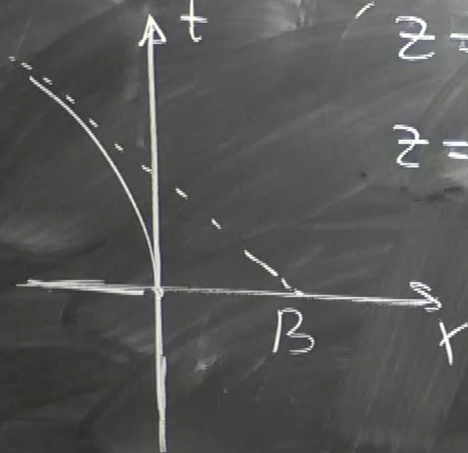
$$\beta_{\tilde\omega\omega} = -\frac{1}{2\pi}\sqrt{\frac{\omega}{\tilde\omega}}\left(\int_0^\infty e^{i(\omega+\tilde\omega)x}\,dx + \int_0^{B-Ae^{-\kappa u_f}} dx\, e^{i(\tilde\omega-\omega)x + \frac{i\tilde\omega}{2\kappa}\ln\left(\frac{B-x}{A}\right)}\right)$$

$$\omega < \frac{1}{B-x_f} = \frac{e^{\kappa u_f}}{A}$$

$$\approx -\frac{1}{2\pi}\sqrt{\frac{\omega}{\tilde\omega}}\int_{\infty}^{\infty} dy\, e^{i(\tilde\omega-\omega)(B-y)}\left(\frac{y}{A}\right)^{\frac{i\tilde\omega}{\kappa}} =$$

$$\frac{dN}{d\tilde{\omega}\, du_+} = \frac{1}{2\pi} \cdot \frac{1}{e^{\frac{2\pi \tilde{\omega}}{\varkappa}} - 1}$$

Ex 2



$z = 0, \quad t < 0$

$z = B - \sqrt{B^2 + t^2}, \quad t > 0$

$d\tau^2 = dt^2 - dz^2$

$t = B\,\mathrm{sh}\,\frac{\tau}{B}, \quad z = -B\cdot\mathrm{ch}\,\frac{\tau}{B} + B$

$a^\mu = \frac{d^2 x^\mu}{d\tau^2} = \left(\frac{1}{B}\,\mathrm{sh}\,\frac{\tau}{B}, -\frac{1}{B}\,\mathrm{ch}\,\frac{\tau}{B}\right)$

$a^\mu a_\mu = \frac{1}{B^2}$

$\frac{dN}{d\tilde{\omega}}$

$B \quad r$

$t = B\,\mathrm{sh}\frac{\tau}{B}\,, \quad z = -B\cdot\mathrm{ch}\frac{\tau}{B} + B$

$\frac{dN}{d\tilde{\omega}} \quad \boxed{\int_0^{\infty} dy\, e^{-y-\frac{z^2}{2y}} = z K_1(z)}$

$a^\mu = \frac{d^2 x^\mu}{d\tau^2} = \left(\frac{1}{B}\,\mathrm{sh}\frac{\tau}{B}, -\frac{1}{B}\,\mathrm{ch}\frac{\tau}{B}\right)$

$a^\mu a_\mu = \frac{1}{B^2}$

https://dlmf.nist.gov

$\langle \varphi(x)\varphi(x')\rangle$

Ex 3.

$z = B - \sqrt{B^2+t^2}$

$\int_0^{\infty} \frac{\ }{2\pi} \sqrt{\frac{\omega}{\tilde{\omega}}}\left(e^{i(\tilde{\omega}+\omega)x} + e^{i(\tilde{\omega}-\omega)x - 2i\tilde{\omega}\tau_x}\right)$

$\tau_x + z(\tau_x) = x$