Title: Quantum Field Theory in Curved Spacetime (PM) - 2023-03-03

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Collection: Quantum Field Theory in Curved Spacetime

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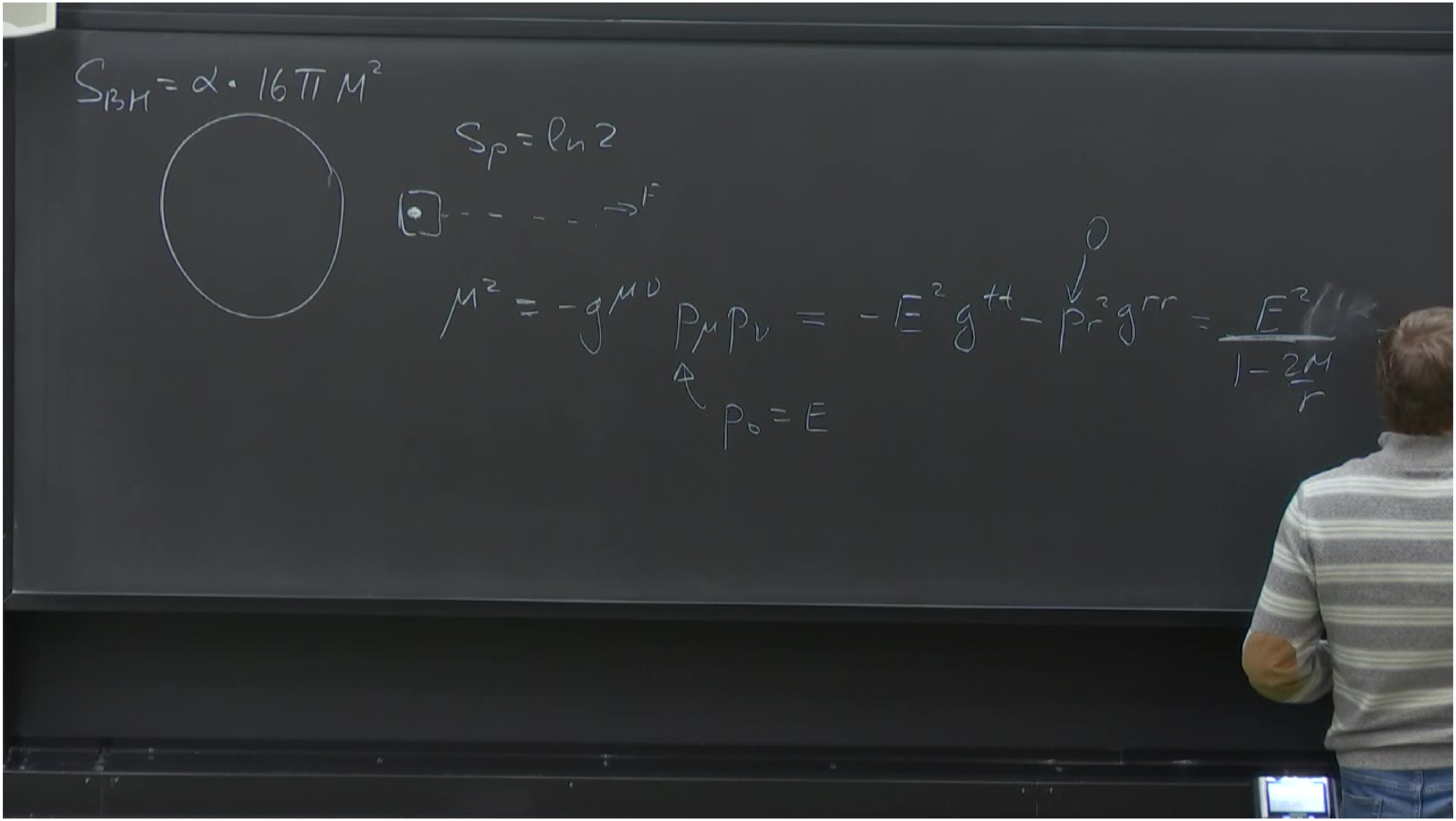
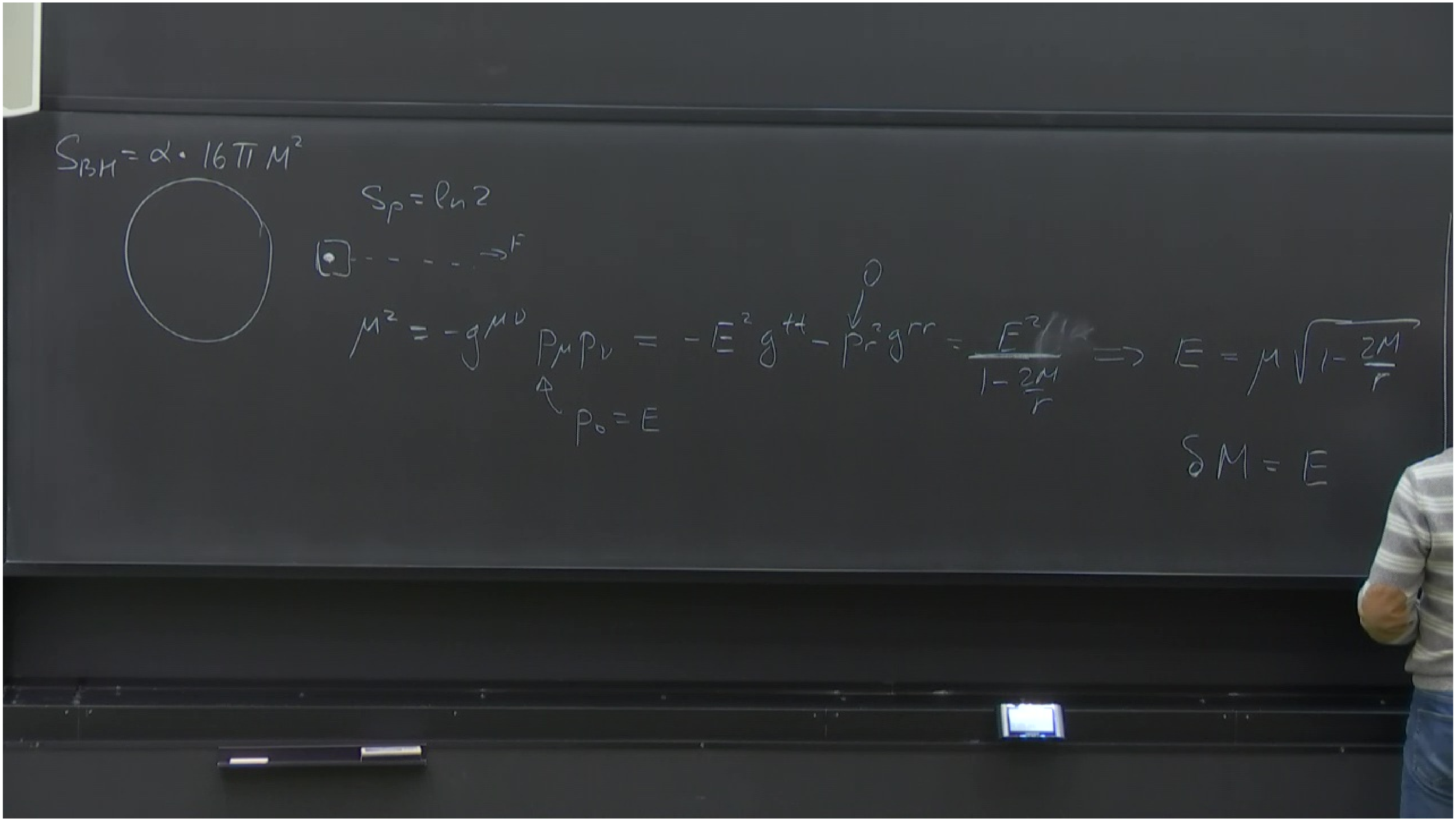
URL: https://pirsa.org/23030094

Abstract: https://pitp.zoom.us/j/95867663528?pwd=dmd5Y3FaVHJ0SnJWSlFscEY4cmhYUT09


$S_{BH} = \alpha \cdot 16\pi M^2$
$S_p = \ln 2$
E
0
$\mu^2 = -g^{\mu\nu} p_\mu p_\nu = -E^2 g^{tt} - p_r^2 g^{rr} = \frac{E^2}{1-\frac{2M}{r}}$
$p_0 = E$

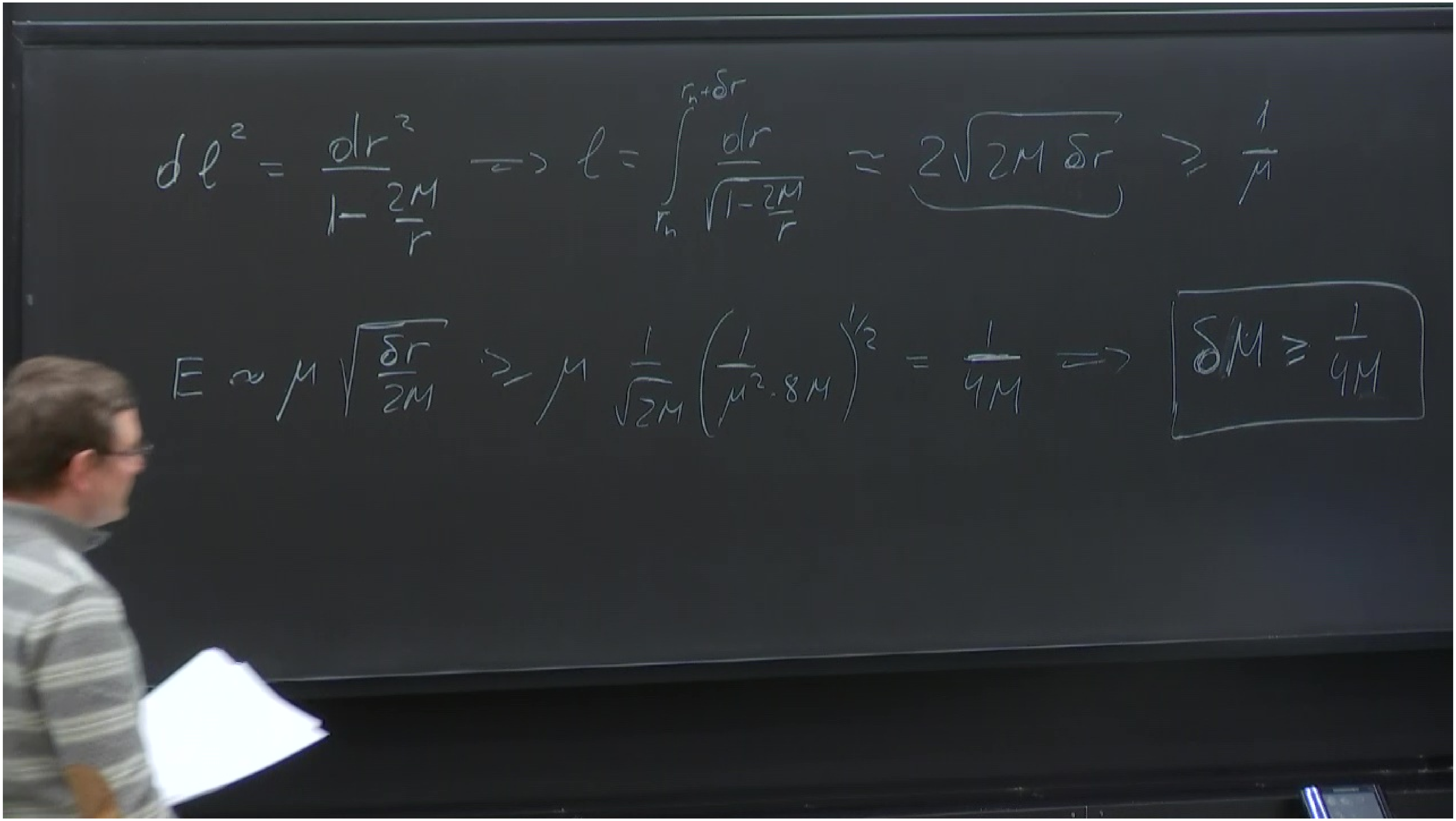

$S_{BH} = \alpha \cdot 16\pi M^2$
$S_p = \ln 2$
F
0
$\mu^2 = -g^{\mu\nu} p_\mu p_\nu = -E^2 g^{tt} - p_r^2 g^{rr} = \frac{E^2}{1-\frac{2M}{r}} \Longrightarrow E = \mu\sqrt{1-\frac{2M}{r}}$
$p_0 = E$
$\delta M = E$

$$\Rightarrow E = \mu\sqrt{1-\frac{2M}{r}}$$

$$\delta M = E$$

$$\delta \ell \geq \frac{1}{\delta p} \;;\; \delta p \sim \delta E < \mu$$

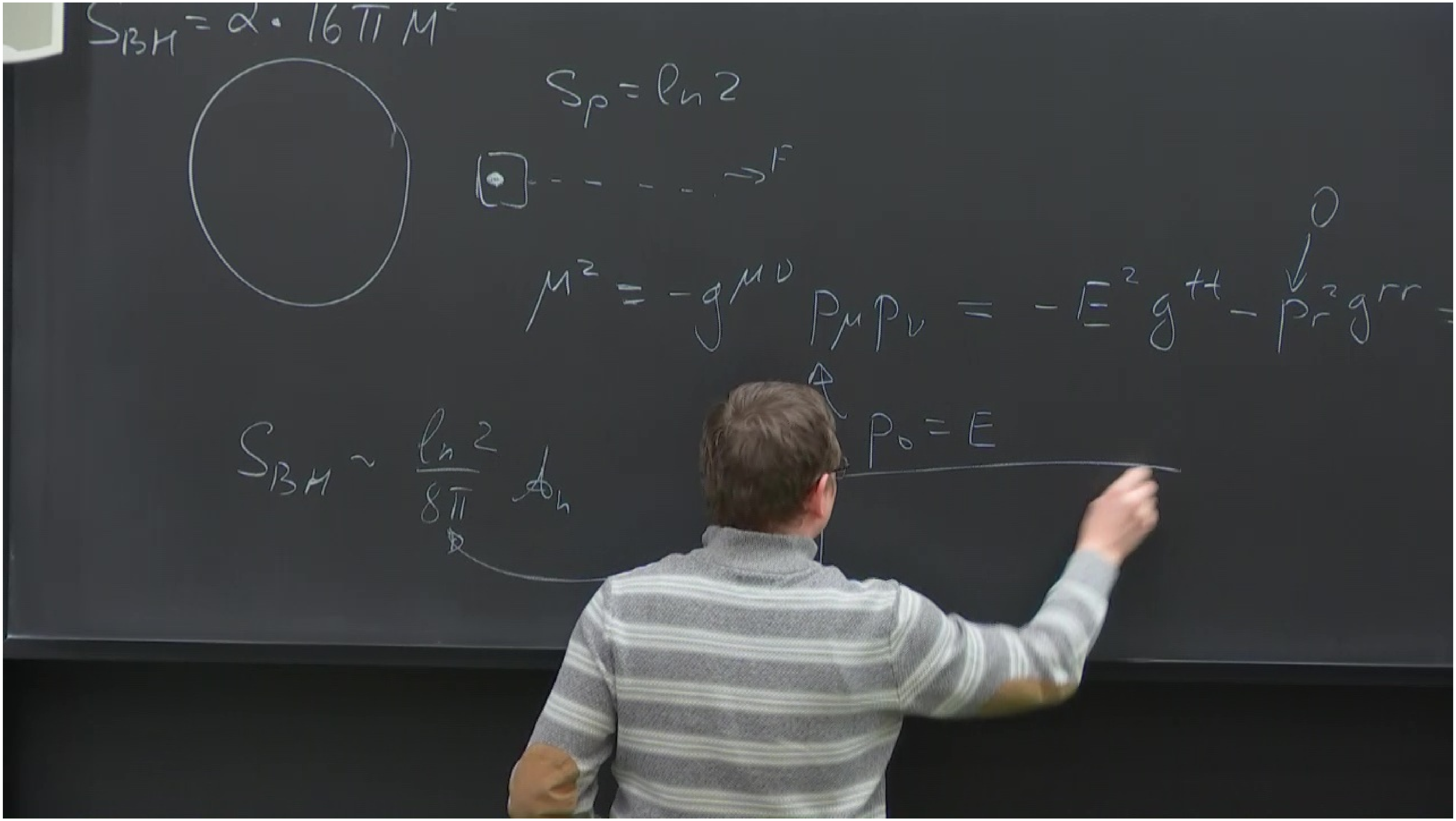
$$\delta \ell \geq \frac{1}{\mu} \Rightarrow \delta r > ?$$

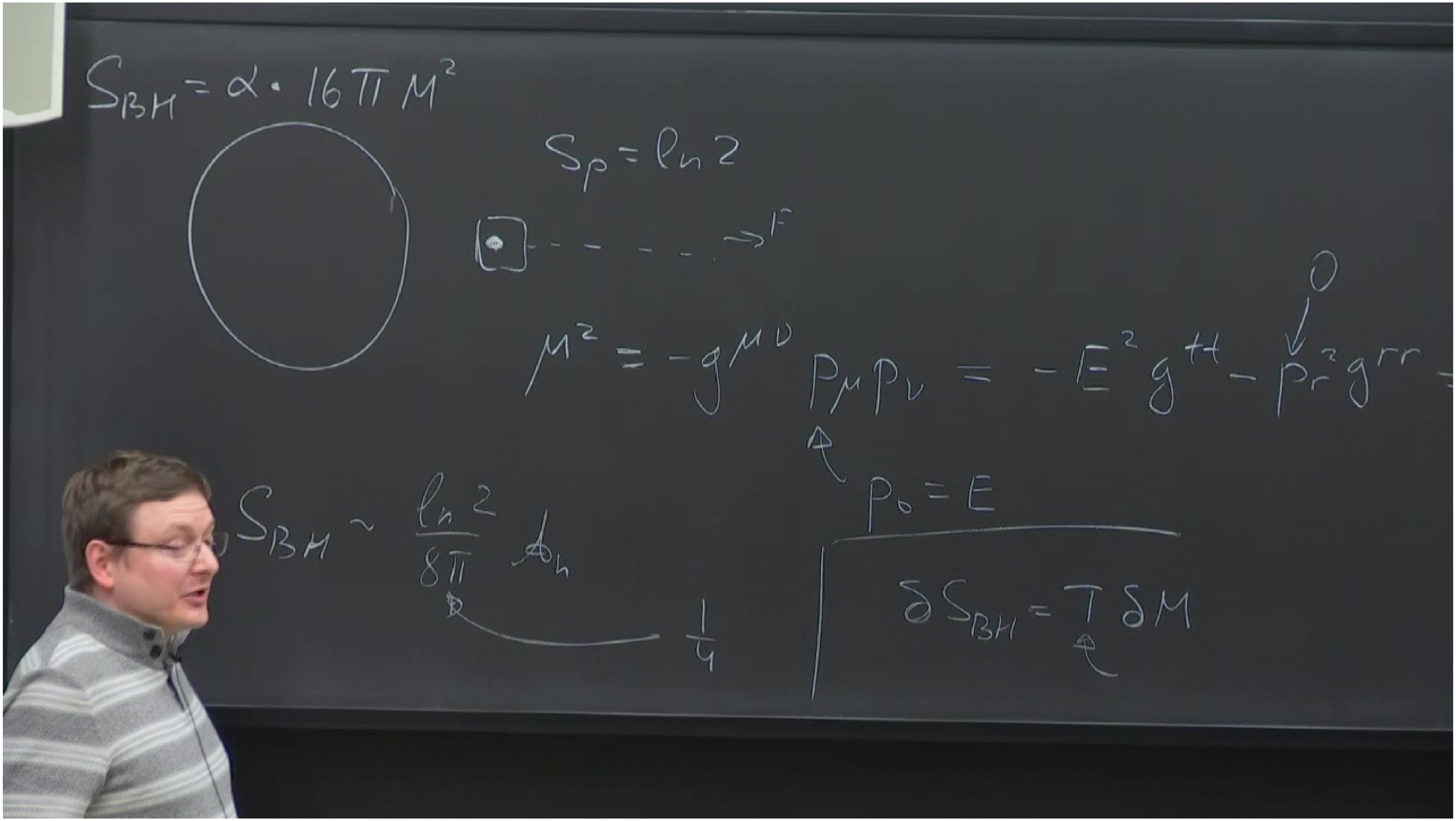

$d\ell^2 = \frac{dr^2}{1-\frac{2M}{r}} \Rightarrow \ell = \int_{r_h}^{r_h+\delta r} \frac{dr}{\sqrt{1-\frac{2M}{r}}} \simeq 2\sqrt{2M\,\delta r} \gtrsim \frac{1}{\mu}$
$E \approx \mu \sqrt{\frac{\delta r}{2M}} \gtrsim \mu \frac{1}{\sqrt{2M}} \left(\frac{1}{\mu^2 \cdot 8M}\right)^{1/2} = \frac{1}{4M} \Rightarrow \boxed{\delta M \geq \frac{1}{4M}}$

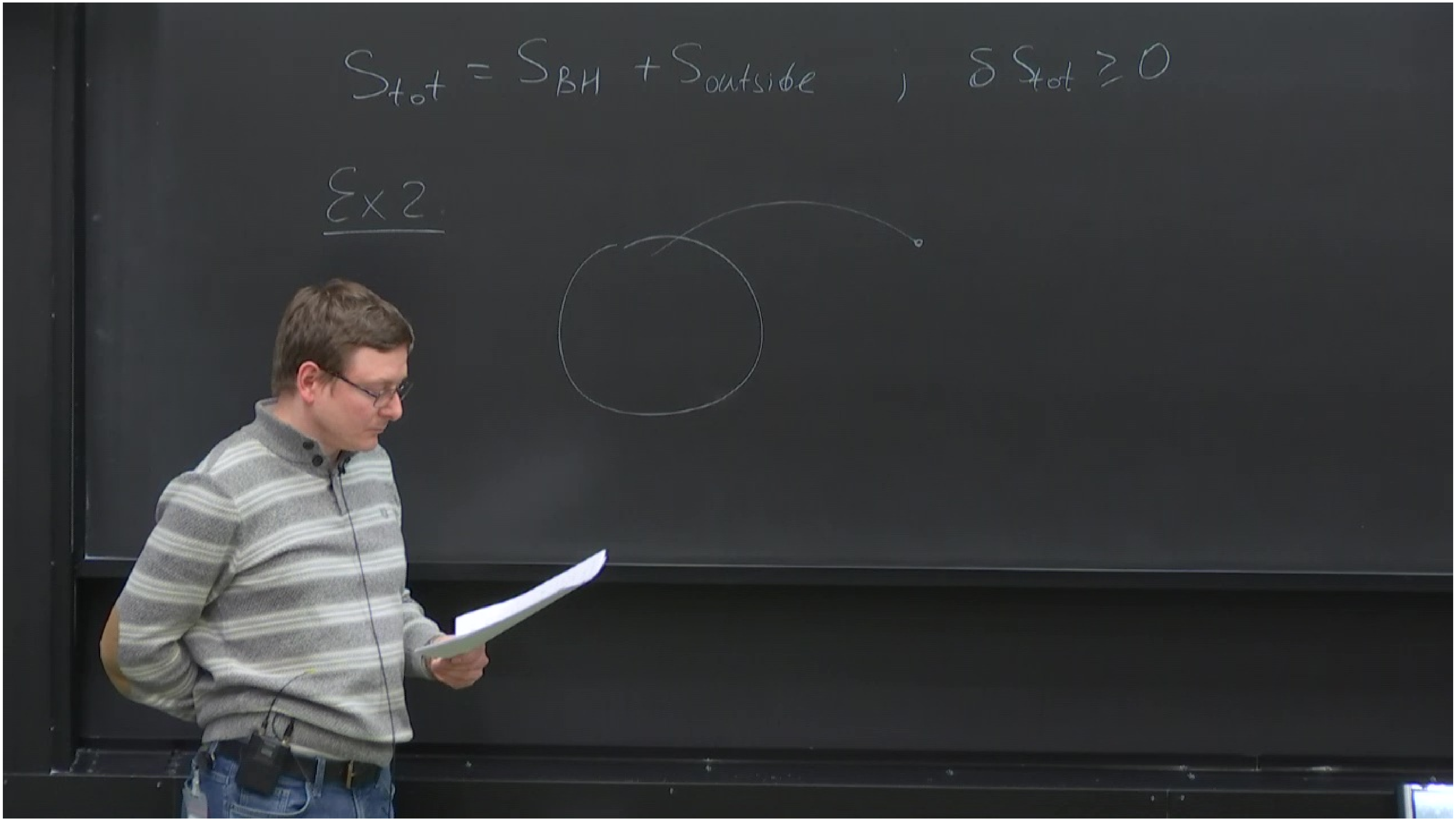
$$dl^2 = \frac{dr^2}{1-\frac{2M}{r}} \Longrightarrow l = \int_{r_h}^{r_h+\delta r} \frac{dr}{\sqrt{1-\frac{2M}{r}}} \simeq \underbrace{2\sqrt{2M\delta r}} \gtrsim \frac{1}{\mu}$$

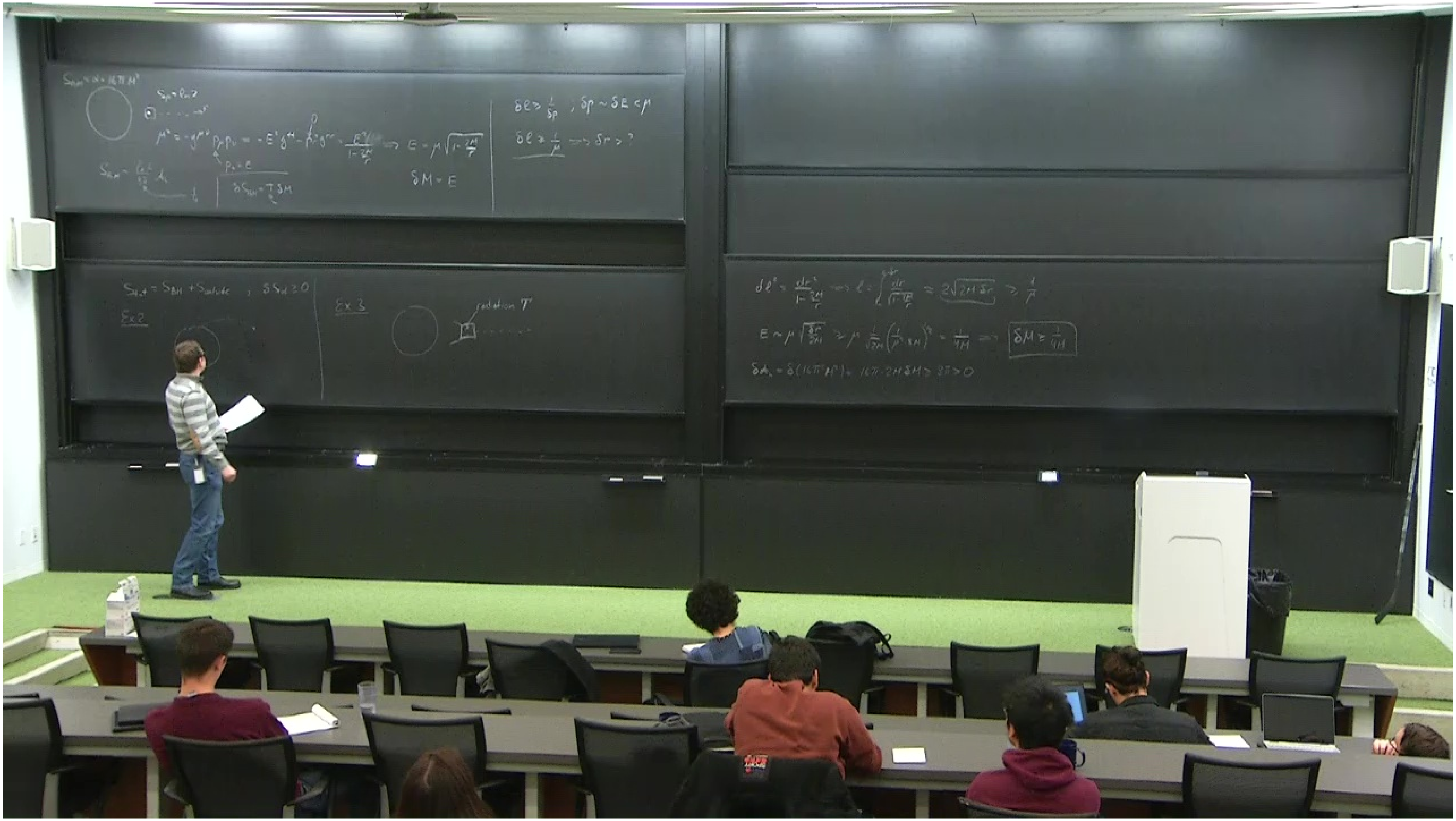
$$E \sim \mu \sqrt{\frac{\delta r}{2M}} \gtrsim \mu \frac{1}{\sqrt{2M}} \left(\frac{1}{\mu^2 \cdot 8M}\right)^{1/2} = \frac{1}{4M} \Longrightarrow \boxed{\delta M \geq \frac{1}{4M}}$$

$$\delta A_h = \delta(16\pi M^2) = 16\pi \cdot 2M\delta M \geq 8\pi > 0$$


$S_{BH} = \alpha \cdot 16\pi M^2$
$S_p = \ln 2$
F
0
$\mu^2 = -g^{\mu\nu} p_\mu p_\nu = -E^2 g^{tt} - p_r^2 g^{rr}$
$p_0 = E$
$S_{BH} \sim \frac{\ln 2}{8\pi} A_h$


$S_{BH} = \alpha \cdot 16\pi M^2$
$S_p = \ln 2$
0
$\mu^2 = -g^{\mu\nu} p_\mu p_\nu = -E^2 g^{tt} - p_r^2 g^{rr} =$
$p_0 = E$
$S_{BH} \sim \frac{\ln 2}{8\pi} A_h$
$\frac{1}{4}$
$\delta S_{BH} = T \delta M$


$S_{tot} = S_{BH} + S_{outside}$; $\delta S_{tot} \geq 0$
Ex 2


Ex 2
Ex 3
radiation T

$$A_0 = \frac{Q}{r} \quad , \quad A_i = 0$$

a) Horizon area?

b)

General formalism

$$S = \int d^dx \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} (m^2 - \xi R) \varphi^2 \right]$$

action

$$\xi = \frac{d-2}{4(d-1)}$$

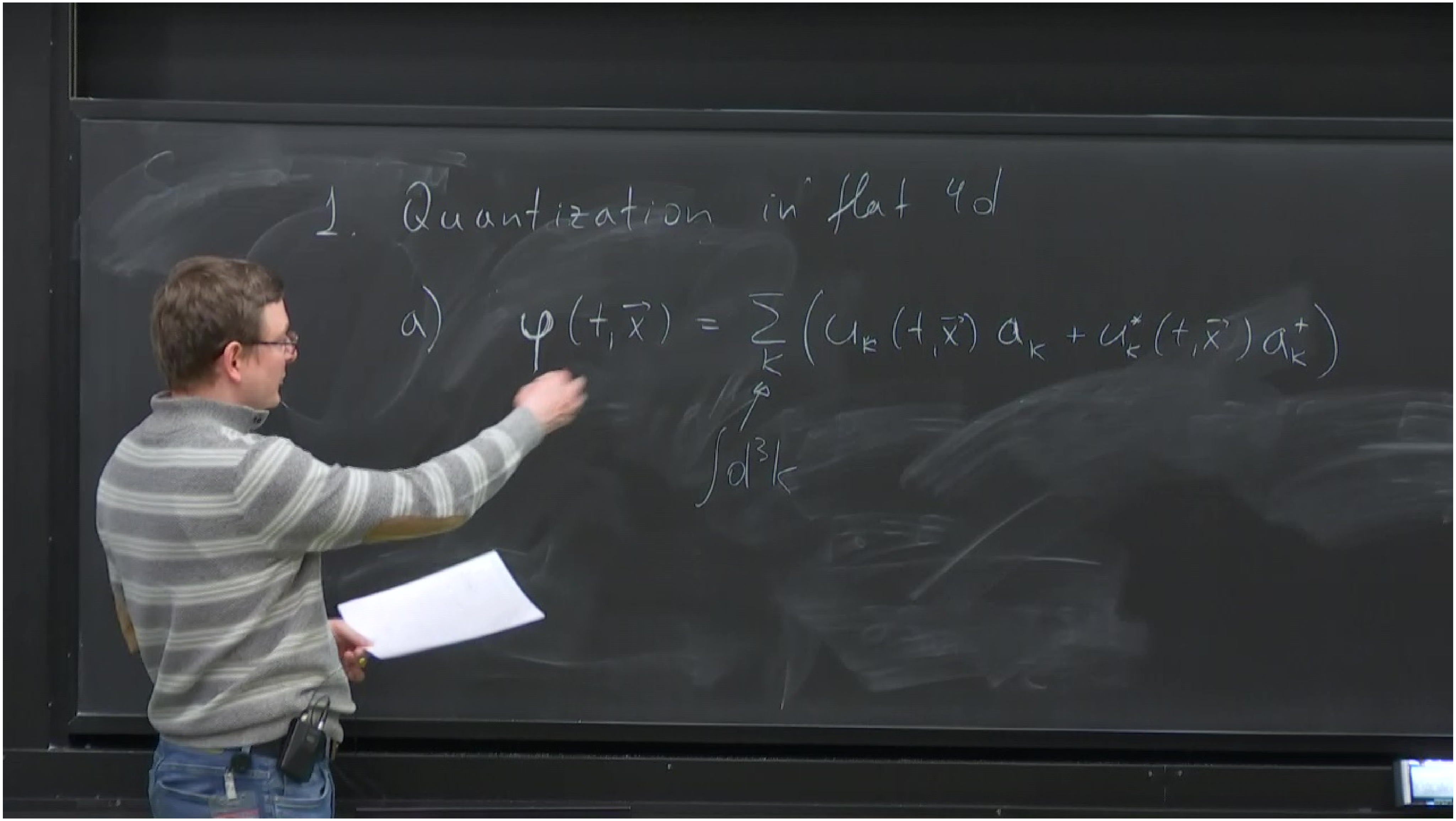
General formalism

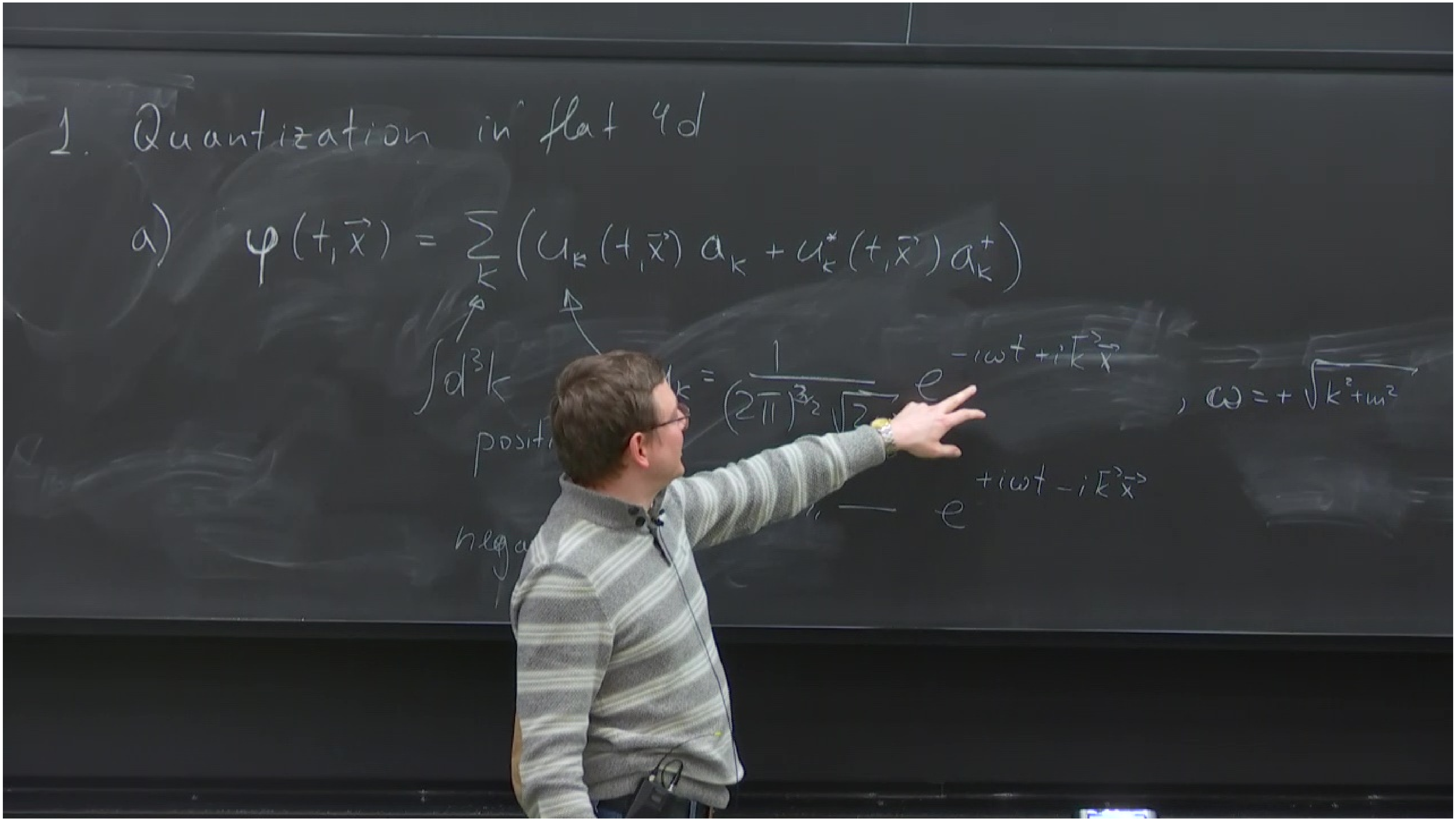
$$S = \int d^dx \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2}\left(m^2 - \xi R\right)\varphi^2\right] \Longrightarrow \left(\Box - m^2 + \xi R\right)\varphi = 0$$

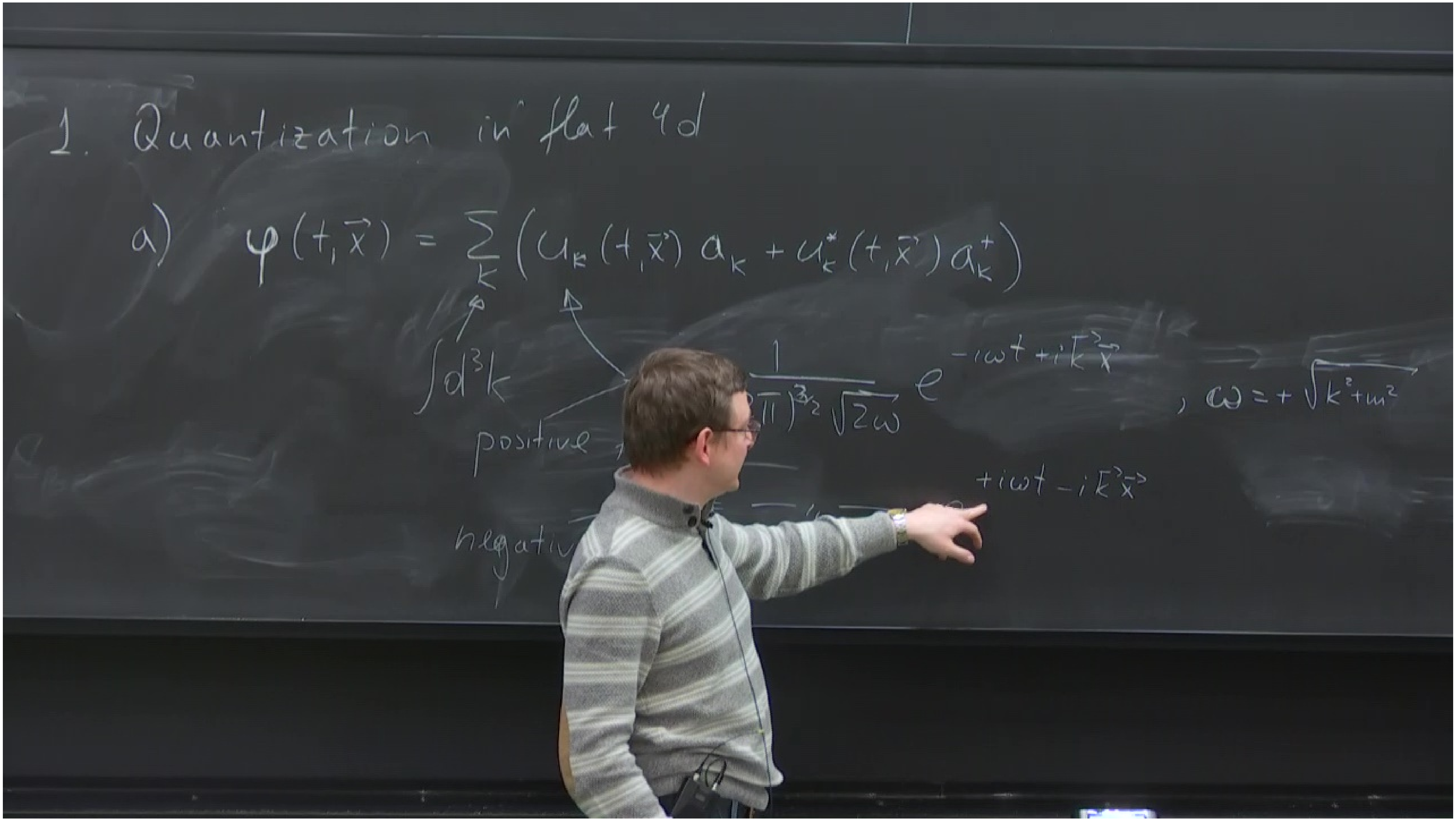
action

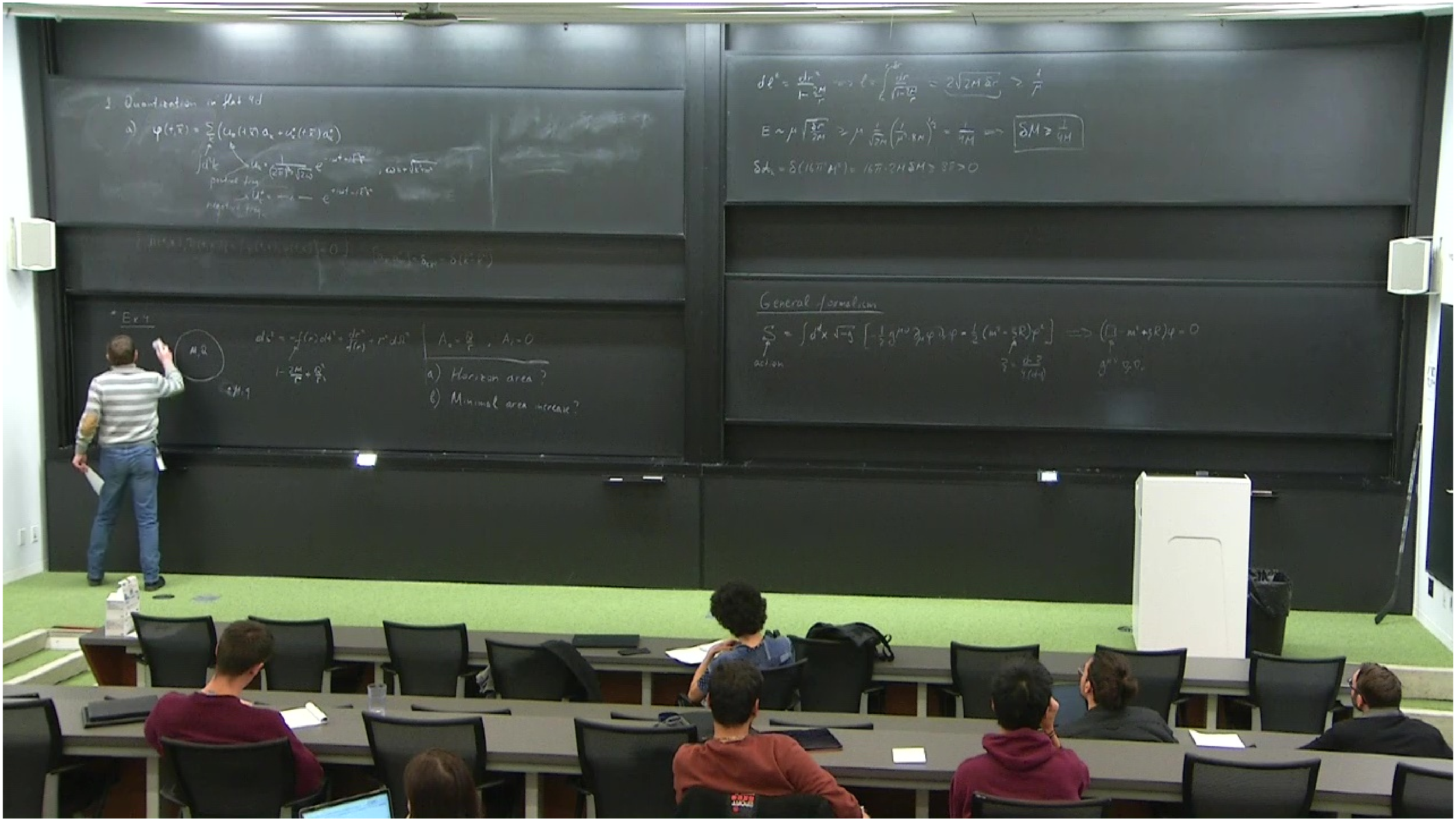
$$\xi = \frac{d-2}{4(d-1)}$$

$$g^{\mu\nu}\nabla_\mu\nabla_\nu$$


1. Quantization in flat 4d
a) $\varphi(t,\vec{x}) = \sum_k \left(u_k(t,\vec{x})\, a_k + u_k^*(t,\vec{x})\, a_k^+ \right)$
$\int d^3k$


1. Quantization in flat 4d
a) $\varphi(t,\vec{x}) = \sum_k \left(u_k(t,\vec{x})\, a_k + u_k^*(t,\vec{x})\, a_k^+ \right)$
$\int d^3k$
$= \frac{1}{(2\pi)^{3/2}\sqrt{2}} e^{-i\omega t + i\vec{k}\vec{x}}$
$, \omega = +\sqrt{k^2+m^2}$
$e^{+i\omega t - i\vec{k}\vec{x}}$


1. Quantization in flat 4d
a) $\varphi(t,\vec{x}) = \sum_k \left(u_k(t,\vec{x})\, a_k + u_k^*(t,\vec{x})\, a_k^+ \right)$
$\int d^3k$
$e^{-i\omega t + i\vec{k}\vec{x}}$
$\omega = +\sqrt{k^2+m^2}$
positive
$+i\omega t - i\vec{k}\vec{x}$
negativ


1. Quantization in flat 4d
General formalism
action
a) Horizon area ?
b) Minimal area increase ?

$$\langle \varphi_1, \varphi_2 \rangle = -i \int_{\text{fixed } t} d^3x \left(\varphi_1 \partial_t \varphi_2^* - \partial_t \varphi_1 \cdot \varphi_2^* \right)$$

Ex 5

$$\langle u_k, u_{k'} \rangle = \delta(\vec{k} - \vec{k}')$$

$$\langle u_k, u_k^* \rangle = 0$$

$$\langle u_k^*, u_{k'}^* \rangle = -\delta(\vec{k} - \vec{k}')$$

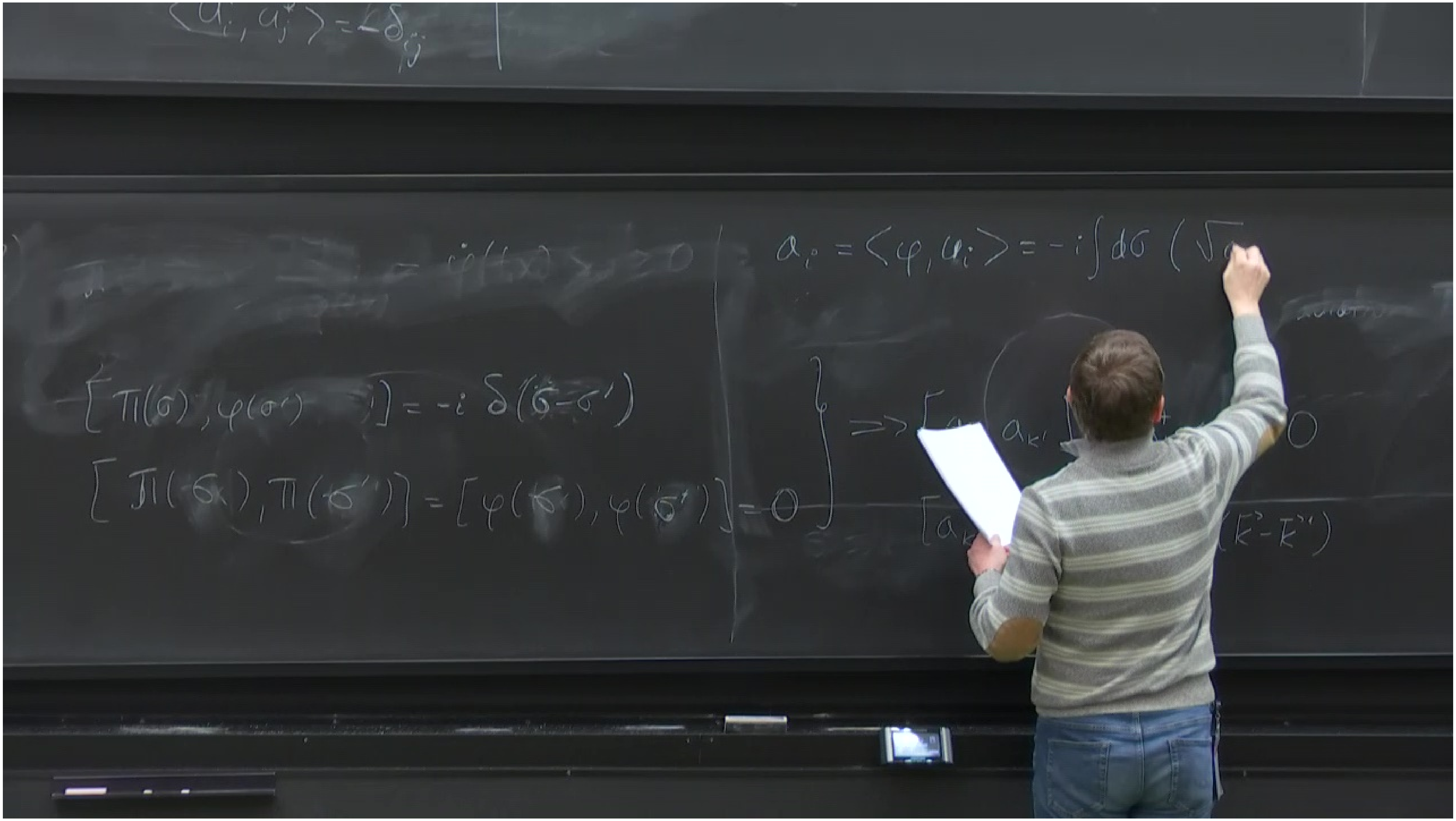
b) u_i, u_i^*

$$\varphi(t,\vec{x}) = \sum_k \left(u_k(t,\vec{x})\, a_k + u_k^*(t,\vec{x})\, a_k^+ \right)$$

$$\langle u_i, u_j \rangle = \delta_{ij}$$

$$\langle u_i, u_j^* \rangle = 0$$

$$\langle u_i^*, u_j^* \rangle = -\delta_{ij}$$



<a_i, a_j^†> = -δ_ij
a_i = <φ, u_i> = -i ∫ dσ (√
[π(σ), φ(σ')] = -i δ(σ - σ')
[π(σ), π(σ')] = [φ(σ), φ(σ')] = 0

$i\rangle$

$u_i^*\rangle$

Ex 7

$$\sum_j \left(\alpha_{ji}^* \alpha_{jk} - \beta_{ji}\beta_{jk}^*\right) = \delta_{ik}$$

$$\sum_j \left(\alpha_{ji}^* \beta_{jk} - \beta_{ji}\alpha_{jk}^*\right) = 0$$

$$\sum_i \left(\alpha_{ji}\alpha_{ki}^* - \beta_{ji}\beta_{ki}^*\right) = \delta_{jk}$$

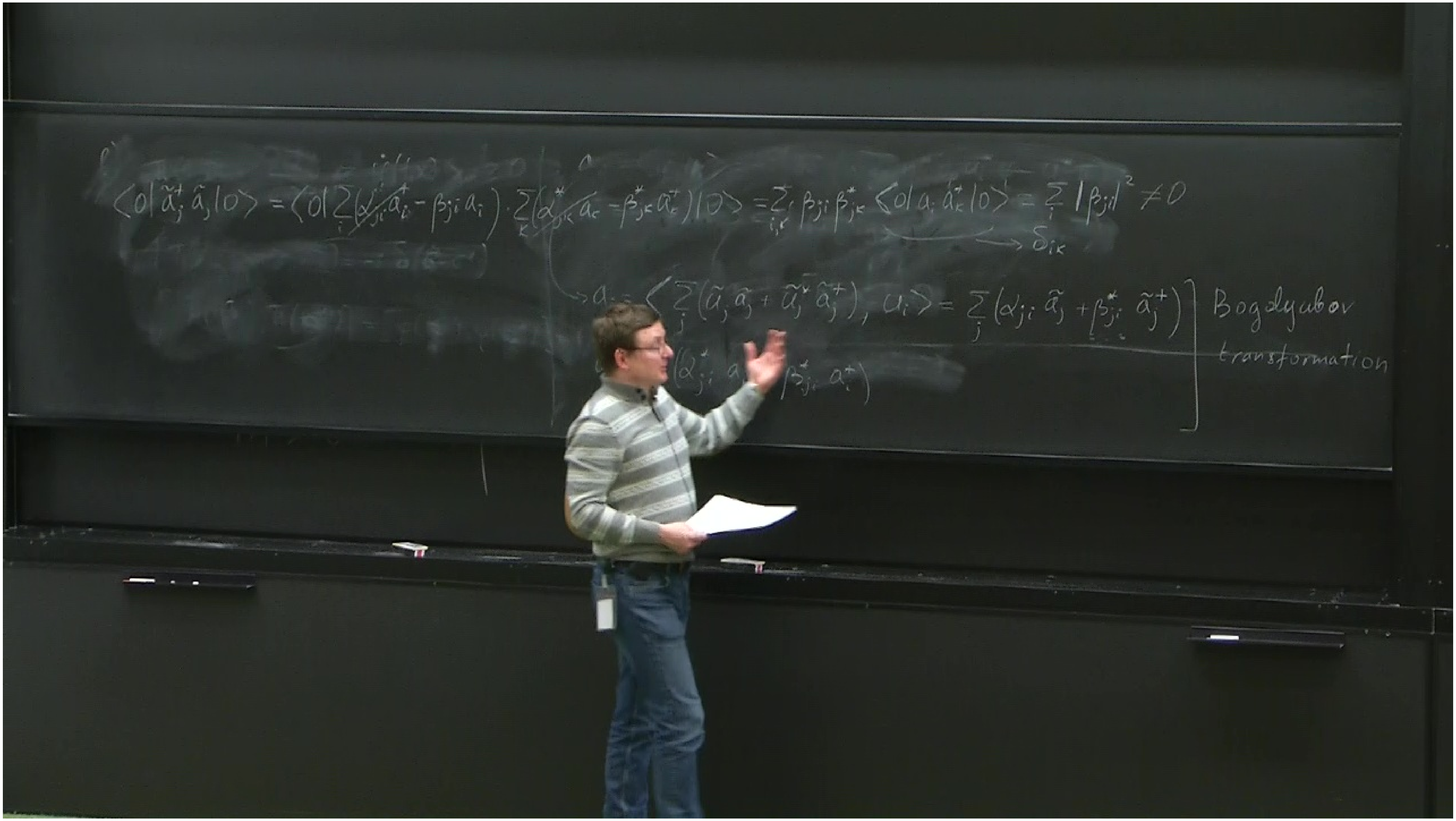
$$\sum_i \left(\alpha_{ij}\beta_{ki} - \beta_{ji}\alpha_{ki}\right) = 0$$

$$a_i = \langle \varphi, u_i \rangle = -i \int d\sigma \left(\sqrt{g_\Sigma}\, n^\mu \partial_\mu u_i^* \varphi - u_i^* \pi \right)$$

$$a_i^+ = -\langle \varphi, u_i^* \rangle = i \int d\sigma \left(\sqrt{g_\Sigma}\, n^\mu \partial_\mu u_i \varphi - u_i \pi \right)$$

$$a_i = \left\langle \sum_j \left(\tilde{u}_j \tilde{a}_j + \tilde{u}_j^* \tilde{a}_j^+ \right), u_i \right\rangle = \sum_j \left(\alpha_{ji} \tilde{a}_j + \beta_{ji}^* \tilde{a}_j^+ \right)$$

$$\tilde{a}_j = \sum_i \left(\alpha_{ji}^* a_i - \beta_{ji}^* a_i^+ \right)$$


Bogolyubov
transformation