Title: Scalar fields in strong gravity: black holes, neutron stars and wormholes

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Series: Strong Gravity

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Abstract: No-hair theorems prevent black holes in General Relativity (GR) from being characterized by any property other than their mass, electric charge, and spin. Scalar fields provide perhaps the simplest way to generalize GR, and it turns out that cases exist where the no-hair theorems can be evaded and black holes with scalar hair may emerge. In this talk I will examine the notion of nontrivial scalar field configurations arising in the strong regime of gravity near black holes, but also compact neutron stars and wormholes. I will go over the concept of spontaneous scalarization of compact objects and discuss the viability of scalarized solutions against observations.

Zoom link: https://pitp.zoom.us/j/92273469560?pwd=elpDRzliOHFaaUh3YVhzbC92NFlmUT09

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Scalar fields in strong gravity

Strong Gravity Seminar, Perimeter Institute

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March 3, 2022

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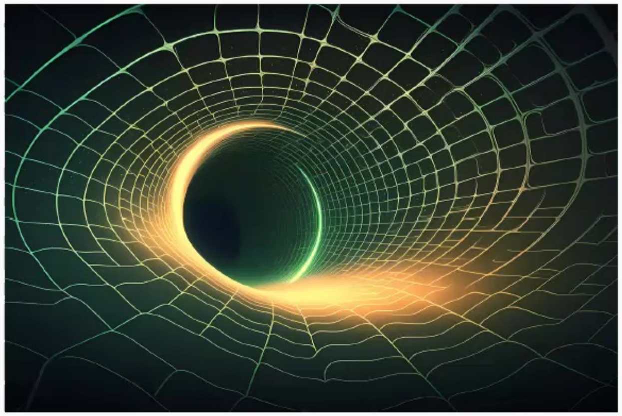
Motivation

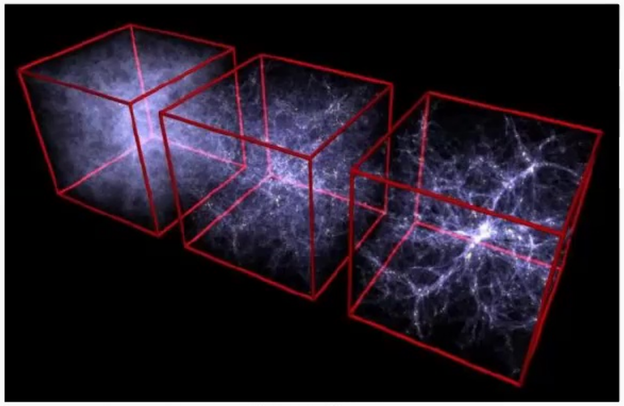
→ Gravity in **General Relativity (GR)** can be expressed through the Einstein-Hilbert action.

$$S_{GR} = \frac{1}{2\kappa}\int d^4x\,\sqrt{-g}\,R \tag{1}$$

→ GR faces challenges:

- Non renormalizable
- Issues with singularities
- Unconstrained in the strong field
- Dark energy and dark matter



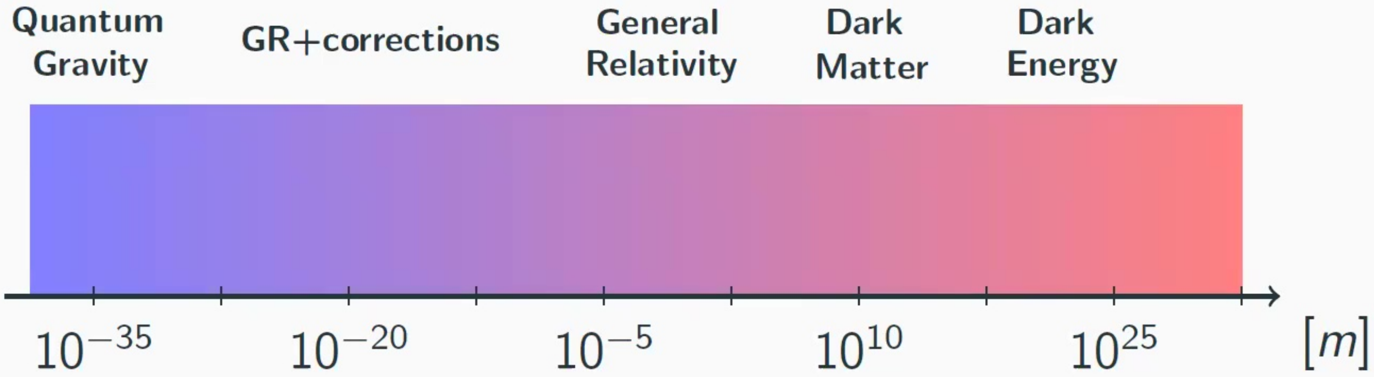


Volker Springel/ MPE/Kavli Foundation

Motivation

→ To tackle these issues new terms are introduced

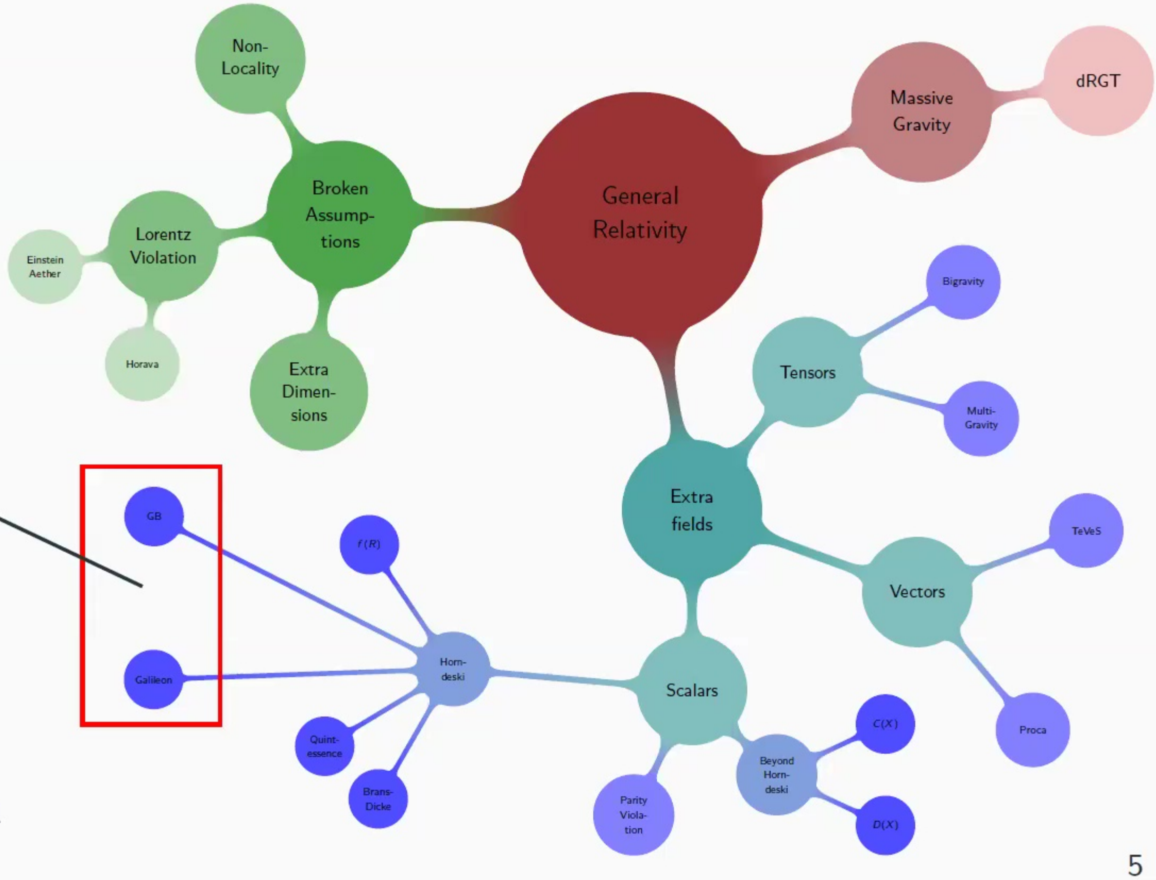
- scalar or vector fields (ϕ, $\vec{A}$...)
- curvature corrections (R^2, $R^{ab}R_{ab}$, $R^{abcd}R_{abcd}$, ${}^*R\,R$...)



Motivation

Roadmap of modified gravity[1]

New physics in strong gravity?



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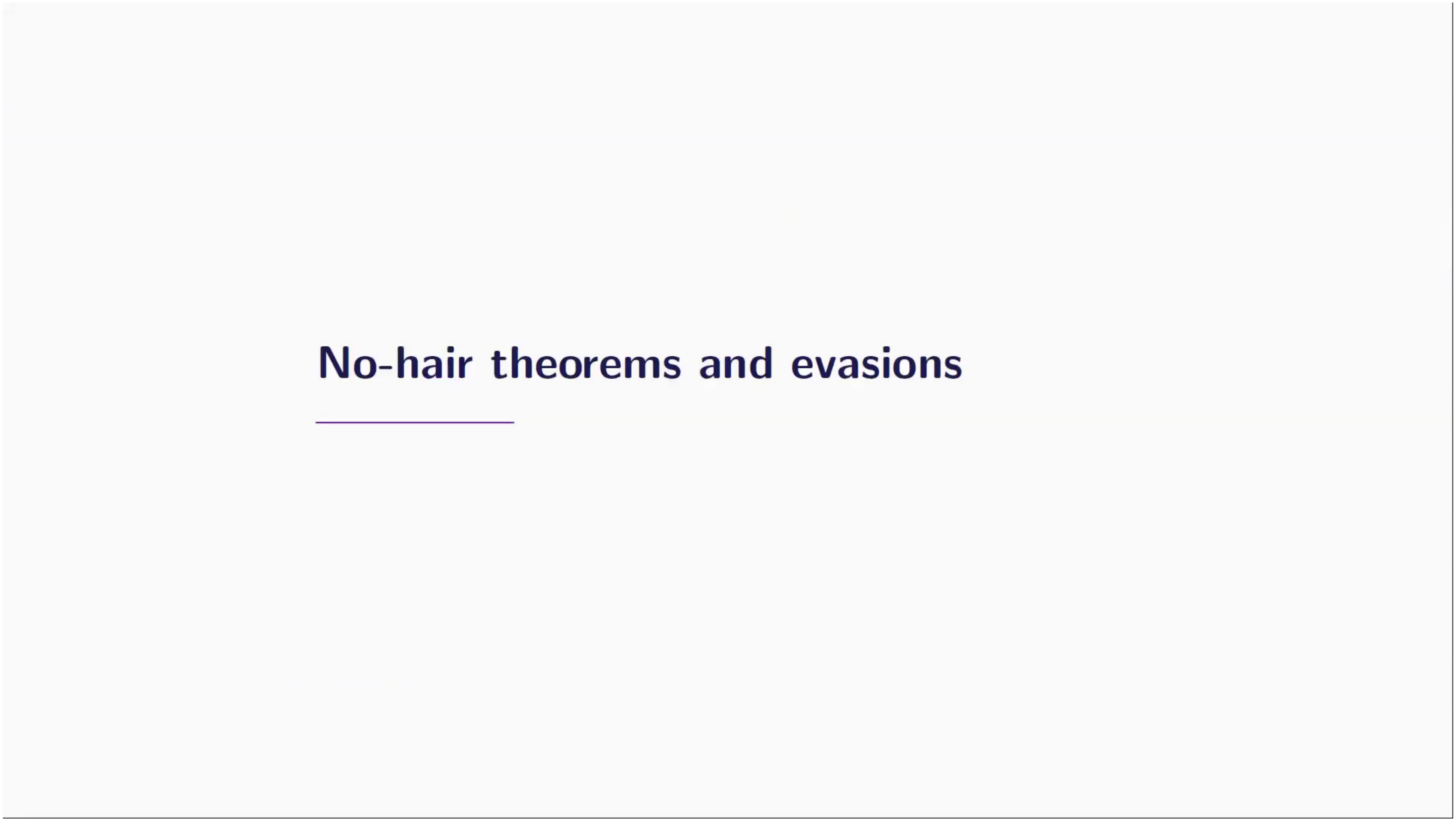
[1] Front. Astron. Space Sci. **5**, 44 (2018), arXiv:1807.09241



No-hair theorems and evasions

No-hair theorems

If we assume:

1. isolated configuration - asymptotic flatness
2. stationary spacetime, at the endpoint of gravitational collapse
3. local stability, *i.e.* $U'' > 0$ $(m^2_{\text{eff}} > 0)$

The scalar equation

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = U'(\phi)\,. \tag{2}$$

admits only the trivial solution, ϕ =constant GR.

Evading the no-hair theorem

Nonminimal couplings in Horndeski:

- Horndeski theory includes a metric and a scalar (+derivatives) and leads to 2nd order equations of motion

- It contains nonminimal couplings of the scalar with curvature

$$X \equiv -\frac{(\nabla\phi)^2}{2}, \quad -G_3(\phi, X)\Box\phi \,, \quad G_4(\phi, X)R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2],$$

$$G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

- It leads to far more complicated scalar equations of motion

Shift symmetry

Can we narrow down the space of theories?

Let's start with **shift symmetry**, $\phi \to \phi + c$, which

- protects ϕ from acquiring a mass (massive scalars would decay exponentially around compact objects)
- calculations are simpler and there exists a conserved current $\nabla_\mu J^\mu = 0$

A **no-hair theorem** was derived in this case provided that:

1. staticity, asymptotic flatness, spherical symmetry are assumed
2. J^2 is finite at the horizon and $J \propto \nabla\phi$

Shift symmetry

It turns out that the only way to accomplish that while respecting shift symmetry corresponds to the choice $G_5 = -4 \ln |X|$. This choice corresponds to the following theory

$$S \sim \int d^4x \sqrt{-g} \left[R + X + \alpha \, \phi \, \mathscr{G} \right] \quad , \quad \mathscr{G} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2 \quad (3)$$

The scalar equation yields

$$\nabla^2 \phi = \alpha \mathscr{G} \quad (\neq 0 \quad \text{for curved spacetimes}) \quad (4)$$

$\Rightarrow \phi \neq$ *constant*, for curved backgrounds.

Shift symmetry

In shift-symmetric Horndeski we can define 3 classes of theories, namely

$$\textbf{Class 1:}\ \ \mathcal{E}_\phi[\phi = 0,\, g] = 0, \qquad \forall g, \tag{5}$$

$$\textbf{Class 2:}\ \ \lim_{g\to\eta} \mathcal{E}_\phi[\phi = 0,\, g] = 0. \tag{6}$$

$$\textbf{Class 3:}\ \ \text{All the rest.} \tag{7}$$

A minimally coupled scalar field belongs in **class 1**, while sGB belongs in **class 2**.

In general for shift symmetry

$$\mathcal{L}_{(2)} = \mathcal{L}_{(1)} + \alpha\phi\mathscr{G} \quad , \quad 4\pi Q = \alpha \int_{\mathcal{H}} n_a \mathscr{G}^a \tag{8}$$

Shift symmetry

From an EFT point of view, additional terms eventually will contribute, *i.e.*

$$S \sim \int d^4x \sqrt{-g} \left[R + X\left(1 + \sigma \Box \phi + \kappa X\right) + \alpha \phi \mathscr{G} + \gamma \, G_{\mu\nu} \nabla^{\mu} \phi \nabla^{\nu} \phi + \dots \right] \quad (9)$$

Black holes with a regular scalar field profile are found under the conditions:

$$\textbf{I:} \quad r_h^6 - 576\alpha^2\gamma - 24\alpha r_h^2(8\alpha + \sigma) \geq 0\,,$$

$$\textbf{II:} \quad \left[24\alpha\gamma r_h + (4\alpha + \sigma) r_h^3\right] \sqrt{r_h^6 - 576\alpha^2\gamma - 24\alpha(8\alpha + \sigma) r_h^2} + 4\alpha \left[r_h^6 - 576\alpha^2\gamma - 24\alpha(8\alpha + \sigma) r_h^2\right] \neq 0\,.$$

$$\boxed{\textbf{Minimum black hole mass} \;\Rightarrow r_h \geq r_h^{\text{min}}}$$

Shift symmetry

In shift symmetry we can employ a perturbative approach

$$g_{tt} = \left(1 - \frac{2M}{r}\right)\left(1 + \sum_{n=1}^{\infty} g_{tt}^{n}(r)\tilde{\alpha}^{n}\right)^{2} \tag{10}$$

$$\phi = \phi_0 + \sum_{n=1}^{\infty} \phi_n \tilde{\alpha}^{n} \tag{11}$$

- which traces the emergence of a finite radius singularity in the black hole interior.
- saturation of the existence conditions is associated with naked singularities!

Scalarization

Scalarization in Neutron Stars

$$S = \frac{c^3}{16\pi G}\int d^4x\sqrt{-g}\,\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) + S_m\left[\psi_m, e^{2a(\varphi)}g_{\mu\nu}\right] . \qquad (12)$$

- Scalar eq. yields $\Box\phi \propto \ldots a'(\varphi)\,T$.
- If, for some $\phi = \phi_0$, $a'(\phi_0) = 0 \Rightarrow$ GR.
- Take $a(\varphi) = \beta_0\varphi^2/2$.
- Estimating the energy

$$\text{Energy} \approx mc^2\left(\frac{\varphi_c^2/2}{Gm/Rc^2} + e^{\beta_0\varphi_c^2/2}\right)$$

For large M/R minima start forming.

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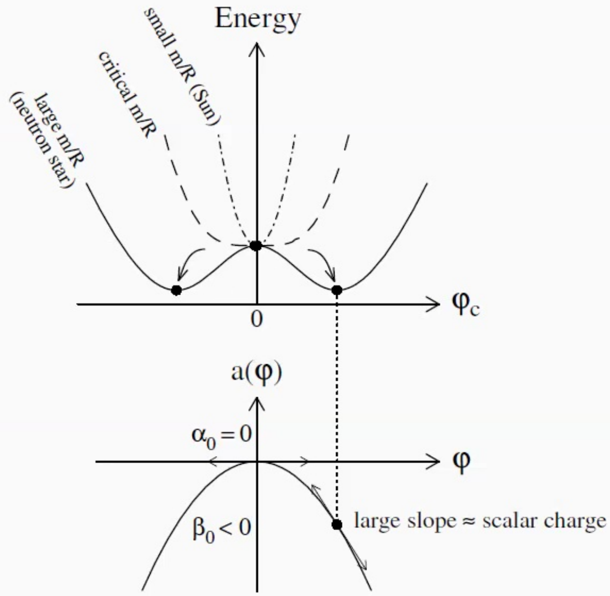

Fig. taken from arXiv:gr-qc/0402007

Scalarization in Black Holes

We may promote the linear coupling to a generic one

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + f(\phi)\mathscr{G}\right]. \tag{13}$$

→ The scalar eq. $\Box\phi = -f'(\phi)\mathscr{G} \Rightarrow [\Box + \overbrace{f''(\phi_0)\mathscr{G}}^{-m^2_{\rm eff}}]\delta\phi = 0.$

→ We are interested in theories that are connected to GR and therefore accept GR as a solution → $f'(\phi_0) = 0$, $-f''(\phi_0)\mathscr{G} > 0$ [1,2,3] .

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[1] Antoniou et al. Phys. Rev. Lett. 120 (2018) 13, 131102

[2] Silva et al. Phys. Rev. Lett. 120 (2018) 13, 131104

[3] Doneva et al. Phys. Rev. Lett. 120 (2018) 13, 131103

Scalarization in Black Holes

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If $m^2_{\text{eff}} < 0 \longrightarrow$ non-trivial scalarized solutions

Scalarization in Black Holes

$$\mathscr{G}_{\text{Kerr}} = \frac{48M^2}{(r^2+\chi^2)^6}\left(r^6 - 15r^4\chi^2 + 15r^2\chi^4 - \chi^6\right)$$

- For $\chi = 0 \Rightarrow \mathscr{G} > 0$ scalarization requires $f''(\phi_0) > 0$.
- For $\chi \neq 0 \rightarrow \mathscr{G} \lesseqgtr 0$ *spin-induced* scalarization for $f''(\phi_0) < 0$.

→ The **minimal model** satisfies the conditions above and contains all terms that contribute to the instability at a linear level

$$\mathcal{L} = \left(1 + \frac{\beta\phi^2}{4}\right) R + X + \gamma\, G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - \frac{\alpha\phi^2}{2}\mathscr{G} - \frac{1}{2}m_\phi^2\phi^2 \qquad (14)$$

Issues of scalarization models

→ Models of scalarization usually face a number of problems:

1. Cosmological consistency
2. Neutron-star constraints
3. Stability (the exponential coupling yields stable solutions but does not satisfy our conditions)
4. Well posedness

Cosmological attractor

Evolution of scalars in a cosmological background:

- $\mathcal{L} = R + X - m_{\text{eff}}^2\phi^2/2 \implies \mathcal{E}_\phi \propto e^{-t(3H-2\omega)}$
- $\mathcal{L} = R + X + f(\phi)\mathcal{G} \implies \mathcal{E}_\phi \propto e^{-\frac{3Ht}{2}}(C_1 e^{-\omega t} + C_2 e^{\omega t})$ for $f \sim \phi^2$
- $\mathcal{L} = R + h(\phi)R + X + f(\phi)\mathcal{G}$, let's see what happens for $f, h \sim \phi^2$

The scalar equation reads[1]

$$\ddot{\phi} + \underbrace{3H\dot{\phi}}_{\text{friction term}} + \underbrace{m_{\text{eff}}^2(t)\phi}_{\text{mass term}} = 0\,, \tag{15}$$

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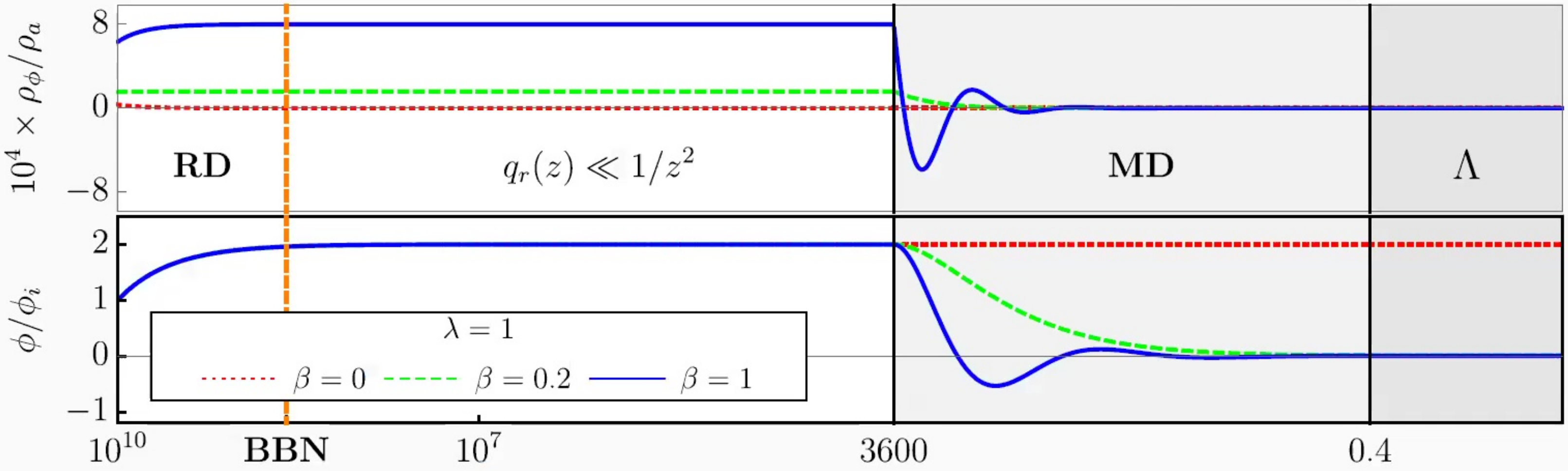
[1] Antoniou et al. Phys. Rev. D **103** (2021) No. 2, 024012

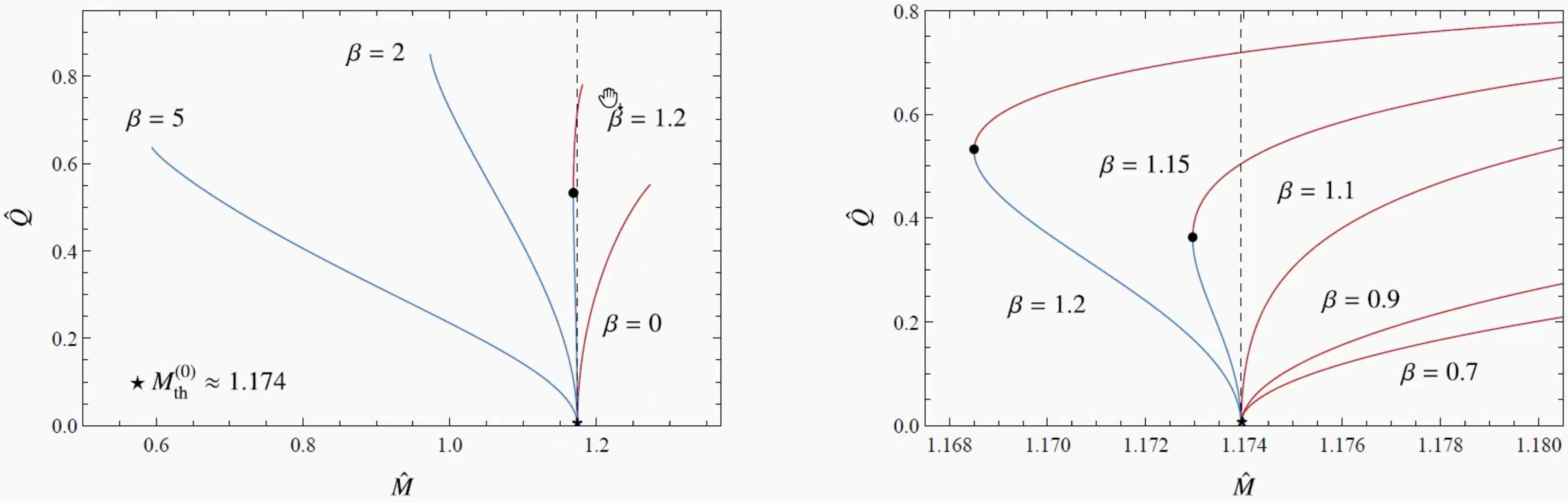
Cosmological attractor

→ We need to keep an eye on the sign of $m^2_{\text{eff}} = \beta R/2 - \alpha\mathcal{G}$:



→ Provided $\beta > \beta_{\text{crit}}$ an attractor is retrieved ✓

Stability of black holes



- Scalarization threshold is unchanged for β
- The properties of solutions present differences

Stability of black holes

Perturbing the scalar field and metric around the scalarized background

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu} \quad , \quad \phi = \phi^{(0)} + \phi^{(1)} \tag{16}$$

In order to study QNMs for stability the master equation used is

$$g(r)^2 \frac{\partial^2 \phi^{(1)}}{\partial t^2} - \frac{\partial^2 \phi^{(1)}}{\partial r^2} + C(r) \frac{\partial \phi^{(1)}}{\partial r} + U(r)\phi^{(1)} = 0, \tag{17}$$

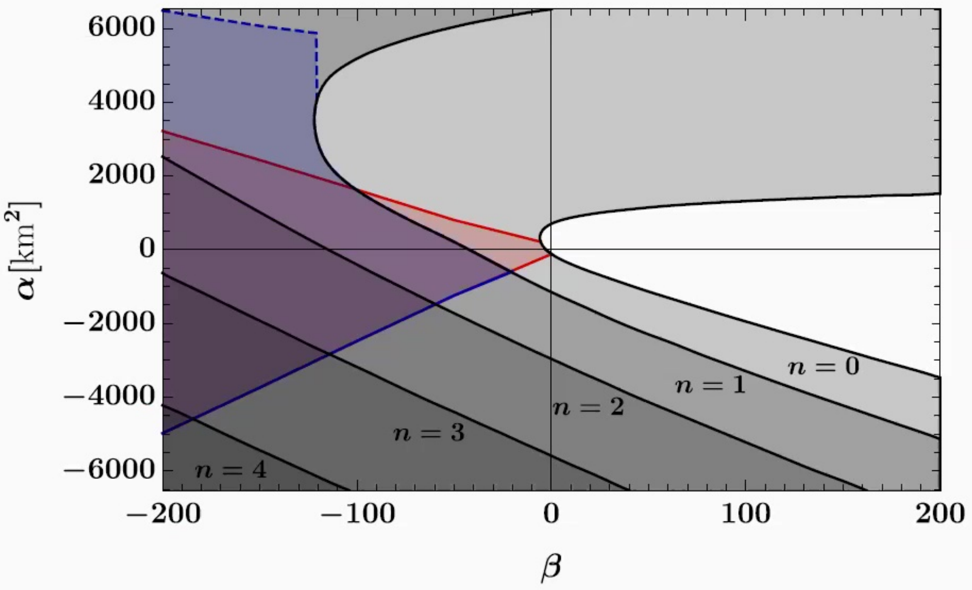
The scalar is decomposed as

$$\phi \sim e^{-i\omega t} \frac{\sigma_l(r,\omega)}{r} \, Y_l^m(\theta,\varphi)\, e^{-im\varphi} \tag{18}$$

→ Unstable modes are only found for $\beta < \beta_{\text{crit}} \sim 1$ ✓

Neutron star constraints

The parameter space for scalarized neutron stars in EsRGB: [2]:



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[2] Ventagli et al. Phys. Rev. D **104** (2021) 12, 124078

Neutron star constraints

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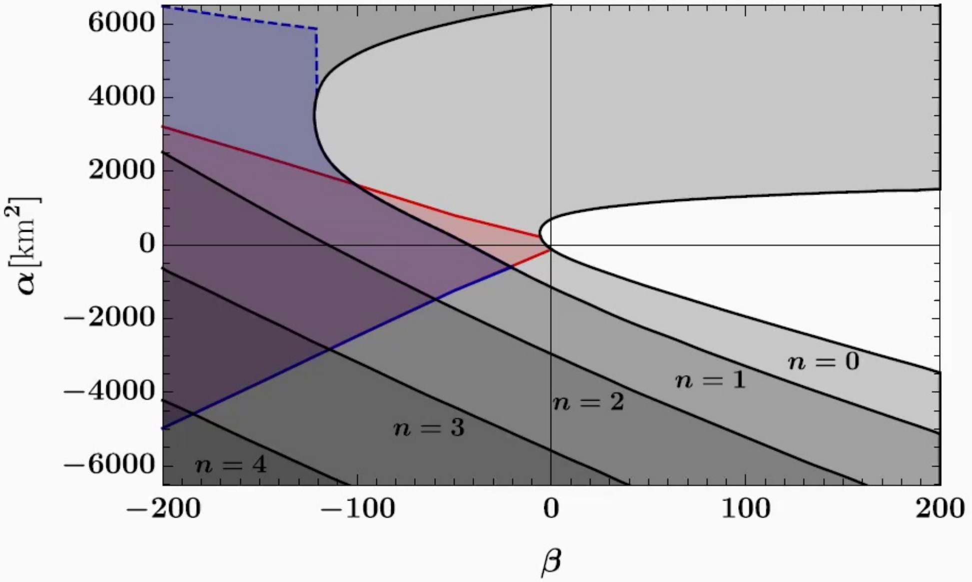
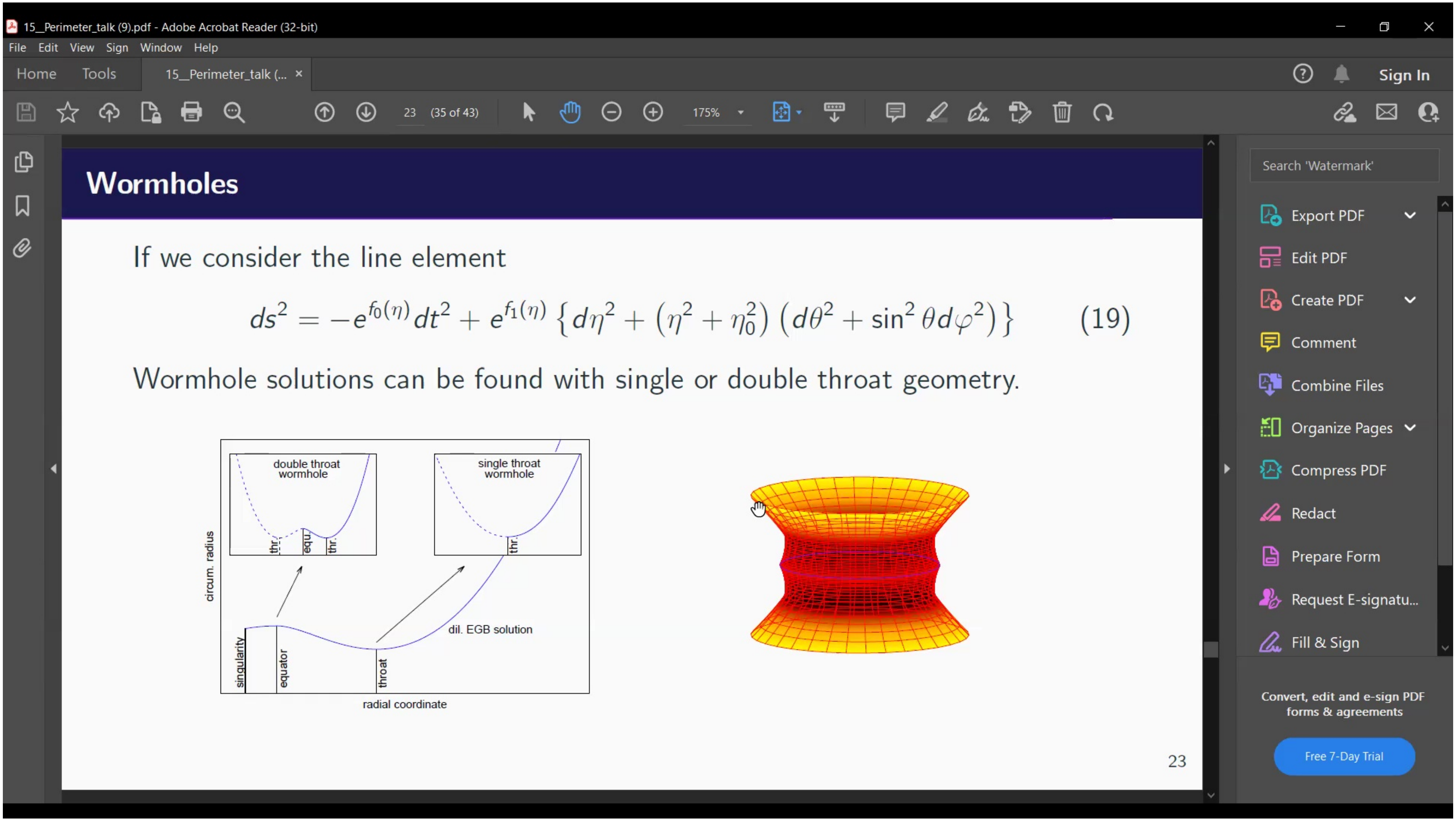


→Neutron star scalarization is suppressed for $\alpha, \beta > 0$ ✓



Wormholes

If we consider the line element

$$ds^2 = -e^{f_0(\eta)}dt^2 + e^{f_1(\eta)}\left\{d\eta^2 + \left(\eta^2 + \eta_0^2\right)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right\} \tag{19}$$

Wormhole solutions can be found with single or double throat geometry.

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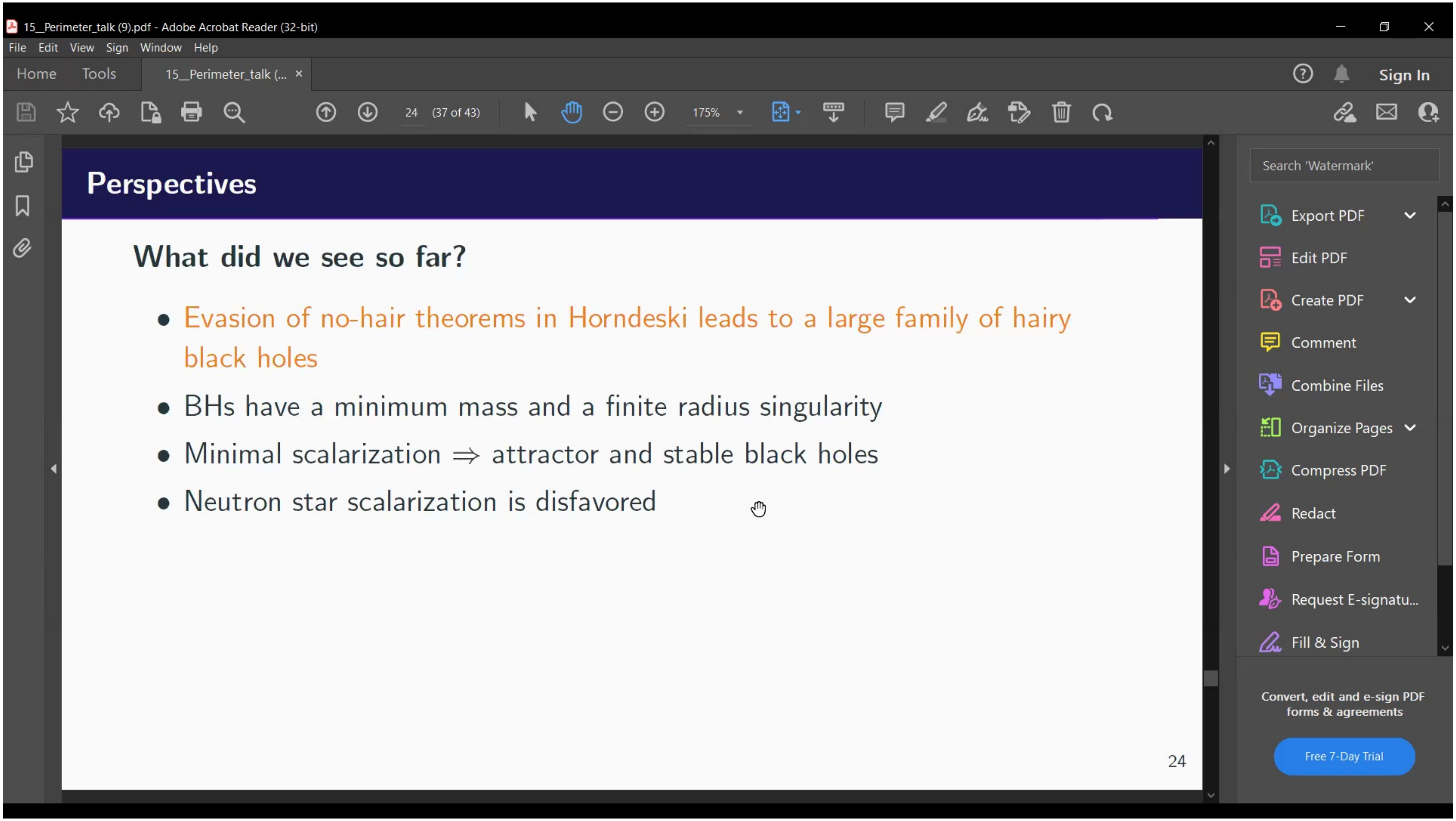

Perspectives

What did we see so far?

- Evasion of no-hair theorems in Horndeski leads to a large family of hairy black holes
- BHs have a minimum mass and a finite radius singularity
- Minimal scalarization $\Rightarrow$ attractor and stable black holes
- Neutron star scalarization is disfavored

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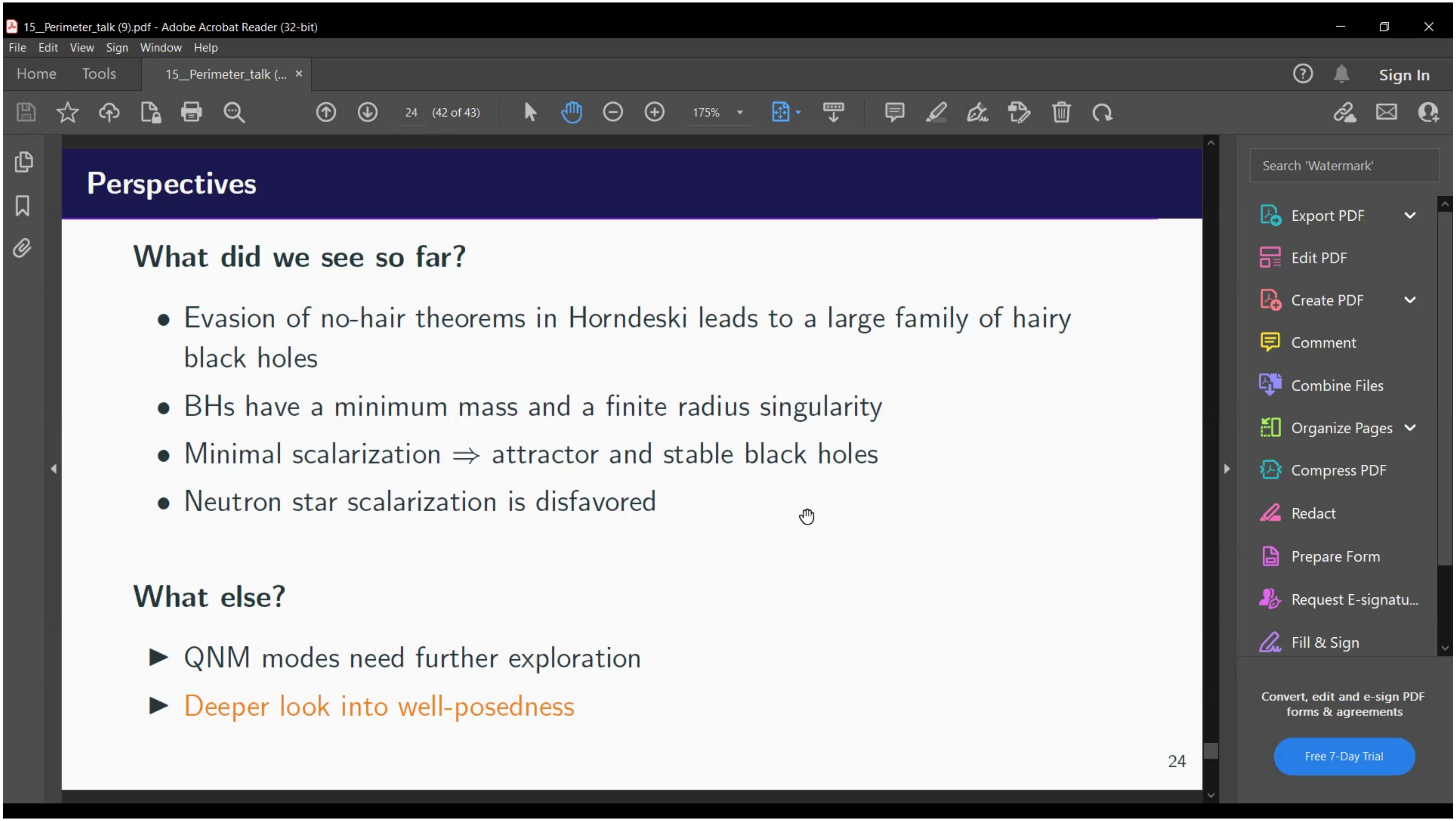

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What else?

- QNM modes need further exploration
- Deeper look into well-posedness

Thank you!

Questions?