

Title: Horizon entropy and the Einstein equation - Lecture 20230302

Speakers: Ted Jacobson

Collection: Horizon entropy and the Einstein equation

Date: March 02, 2023 - 10:00 AM

URL: <https://pirsa.org/23030080>

Abstract: Zoom link: <https://ptp.zoom.us/j/96212372067?pwd=dWVaUFFFFc3c5NTlVTDFHOGhCV2pXdz09>

$$MVEH \rightarrow \delta S_{uv} + \delta S_{IR} = 0$$

$$\downarrow$$

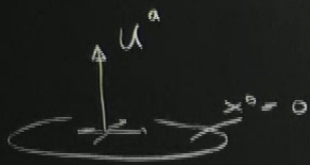
$$\delta A|_v$$

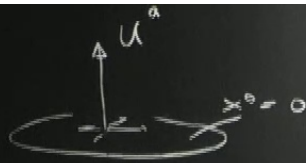
$$\downarrow$$

$$CFT$$

$$-\gamma \frac{\Omega_{d-2} l^d}{d^2-1} G_{00} + \frac{2\pi}{\hbar} \frac{\Omega_{d-2} l^d}{d^2-1} \delta \langle T_{00} \rangle = 0$$

$$G_{00} = \frac{2\pi}{\hbar} \delta \langle T_{00} \rangle$$





$$-\gamma \frac{\Omega_{d-2} \lambda}{d^2-1} G_{00} + \gamma \frac{\Omega_{d-2} \lambda}{d^2-1} \delta \langle T_{00} \rangle = 0$$

$$\forall u^a \Rightarrow G_{ab} = \left( \frac{2\pi}{k\gamma} \right) \delta \langle T_{ab} \rangle_{\text{CFT}}$$

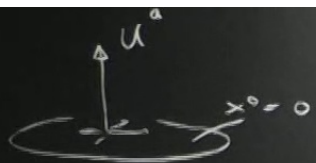
$$8\pi G \rightarrow \gamma = \frac{1}{4kG}$$

$$\textcircled{1} \Phi = \phi_1(\vec{x}_1)e^{i\omega_1 t + i\alpha} + \phi_2(\vec{x}_2)e^{i\omega_2 t + i\alpha}$$

$$F \sim \cos[(\omega_1 + \omega_2)t + \alpha] e^{-D(l_1 + l_2)} \quad D = |\vec{x}_1 - \vec{x}_2|$$

$$\textcircled{2} \text{ Vortex: } \Phi = \phi e^{i\gamma}$$

$$\boxed{\frac{1}{2\pi} \oint_{\gamma} d\epsilon_i D^i \gamma = q}$$

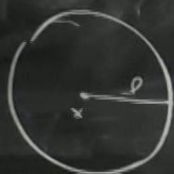


$$-\gamma \frac{\Omega_{d-2} \lambda}{d^2-1} G_{00} + \frac{\Omega_{d-2} \lambda}{d^2-1} \delta \langle T_{00} \rangle = 0$$

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$$8\pi G \rightarrow \gamma = \frac{1}{4kG}$$

In max. sym spacetime,  $G_{\mu\nu} = -\Lambda g_{\mu\nu} \quad (- + \dots +)$



$$\lambda(x, l), \quad G_{ab}^{\text{MSS}} = -\lambda g_{ab}$$

$$\delta A|_{V, \lambda} = -\frac{\Omega_{d-2} l^d}{d^2-1} (G_{00} - G_{00}^{\text{MSS}}) = -\frac{\Omega_{d-2} l^d}{d^2-1} (G_{00} + \lambda g_{00})$$

$$\rightarrow G_{00} + \lambda g_{00} = \frac{2\pi}{k\gamma} \delta \langle T_{00} \rangle$$

$$\left( G_{ab} + \lambda g_{ab} = \frac{2\pi}{k\gamma} \delta \langle T_{ab} \rangle \right) u^a u^b$$

$$\nabla^a G_{ab} = 0 = \nabla^a \delta \langle T_{ab} \rangle \Rightarrow \nabla_a \lambda = 0 \rightarrow \lambda = \text{space-time const.}$$


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$$\rightarrow G_{00} + \lambda g_{00} = \frac{2\pi}{\hbar\gamma} \delta\langle T_{00} \rangle$$

$$\left( G_{ab} + \lambda g_{ab} = \frac{2\pi}{\hbar\gamma} \delta\langle T_{ab} \rangle \right) u^a u^b$$

$$\text{holds } \forall l \Rightarrow \lambda(x, l) = \lambda(x)$$

$$\nabla^a G_{ab} = 0 = \nabla^a \delta\langle T_{ab} \rangle \Rightarrow \nabla_a \lambda = 0 \rightarrow \lambda = \text{space-time const.}$$

What about QFT not a CFT?

DR. W. P. L.



What about QFT not a CFT?

Any state  $\rho$  in QFT,  $\rho = e^{-\frac{2\pi}{\hbar} K}$  for some  $K$   
 (modular hamiltonian)

$$\delta S = \delta(-\text{Tr} \rho \ln \rho)$$

$$= \frac{2\pi}{\hbar} \delta \langle K \rangle \quad \text{for Rindler wedge} \quad K = \int_{\Sigma_R} T_{ab} \chi^a d\Sigma^b$$

DRTWALL

What about QFT not a CFT?

Any state  $\rho$  in QFT,  $\rho = e^{-\frac{2\pi}{h} K}$  for some  $K$   
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$$\delta S = \delta(-\text{Tr} \rho \ln \rho)$$

$$= \frac{2\pi}{h} \underbrace{\delta \langle K \rangle}_{\text{Tr}(\delta \rho K)} \text{ for Rindler wedge } K = \int_{\Sigma_R} T_{ab} X^a d\Sigma^b$$

DRTWALL

guess about  $\delta\langle K \rangle$  of non-CFT

$$\delta\langle K \rangle = \int (\delta\langle T_{ab} \rangle + \Phi g_{ab}) \zeta^a d\zeta^b$$

unknown at this point.

(in sufficiently small balls)

pace time const.

(in sufficiently small balls)

$$\rightarrow G_{ab} + \lambda g_{ab} = \frac{2\pi}{\hbar\gamma} \left( \delta \langle T_{ab} \rangle + \Phi^{(x,R)}_{\perp} g_{ab} \right)$$

CAUTION  
DO NOT TOUCH THE BOARD OR THE BOARDER  
IT IS PROHIBITED TO TOUCH  
THE BOARD OR THE BOARDER  
ANYONE VIOLATING THIS RULE  
WILL BE PUNISHED

$$\text{if balls} \Rightarrow \lambda - \frac{2\pi}{k\gamma} \Phi = \Lambda = \text{const.}$$

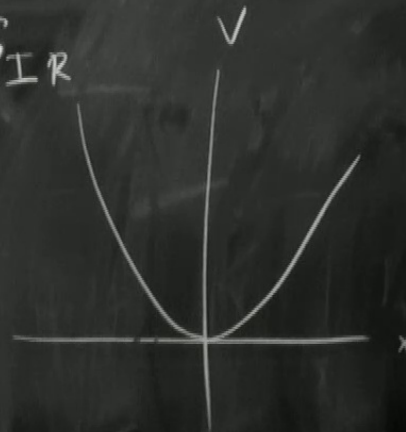
Casini, Calante, Myers : using holographic QFT & RT formula

Speranza : using conformal perturbation theory-  
(2016)

Varadarajan: a coherent state (e.g. of em field)  
can have  $\langle T_{xx} \rangle \neq 0$  but no  $\delta S_{IR}$

coherent state of h.o.

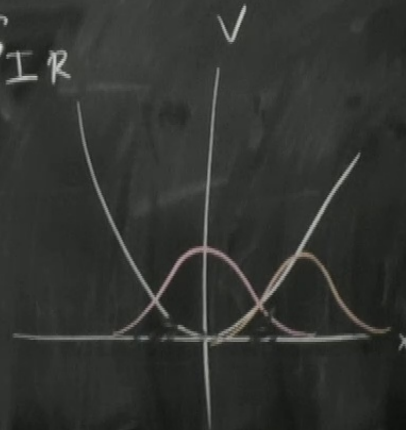
$$|0\rangle, |n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}} |0\rangle,$$



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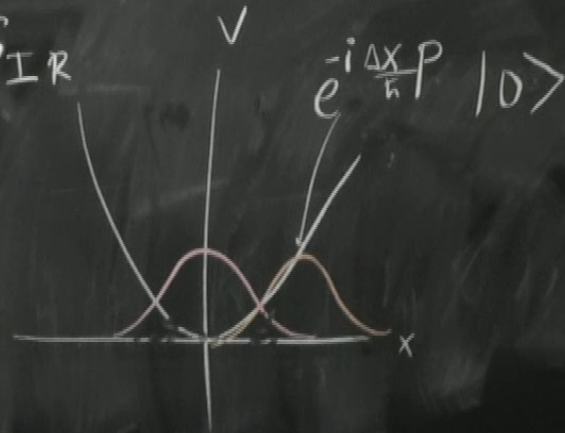


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coherent state of h.o.

$$|0\rangle, |n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}} |0\rangle,$$

in QFT

$$e^{-i\frac{\Delta x}{\hbar}P} |0\rangle$$

$$e^{-\frac{i}{\hbar} \int dx' \phi(x') \hat{\pi}(x)} |0\rangle$$

$$\int_{\text{inside}} + \int_{\text{outside}}$$

$$\rightarrow G_{00} + \lambda g_{00} = \frac{2\pi}{4\pi} \delta \langle T_{00} \rangle$$

$$\left( G_{ab} + \lambda g_{ab} = \frac{2\pi}{4\pi} \delta \langle T_{ab} \rangle \right) u^a u^b$$

$$\text{holds } \forall l \Rightarrow \lambda(x, l) = \lambda(x)$$

$$\nabla^a G_{ab} = 0 = \nabla^a \delta \langle T_{ab} \rangle \Rightarrow \nabla_a \lambda = 0 \rightarrow \lambda = \text{space time const.}$$

What about QFT not a CFT?

$$T(x)|0\rangle$$

outside

$$l \Rightarrow$$

$$= \lambda |x\rangle$$

space time const.

$$\rightarrow G_{ab} + \lambda g_{ab} = \frac{\delta H}{\delta \gamma} (\delta \langle T_{ab} \rangle + \Phi g_{ab})$$

$$\delta |0\rangle = |\delta\psi\rangle$$

$$|\psi\rangle = |0\rangle + |\delta\psi\rangle \rightarrow \langle \psi | T_{ab} | \psi \rangle \propto \langle \psi | T_{ab} | 0 \rangle + \langle \delta\psi | T_{ab} | 0 \rangle + \langle 0 | T_{ab} | \delta\psi \rangle + \mathcal{O}(\delta\psi^2)$$

What about QFT not a CFT?

$$S \rightarrow U_\epsilon S U_\epsilon^{-1} \quad \delta S = \frac{2\pi}{h} \delta \langle K \rangle \quad U_\epsilon = e^{i\epsilon A}$$

$U_\epsilon$

$$\langle K \rangle_\epsilon = \text{Tr} \rho_\epsilon K = \text{Tr} (U_\epsilon \rho U_\epsilon^{-1} K)$$

$$\frac{d}{d\epsilon} \langle K \rangle_\epsilon = i \text{Tr} (A \rho K - \rho A K)$$

What about QFT not a CFT?

$$S \rightarrow U_\epsilon S U_\epsilon^{-1} \quad \delta S = \frac{2\pi}{h} \delta \langle K \rangle \quad U_\epsilon = e^{i\epsilon A}$$

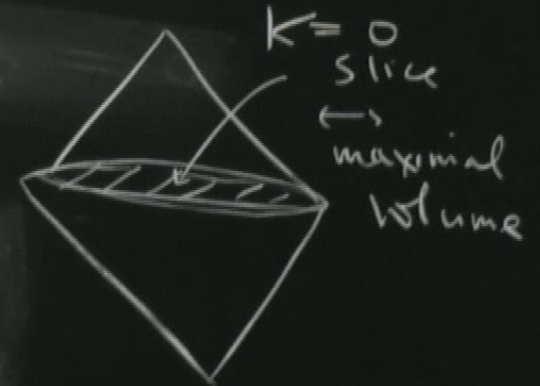
$U_\epsilon$

$$\langle K \rangle_\epsilon = \text{Tr} \rho_K K = \text{Tr} (U_\epsilon \rho U_\epsilon^{-1} K)$$

$$\left. \frac{d}{d\epsilon} \langle K \rangle_\epsilon \right|_{\epsilon=0} = i \text{Tr} (A \rho K - \rho A K) = 0$$

Why fixed volume?

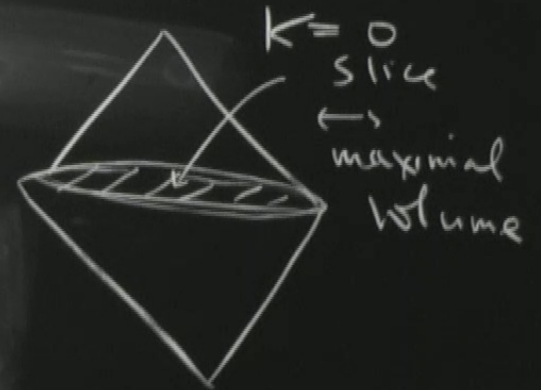
- Volume  $\propto$  Hamiltonian in CMC gauge (York)
- volume  $\longleftrightarrow$  complexity?



- Volume  $\propto$  Hamiltonian in CMC gauge (York)

- volume  $\longleftrightarrow$  complexity ?

- dim of Hilbert space @ fixed volume  
 $=$  area of boundary saddle ball  
 $\div 4\pi G$

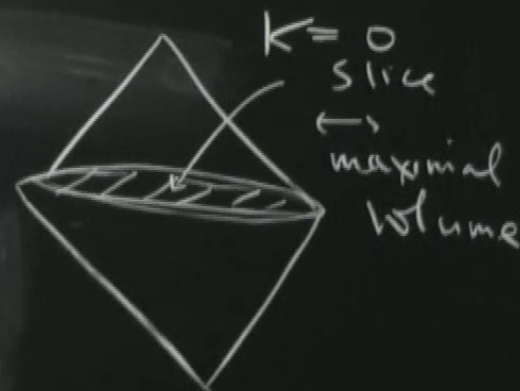


$$\delta A|_{V,\lambda} = -\frac{\int d^d x}{d-1} (G_{00} - G_{00}^{MSS}) = -\frac{\int d^d x}{d-1} (G_{00} + \lambda g_{00})$$

- Volume  $\propto$  Hamiltonian in CMC gauge (York)

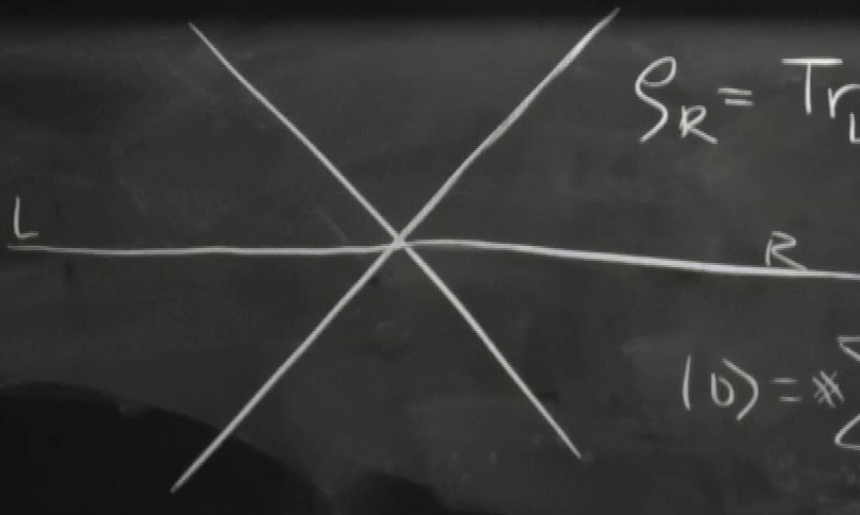
- volume  $\longleftrightarrow$  complexity ?

- dim of Hilbert space @ fixed volume  
 $= \exp \left( \frac{\text{area of boundary saddle ball}}{4\pi G} \right)$



$$\delta A|_{V,\lambda} = -\frac{\Omega_{d-2} l}{d-1} (G_{00} - G_{00}^{MSS}) = -\frac{\Omega_{d-2} l}{d-1} (G_{00} + \lambda g_{00})$$

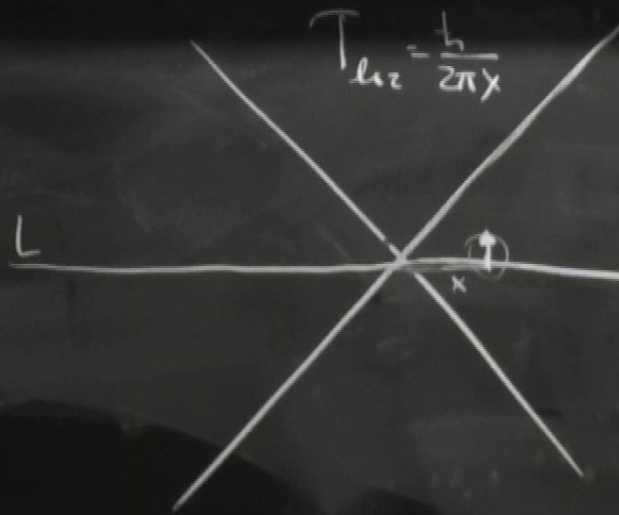
$$\exp \left( - \frac{1}{4 \hbar G} \right)$$



$$S_R = \text{Tr}_L |0\rangle\langle 0| = \frac{e^{-\frac{2\pi}{\hbar} H_B}}{\mathcal{Z}}$$

$$|0\rangle = \sum_n e^{-\frac{\pi}{\hbar} E_n} |n_R\rangle |\bar{n}_L\rangle$$

$$\exp \left( -\frac{1}{4\hbar G} \right)$$



$$T_{LR} = \frac{\hbar}{2\pi\gamma}$$

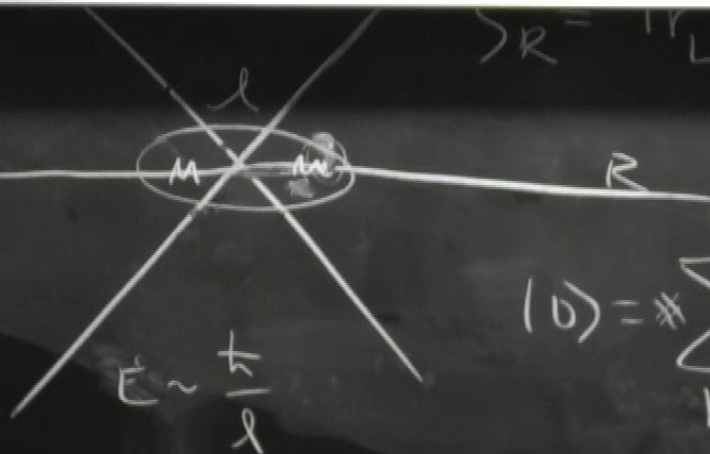
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$$= \exp \left( \frac{\text{area of boundary saddle surface}}{4\pi G} \right)$$

$$\langle R = |r_L| \rangle \langle 0| = e^{-\frac{\pi}{4} \frac{R}{l}}$$

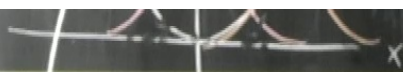
4d  
Spacetime



$$|0\rangle = \sum_n e^{-\frac{\pi}{4} E_n} |n_R\rangle |\bar{n}_L\rangle$$

$$R_S \sim \frac{Gh}{l} > l \longleftrightarrow l < \sqrt{tG} = l_{Pl}$$

$$|0\rangle, |n\rangle = (a^\dagger)^n |0\rangle$$



$$S_H \sim \int dx dA \underbrace{T_{loc}^3}_{\text{entropy density}}$$

$$\sim \int_{-\epsilon}^{\epsilon} dx dA \frac{1}{x^3}$$

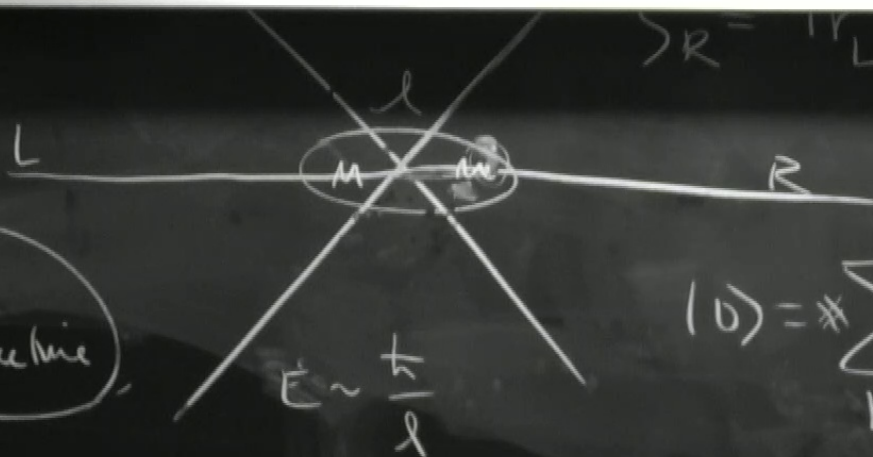
$$\sim \boxed{A} \frac{1}{\epsilon^2}$$

$$\epsilon=0$$

$$= \exp \left( \frac{\text{area of boundary saddle surface}}{4\pi G} \right)$$

$$\langle R = |r_L| \rangle \langle 0| = e^{-\frac{\pi}{4} \frac{R}{l}}$$

4d  
Spacetime



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$$R_S \sim \frac{G\hbar}{l} > l \leftrightarrow l < \sqrt{\hbar G} = l_{Pl}$$