Title: VOAs and Twisted Chern-Simons Matter Theories

Speakers: Niklas Garner

Series: Mathematical Physics

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Abstract: The rich interplay between three-dimensional topological quantum field theories (TQFTs) and vertex operator algebras (VOAs) has been a useful bridge in understanding aspects of both subjects. In this talk, I will describe some aspects of this correspondence focusing on the simple, yet surprisingly rich, examples of Chern-Simons theories based on the Lie superalgebra gl(1|1).

Zoom Link: https://pitp.zoom.us/j/97698409749?pwd=ZkdzUHI2YXJEYk53VDQyZm1ERzJHUT09

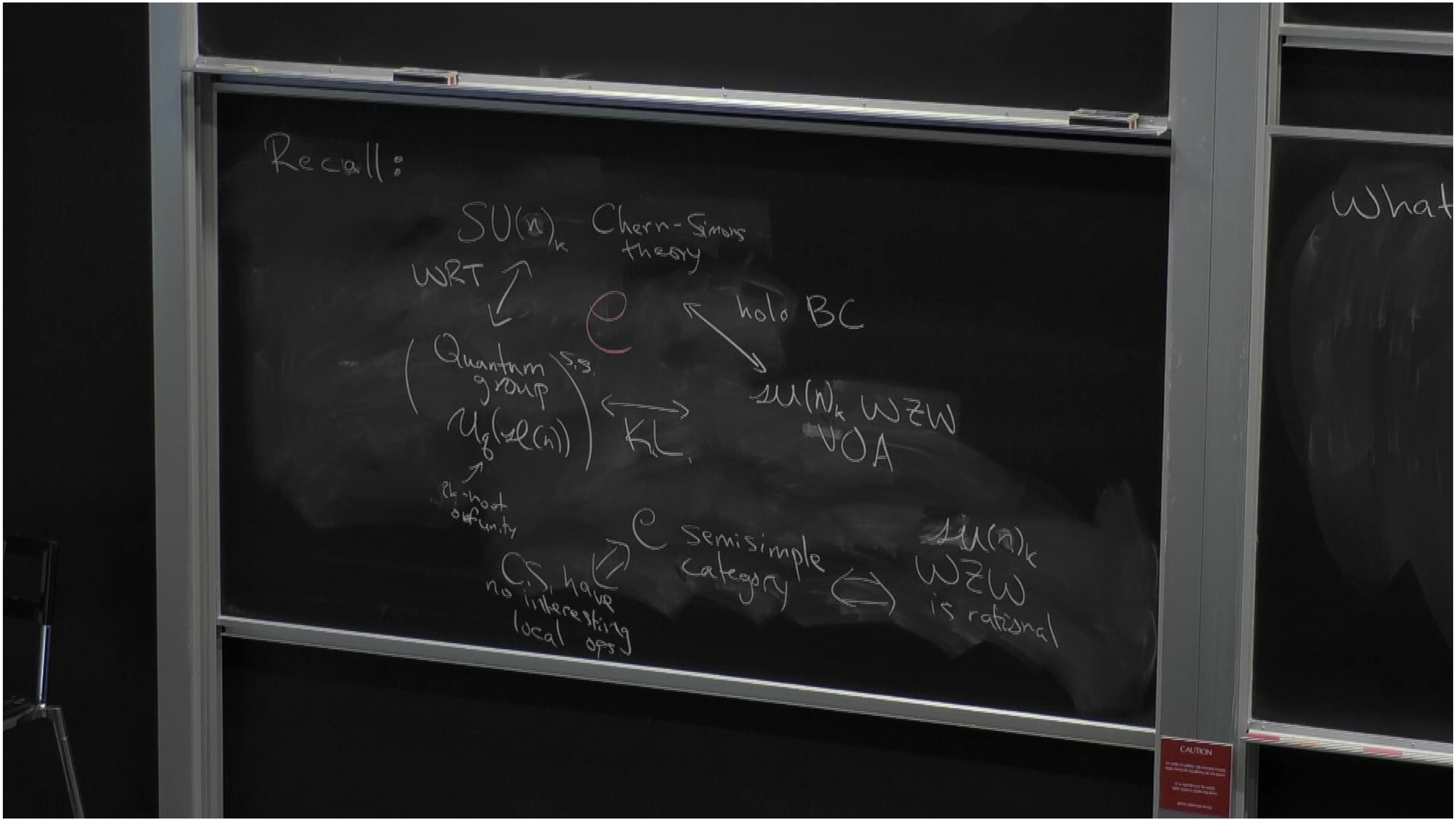
VOAs & Twisted Chern-Simons-matter TQFTs

N. Garner U. Washington

based on jt work:

arXiv: 2112.01559 w/ T. Creutzig, T. Dimofte, N. Geer

WIP w/ W. Niu



Recall:
SU(n)_k Chern-Simons theory
WRT
C
holo BC
Quantum group U_q(sl(n)) s.s.
K.L.
su(n)_k WZW VOA
rk-root of unity
C semisimple category
su(n)_k WZW is rational
C.S. have no interesting local ops
What

What about the rest of $U_q(\mathfrak{sl}(n))$-mod?

WZW $\rightsquigarrow$ Feigin–Tipunin algebra $FT_k(\mathfrak{sl}(n))$ Log. K.C.

FGST

Axiomatic TQFT: CGP TQFT

2112.01559: $SU(n)_k \rightsquigarrow \left(T[SU(n)] / SU(n)_k \right)^A$

TQFTs arising from twisting SUSY theories
often non-semisimple, which makes them hard.

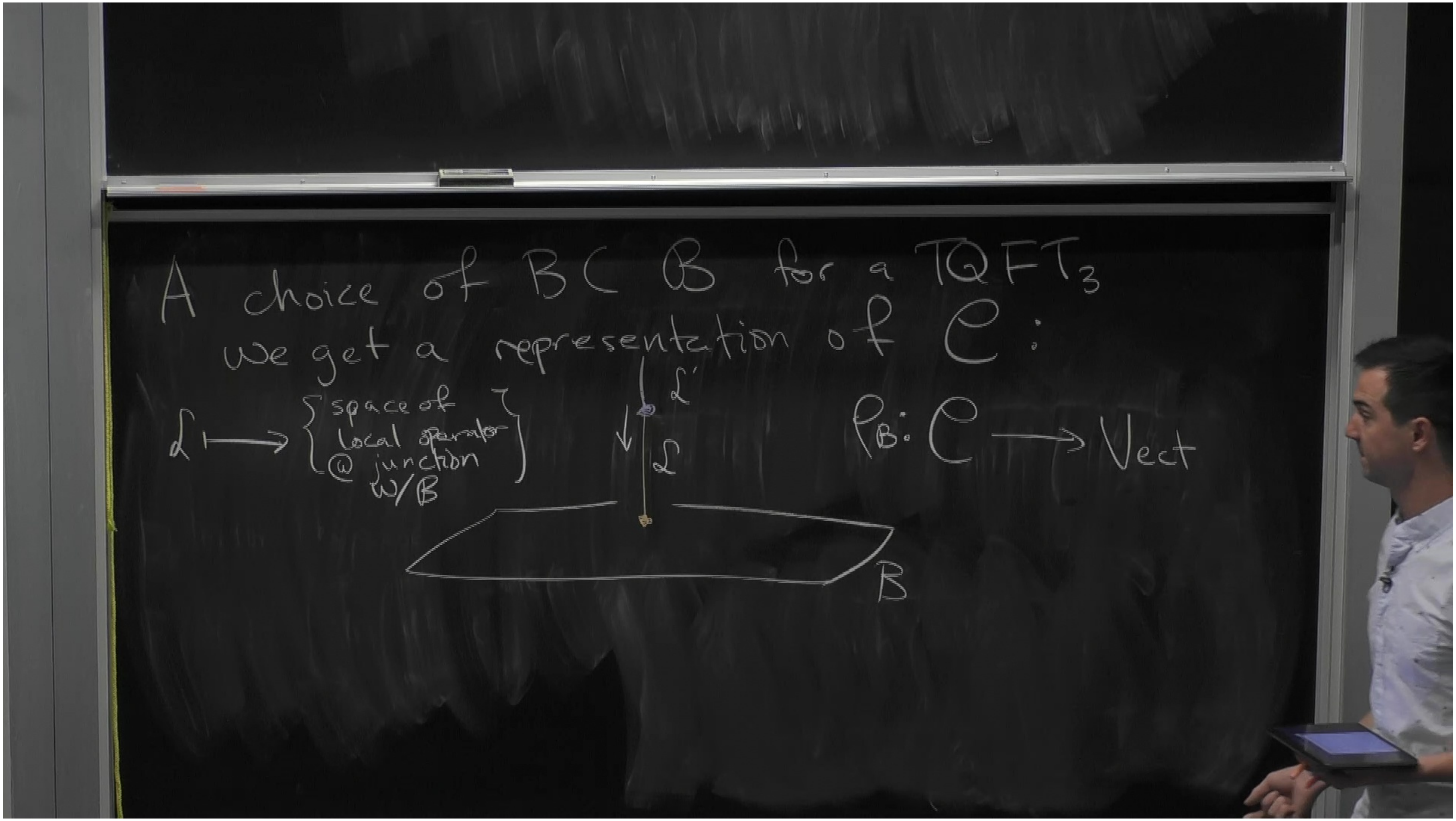
- partition fcns need subtle regularization
- often ∞-dim'l state spaces
- non-trivial alg. of local ops
- log. VOAs

Today:

0) Reminder of $TQFT_3$/VOA

1) Consequences of $TQFT_3$/VOA

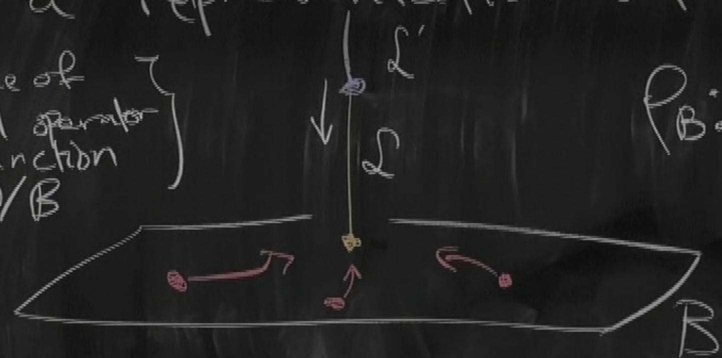
2) Illustrate simple examples based on C.S. for $\mathfrak{gl}(1|1)$


A choice of BC B for a TQFT3
we get a representation of C:
L ⟼ {space of local operator @ junction w/ B}
L'
L
ρB: C ⟶ Vect
B

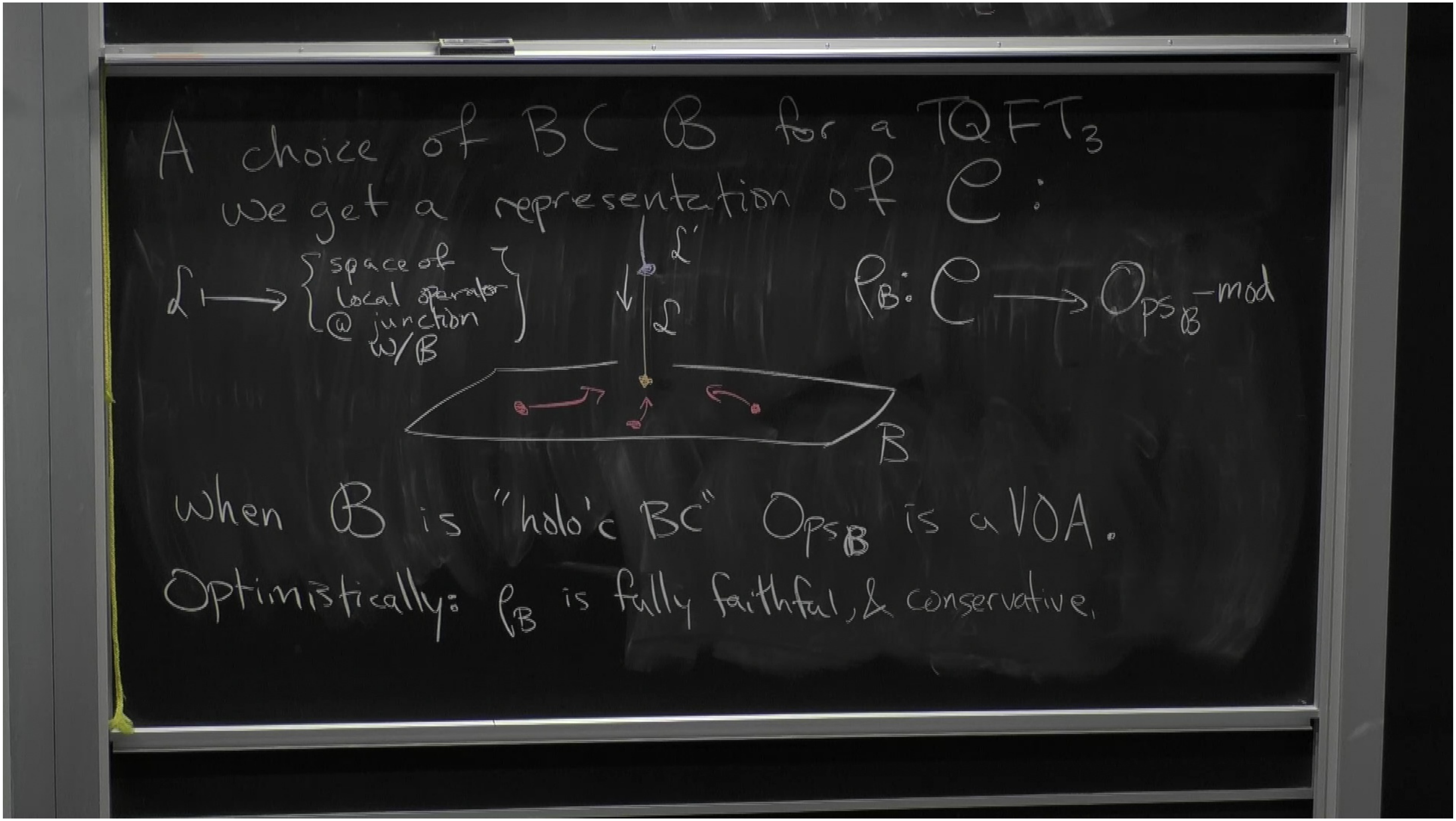
A choice of BC $\mathcal{B}$ for a TQFT$_3$

we get a representation of $\mathcal{C}$:

$$\mathcal{L} \longmapsto \left\{ \begin{array}{l} \text{space of} \\ \text{local operator} \\ \text{@ junction} \\ \text{w/ } \mathcal{B} \end{array} \right\}$$



$$\rho_{\mathcal{B}}: \mathcal{C} \longrightarrow \mathcal{O}_{ps_{\mathcal{B}}}\text{-mod}$$

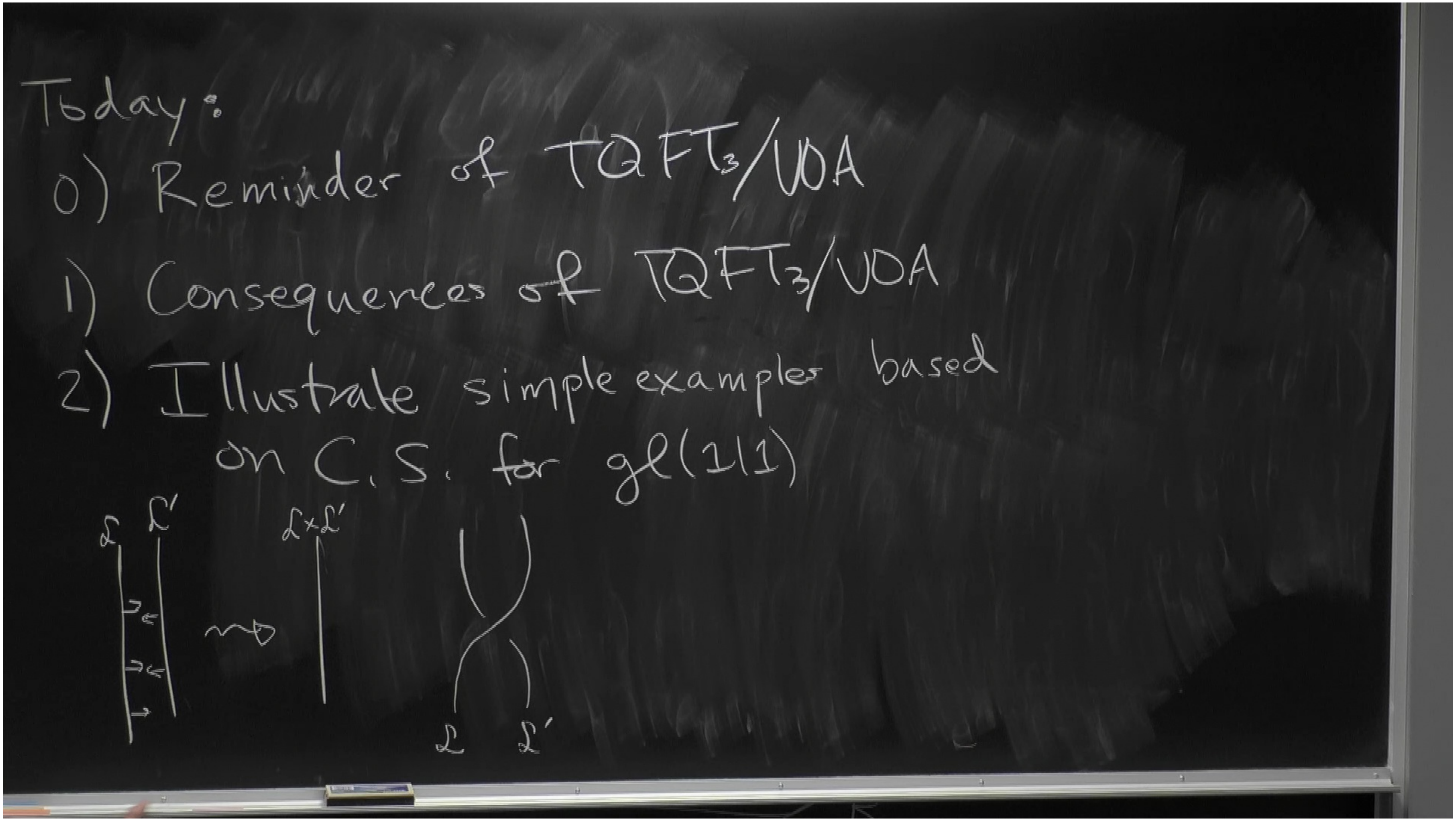

A choice of BC $\mathcal{B}$ for a $TQFT_3$
we get a representation of $\mathcal{C}$:
$\mathcal{L} \longmapsto$ {space of local operator @ junction w/ $\mathcal{B}$}
$\mathcal{L}'$
$\mathcal{L}$
$\rho_{\mathcal{B}}: \mathcal{C} \longrightarrow \mathcal{O}ps_{\mathcal{B}}\text{-mod}$
$\mathcal{B}$
when $\mathcal{B}$ is "holo'c BC" $\mathcal{O}ps_{\mathcal{B}}$ is a VOA.
Optimistically: $\rho_{\mathcal{B}}$ is fully faithful, & conservative.

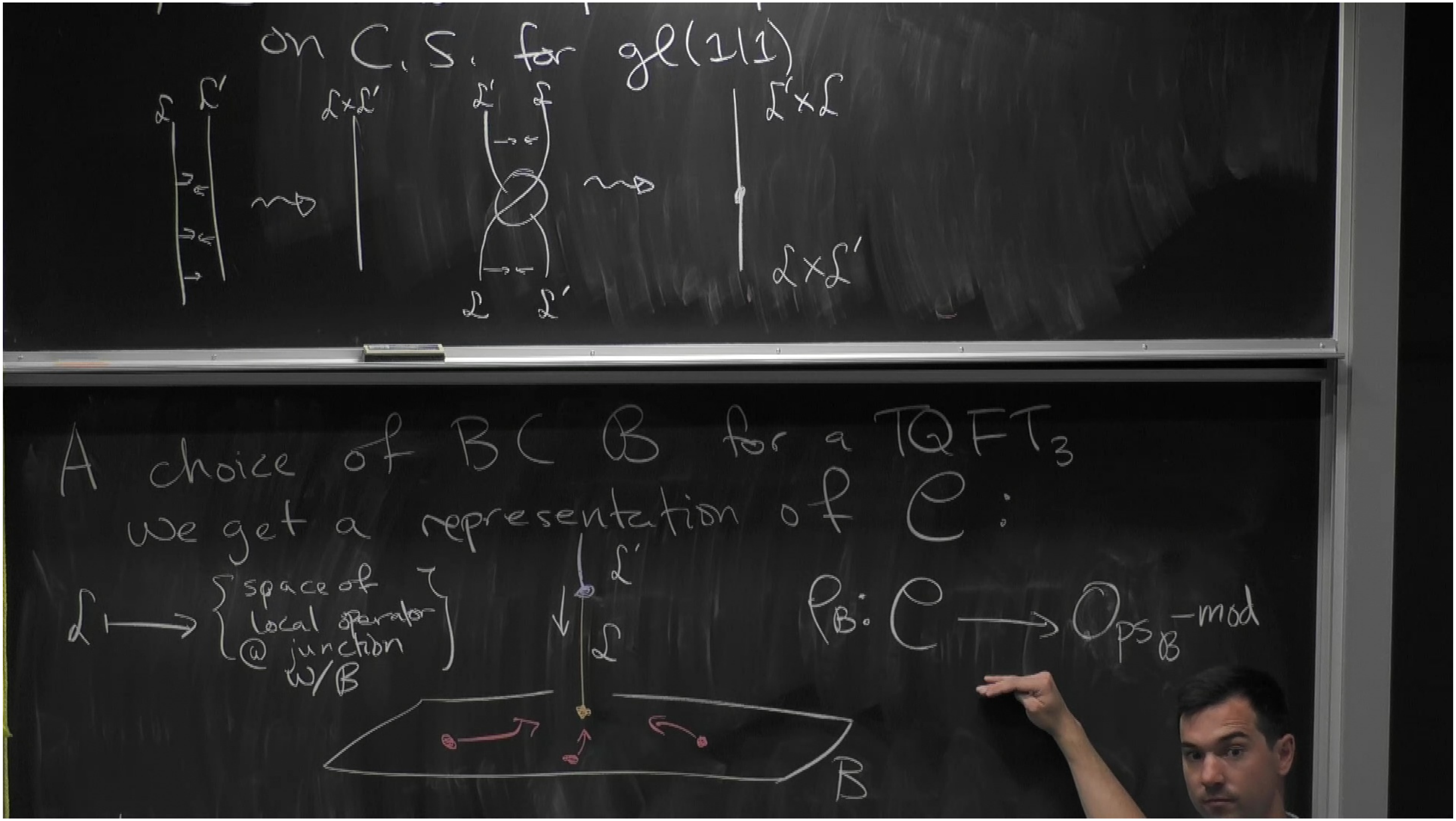
Dictionary/expectations:

$\mathcal{L} \longleftrightarrow M_{\mathcal{L}}$ VOA module chain cx's $\sim$ quasi.

$\mathcal{O} \in \mathrm{Hom}(\mathcal{L}, \mathcal{L}') \longleftrightarrow f_{\mathcal{O}} : M_{\mathcal{L}} \to M_{\mathcal{L}'}$ VOA module derived morphism

$\mathcal{H}(\Sigma) \longleftrightarrow$ derived conformal blocks of VOA


Today:
0) Reminder of TQFT3/VOA
1) Consequences of TQFT3/VOA
2) Illustrate simple examples based on C.S. for gl(1|1)
L L'
L×L'
L L'


on C.S. for gl(1|1)
L L'
L×L'
L' L
L'×L
L×L'
L L'
A choice of BC B for a TQFT3
we get a representation of C:
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L
ρB: C ⟶ Ops B-mod
B

obvious: algebraic description of TQFT

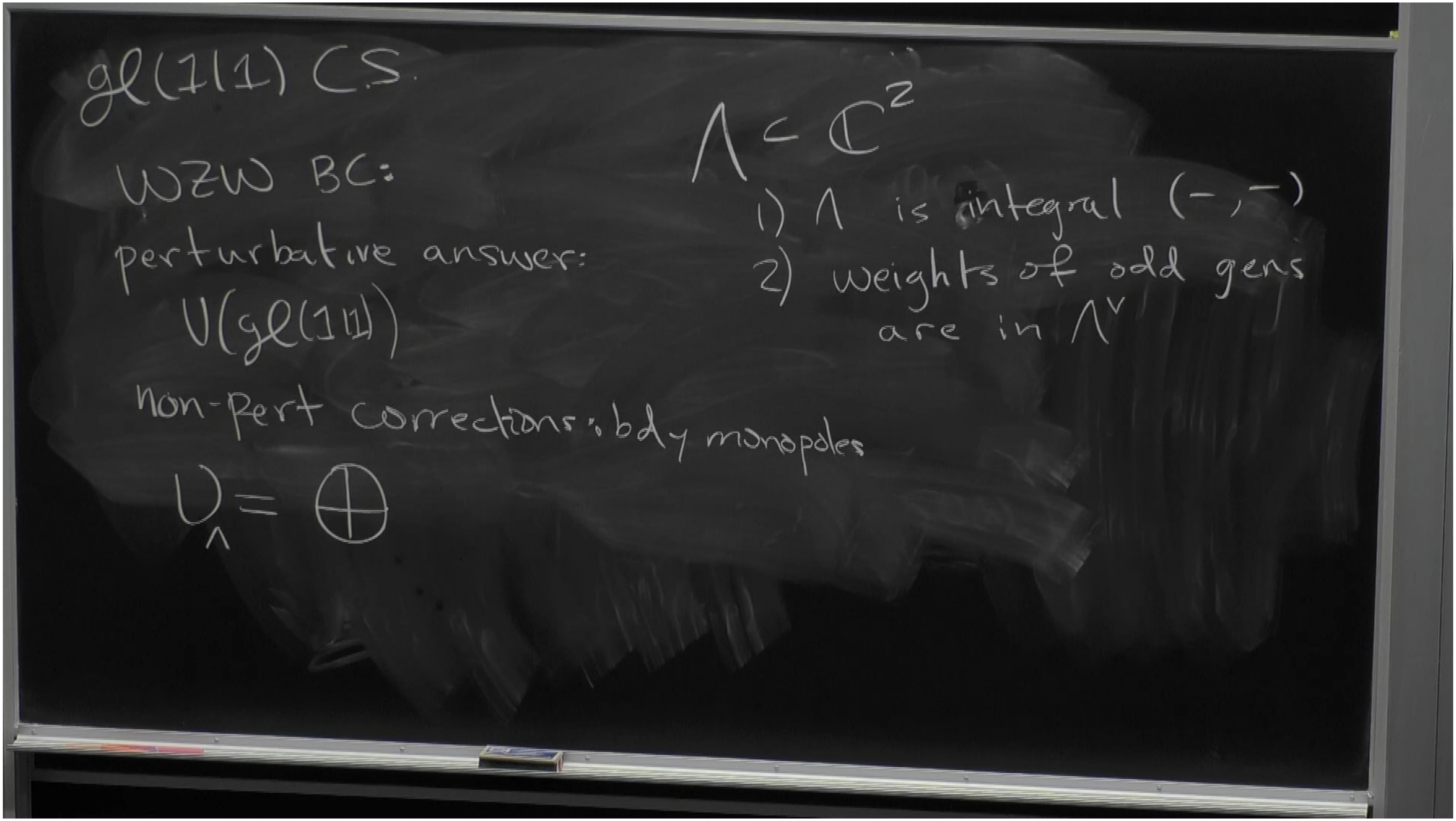
can access non-pert. aspects

also obv: symmetries of TQFT

provide structure to V-mod

more subtle: if we have $V_1, \dots, V_n$ describe TQFT

$\rightsquigarrow V_1\text{-mod} \simeq \dots \simeq V_n\text{-mod}$


gl(1|1) C.S.
WZW BC:
perturbative answer:
U(gl(1|1))
non-pert corrections: bdy monopoles
U_Λ = ⊕
Λ ⊂ ℂ²
1) Λ is integral (−,−)
2) weights of odd gens
are in Λ^∨

$\mathfrak{gl}(1|1)$ C.S.

WZW BC:

perturbative answer:

$$V(\mathfrak{gl}(1|1))$$

non-pert corrections: bdy monopoles

$$U_\Lambda = \bigoplus_{\lambda \in \Lambda} \sigma_\lambda \left(V(\mathfrak{gl}(1|1)) \right)$$

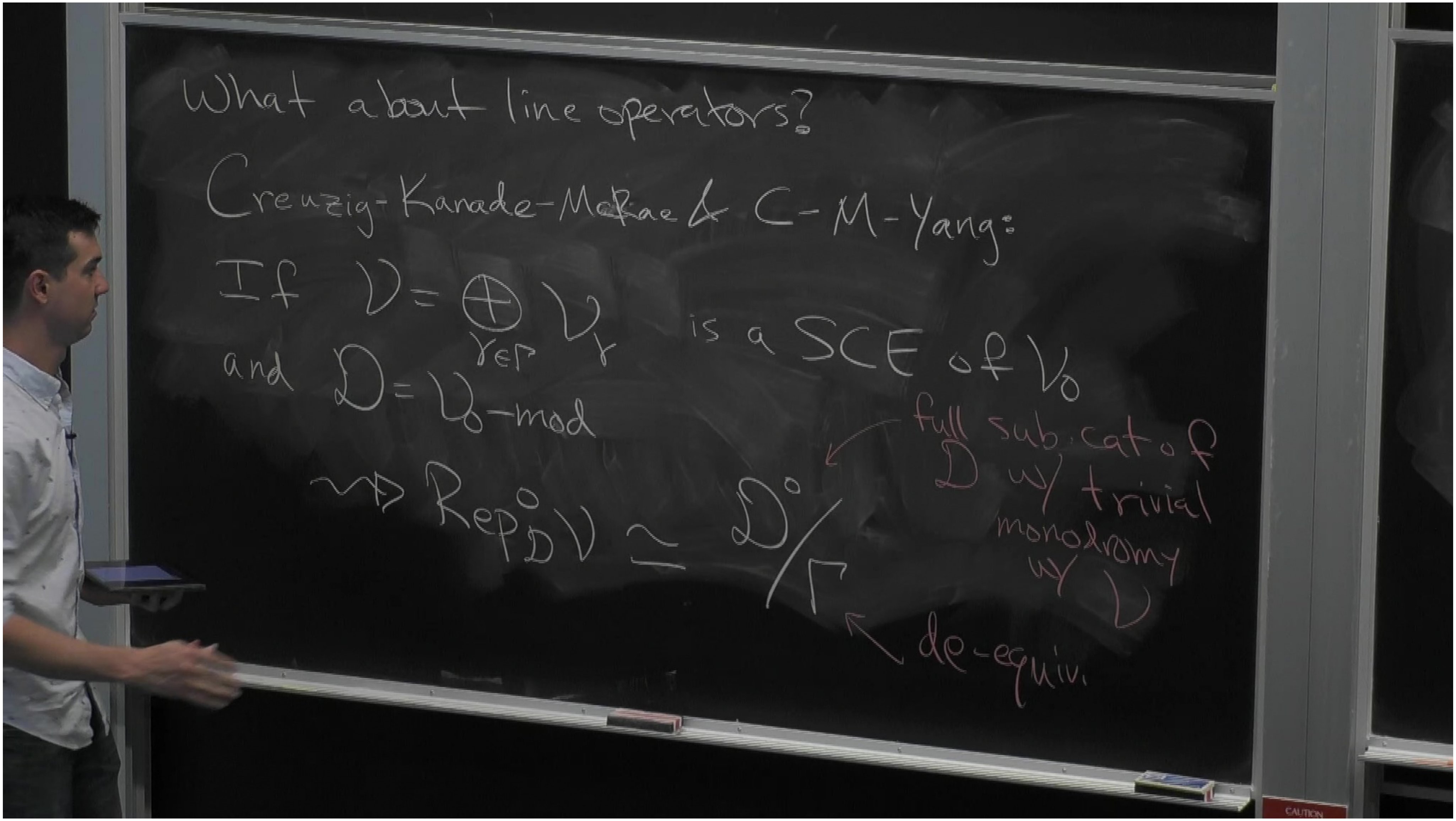
$\Lambda \subset \mathbb{C}^2$

1) Λ is integral $(-,-)$
2) weights of odd gens are in $\Lambda^\vee$

The rank 1 examples were studied by Creutzig-Ridout $W_{n+\frac{1}{2},\ell}$

Thm: (C-R)

$W_{0,1} \simeq \underbrace{\text{symplectic bosons}}^{\text{A-twist VOA for hyper}} \otimes \text{complex fermion}$


What about line operators?
Creuzig-Kanade-McRae & C-M-Yang:
If $V = \bigoplus_{\gamma \in \Gamma} V_\gamma$ is a SCE of V_0
and $\mathcal{D} = V_0$-mod
$\rightsquigarrow \mathrm{Rep}^0_{\mathcal{D}} V \simeq \mathcal{D}^0 / \Gamma$
full sub. cat of $\mathcal{D}$ w/ trivial monodromy w/ V
de-equiv.

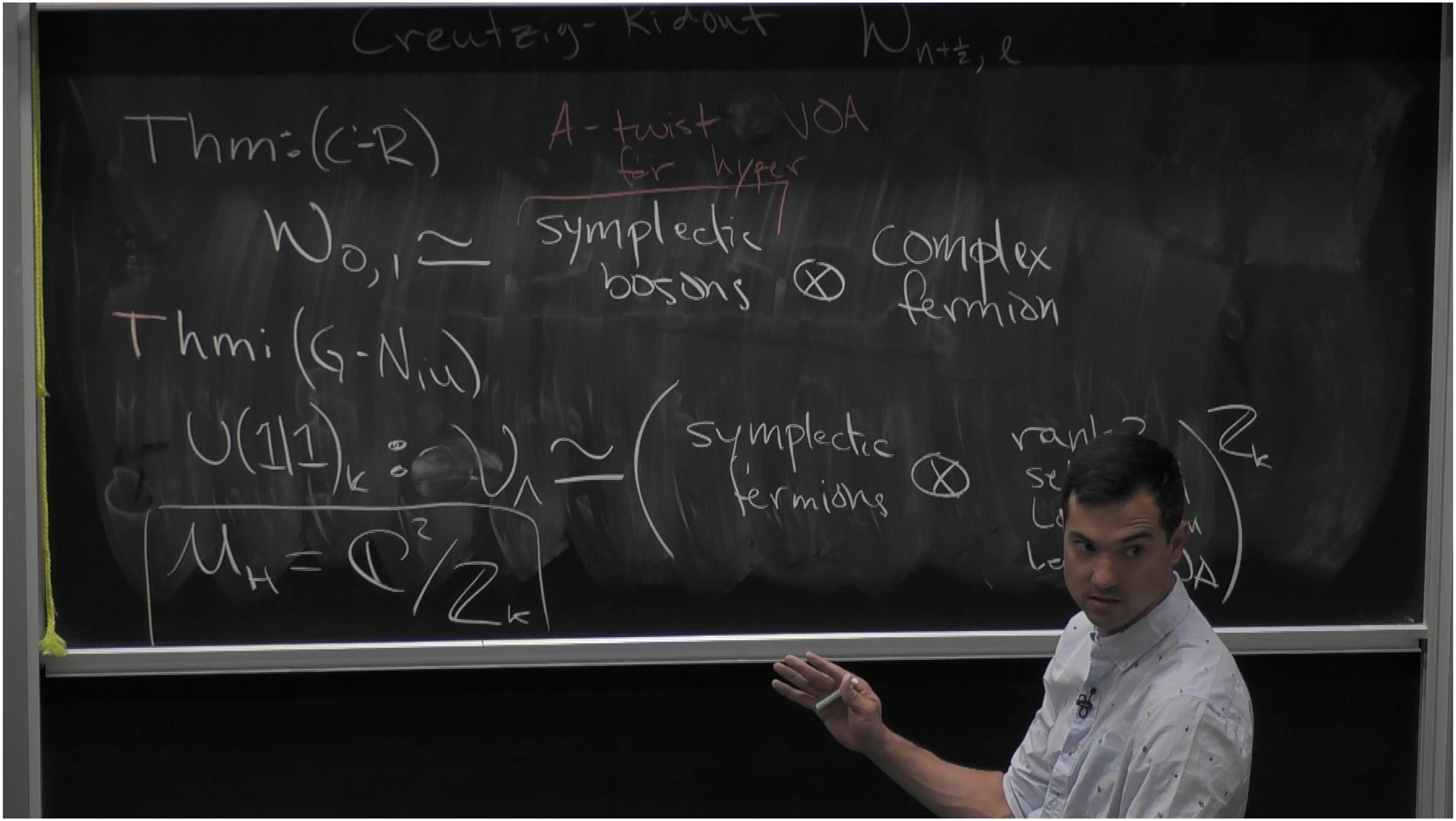
For V_1:

$\mathcal{D}$ = Kazhdan Lusztig category for $V(\mathfrak{gl}(1|1))$

$\mathcal{D}^0$ = modules where $V_1, \cancel{V_2}$ act s.s. w/ $\mathbb{Z}$ eigenvalues

Thm:

$$\mathcal{C}_1 \simeq KL^{V_1, \cancel{V_2}} / \Lambda$$


Creutzig-Ridout $W_{n+\frac{1}{2}, \ell}$
Thm: (C-R)
A-twist VOA
for hyper
$W_{0,1} \simeq$ symplectic bosons $\otimes$ complex fermion
Thm: (G-N, iu)
$U(1|1)_k : V_\Lambda \simeq$ (symplectic fermions $\otimes$ ... $)^{\mathbb{Z}_k}$
$\mathcal{M}_H = \mathbb{C}^2/\mathbb{Z}_k$