Title: Causal Inference Lecture - 230306

Speakers: Robert Spekkens

Collection: Causal Inference: Classical and Quantum

Date: March 06, 2023 - 10:00 AM

URL: https://pirsa.org/23030069

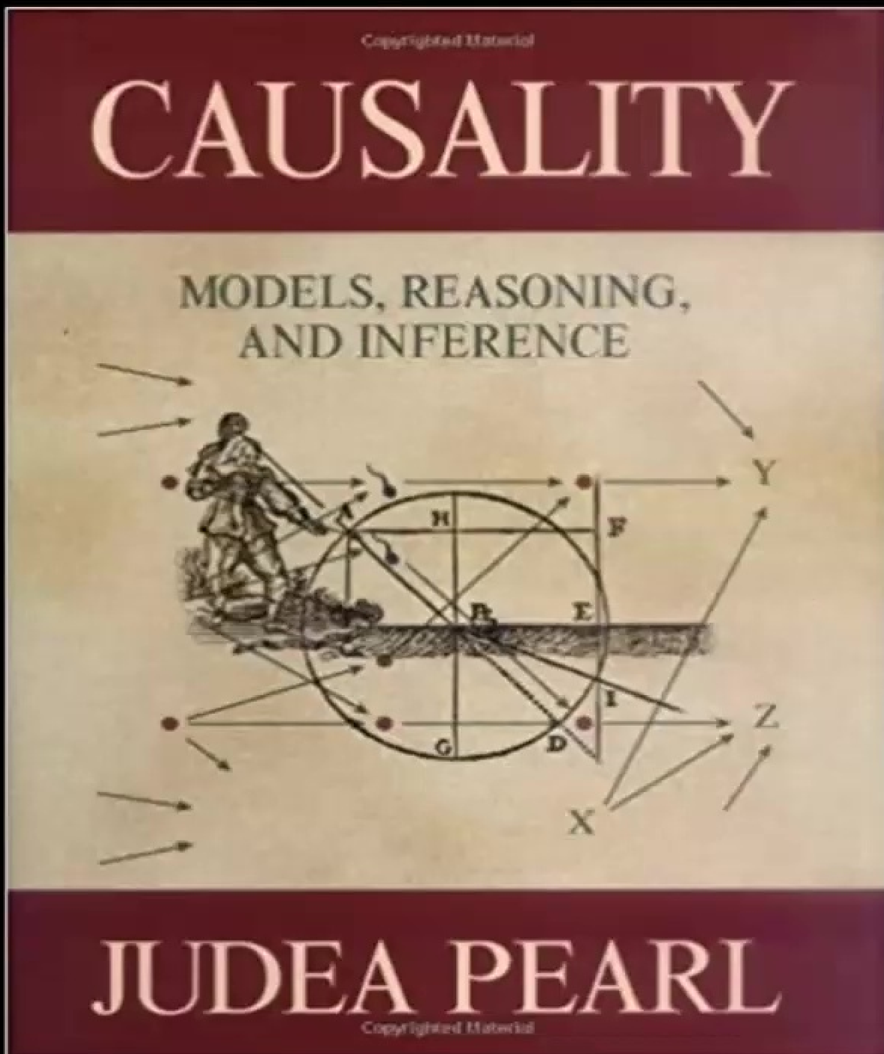
Abstract: zoom link: https://pitp.zoom.us/j/94143784665?pwd=VFJpajVIMEtvYmRabFYzYnNRSVAvZz09

Causal Inference: Classical and Quantum

PHYS 777-007
Lecturer: Robert Spekkens
TA: Marina Ansanelli

March 6, 2023

Causarum Investigatio
“Investigate the causes”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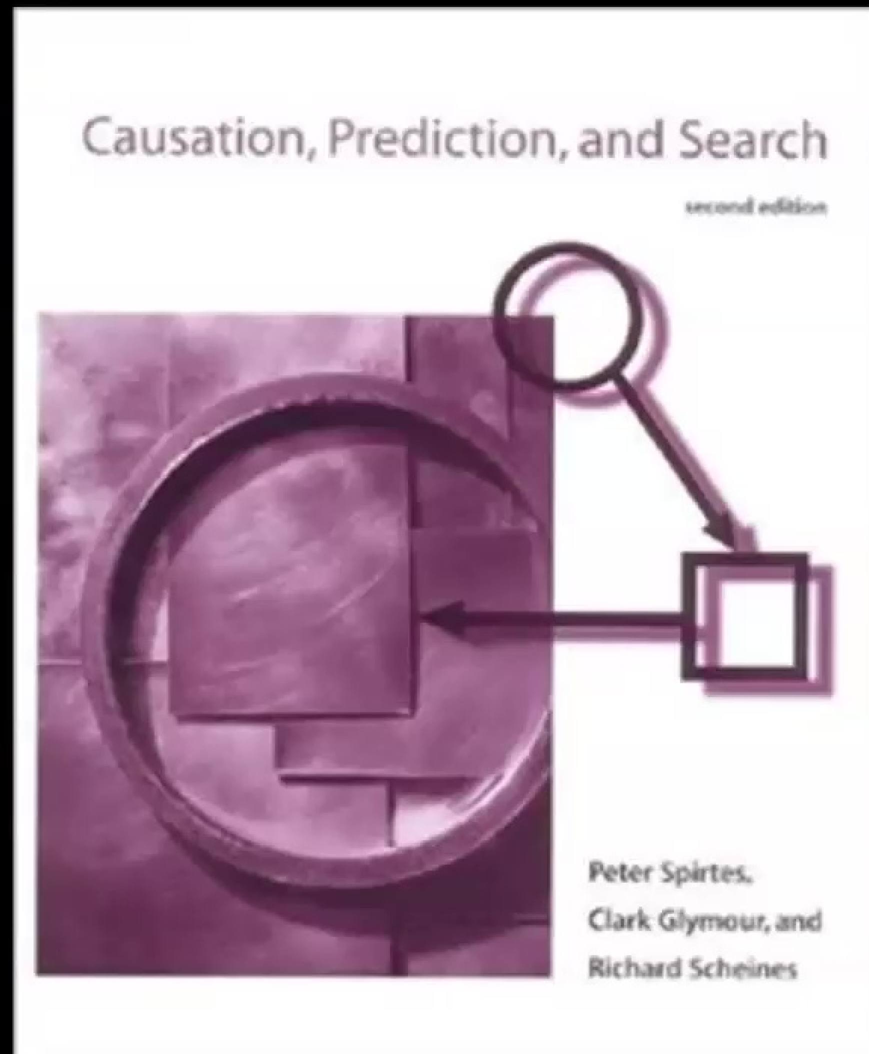
Causation, Prediction, and Search
second edition
Peter Spirtes,
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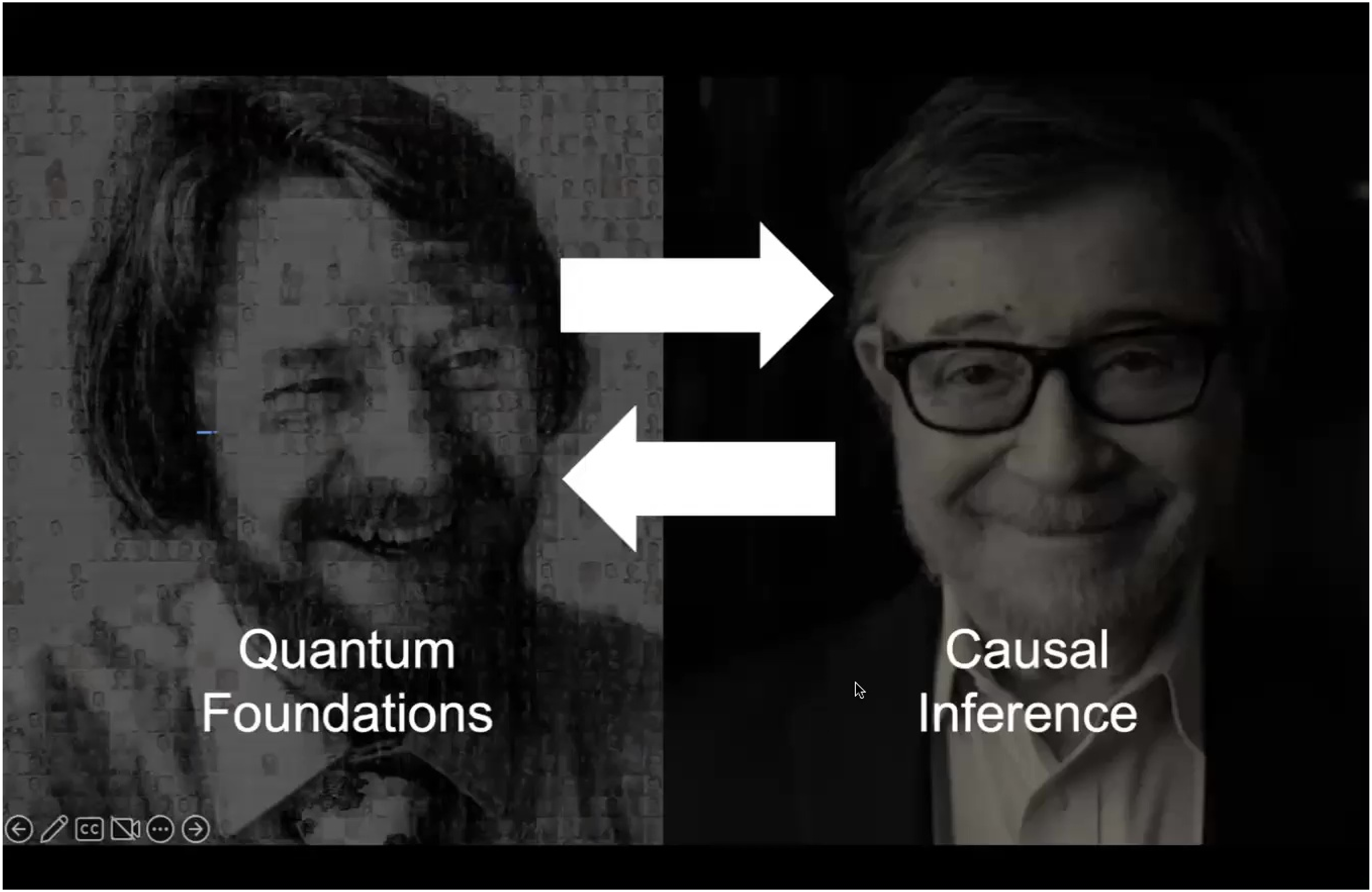
CAUSAL INFERENCE IN STATISTICS
A Primer
Judea Pearl
Madelyn Glymour
Nicholas P. Jewell
WILEY
Jonas Peters, Dominik Janzing, and Bernhard Schölkopf
Elements of Causal Inference
Foundations and Learning Algorithms
EXPLANATION IN CAUSAL INFERENCE
Methods for Mediation and Interaction
TYLER J. VANDERWEELE
OXFORD
CAUSAL INFERENCE FOR STATISTICS, SOCIAL, AND BIOMEDICAL SCIENCES
AN INTRODUCTION
GUIDO W. IMBENS
DONALD B. RUBIN
ICSA Book Series in Statistics
Hua He
Pan Wu
Ding-Geng (Din) Chen Editors
Statistical Causal Inferences and Their Applications in Public Health Research
Springer
ANALYTICAL METHODS FOR SOCIAL RESEARCH
Counterfactuals and Causal Inference
Methods and Principles for Social Research
SECOND EDITION
STEPHEN L. MORGAN
CHRISTOPHER WINSHIP

“statisticians are invariably motivated by causal questions but the peculiar nature of these questions is that they cannot be answered, or even articulated, in the traditional language of statistics.

[…]

causation is not merely an aspect of statistics; it is an addition to statistics, an enrichment that allows statistics to uncover workings of the world that traditional methods cannot”

From: J. Pearl, M. Glymour and N. Jewell, ‘Causal Inference in Statistics’


Quantum
Foundations
Causal
Inference

$$P(\text{recovery} \mid \text{drug}) > P(\text{recovery} \mid \text{no drug})$$

$$P(\text{recovery} \mid \text{drug, male}) < P(\text{recovery} \mid \text{no drug, male})$$

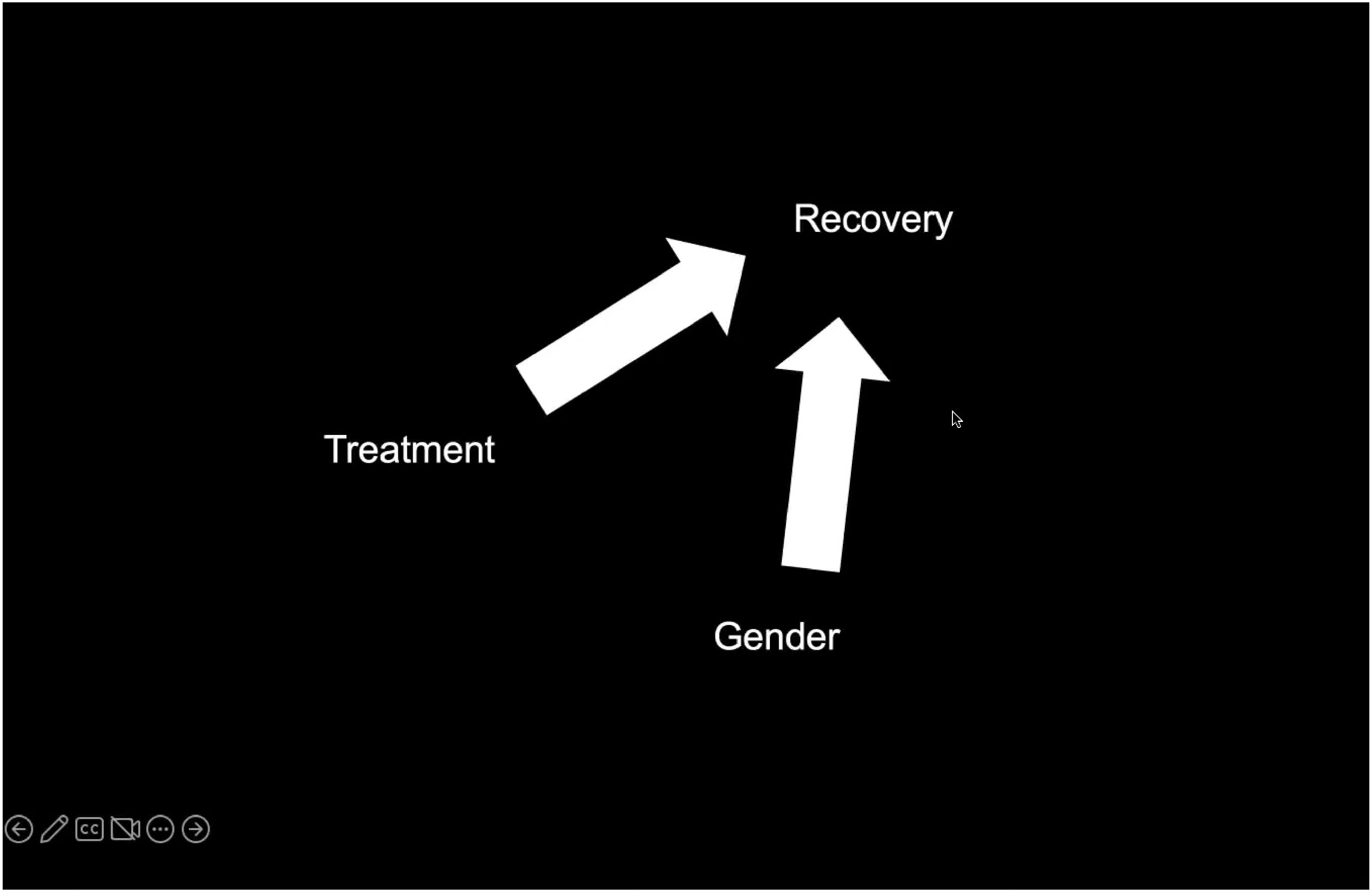
$$P(\text{recovery} \mid \text{drug}) > P(\text{recovery} \mid \text{no drug})$$

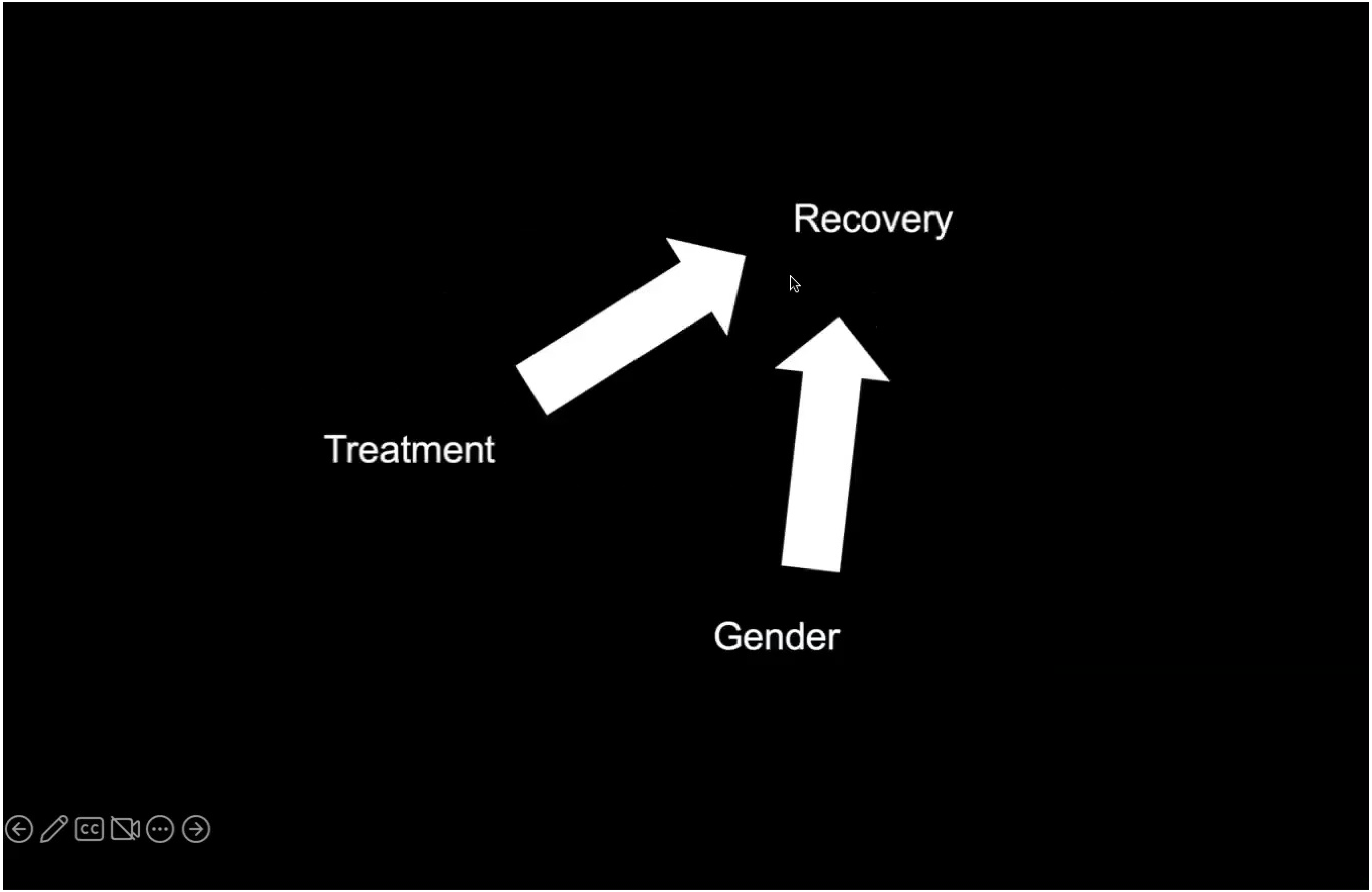
$$P(\text{recovery} \mid \text{drug, male}) < P(\text{recovery} \mid \text{no drug, male})$$

$$P(\text{recovery} \mid \text{drug, female}) < P(\text{recovery} \mid \text{no drug, female})$$

Recovery probability

	drug	no drug
male	180/300 = 60%	70/100 = 70%
female	20/100 = 20%	90/300 = 30%
combined	200/400 = 50%	160/400 = 40%

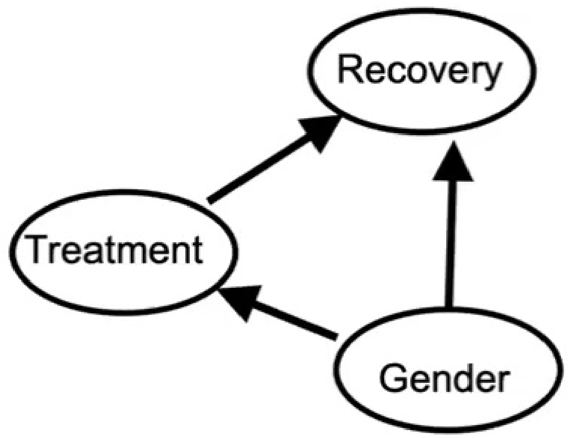

Recovery
Treatment
Gender


Recovery
Treatment
Gender

P(recovery | drug) > P(recovery | no drug)

P(recovery | drug, male) < P(recovery | no drug, male) ✓

P(recovery | drug, female) < P(recovery | no drug, female) ✓

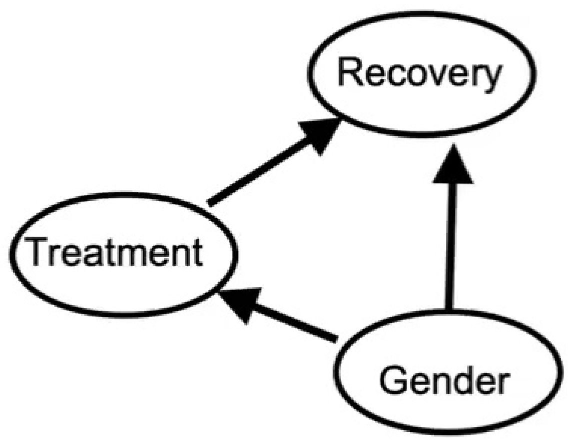


Therefore: stratify the data by the common cause

$$P(\text{recovery} \mid \text{drug}) > P(\text{recovery} \mid \text{no drug})$$

$$P(\text{recovery} \mid \text{drug, male}) < P(\text{recovery} \mid \text{no drug, male}) \checkmark$$

$$P(\text{recovery} \mid \text{drug, female}) < P(\text{recovery} \mid \text{no drug, female}) \checkmark$$



Therefore: stratify the data by the common cause

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$$P(\text{recovery} \mid \text{drug, male}) < P(\text{recovery} \mid \text{no drug, male})$$

$$P(\text{recovery} \mid \text{drug, female}) < P(\text{recovery} \mid \text{no drug, female})$$

Recovery probability

	drug	no drug
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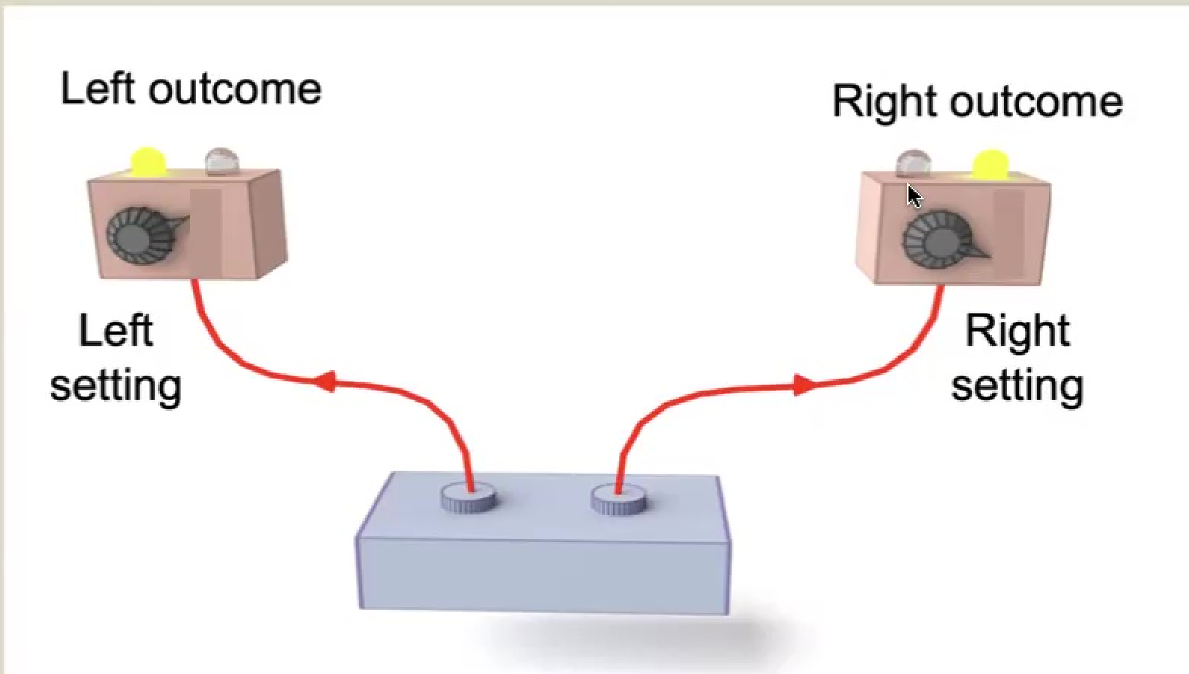
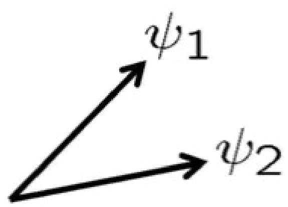

Left outcome
Right outcome
Left setting
Right setting

Left outcome and Right outcome

Left setting and Right setting

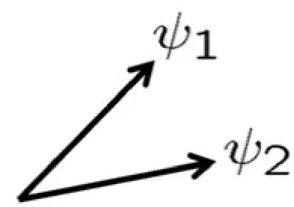
	0 and 0	0 and 1	1 and 0	1 and 1
0 and 0	50%	0%	0%	50%
0 and 1	25%	25%	25%	25%
1 and 0	25%	25%	25%	25%
1 and 1	50%	0%	0%	50%

ψ is ontic vs. ψ is epistemic



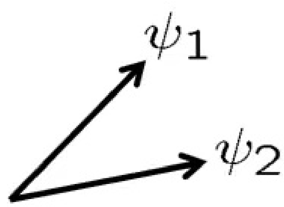
= two different ways the world could be

vs.



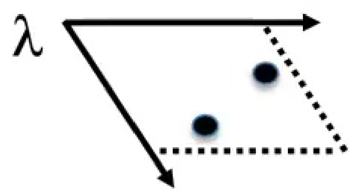
= two different ways of having incomplete information about the world

ψ is ontic



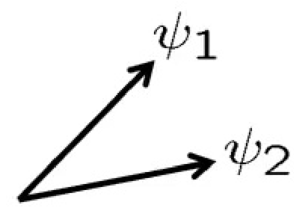
= two different ways the world could be

Example:



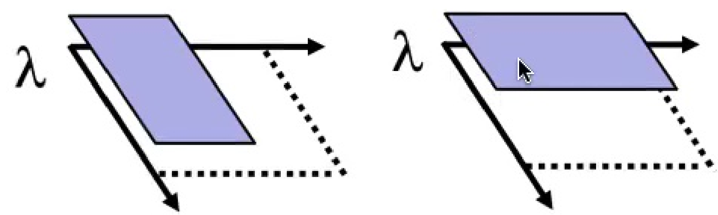
vs.

ψ is epistemic



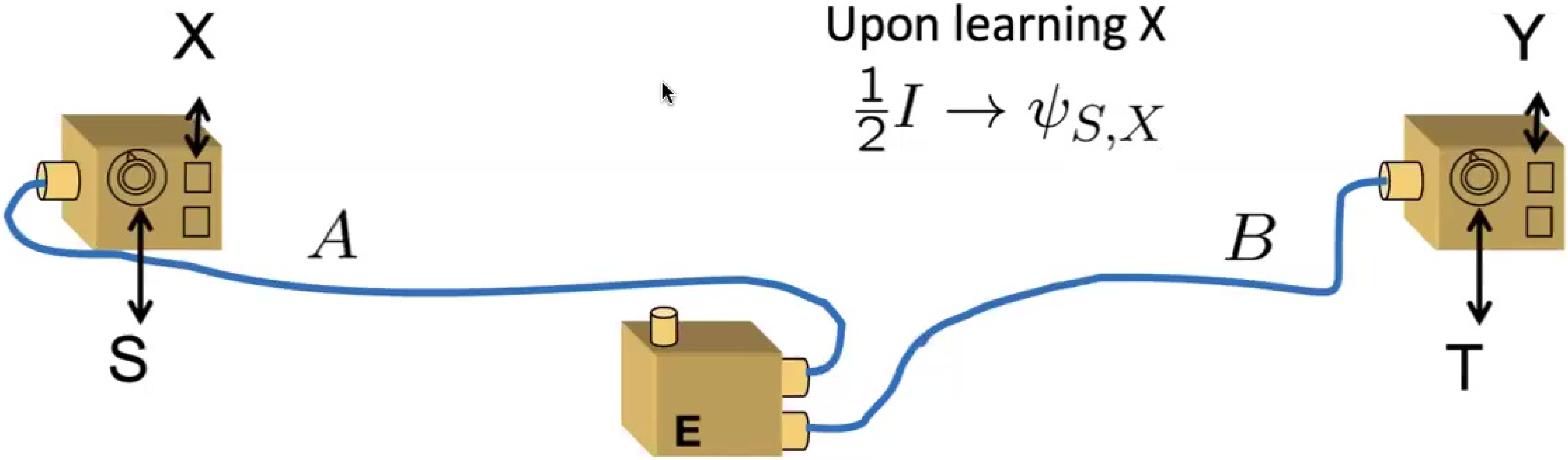
= two different ways of having incomplete information about the world

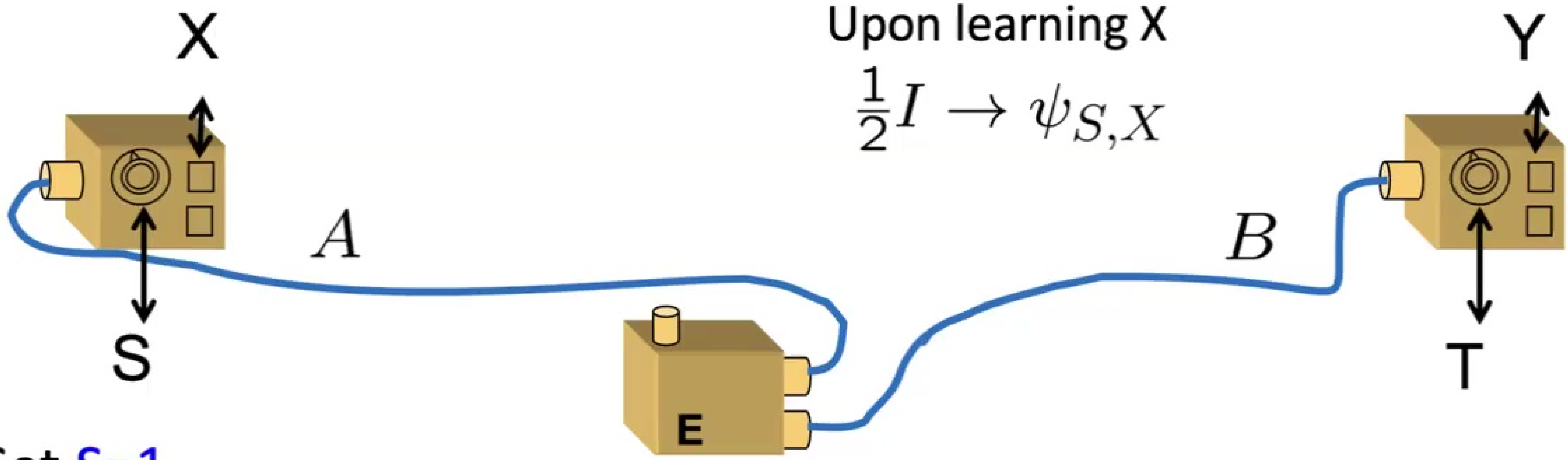
Example:



Upon learning X

$$\frac{1}{2}I \to \psi_{S,X}$$





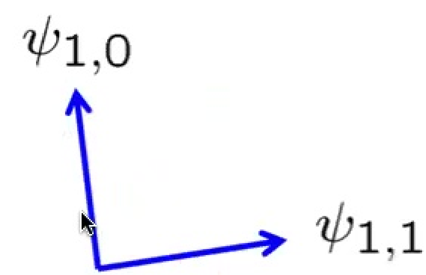
Upon learning X

$$\frac{1}{2}I \to \psi_{S,X}$$

Set S=1

get X=0 $\quad \frac{1}{2}I \to \psi_{1,0}$

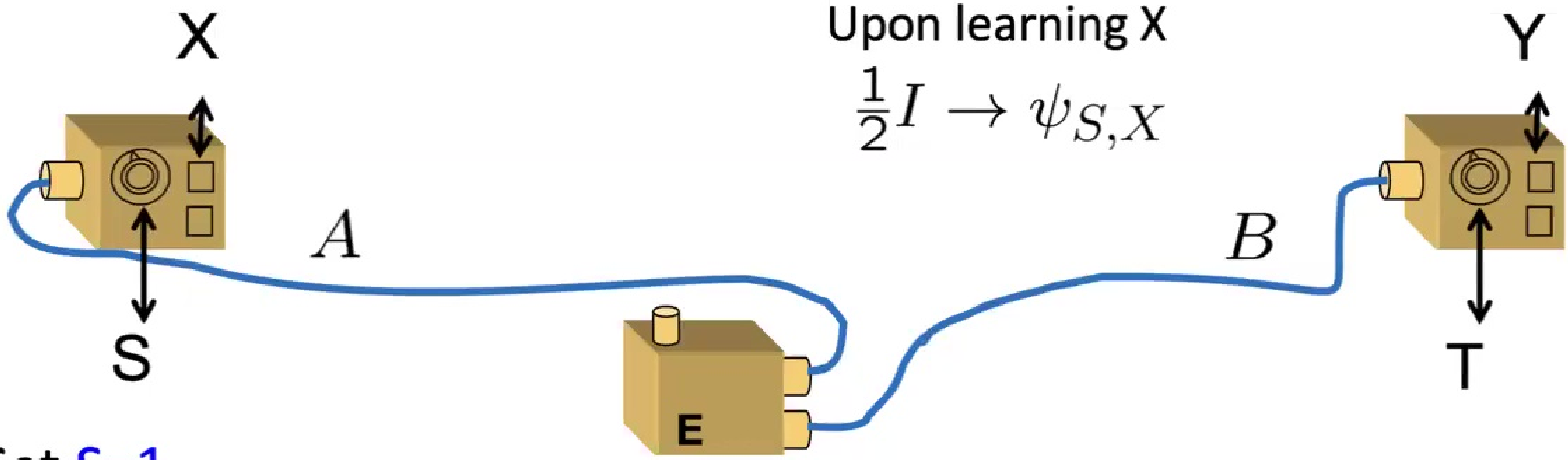
get X=1 $\quad \to \psi_{1,1}$



Pauli X

Upon learning X

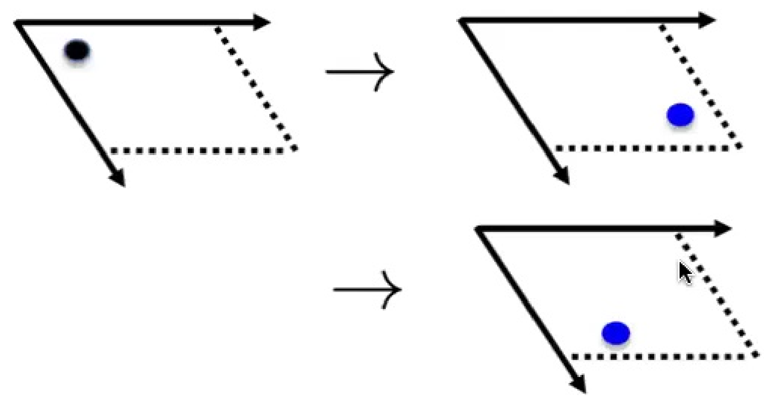
$$\frac{1}{2}I \to \psi_{S,X}$$



Set S=1

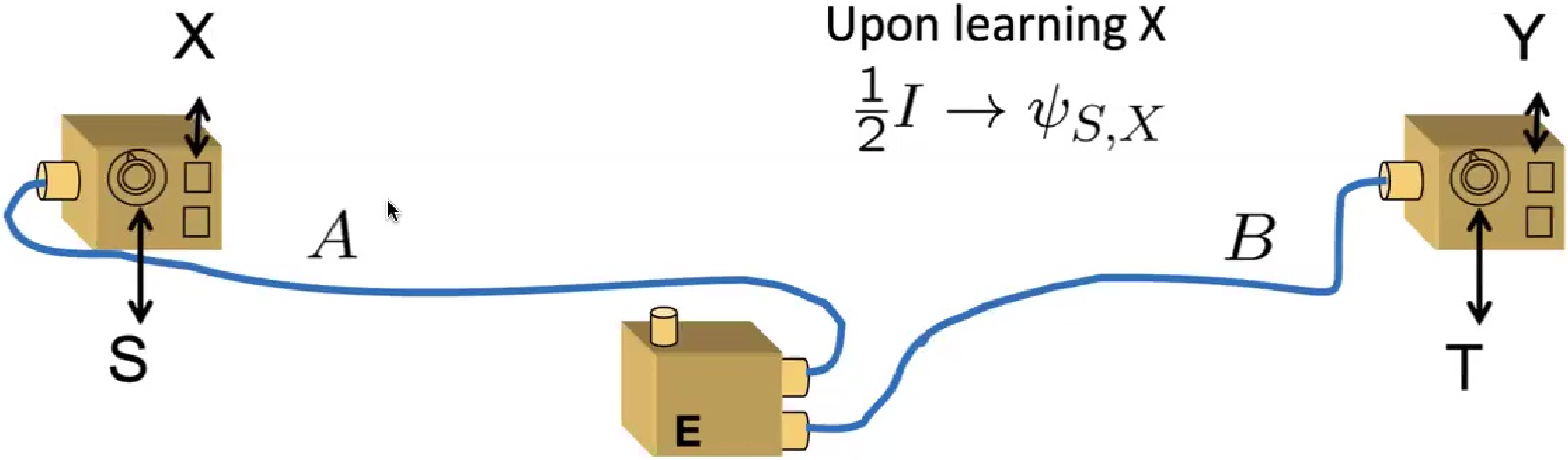
get X=0

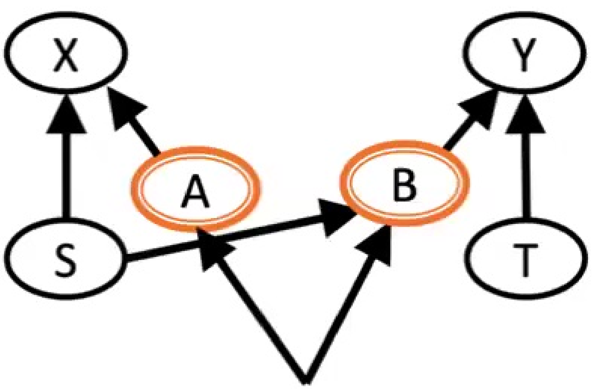
get X=1



Upon learning X

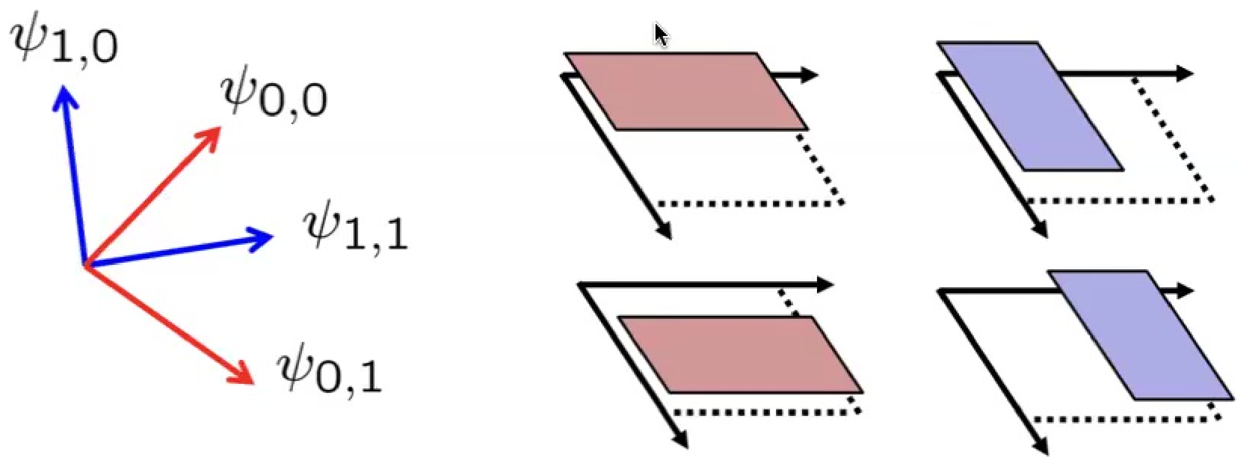
$$\frac{1}{2}I \to \psi_{S,X}$$





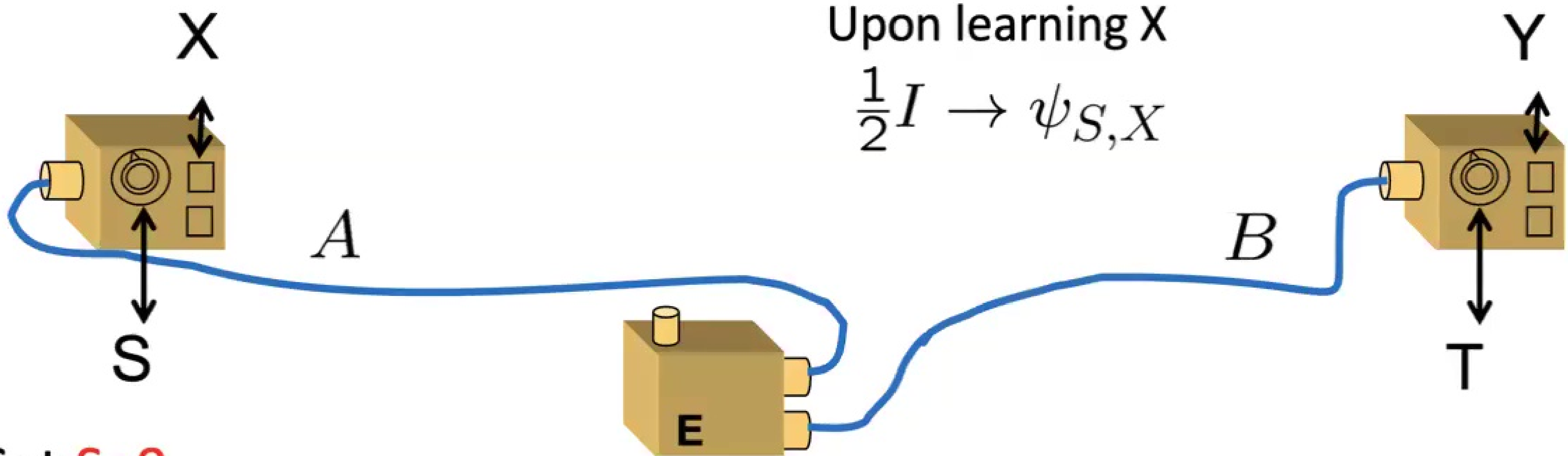
"Spooky action at a distance"

ψ is epistemic



Upon learning X

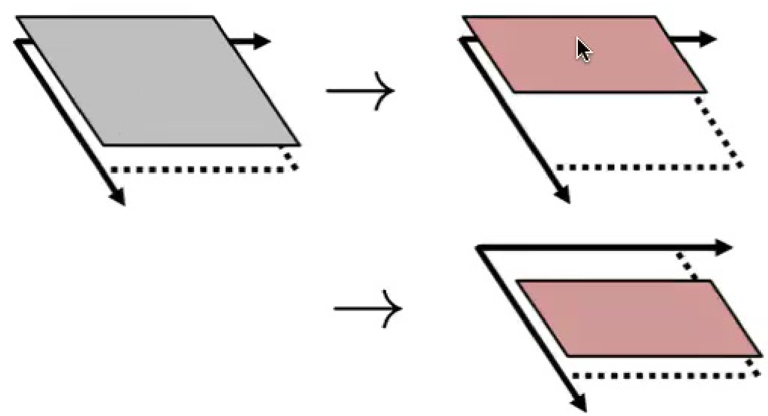
$$\frac{1}{2}I \to \psi_{S,X}$$



Set S=0

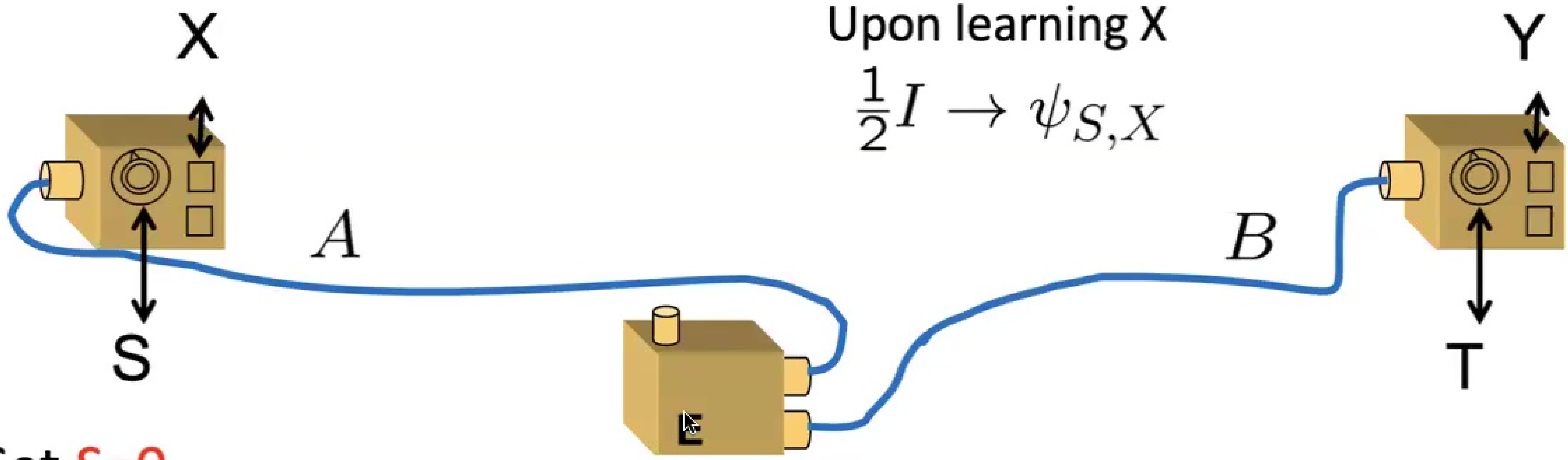
get X=0 $\to$

get X=1 $\to$



Upon learning X

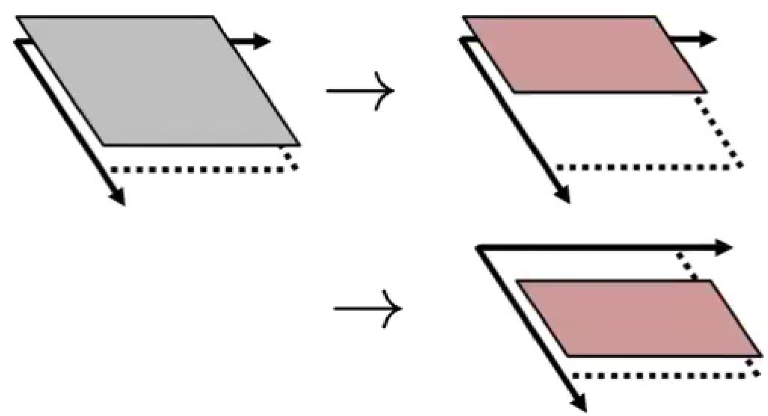
$$\frac{1}{2}I \rightarrow \psi_{S,X}$$

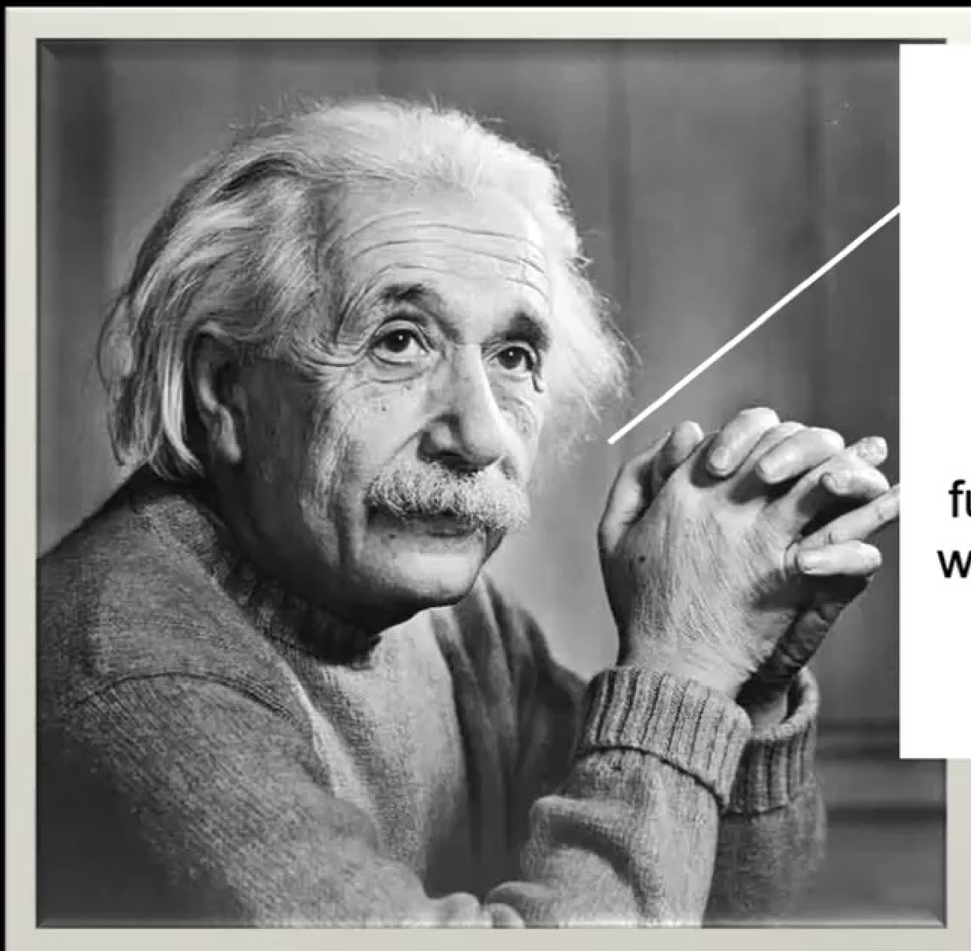


Set S=0

get X=0 $\rightarrow$

get X=1 $\rightarrow$





"ψ_2 does not describe the totality of what "really" pertains to the partial system 2, rather only what we know about it in this particular case."

"I incline to the opinion that the wave function does not (completely) describe what is real, but only a to-us-empirically-accessible maximal knowledge regarding that which really exists."

Birth weight paradox

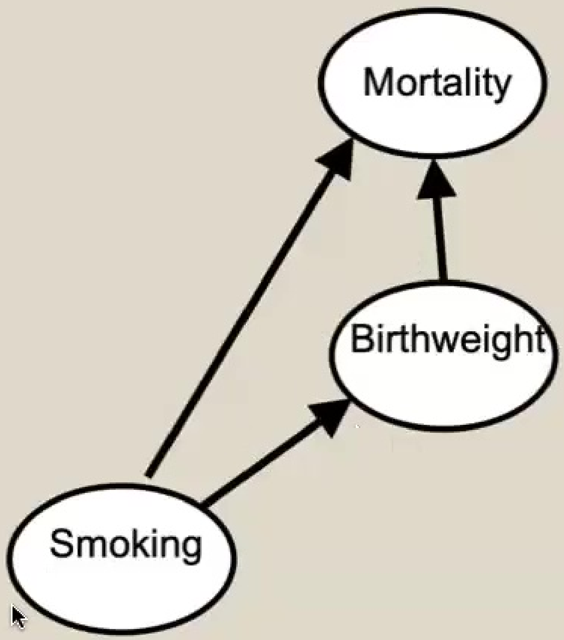
P(mortality | born to smoker) > P(mortality | born to nonsmoker)

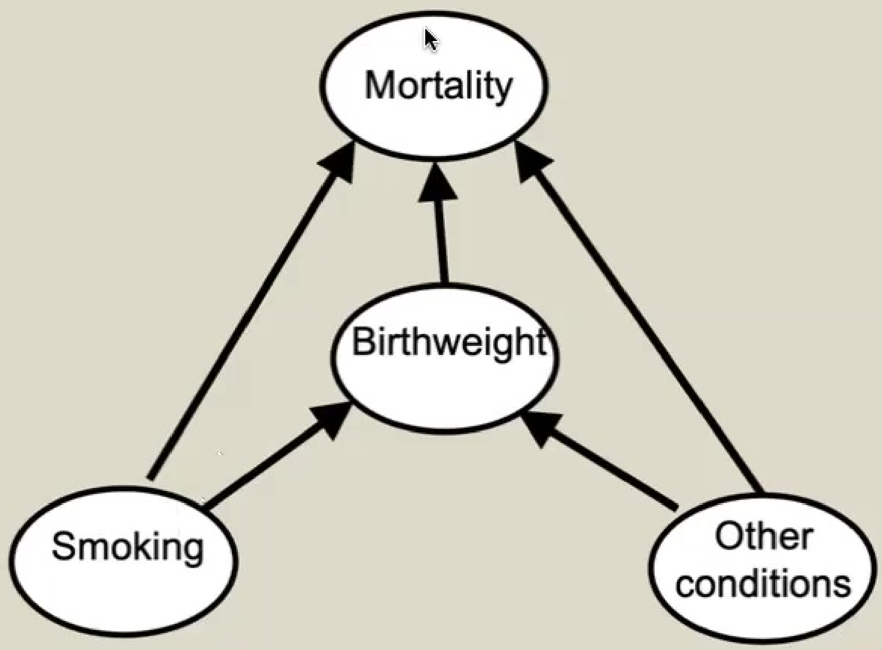
Birth weight paradox

P(mortality | born to smoker) > P(mortality | born to nonsmoker)

P(mortality | born to smoker, LBW) < P(mortality | born to nonsmoker, LBW)

Can a mother being a smoker really reduce the risk of mortality for low birth weight children?

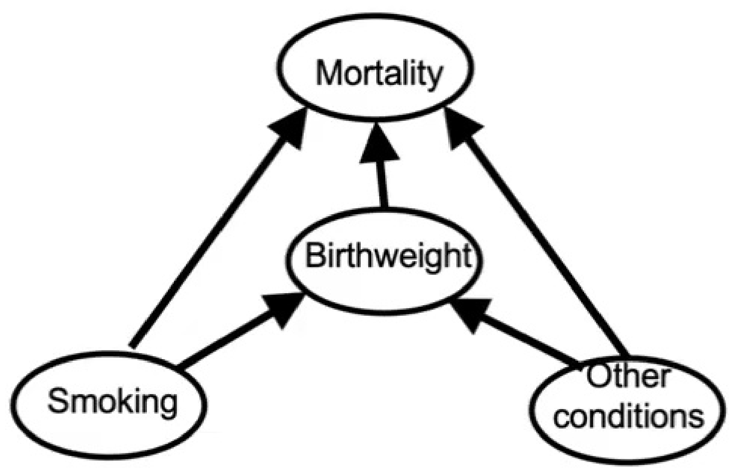

Mortality
Birthweight
Smoking


Mortality
Birthweight
Smoking
Other
conditions

Birth weight paradox

P(mortality | born to smoker) > P(mortality | born to nonsmoker) ✓

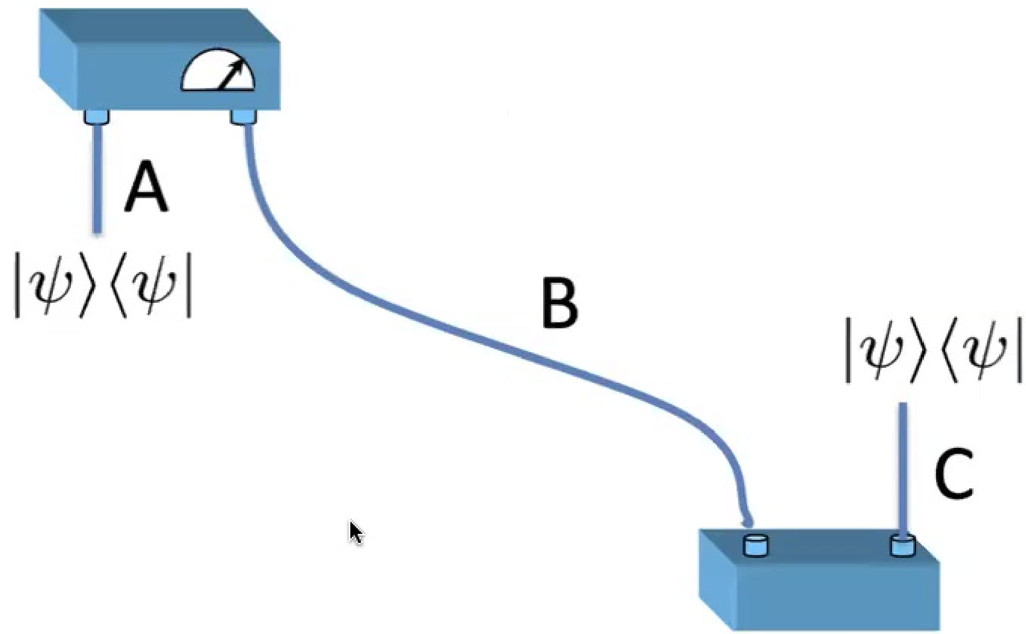
P(mortality | born to smoker, LBW) < P(mortality | born to nonsmoker, LBW)



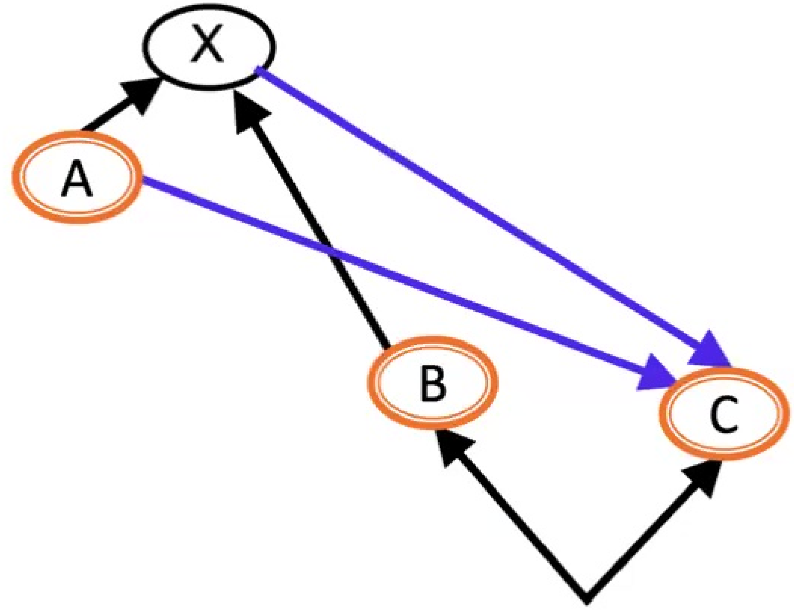
Therefore: *marginalize* over colliders on the "backdoor path"

The paradox of Quantum Teleportation

Post-select on outcome X

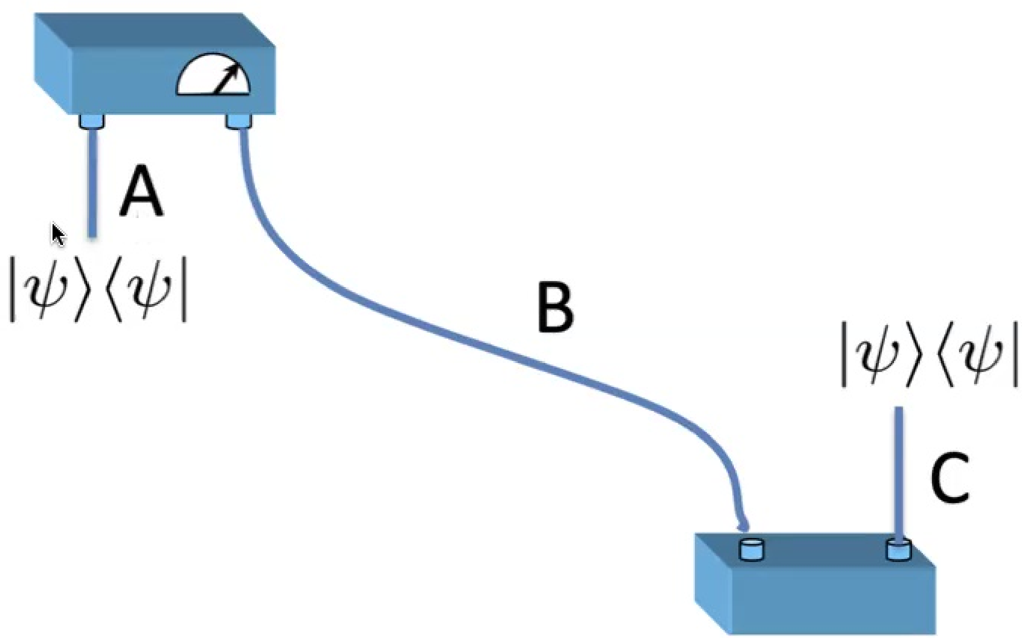


ψ is ontic

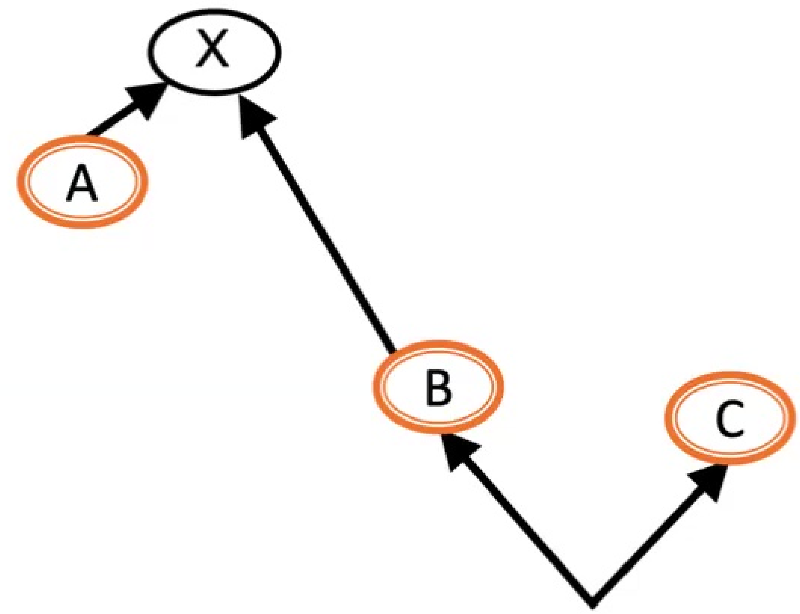


Backward-in-time causation!

Post-select on outcome X



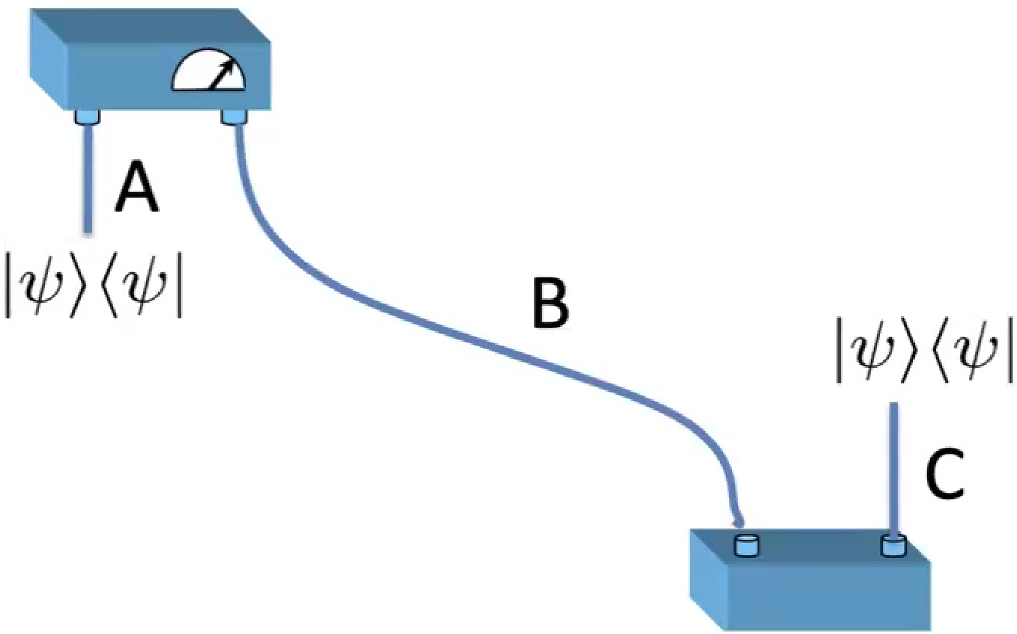
ψ is epistemic



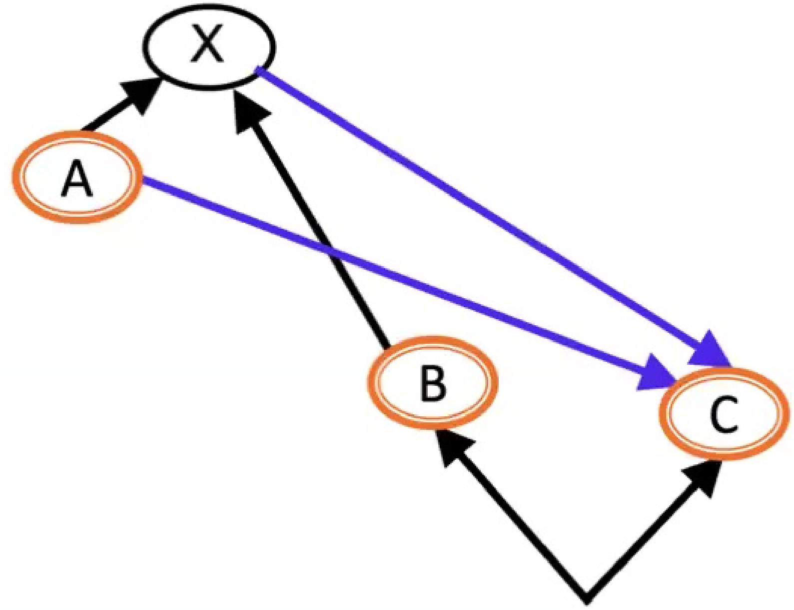
Given post-selection, your posterior about C tracks your prior about A

What hope do we have of making sense of quantum theory if we do not understand how to resolve Simpson’s or Berkson’s paradox?

Post-select on outcome X



ψ is ontic



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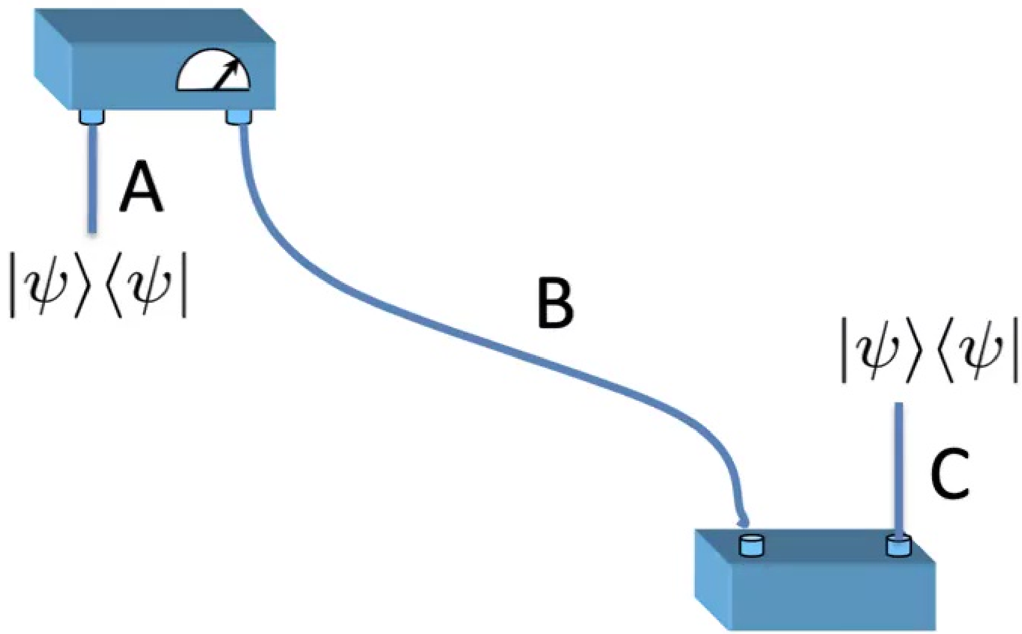
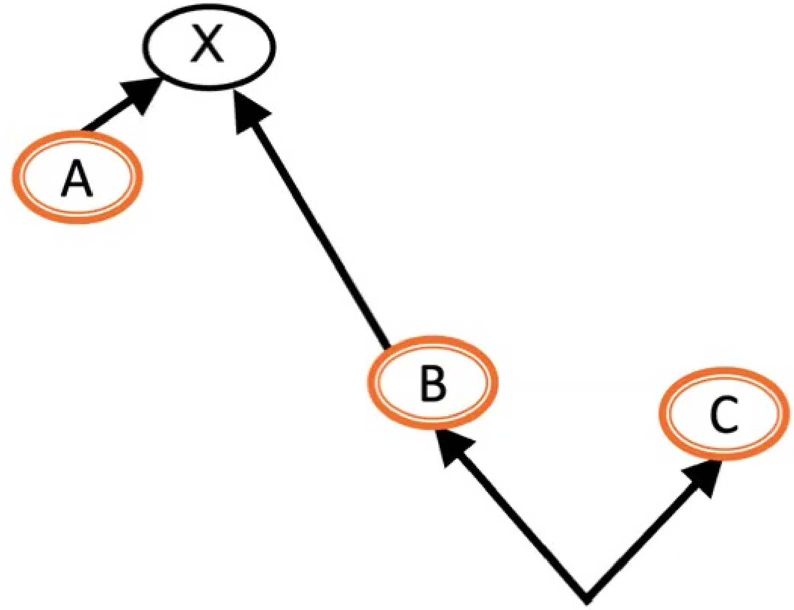
Post-select on outcome X



ψ is epistemic



Given post-selection, your posterior about C tracks your prior about A

"[...] our present Quantum Mechanical formalism [...] is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble."

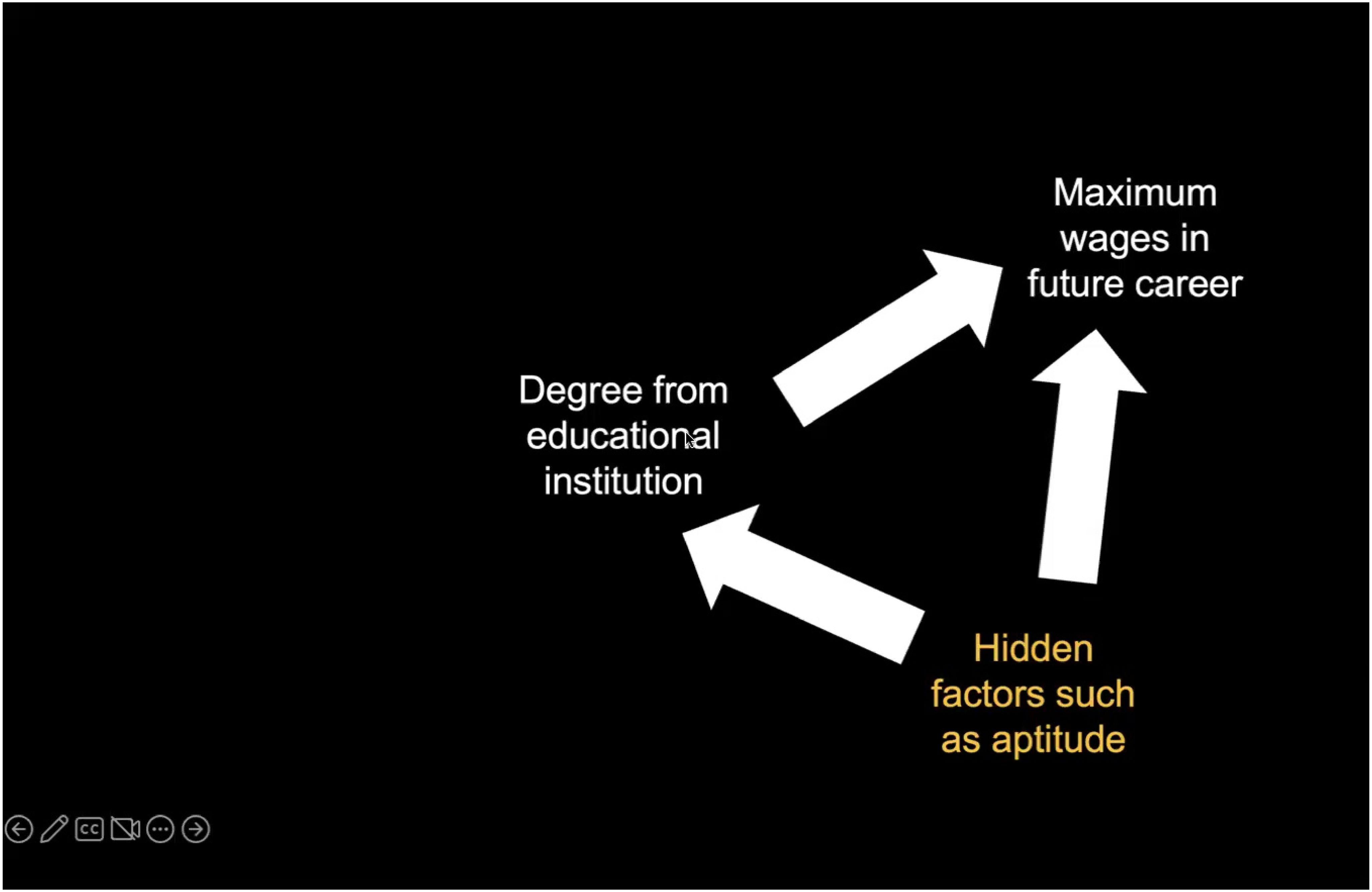
E.T. Jaynes, 1989

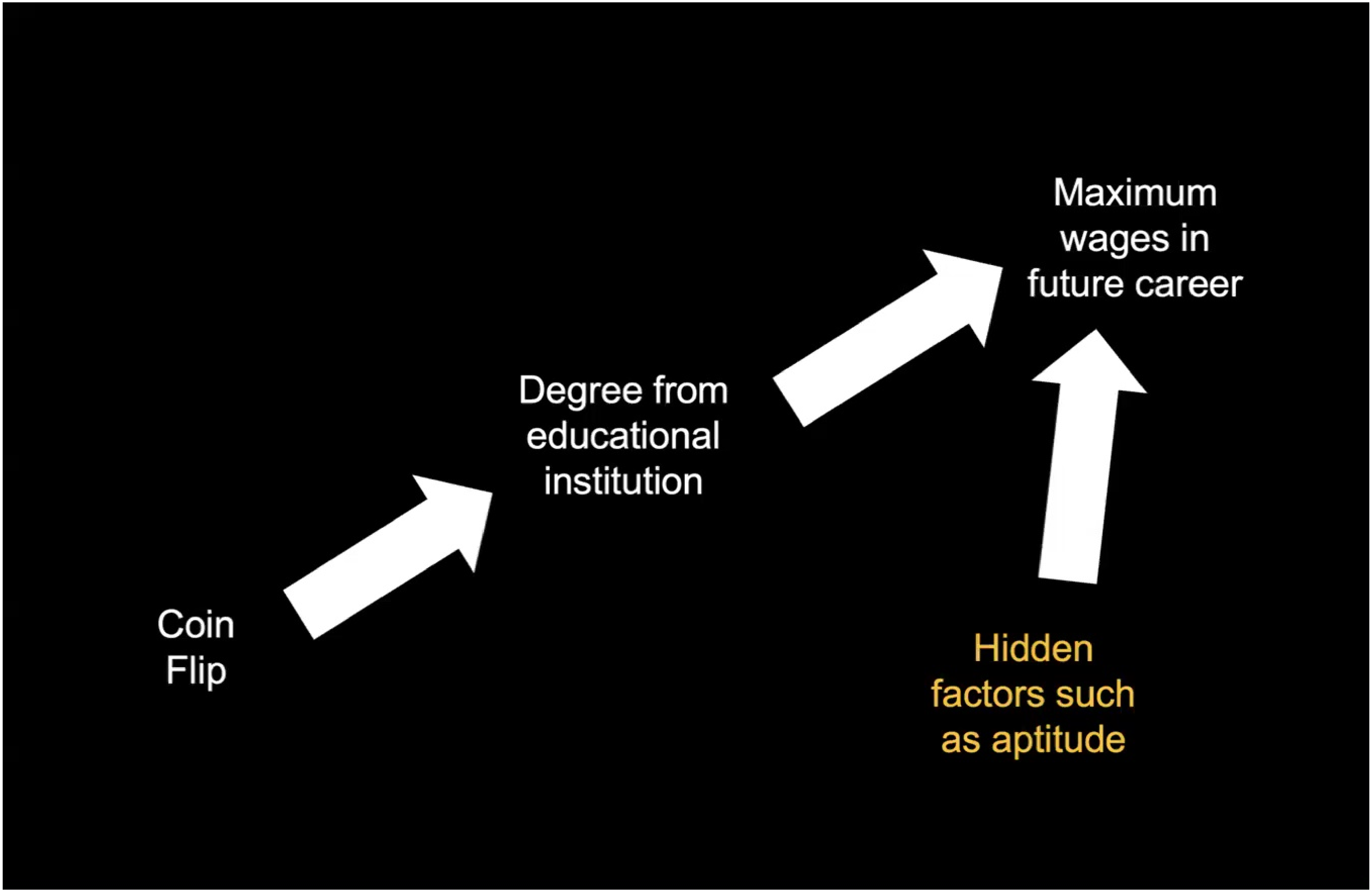
Causal Inference in the presence of hidden variables

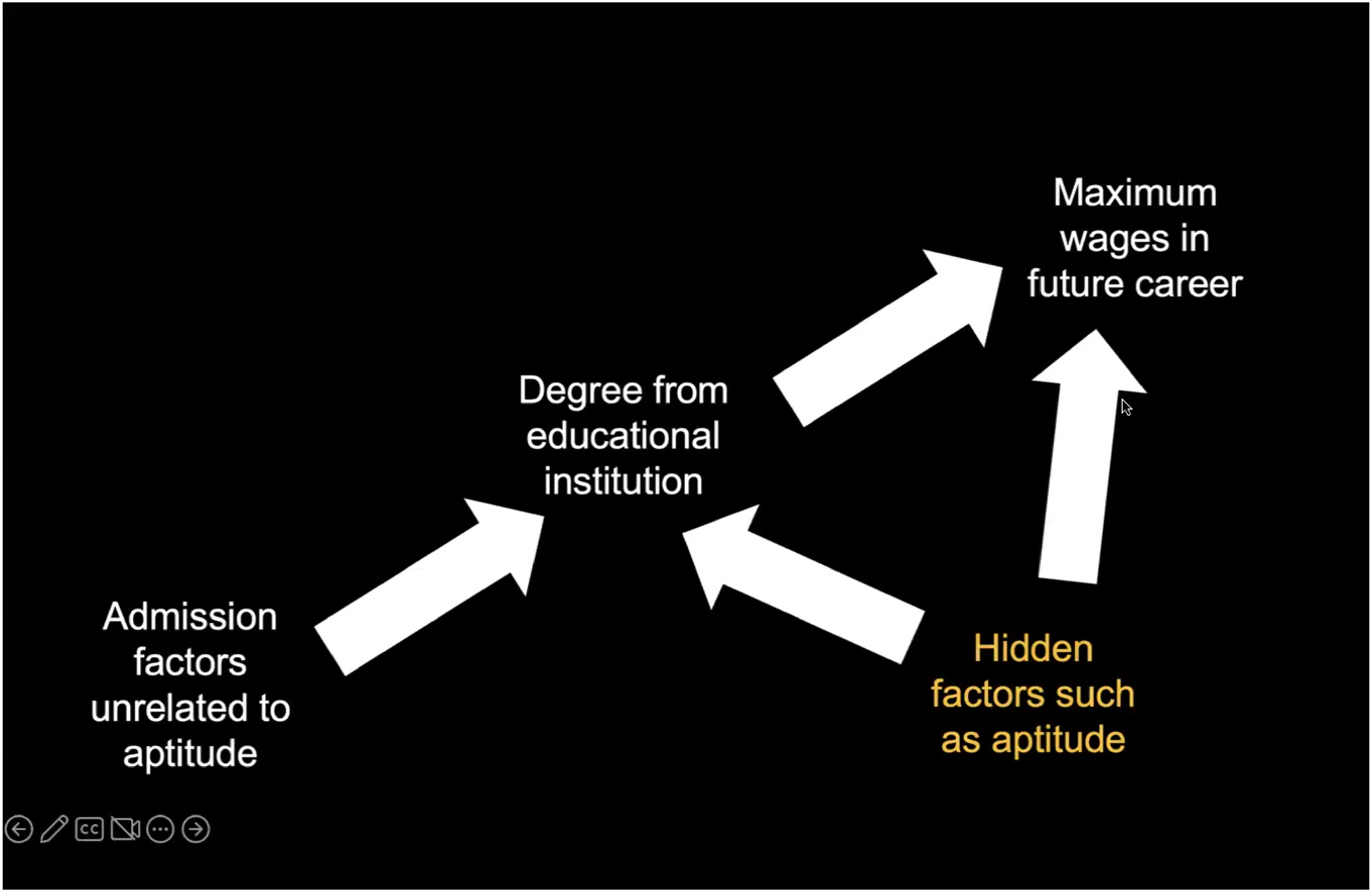
Maximum wages in future career above some threshold?

Degree from educational institution?

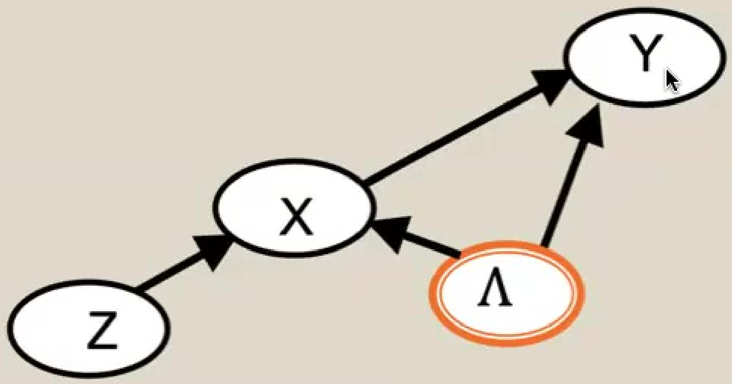
	Yes	No
Yes	79%	21%
No	43%	57%


Maximum wages in future career
Degree from educational institution
Hidden factors such as aptitude


Maximum
wages in
future career
Degree from
educational
institution
Coin
Flip
Hidden
factors such
as aptitude


Maximum wages in future career
Degree from educational institution
Admission factors unrelated to aptitude
Hidden factors such as aptitude

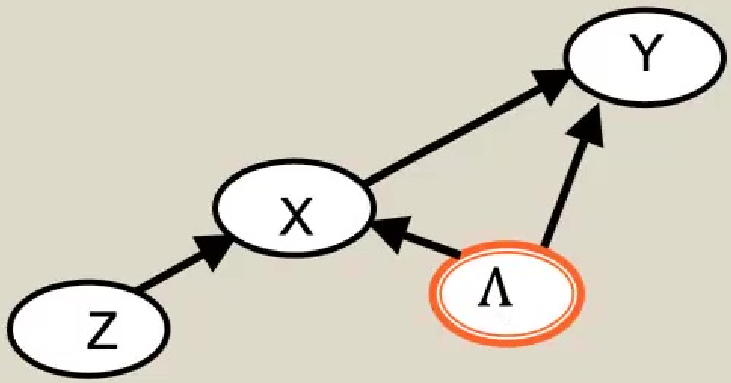
Causal structure



Parameters

$$P_{X|\Lambda Z}$$
$$P_{Y|\Lambda X}$$
$$P_{\Lambda}$$
$$P_Z$$

Causal structure



Parameters

$P_{X|\Lambda Z}$

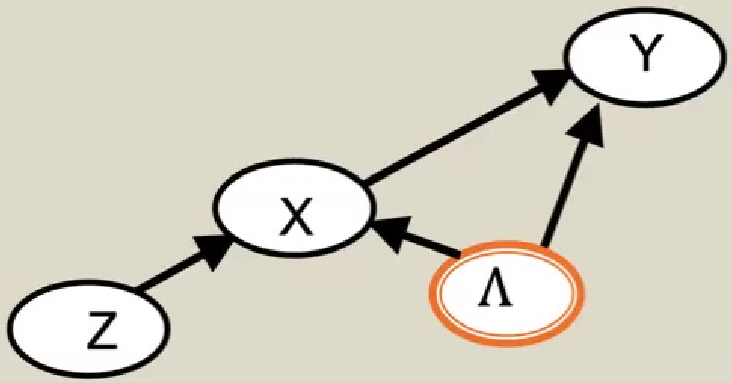
$P_{Y|\Lambda X}$

P_{Λ}

P_{Z}

$$P_{XYZ} = \sum_{\Lambda} P_{Y|X\Lambda} P_{X|Z\Lambda} P_{\Lambda} P_{Z}$$

Causal structure



Parameters

$P_{X|\Lambda Z}$

$P_{Y|\Lambda X}$

P_{Λ}

P_Z

$$P_{XYZ} = \sum_{\Lambda} P_{Y|X\Lambda} P_{X|Z\Lambda} P_{\Lambda} P_Z$$

Example of causal compatibility constraint:

$$\boxed{P_{XY|Z}(00|0) + P_{XY|Z}(01|1) \leq 1}$$

Pearl, 1993

The evidence

Z=0

	Y=0	Y=1
X=0	0.79	0.21
X=1	0.43	0.57

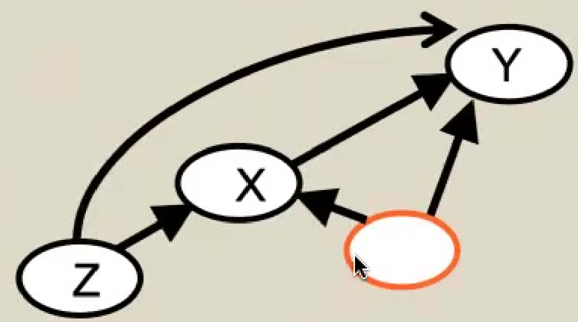
Z=1

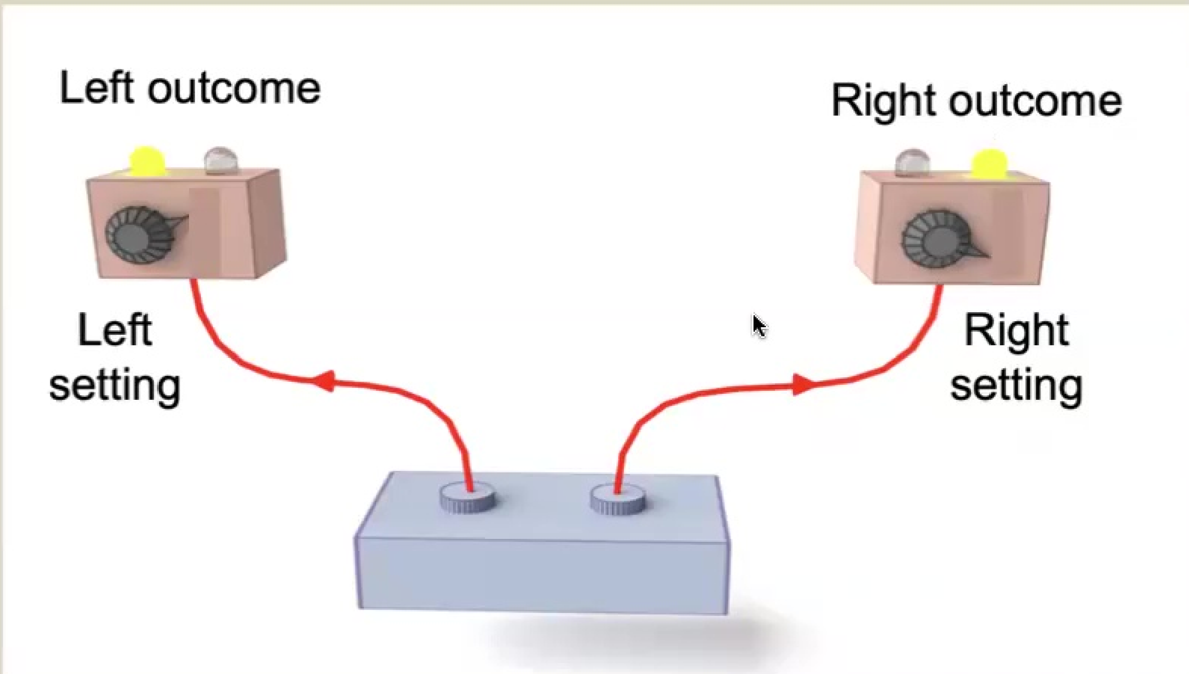
	Y=0	Y=1
X=0	0.59	0.41
X=1	0.39	0.61

Violates the instrumental inequality!

The hypotheses

Implies a constraint:
the instrumental inequality




Left outcome
Right outcome
Left setting
Right setting

Left outcome and
Right outcome

Left setting and Right setting

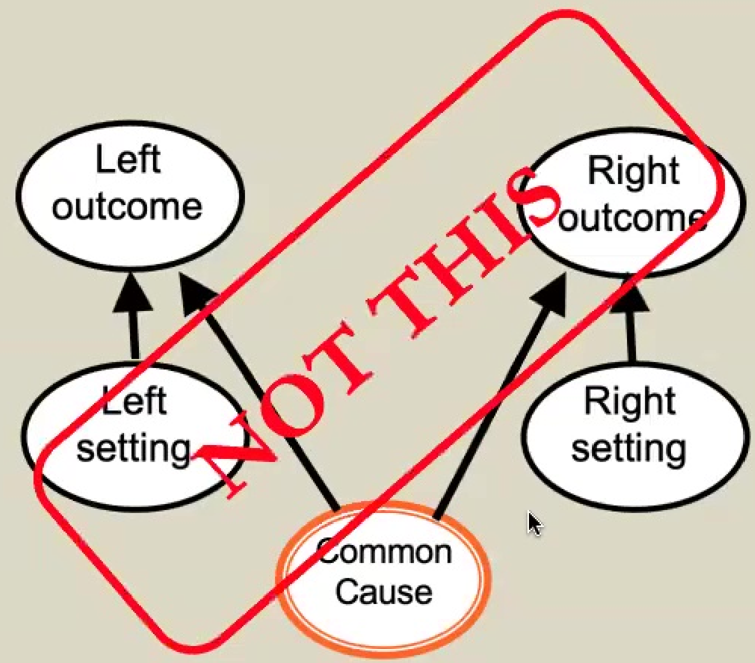
	0 and 0	0 and 1	1 and 0	1 and 1
0 and 0	43%	7%	7%	43%
0 and 1	43%	7%	7%	43%
1 and 0	43%	7%	7%	43%
1 and 1	7%	43%	43%	7%

The evidence

Left setting and Right setting	Left outcome and Right outcome			
	0 and 0	0 and 1	1 and 0	1 and 1
0 and 0	43%	7%	7%	43%
0 and 1	43%	7%	7%	43%
1 and 0	43%	7%	7%	43%
1 and 1	7%	43%	43%	7%

Violates Bell inequalities
(up to Tsirelson bound)

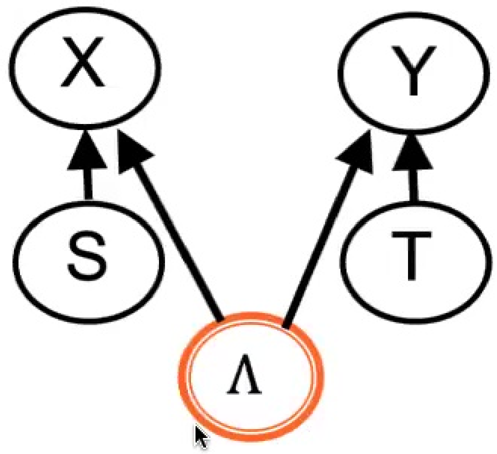
The natural hypothesis



Implies Bell inequalities

Incompatible

Causal structure



Parameters

$P_{X|S\Lambda}$

$P_{Y|T\Lambda}$

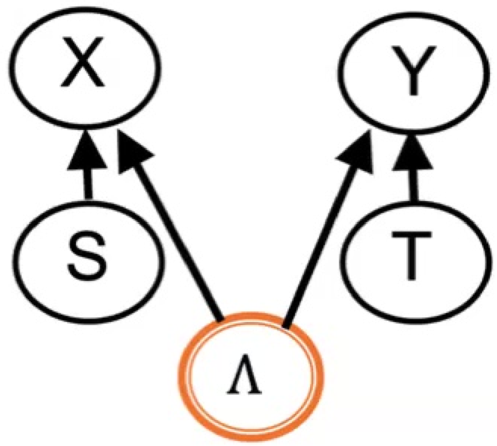
P_{Λ}

P_S

P_T

$$P_{XYST} = \sum_{\Lambda} P_{Y|T\Lambda} P_{X|S\Lambda} P_{\Lambda} P_S P_T$$

Causal structure



Parameters

$P_{X|S\Lambda}$

$P_{Y|T\Lambda}$

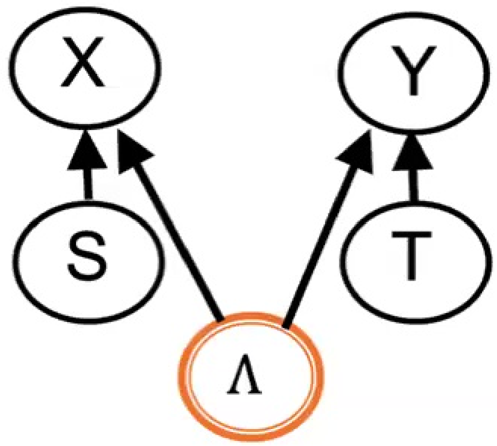
P_{Λ}

$$P_{XY|ST} = \sum_{\Lambda} P_{Y|T\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Examples of causal compatiblity constraints:

$$P_{X|ST} = P_{X|S}$$
$$P_{Y|ST} = P_{Y|T}$$

Causal structure | Parameters



$P_{X|S\Lambda}$
$P_{Y|T\Lambda}$
P_{Λ}

$$P_{XY|ST} = \sum_{\Lambda} P_{Y|T\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Examples of causal compatiblity constraints:

$$P_{X|ST} = P_{X|S}$$
$$P_{Y|ST} = P_{Y|T}$$

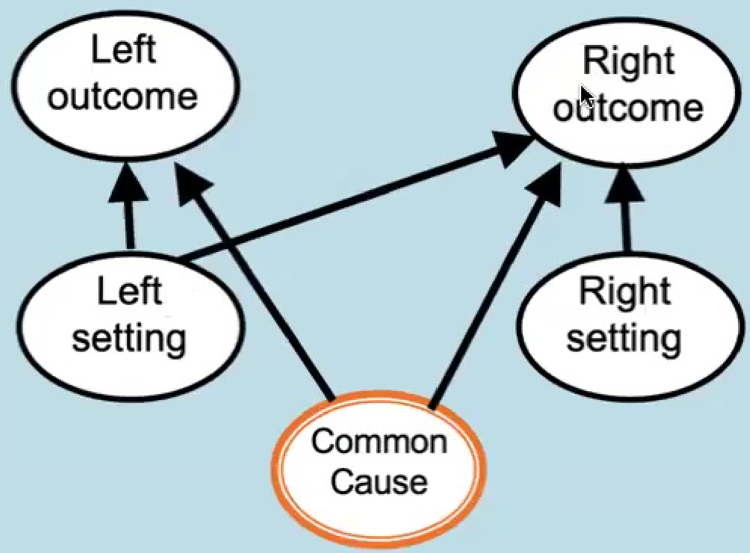
$$\boxed{\begin{aligned}&\tfrac{1}{4}\textstyle\sum_{x=y} P_{XY|ST}(xy|00) + \tfrac{1}{4}\sum_{x=y} P_{XY|ST}(xy|01)\\ &\quad \tfrac{1}{4}\textstyle\sum_{x=y} P_{XY|ST}(xy|10) + \tfrac{1}{4}\sum_{x\neq y} P_{XY|ST}(xy|11) \leq \tfrac{3}{4}\end{aligned}}$$

Clauser, Horne, Shimony and Holte, Phys. Rev. Lett.23, 880 (1967)

The evidence

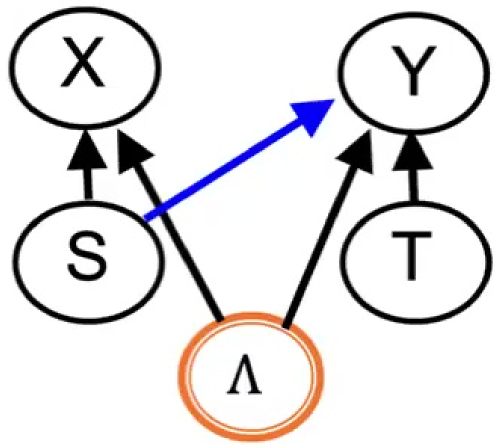
	Left outcome and Right outcome			
Left setting and Right setting	0 and 0	0 and 1	1 and 0	1 and 1
0 and 0	43%	7%	7%	43%
0 and 1	43%	7%	7%	43%
1 and 0	43%	7%	7%	43%
1 and 1	7%	43%	43%	7%

The 2nd possibility



Compatible

Causal structure



Parameters

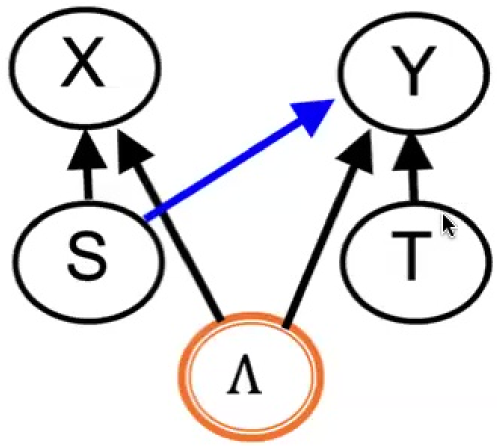
$P_{X|S\Lambda}$

$P_{Y|ST\Lambda}$

P_{Λ}

$$P_{XY|ST} = \sum_{\Lambda} P_{Y|ST\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Causal structure



Parameters

$P_{X|S\Lambda}$

$P_{Y|ST\Lambda}$

P_{Λ}

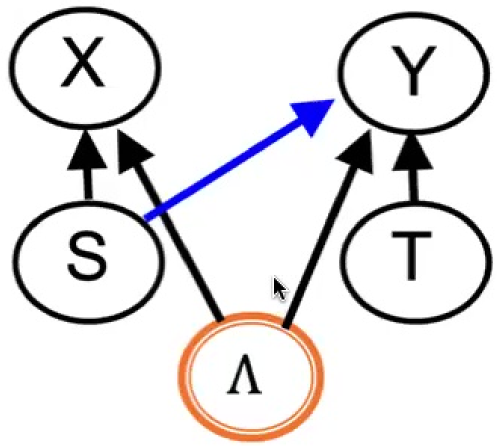
$$P_{XY|ST} = \sum_{\Lambda} P_{Y|ST\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Causal compatibility constraints:

$P_{X|ST} = P_{X|S}$

But the data *also* satisfies $P_{Y|ST} = P_{Y|T}$

Causal structure



Parameters

$P_{X|S\Lambda}$

$P_{Y|ST\Lambda}$

P_{Λ}

$$P_{XY|ST} = \sum_{\Lambda} P_{Y|ST\Lambda} P_{X|S\Lambda} P_{\Lambda}$$

Causal compatibility constraints:

$P_{X|ST} = P_{X|S}$

But the data *also* satisfies $P_{Y|ST} = P_{Y|T}$

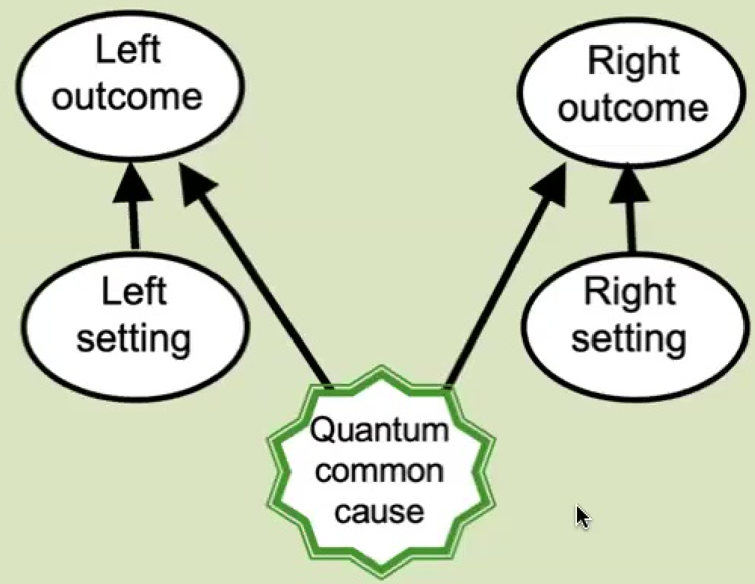
Reproducing this requires fine-tuning

Wood and RWS, New J. Phys. 17, 033002 (2015)

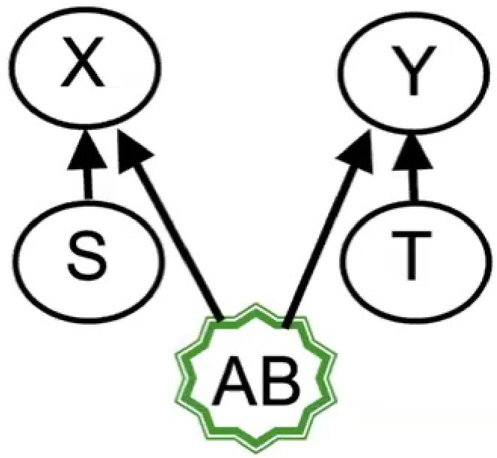
The evidence

Left setting and Right setting	Left outcome and Right outcome: 0 and 0	0 and 1	1 and 0	1 and 1
0 and 0	43%	7%	7%	43%
0 and 1	43%	7%	7%	43%
1 and 0	43%	7%	7%	43%
1 and 1	7%	43%	43%	7%

A new possibility



Causal structure



Parameters

$\rho_{X|SA}$

$\rho_{Y|TB}$

ρ_{AB}

$$P_{XY|ST} = \mathrm{Tr}_{AB}(\rho_{X|SA}\rho_{Y|TB}\rho_{AB})$$

Causal compatibility constraints:

$$P_{X|ST} = P_{X|S}$$
$$P_{Y|ST} = P_{Y|T}$$

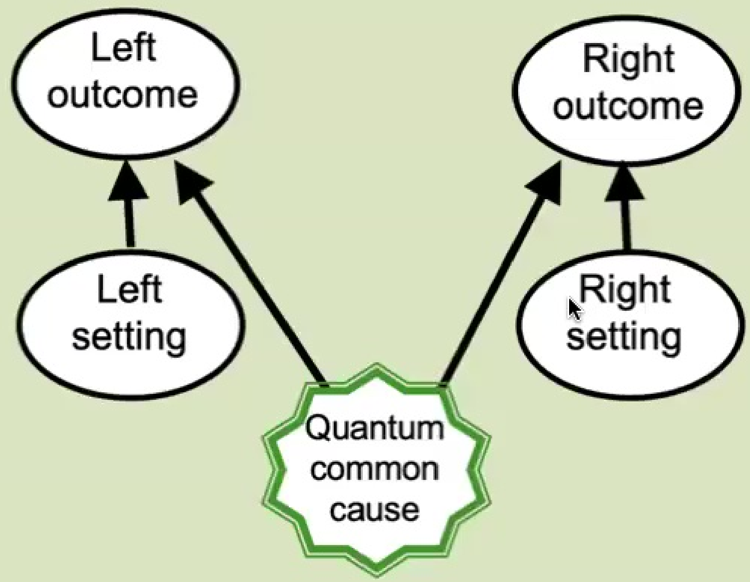
$$\boxed{\begin{array}{l} \frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|00) + \frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|01) \\ \quad \frac{1}{4}\sum_{x=y} P_{XY|ST}(xy|10) + \frac{1}{4}\sum_{x\neq y} P_{XY|ST}(xy|11) \leq \underline{0.85} \end{array}}$$

Tsirelson, Lett. Math. Phys. 4, 93 (1980)

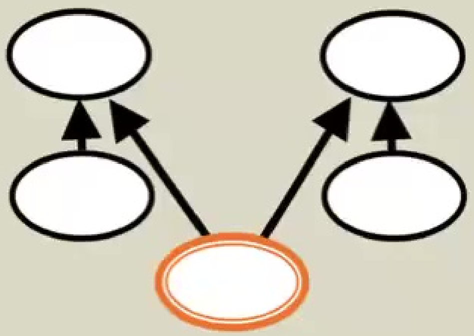
The evidence

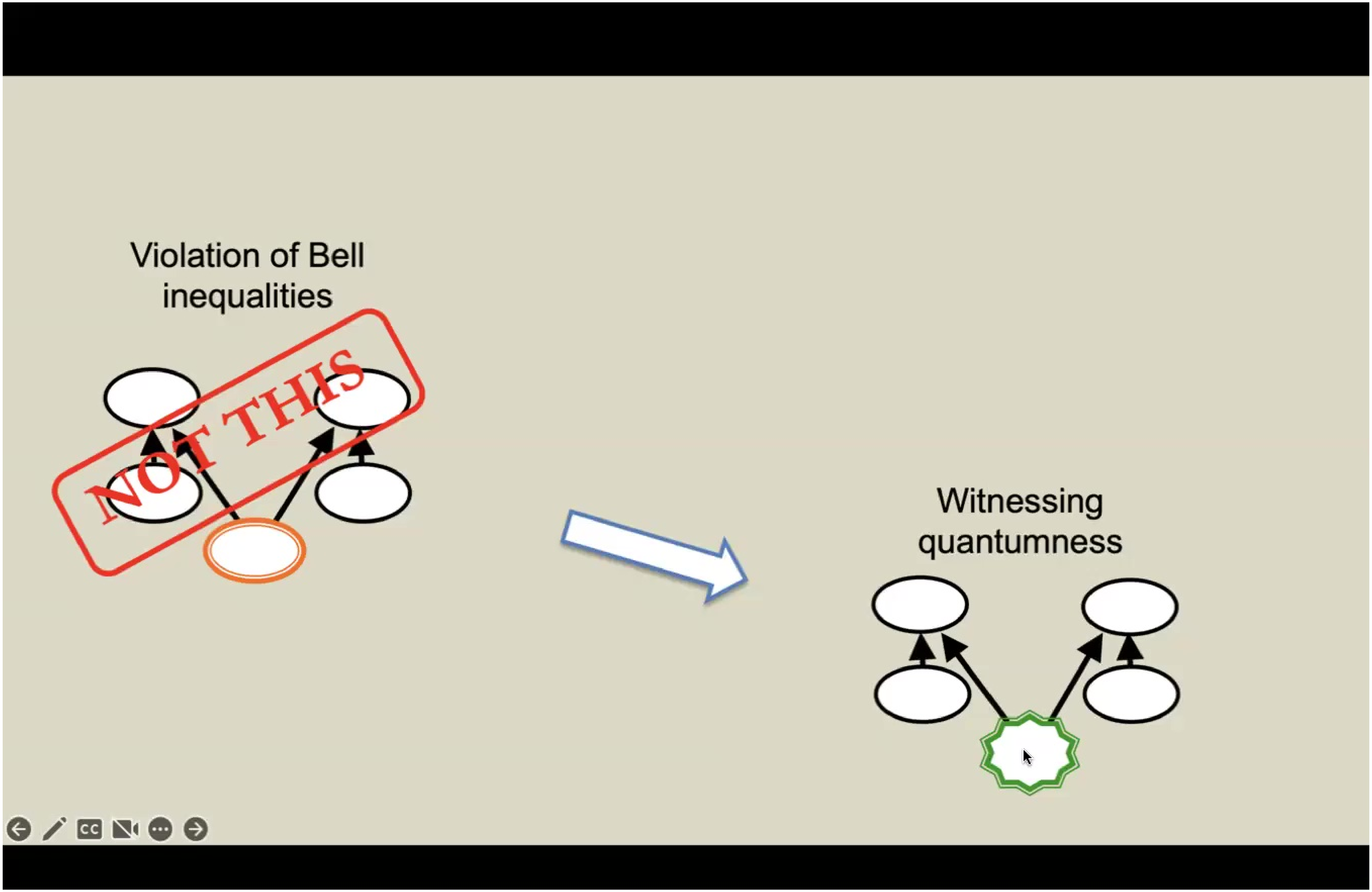
		Left outcome and Right outcome			
		0 and 0	0 and 1	1 and 0	1 and 1
Left setting and Right setting	0 and 0	43%	7%	7%	43%
	0 and 1	43%	7%	7%	43%
	1 and 0	43%	7%	7%	43%
	1 and 1	7%	43%	43%	7%

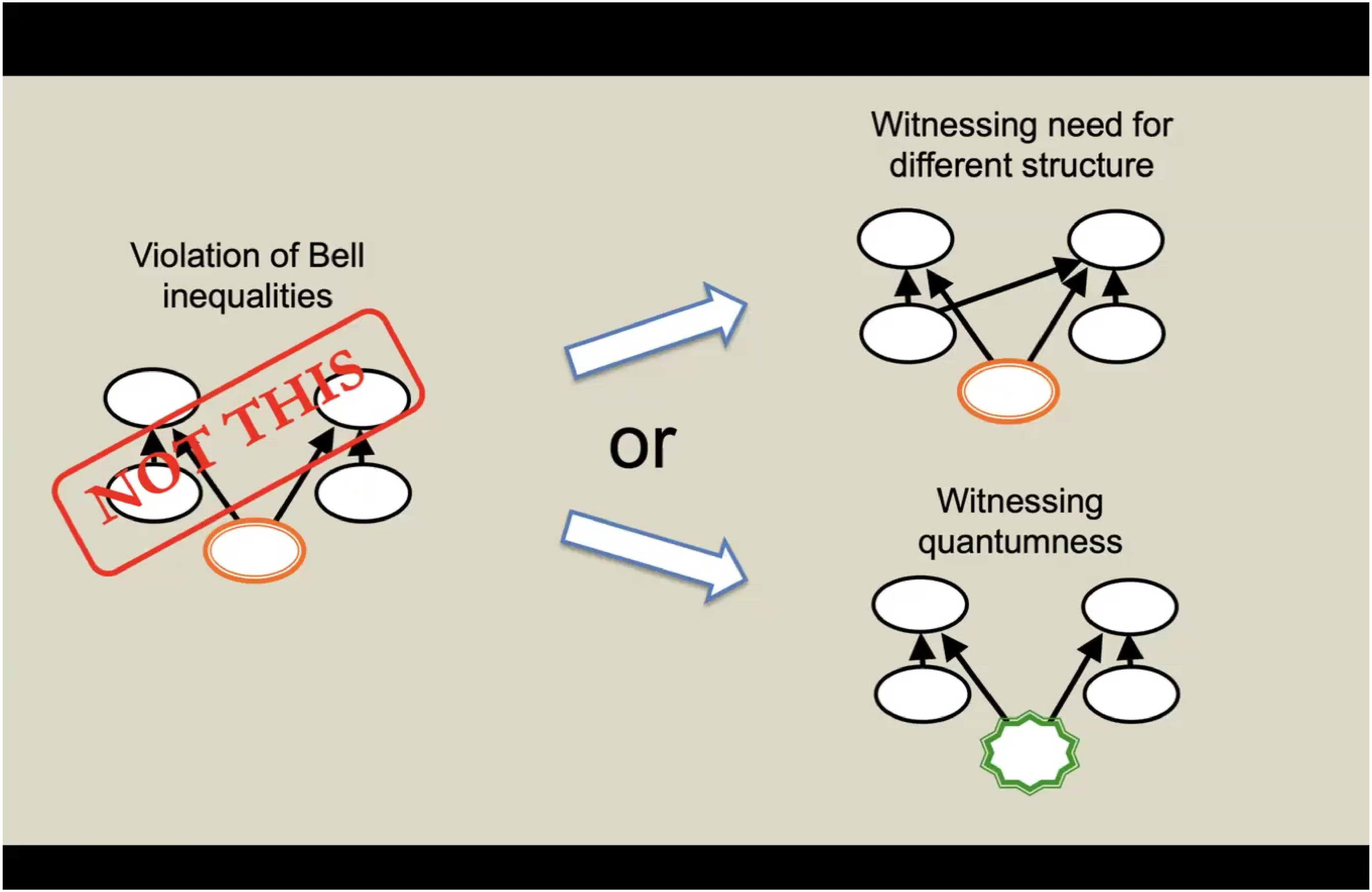
A new possibility



Compatible

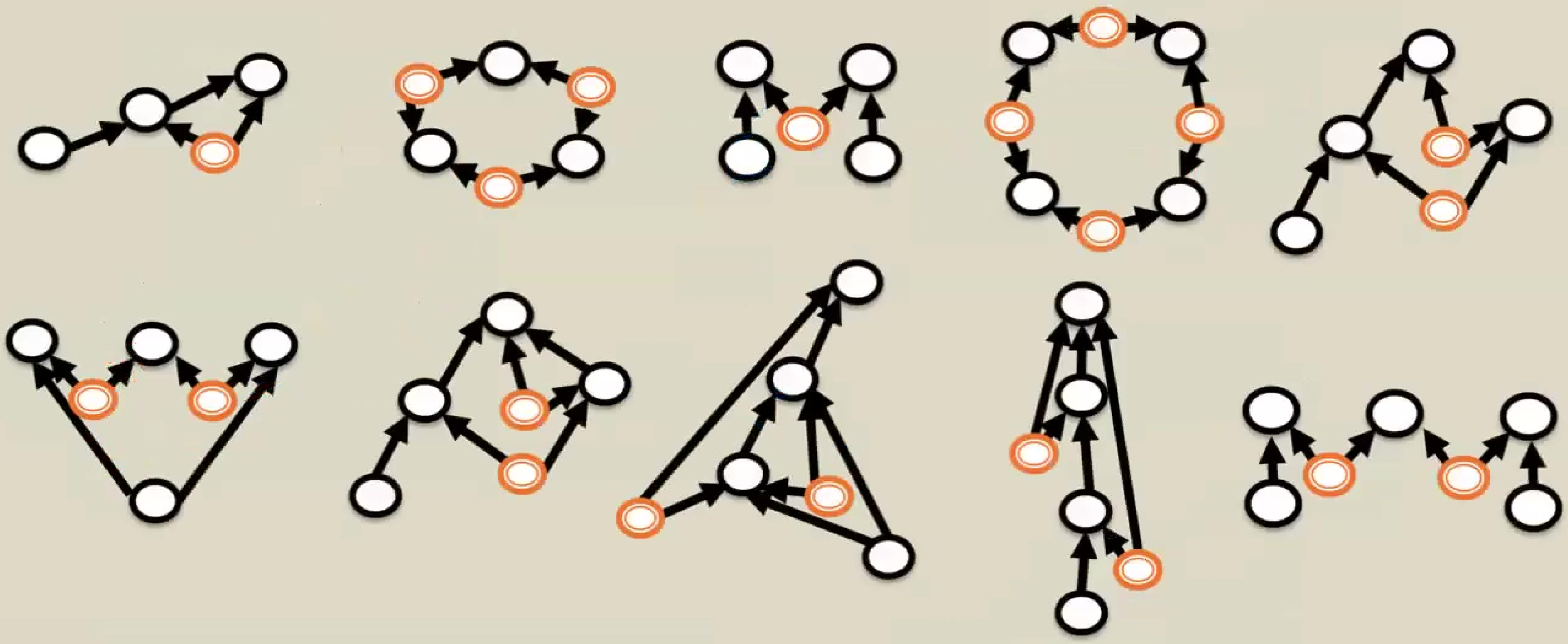



Violation of Bell inequalities
NOT THIS
Witnessing quantumness

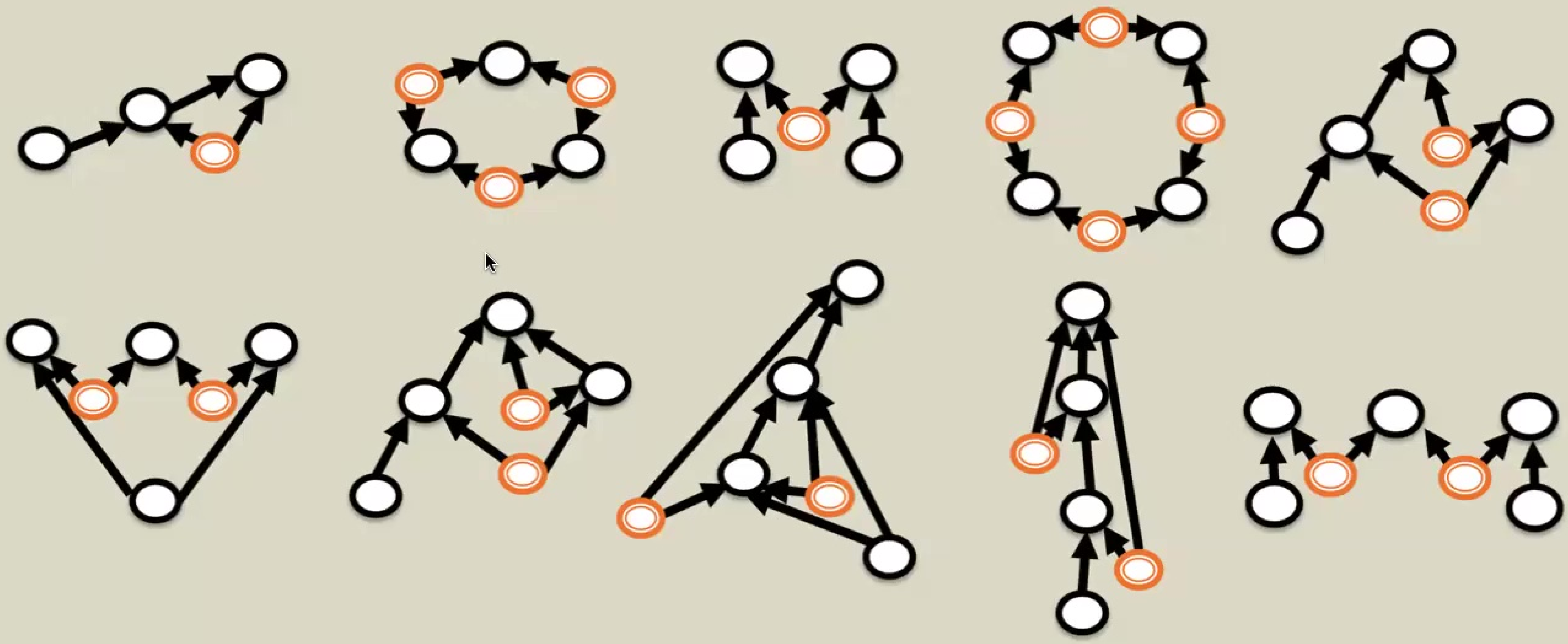


Violation of Bell inequalities
NOT THIS
or
Witnessing need for different structure
Witnessing quantumness

Some causal structures that admit of quantum-classical gaps:

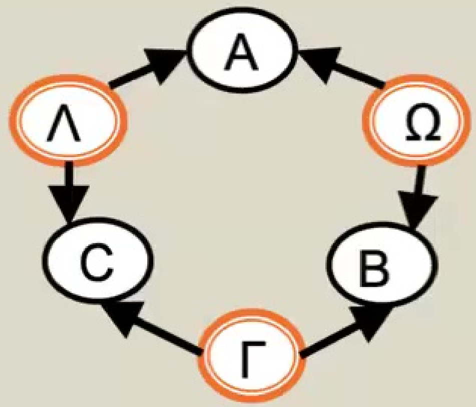


Some causal structures that admit of quantum-classical gaps:



It is likely that such gaps are generic!

Causal structure



Parameters

$\mathrm{P}_{A|\Lambda\Omega}$

$\mathrm{P}_{B|\Omega\Gamma}$

$\mathrm{P}_{C|\Gamma\Lambda}$

P_{Λ}

P_{Ω}

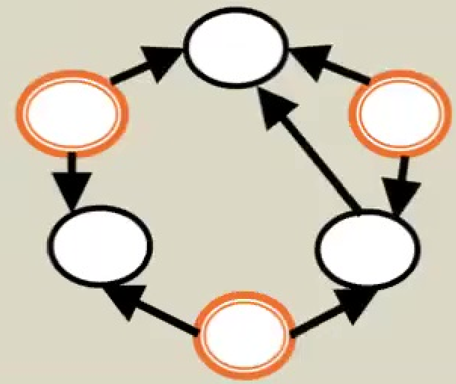
P_{Γ}

$$P_{ABC} = \sum_{\Lambda\Omega\Gamma} \mathrm{P}_{A|\Lambda\Omega}\mathrm{P}_{B|\Omega\Gamma}\mathrm{P}_{C|\Gamma\Lambda}\mathrm{P}_{\Lambda}\mathrm{P}_{\Omega}\mathrm{P}_{\Gamma}$$

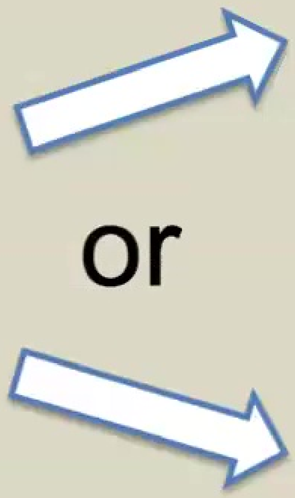
Example of a causal compatibility inequality:

$$\boxed{\begin{aligned} P_A(1)P_B(1)P_C(1) \quad &\leq P_{AB}(11)P_C(1) + P_{BC}(11)P_A(1) \\ &+ P_{AC}(11)P_B(1) + P_{ABC}(000) \end{aligned}}$$

Witnessing need for different structure

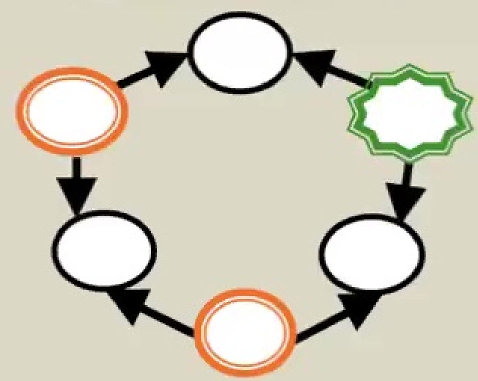


Violation of certain causal compatibility inequalities



or

Witnessing quantumness



Fritz, New J. Phys. 14, 103001 (2012)

Quantum
Causation and Inference

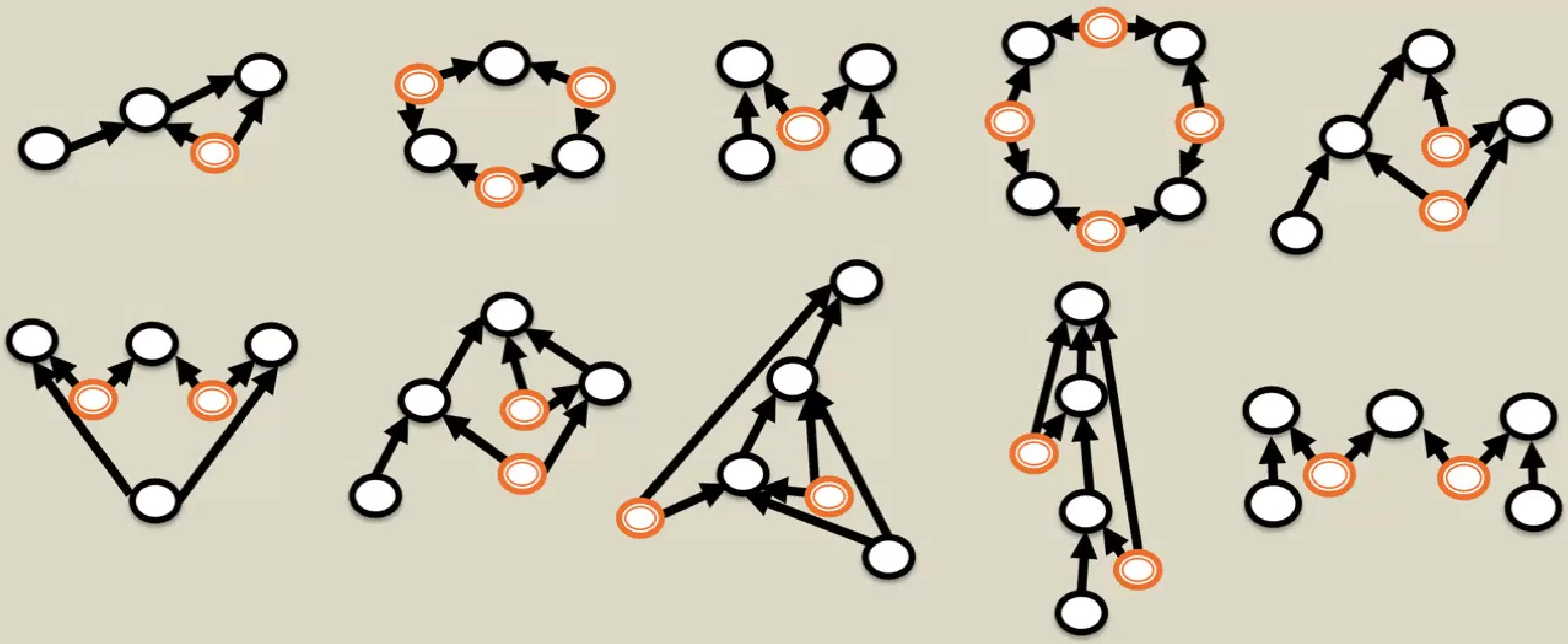
Classical
Causation and Inference

≈

Relativistic
Notions of Space
and Time

PreRelativistic
Notions of Space
and Time

Some causal structures that admit of quantum-classical gaps:



It is likely that such gaps are generic!

Next lecture:
causal theories and inferential theories