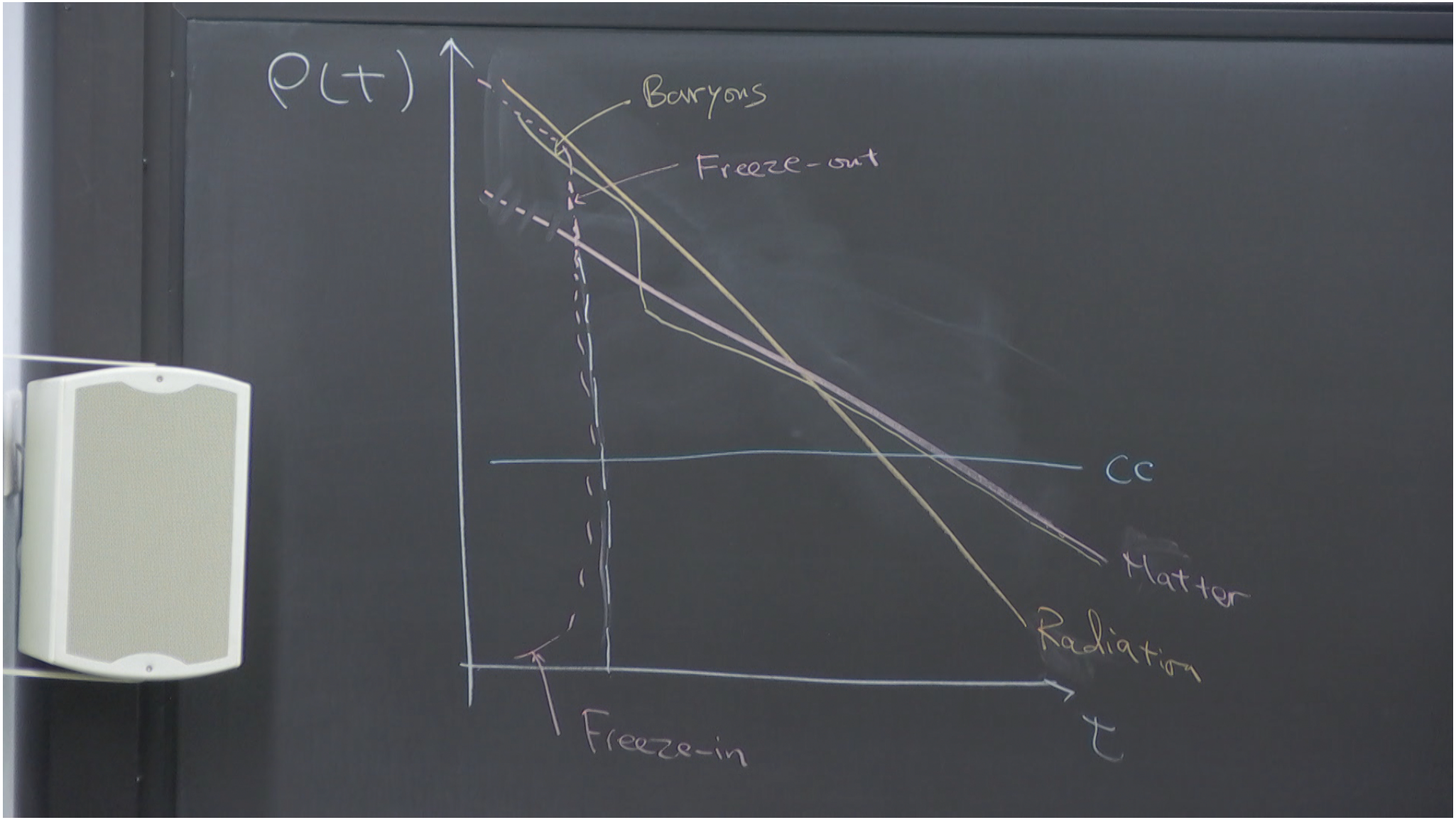
Title: Particle Physics Lecture - 230327

Speakers: Junwu Huang

Collection: Particle Physics (2022/2023)

Date: March 27, 2023 - 11:30 AM

URL: https://pirsa.org/23030066


ρ(t)
Baryons
Freeze-out
CC
Matter
Radiation
Freeze-in
t

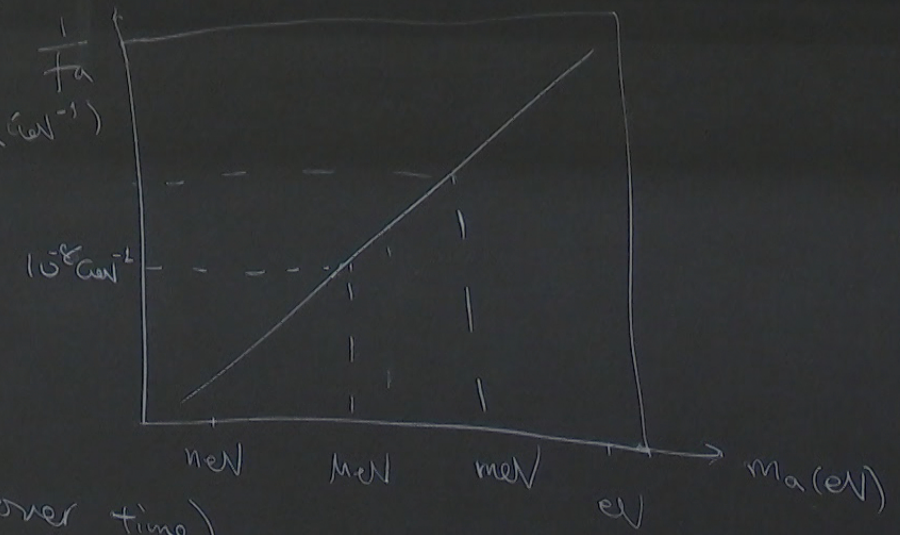
Lecture 11: QCD axion dark matter

$$E(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \left(\frac{4 m_u m_d}{(m_u+m_d)^2}\right) \sin^2\frac{\theta}{2}}$$

- $\theta = 0$ is the minimum
- $\theta \to \boxed{\frac{a}{f_a}}$ is dynamical (Damped over time)

Axion

$$E''(\theta) \Rightarrow m_a^2 = \left(\frac{m_u m_d}{(m_u+m_d)^2}\right) \cdot m_\pi^2 f_\pi^2 / f_a^2$$



$\ddot{\theta} + \gamma\dot{\theta} + \omega_0^2(\theta - \theta_{min}) = 0$

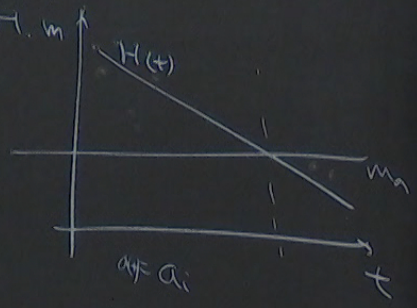
$V''(\theta)$ — H — a/f_a

$\begin{cases} \gamma \gg \omega_0, & \theta(t) = \text{constant} \\ \gamma \ll \omega_0, & \theta(t) \propto e^{-\gamma t}\cos\omega_0 t \end{cases}$

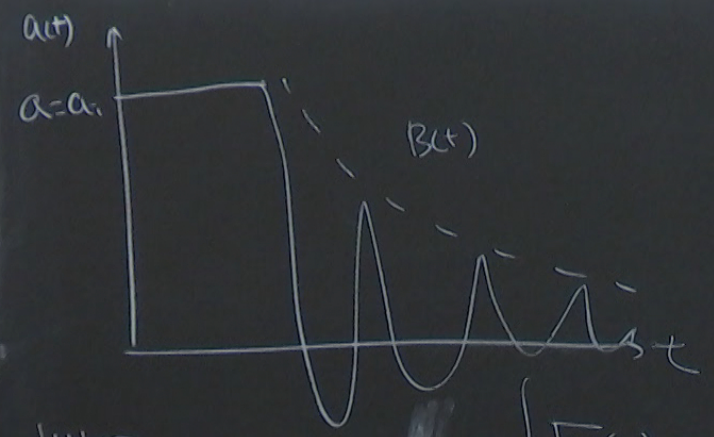
$S = \int \sqrt{-g}\, d^4x \left[\partial_\mu a\, \partial^\mu a - m^2 a^2 \right]$

EOM: $\ddot{a} + 3H\dot{a} + m_a^2 a^2 = 0$ $\qquad \boxed{H = \frac{1}{2t}}$

$\begin{cases} H \gg m_a, & a(t) = \text{constant} \\ H \ll m_a & \boxed{B(t)}\cos m_a t \end{cases}$

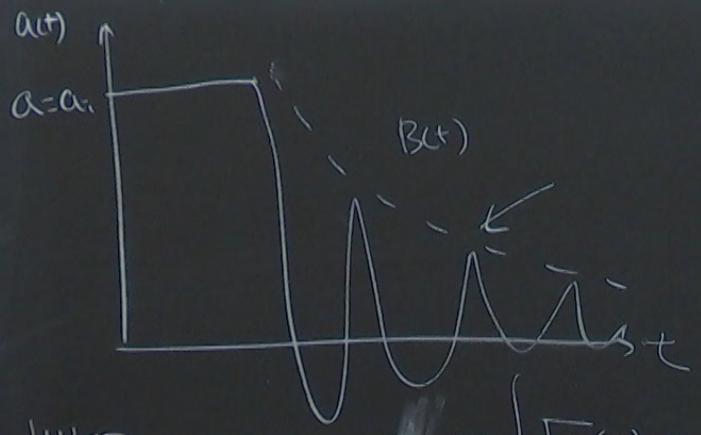


$\rho \propto m^2 a^2 = \text{constant}$ for $H \gg m$.



WKB: $a(t) = A e^{-\int \Gamma(t)\,dt} \cos \omega t$

$-2\Gamma(t) \sin \omega t \cdot \dot{A} + 3H(t) \sin \omega t \, A = 0$



WKB: $a(t) = A e^{-\int \Gamma(t)\,dt} \cos mat$

$-2\Gamma(t) \sin mat \cdot \dot{A} + 3H(t) \sin mat \, A = 0$

$\Gamma(t) = \frac{3}{2} H(t) = \frac{3}{2} \frac{\dot{a}}{a}$

$\Gamma(t)\,dt = \frac{3}{2} \frac{da}{a} = \frac{3}{2} d \ln a$

$$\boxed{a(t) = \left(\frac{a(H = m_a)}{a(t)} \right)^{3/2}}$$

$\rho(t) = m_a^2 a^2(t) \sim$

This is redshifting of

$$\Gamma(t)\,dt = \frac{3}{2}\frac{da}{a} = \frac{3}{2}\,d\ln a$$

$$\boxed{a(t)} = \left(\frac{a(H=m_a)}{a(t)}\right)^{3/2} \cos m_a t.$$

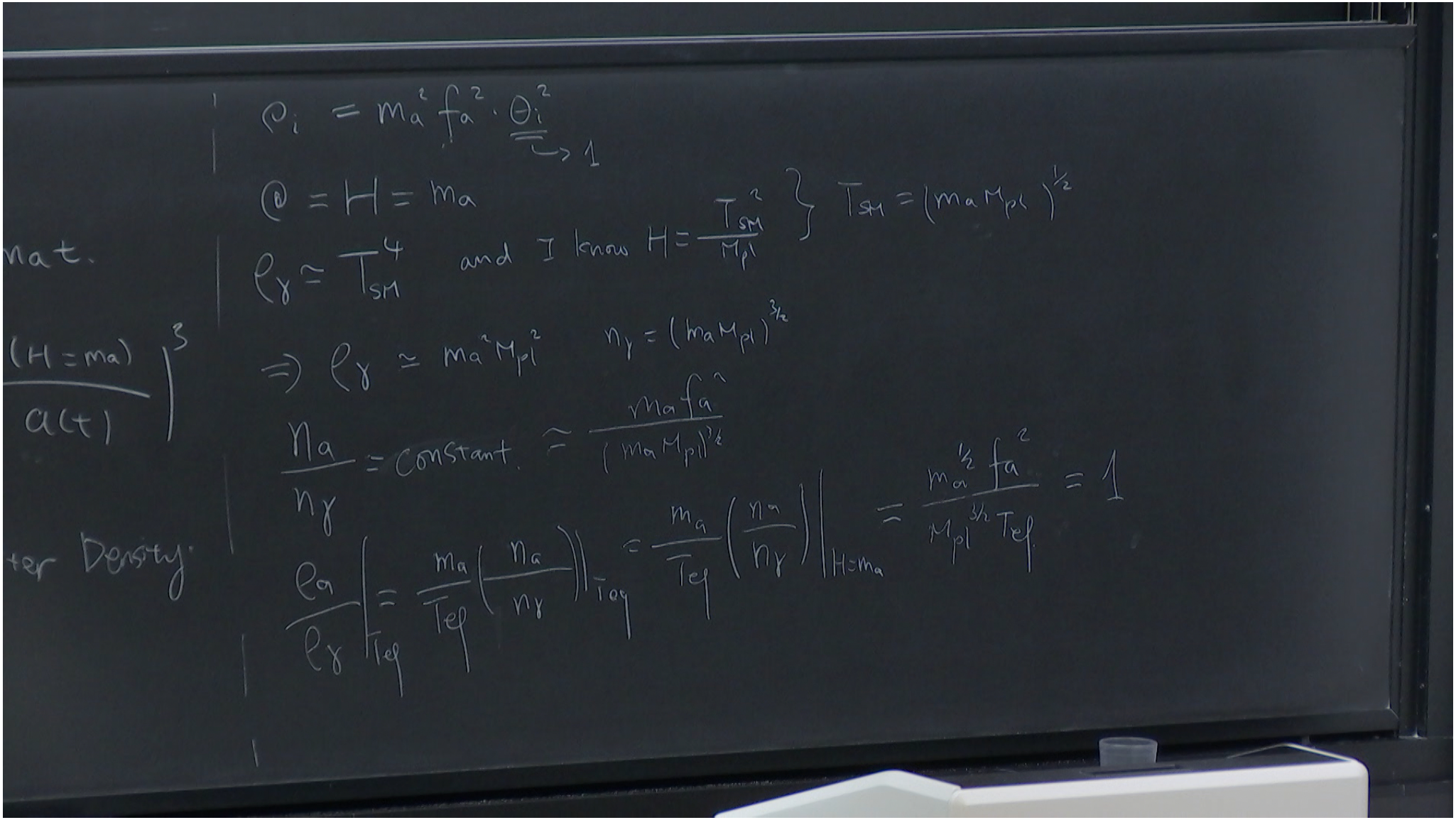
$$\rho(t) = m_a^2 a^2(t) \sim \left(\frac{a(H=m_a)}{a(t)}\right)^3$$

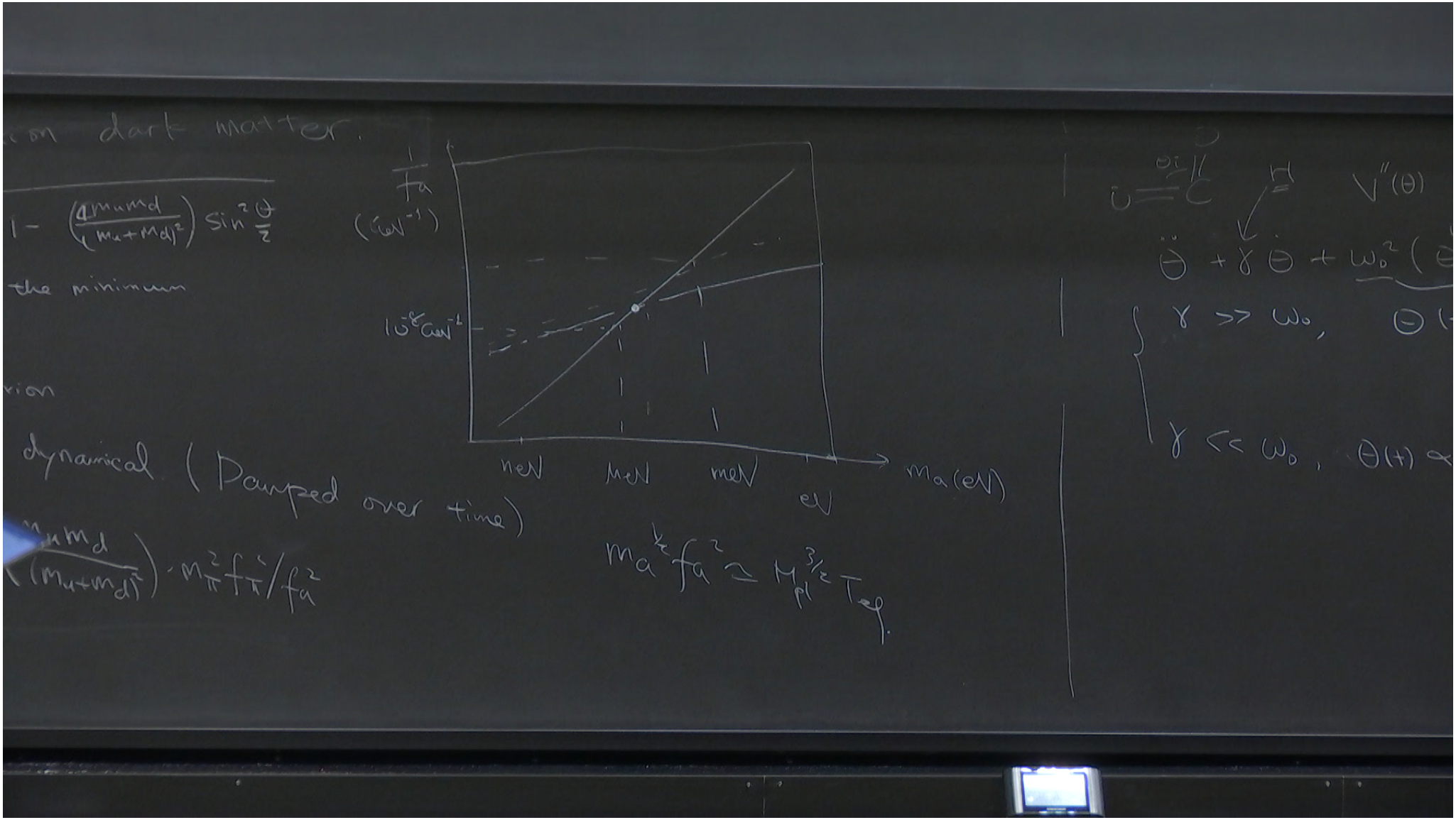
This is redshifting of Matter Density.

$\rho_i =$

$\cos m_a t$

$t)\sin m_a t$ $\Lambda = 0$

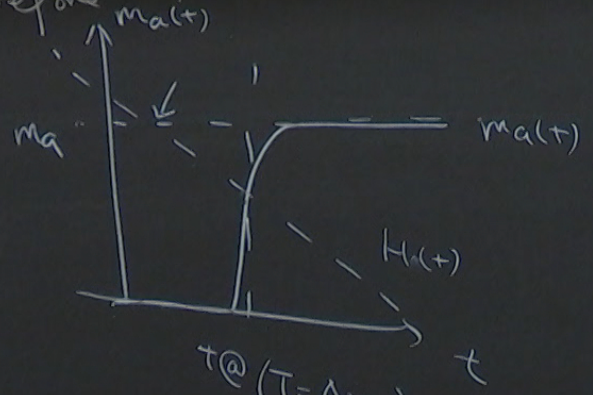

$\rho_i = m_a^2 f_a^2 \cdot \theta_i^2$
$\theta_i^2 \to 1$
$@ = H = m_a$
$\rho_\gamma \simeq T_{SM}^4$ and I know $H = \frac{T_{SM}^2}{M_{pl}}$
$T_{SM} = (m_a M_{pl})^{1/2}$
$\Rightarrow \rho_\gamma \simeq m_a^2 M_{pl}^2$
$n_\gamma = (m_a M_{pl})^{3/2}$
$\frac{n_a}{n_\gamma} = \text{constant} \simeq \frac{m_a f_a}{(m_a M_{pl})^{3/2}}$
$\frac{\rho_a}{\rho_\gamma}\Big|_{T_{eq}} = \frac{m_a}{T_{eq}}\left(\frac{n_a}{n_\gamma}\right)\Big|_{T_{eq}} = \frac{m_a}{T_{eq}}\left(\frac{n_a}{n_\gamma}\right)\Big|_{H=m_a} = \frac{m_a^{1/2} f_a^2}{M_{pl}^{3/2} T_{eq}} = 1$
nat.
$\left(\frac{(H=m_a)}{a(t)}\right)^3$
ter Density


dark matter.
$1 - \left(\frac{4 m_u m_d}{(m_u+m_d)^2}\right) \sin^2\frac{\theta}{2}$
the minimum
dynamical (Damped over time)
$\left(\frac{m_u m_d}{(m_u+m_d)^2}\right) \cdot m_\pi^2 f_\pi^2 / f_a^2$
$\frac{1}{f_a}$ (GeV^{-1})
$10^{-6}\,\mathrm{GeV}^{-2}$
neV
μeV
meV
eV
m_a(eV)
$m_a^{1/2} f_a^2 \simeq M_{Pl}^{3/2} T_{eq}$
$V''(\theta)$
$\ddot{\theta} + \gamma\dot{\theta} + \omega_0^2(\theta$
$\gamma >> \omega_0$,
$\gamma << \omega_0$, $\theta(t) \propto$

m_a of axion is not a constant for QCD axion

Before $T = \Lambda_{QCD}$

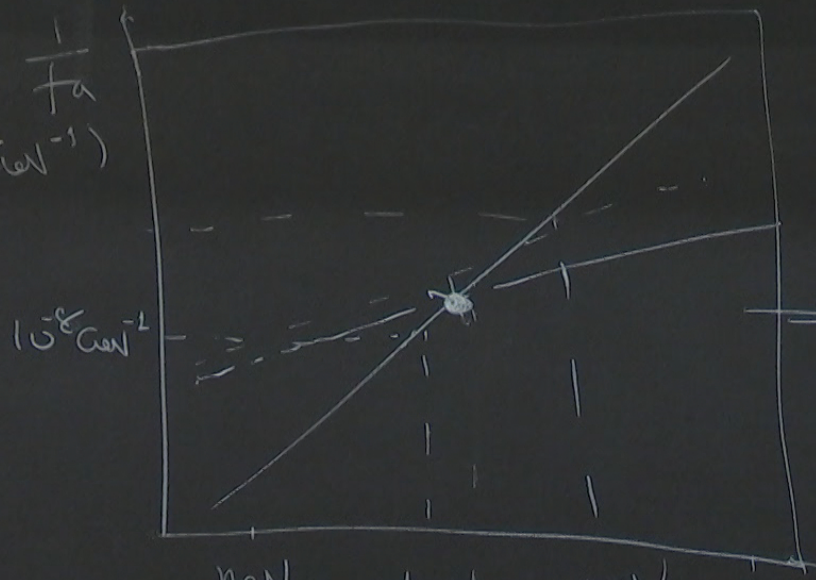
$$m_a f_a^2 = T_{QCD} \cdot T_{eq} \cdot M_{pl}$$



Means axion oscillate later than we expect.

matter.

$\sin^2\frac{\theta}{2}$



$\lambda_a \sim \frac{1}{m_a} \sim$ meter

$\begin{cases} m_a \sim 10\,\mathrm{meV} \\ f_a \sim 10^{11}\,\mathrm{GeV} \end{cases}$

$\ddot{\theta} + \gamma\dot{\theta} +$

$\begin{cases} \gamma \gg \omega_0, \\ \gamma \ll \omega_0, \end{cases}$

(Pumped over time)

$m_\pi^2 f_\pi^2 / f_a^2$

$m_a^{1/2} f_a^2 \simeq M_{Pl}^{3/2} T_{eq}$

nstant for QCD axion

$a f_a^2 = T_{QCD} \cdot T_{eq} M_{pl}$

Production of QCD axion 2:

Cosmic string & Domain Walls.

$V(\Phi,$

$\theta \to \frac{a}{f_a}$

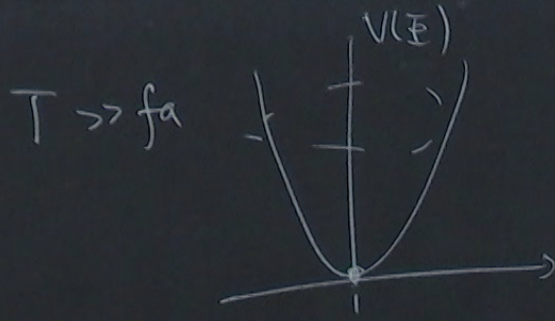
$V(\Phi) = \lambda \left(\Phi^* \Phi - f_a^2 \right)^2$

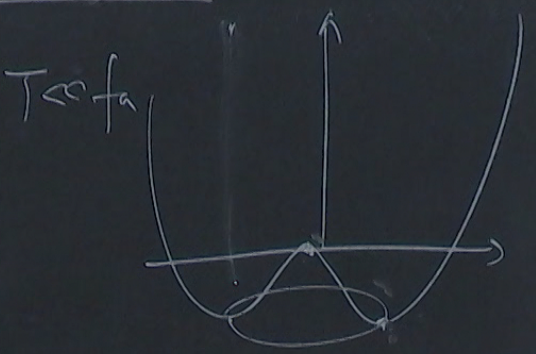
@ $T=0$ $\langle \Phi \rangle = f_a$

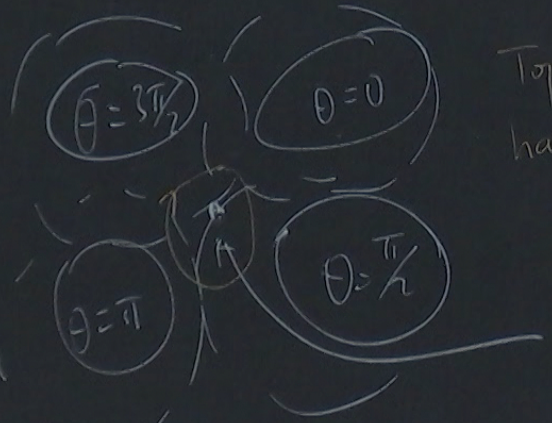
around this minimum. $\Phi = (f_a + \rho) e^{ia/f_a}$

@ $T > f_a$ $V(\Phi) = \lambda \cdot T^2 \cdot |\Phi|^2$

$$V(\Phi, T) = -2\lambda f_a^2 |\Phi|^2 + \lambda T^2 |\Phi|^2 + \lambda |\Phi|^4 +$$







Topological Defect has formed (Kibble)

Cosmic String (Axion String)

String tension
$\mu \sim f_a^2$

; Around the string
θ goes from 0 to $2\pi f_a$

① Individual String $\rho_s \propto \frac{1}{a^2(t)}$

$\frac{\rho_s}{\rho_\gamma} \propto a^2(t)$

② Radiates

$P \sim \mu \sim f_a^2$

$t_{life} \sim \frac{E}{P} = \frac{\mu \cdot L}{\mu} \sim L$

If $L < \frac{1}{H}$, over a Hubble time
it will disappear

③

sion

ng
0. to $2\pi f_a$

g $\underline{P_s \propto \frac{1}{a^2(t)}}$

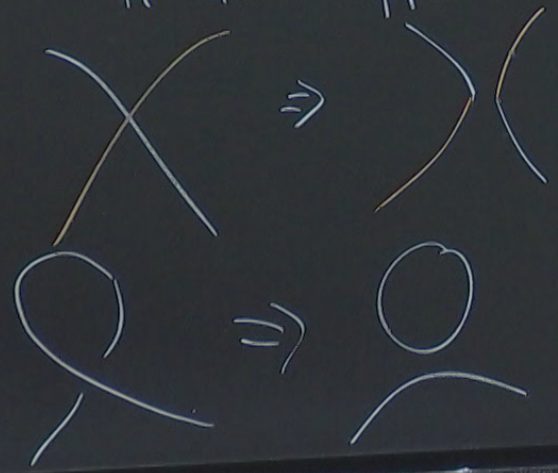
② Radiates

$P \sim M \sim f_a^2$

$t_{life} \doteq \frac{E}{P} = \frac{M \cdot L}{M} \sim L$

If $L < \frac{1}{H}$, over a Hubble time it will disappear.

③.



$\sigma \sim \eta \cdot L_1 \cdot L_2$

$$\frac{\Gamma}{H} = \frac{n_{string} \cdot L^2}{1/L}\Bigg|_{L \sim \frac{1}{H}} \Rightarrow n_{string} \sim \frac{1}{L^3}, \text{ we have } \frac{\Gamma}{H} \sim 1$$

$$\sim H^3$$

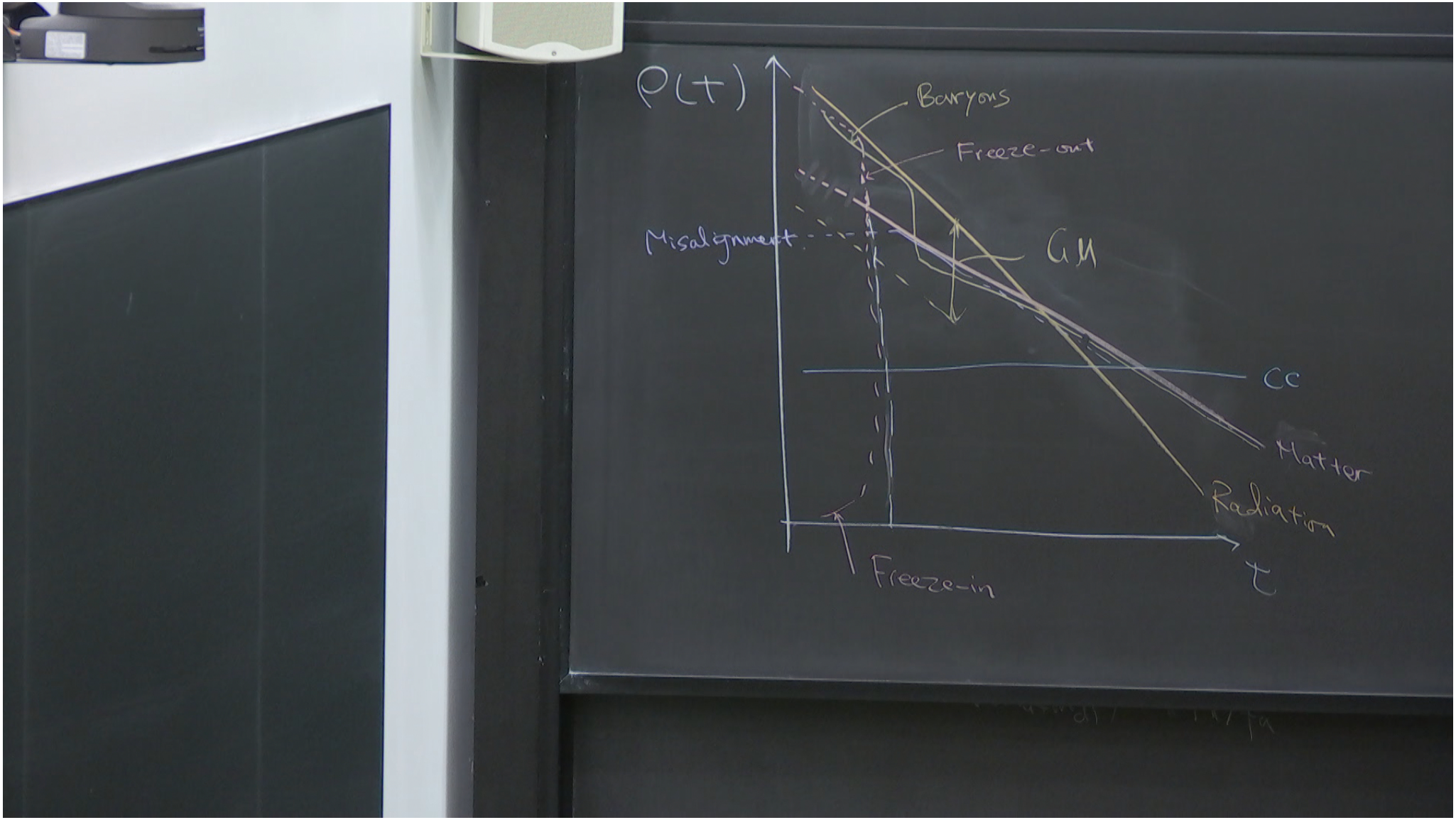
L

sible time

Scaling solution: $\boxed{n_{string} \sim H^3}$

$$\rho_s \simeq \mu L \cdot H^3 \simeq \mu \cdot H^2$$

$$\frac{\rho_s}{\rho_\gamma} \simeq \frac{\mu H^2}{H^2 M_{pl}^2} \simeq \boxed{G\mu}$$


ρ(t)
Baryons
Freeze-out
Misalignment
GM
CC
Matter
Radiation
Freeze-in
t