Title: Strong Gravity Lecture - 230327

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Collection: Strong Gravity (2022/2023)

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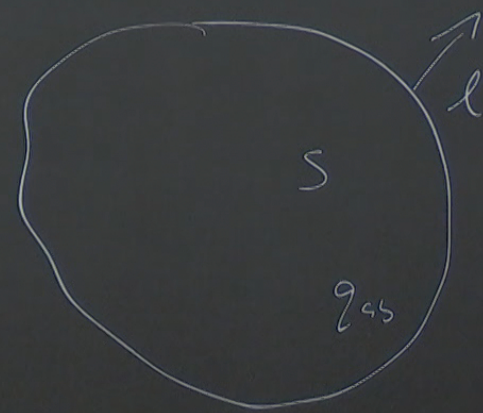
Apparent horizons

Σ_t is 3-dim timeslice

Let S be 2-dim surface

q_{ab} be the induced metric S

ℓ be a normal to S



Define expansion:

$$\boxed{\Theta = q^{ab}\nabla_a l_b} = \mathcal{L}_l\left[\ln(\sqrt{q})\right]$$

Take l is null

$$(l^{\pm})^a = n^a \pm s^a$$

unit normal to Σ_t

unit normal to S

$$n_a n^a = -1$$

$$s^a s_a = 1$$

$$l^a_{\pm} l^{\pm}_a = 0$$

$$l^-_a l^a_+ = -2$$

Outward null expansion

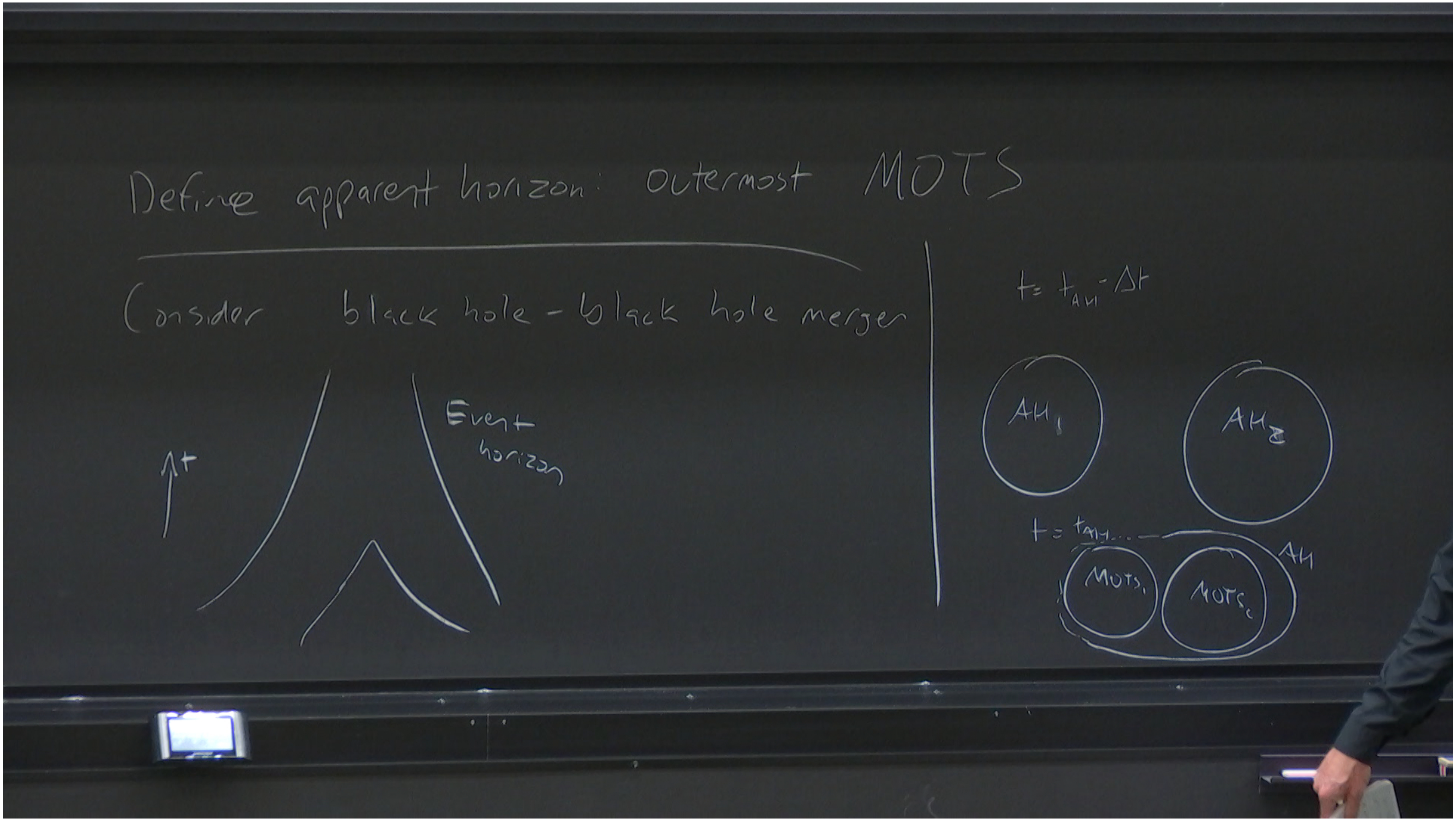
$$\theta_{\ell^+} = q^{ab} \nabla_a \ell_b^+$$

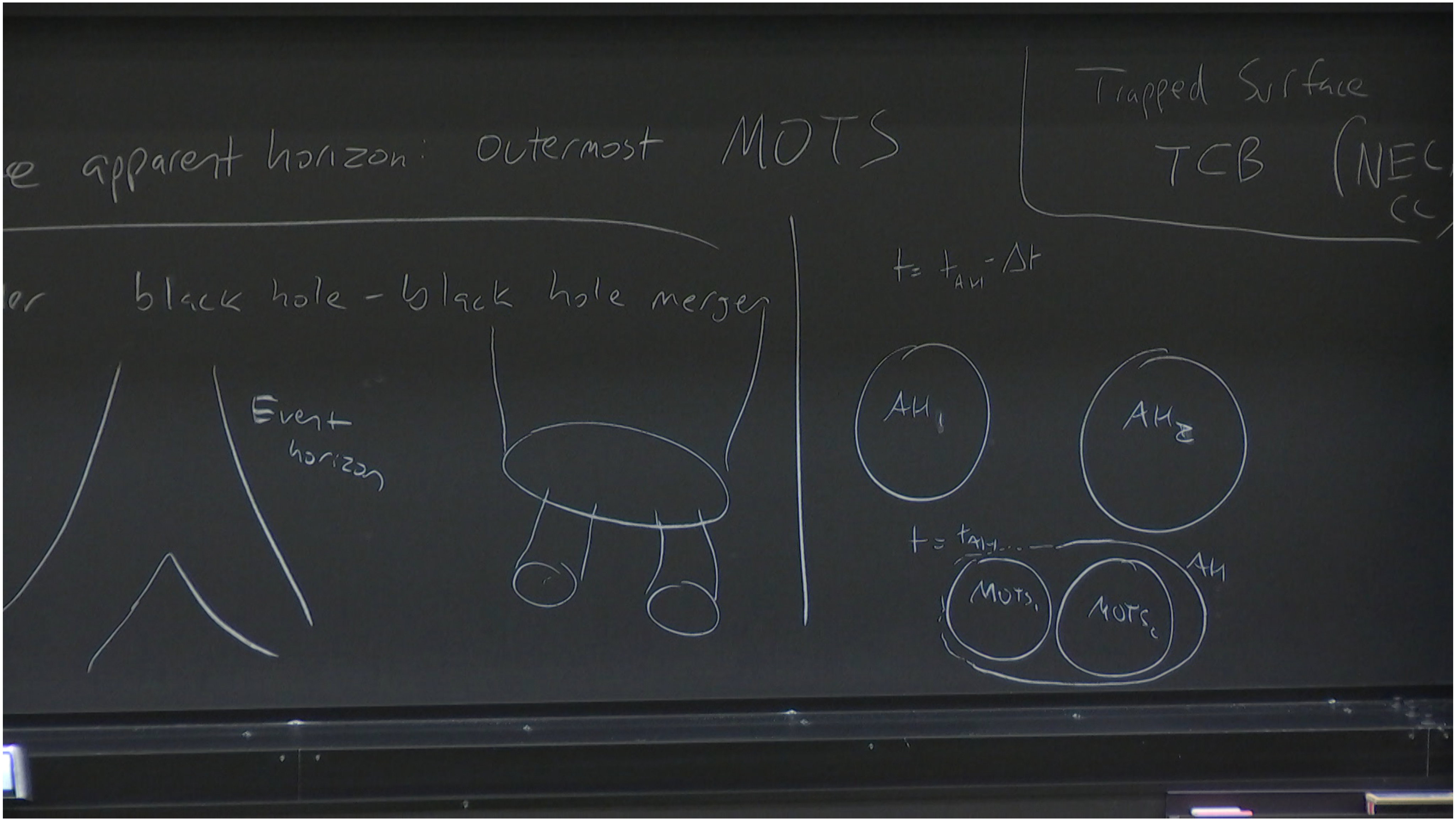
$$= D_i s^i + K_{ij} s^i s^j - K \quad \text{(in 3+1 variables)}$$

If $\theta_{\ell^+} \leq 0$ everywhere on S, S is outer trapped

$= 0$ S is marginally trapped

MOTS


Define apparent horizon: outermost MOTS
Consider black hole - black hole merger
t
Event horizon
t = t_AH - Δt
AH1
AH2
t = t_AH
AH
MOTS1
MOTS2


e apparent horizon: outermost MOTS
Trapped Surface
TCB (NEC
cc
or black hole - black hole merger
Event horizon
t= t_AH - Δt
AH1
AH2
t = t_AH
AH
MOTS1
MOTS2

Gravitational Waves

Weak field regime

$$g_{ab} = \eta_{ab} + h_{ab} \qquad |h_{ab}| << |\eta_{ab}|$$

$$g^{ab} = \eta^{ab} - h^{ab} \quad \text{(to linear order)}$$

$$| \ll |\eta_{ab}|$$

rder)

Use harmonic gauge

$$\Box x^a = 0$$

$$\underbrace{\eta^{cd}\partial_c\partial_d}_{\Box} h_{ab} = -16\pi\left(T_{ab} - \frac{1}{2} g_{ab} T\right)$$

$$\Box\underbrace{\left(h_{ab} - \frac{1}{2}\eta_{ab} h^{cd}\eta_{cd}\right)}_{\bar{h}_{ab}} = -16\pi T_{ab}$$

$$\Box x^a = 0 \Rightarrow \partial_a \bar{h}^{ab} = 0 \quad \text{(Lorenz gauge)}$$

In E.M $A_a \to A_a + \partial_a \Phi$ with $\partial_a \partial^a \phi = 0$

leaves $\partial_a A^a$ unchanged

In G.R. $x^a \to x^a + \xi^a$ $\Box \xi^a = 0$ still have harmonic gauge

require ξ^a to be "small"

In wavezone use 4 gauge DOFs to set $h_{0i}^{TT} = 0$

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$x^a \to x^a + \xi^a \qquad \Box \xi^a = 0 \qquad$ still have harmonic gauge

require ξ^a to be "small"

$\eta^{ab} h^{TT}_{ab} = 0$ (traceless)

In wavezone use 4 gauge DOFs to set $h^{TT}_{0i} = 0$

transverse

$h^{TT}_{ab} = \bar{h}_{ab}$

Lorenz condition

$$0 = \partial_a h^{a0}_{TT} = -\partial_0 h^{00}_{TT} + \overset{=0}{\cancel{\partial_i h^{i0}_{TT}}}$$

In vacuum

$$\partial_i \partial^i h^{TT}_{00} = 0$$

$$h^{00}_{TT} = 0$$

$$h^{TT}_{0a} = 0 \quad \text{(purely spatial)}$$

$$\Box h^{TT}_{ij} = 0 \,, \quad h^{TT}_{ij} = \underset{\text{constant}}{A_{ij}} \exp(i k_a x^a) \quad \text{(plane waves)}$$

In wavezone ... transverse $h_{ab} = h_{ab}$

$$0 = \Box h_{ij}^{TT} = \eta^{ab}(ik_a)(ik_b)\, h_{ij}^{TT} = -\underbrace{k_a k^a}_{=0} h_{ij}^{TT}$$

$$A_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Want $k_a k^a = 0$ (null)

$$0 = \partial_a h^{ab}_{TT} = ik_a h^{ab}_{TT} = 0 \quad \text{(transverse)}$$

(Choose $k^a = (\omega, 0, 0, \omega)$ propagating in z-direction)

$$A_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$A^{z i} = 0$, $A^{ij} = A^{ji}$, $A^i_i = 0$

$h_+, h_\times$ two propogating GW DOF

Two geodesics

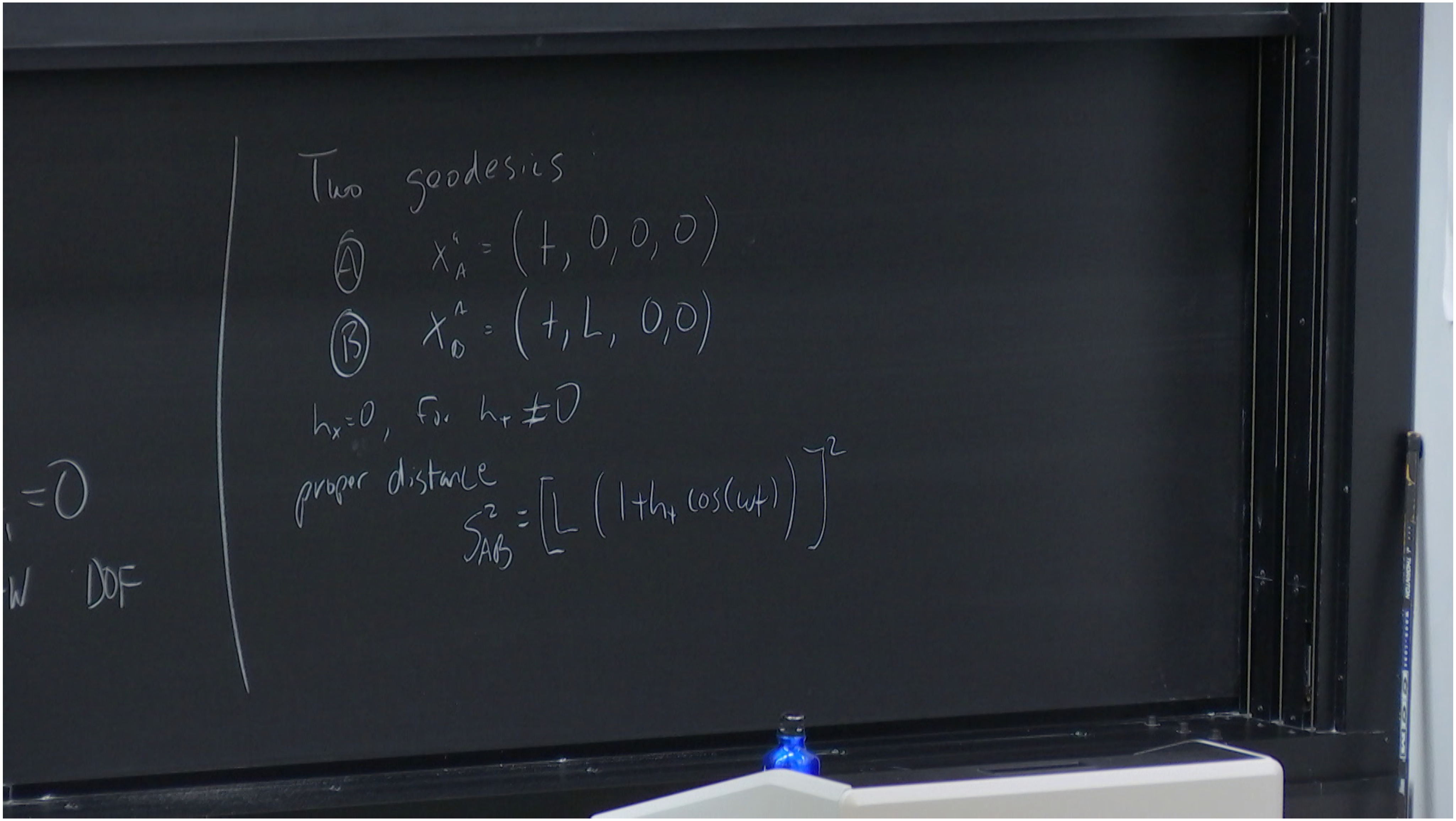
A $x^a_A = (t, 0, 0, 0)$

B $x^a_B = (t, L, 0, 0)$

$h_\times = 0$, for h_+

proper distance

$$S^2_{AB} = \left[L\left(1 + h_+ \cos(\omega t)\right)\right]^2$$



$=0$
W DOF
Two geodesics
(A) $x_A^\mu = (t, 0, 0, 0)$
(B) $x_B^\mu = (t, L, 0, 0)$
$h_\times = 0$, for $h_+ \neq 0$
proper distance
$S_{AB}^2 = \left[L \left(1 + h_+ \cos(\omega t) \right) \right]^2$